Inference for Mean Residual Life Function Under Nonparametric Mixture Modeling of Survival Distribution
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Introduction

The mean residual life function provides the expected remaining life given that the subject has survived (i.e., is event-free) up to a particular time. The mean residual life function characterizes the survival distribution, and thus it can be used in fitting a model to the data. We review the key properties of the mean residual life function and investigate its form for some common distributions. We next develop Bayesian nonparametric inference for mean residual life functions obtained from a flexible model for the corresponding survival distribution, using Dirichlet process mixtures of lognormal or Weibull distributions.

We compare with an exponentiated Weibull model, a parametric survival distribution that allows various shapes for the mean residual life function. The approach is illustrated with two data examples, one involving comparison of lifetimes of experimental subjects under different diets, and one on right censored survival times of liver metastases patients.

Common Survival Distributions

The distributions in the table below may be restrictive in the form of the MRL function. These distributions would require knowledge of the shape the MRL function for the data to ensure the proper model is fit. Moreover, many of these distributions have monotonic MRL functions which is not characteristic for most practical situations.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Hazard Function</th>
<th>MRL Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamma(α, λ)</td>
<td>α &lt; 1 DCR</td>
<td>α &lt; 1 INC</td>
</tr>
<tr>
<td>shape parameter α &gt; 0</td>
<td>scale parameter λ &gt; 0</td>
<td></td>
</tr>
<tr>
<td>Lognormal(α, λ)</td>
<td>∀α INC</td>
<td>∀α DCR</td>
</tr>
<tr>
<td>shape parameter α &gt; 0</td>
<td>scale parameter λ &gt; 0</td>
<td></td>
</tr>
<tr>
<td>Weibull(α, λ)</td>
<td>α &lt; 1 DCR</td>
<td>α &lt; 1 INC</td>
</tr>
<tr>
<td>shape parameter α &gt; 0</td>
<td>scale parameter λ &gt; 0</td>
<td></td>
</tr>
</tbody>
</table>

Model

The Exponentiated Weibull Distribution has an MRL function that can be INC, DCR, UBT, or constant. For t, α, θ, σ > 0, the probability density function is defined as:

\[ f(x; t, α, θ, σ) = \frac{α}{σ} \left( \frac{x}{σ} \right)^{α-1} e^{-\left( \frac{x}{σ} \right)^α} e^{-t \left( \frac{x}{σ} \right)^α} \]

α and θ are shape parameters while σ is a scale parameter. The form of the MRL function depends only on the values of α and θ.

The Dirichlet Process (DP) Mixture Model:

\[ x_i \sim \int \mathcal{G}(x_i; θ) dG(θ) = \sum_{l=1}^{L} \pi_l K(x_i; θ_l) \]

such that \( C = \sum_{l=1}^{L} \pi_l \) is truncated by \( G_l = \sum_{i=1}^{L} \pi_i \), and where \( p_i \) are the weights, obtained, via DP Stick-Breaking (SB) construction, corresponding to the component \( θ_0 \) and \( L \) is the total number of components specified in the model.

The Lognormal (LN) Dirichlet Process Mixture Model is fit to the survival times of the rats under different diets. In this model, the kernel distribution is the Lognormal distribution and the mixing is performed on the location and scale parameters, \( θ = (μ, σ^2) \), respectively.

Future Work

→ Extensions to regression modeling with censored responses.
→ Models that develop priors directly for the MRL function.

Results

We fit an Exponentiated Weibull Model and LN DP Mixture Model to two data sets in an experiment that studied the lifetimes of rats under different diets. The Restricted diet group consisted of 106 rats while the Ad Libitum (free-eating) group consisted of 108 rats.