Bayesian Point Process Modeling for Extreme Value Analysis, with an Application to Systemic Risk Assessment in Correlated Financial Markets

Athanasios Kottas

Department of Applied Mathematics and Statistics, University of California, Santa Cruz

Joint work with Ziwei Wang (*Ask.com*) and Abel Rodriguez (*UC Santa Cruz*)

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- Study of extremes consists of the exploration of events that occur in the tails of probability distributions.
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- Applications in several diverse fields: environmental sciences, finance, actuarial sciences, climatology ...
- Nonparametric Bayesian modeling for extreme value analysis, using the point process approach based on threshold exceedances.
- Application to estimating systematic and idiosyncratic risks on multiple financial markets.

Approaches to modeling extremes: GEV

- **Generalized extreme value (GEV) distribution**
- $X_1, X_2, \ldots,$ i.i.d. sequence of random variables, and $M_n = \max\{X_1, \ldots, X_n\}$.

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- If there exist $a_n > 0$ and b_n such that $\Pr((M_n - b_n)/a_n \le x) \to H(x)$, as $n \to \infty$, for a non-degenerate distribution H, then H is a GEV

$$H(x) = \exp\left\{ - \left(1 + \xi\psi^{-1}(x - \mu)\right)_+^{-1/\xi} \right\}$$

  - $\xi < 0$ corresponds to the Weibull distribution ("short-tailed" case);
  - $\xi > 0$ to the Fréchet distribution ("long-tailed" case);
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- For a sequence of observations in time, inference on the GEV parameters is obtained by assuming that the block-wise maxima (over a given time unit) are distributed as $H(x)$.

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- Modeling approach: the number N of exceedences over the threshold u in any unit of time has a Poisson distribution; and conditionally on $N \geq 1$, the values of the excesses, $y_1, ..., y_N$, are i.i.d. with a GPD distribution.

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- $X_1, \ldots X_r$ i.i.d. sequence of random variables from a distribution for which the GEV limit exists.
- Focus on observations above threshold u, $\{(Z_i, Y_i) : i = 1, ..., N\}$, with $N \leq r$, viewed as a realization from a point process on $\mathcal{A} = \{1, \ldots, r\} \times [u, \infty)$
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  - Z_i: the time at which the i-th exceedance occurs
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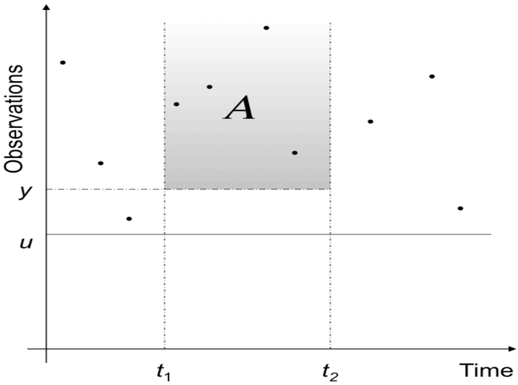
- The limiting form of this point process (as $u \to \infty$) is a bivariate non-homogeneous Poisson process (NHPP) with intensity function

$$\psi^{-1}\left\{1 + \xi\psi^{-1}(y - \mu)\right\}_{+}^{-1/\xi-1}$$

(Pickands, 1971)

Graphical illustration of the point process approach



For any set $A = [t_1, t_2] \times [y, \infty)$, within the support of the NHPP, the number of exceedances in A follows a Poisson distribution with mean

$$\Lambda(A) = (t_2 - t_1)\{1 + \xi\psi^{-1}(y - \mu)\}_{+}^{-1/\xi}$$

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- Inference through the NHPP likelihood applied to the point pattern $\{(t_i, y_i) : i = 1, ..., N\}$, of the N observed excess values y_i paired with the corresponding time points t_i (e.g., Smith, 1989; Coles & Tawn, 1996).

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- Restriction of the point process modeling framework: relies on the parametric form of the (asymptotic) NHPP intensity, which is homogeneous in time.

NPB mixture modeling under the point process approach

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- Assume again that the point pattern $\{(t_i, y_i) : i = 1, ..., N\}$ arises from a NHPP on $\mathcal{A} = [0, T] \times [u, \infty)$
  - $\mathcal{N}(A) \sim \text{Poisson}(\int_A \lambda(t, y)\, \mathrm{d}t\mathrm{d}y)$, for any measurable $A \subset \mathcal{A}$
  - given $\mathcal{N}(A)$, the points within A are i.i.d. $\lambda(t, y)/\{\int_A \lambda(t, y)\, \mathrm{d}t\mathrm{d}y\}$.

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- Replace the restrictive parametric form for the NHPP intensity, $\lambda(t, y)$, with a nonparametric mixture model that balances:
  - general inference for extreme value analysis functionals, and
  - desirable properties for the tail behavior of the marginal distribution F_t of X_t.

NPB mixture modeling under the point process approach

- The likelihood for the NHPP intensity function is proportional to

$$\exp\left(-\int_{\mathcal{A}} \lambda(t,y)\,\mathrm{d}t\mathrm{d}y\right)\prod_{i=1}^{N}\lambda(t_i,y_i) = \exp(-\gamma)\gamma^{N}\prod_{i=1}^{N} f(t_i,y_i)$$

- $\gamma = \int_{\mathcal{A}} \lambda(t,y)\,\mathrm{d}t\mathrm{d}y$ is the total intensity of exceedances.
- $f(t,y) = \lambda(t,y)/\gamma$ is a density function on $\mathcal{A}$ which fully controls the shape of the intensity of exceedances.

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- $f(t,y) = \lambda(t,y)/\gamma$ is a density function on $\mathcal{A}$ which fully controls the shape of the intensity of exceedances.
- A flexible model for $\lambda(t,y)$ can be built through a nonparametric mixture model for density $f(t,y)$ (along with a parametric prior for γ).

NPB mixture modeling under the point process approach

- Dirichlet process (DP) mixture model for the exceedance density:

$$f(t, y \mid G) = \int \text{be}(t \mid \kappa, \tau)\text{gpd}(y \mid \xi, \phi)\,\text{d}G(\kappa, \tau, \xi, \phi), \quad G \sim \text{DP}(\alpha, G_0)$$

  - $\text{be}(t \mid \kappa, \tau)$ is the density of the (rescaled) Beta distribution on $(0, T)$, with mean $\kappa \in (0, T)$, and scale parameter $\tau > 0$
  - $\text{gpd}(y \mid \xi, \phi) = \phi^{-1}\left(1 + \xi\phi^{-1}(y - u)\right)^{-1/\xi - 1}$, for $y > u$, is the GDP density with location defined by the threshold, $\phi > 0$, and with $\xi > 0$ corresponding to heavy-tailed distributions.

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- DP prior for the mixing distribution: $G = \sum_{\ell=1}^{\infty} \omega_\ell \delta_{\eta_\ell}$
  - atoms η_ℓ i.i.d. from centering distribution G_0
  - weights generated through *stick-breaking*: $\omega_1 = \zeta_1$, $\omega_\ell = \zeta_\ell \prod_{r=1}^{\ell-1}(1 - \zeta_r)$ for $\ell \geq 2$, with ζ_ℓ i.i.d. Beta(1,α) (independently of the η_ℓ).

NPB mixture modeling under the point process approach

- Under mild conditions on the underlying process $\{X_t : t \in [0, T]\}$, it can be shown that for any specified time point $t_0 \in (0, T)$,

$$\Pr(X_{t_0} > x \mid X_{t_0} > u) = \int_x^{\infty} f(y \mid t_0)\,\mathrm{d}y, \quad x > u$$

  which enables time-dependent inference for tail probabilities of the marginal distributions from the underlying process.

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- It can also be proved that the marginal distributions of the underlying process, $\Pr(X_t > x)$, belong to the Fréchet domain of attraction
  - for large x, $\Pr(X_t > x) \approx C x^{-\beta} L(x)$, where $C > 0$ and $\beta > 0$ are constants in x
  - $L(x)$ is a slowly varying function, $\lim_{x \to \infty} L(vx)/L(x) = 1$, $\forall v > 0$
  - β is the tail index parameter (β^{-1} can be interpreted as a risk indicator).

NPB mixture modeling under the point process approach

- Posterior simulation methods for DP mixtures can be used for inference on:
  - bivariate intensity, $\lambda(t, y)$, and marginal intensity over time, $\lambda(t)$
  - conditional density of exceedances given specific time points, $f(y \mid t_0)$
  - different types of return level functions.

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- Formulating the modeling and inference within a density estimation context enables different types of practically relevant extensions:
  - risk assessment for extremes recorded over time and space, extending G to a random spatial surface $G_{\mathcal{S}} = \{G_s : s \in \mathcal{S}\}$ (Kottas, Wang & Rodriguez, 2012)
  - hierarchical modeling for extremes recorded from a (finite) number of related processes — a specific application coming up next ...

Motivating application

- Assess the effect of *systematic* and *idiosyncratic* risks on multiple financial markets
  - systematic risk: the vulnerability of a financial market to events that affect all (most) of the agents and products in the market
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- Focus on the time dimension of threshold exceedances.
- Model the point process of occurrence times of extreme losses in each market using a superposition of two NHPPs, one that corresponds to systematic risks, and one that corresponds to idiosyncratic risks.

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- Beta DP mixture models for the NHPP intensities to capture changes in the risk structure over time.

Data example

- Data on *extreme* negative log returns from the Standard & Poor's 500 (S&P500) sector indexes, recorded between January 1, 2000 and December 31, 2011.
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- Companies included in the S&P500 index are commonly grouped into 10 economic sectors
  - focus on 4 sectors: consumer staples; energy; financials; and information technology
  - the other 6 sectors: consumer discretionary; health care; industrials; materials; telecommunication services; and utilities.

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  - the other 6 sectors: consumer discretionary; health care; industrials; materials; telecommunication services; and utilities.
- In addition to the overall S&P500 index, Standard & Poor's publishes separate indexes for each of the sectors (prices for the individual indexes were obtained from Bloomberg financial services).

Data example

- Data on negative daily log returns for the S&P500 sector indexes

$$x_{i,j} = -\log(S_{i,j}/S_{i-1,j})$$

where $S_{i,j}$ is the index value for sector $j = 1, \ldots, J = 10$ at day $i = 1, \ldots, T$.

- Note that large positive values of $x_{i,j}$ indicate a large drop in the price index associated with sector j.

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- For given threshold u, consider point patterns

$$\{t_{j,k} : k = 1, \ldots, N_j\}, \quad \text{for } j = 1, \ldots, J$$

  where $t_{j,k}$ is the date of the k-th extreme drop in the price index for sector j (negative log return in sector j that is larger than u).

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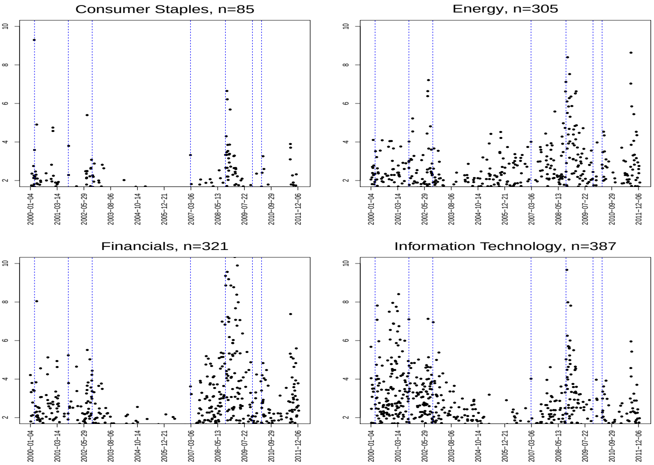
- Threshold $u = 2\%$ (a 2% drop in the market has been historically used as a threshold for trading curbs on program trades).



Negative log returns above 2% for four sectors of the S&P500 index. Vertical dotted lines identify seven events of significance to the markets: the bursting of the .com bubble (03/10/2000), the 09/11 terrorist attacks (09/11/2001), the stock market downturn of 2002 (09/12/2002), the bursting of the Chinese bubble (02/27/2007), the bankruptcy of Lehman Brothers (09/16/2008), Dubai's debt standstill (11/27/2009), and the beginning of the European sovereign debt crisis (08/27/2010).

The modeling approach

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- Construct $\mathcal{N}_j$ as a superposition of two (independent) NHPPs:
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- $\mathcal{N}_j$ is also a NHPP with intensity

$$\begin{aligned} \lambda_j(t) &= \lambda_0^*(t) + \lambda_j^*(t) = \gamma_0^* f_0^*(t) + \gamma_j^* f_j^*(t) \\ &= (\gamma_0^* + \gamma_j^*)\{\varepsilon_j f_0^*(t) + (1 - \varepsilon_j) f_j^*(t)\} \end{aligned}$$

where $\varepsilon_j = \gamma_0^*/(\gamma_0^* + \gamma_j^*)$ is the proportion of exceedances in sector j that are associated with the systematic component.

The modeling approach

- For $j = 0, 1, ..., J$, Beta DP mixture prior model for the densities $f_j^*(t)$

$$f_j^*(t \mid G_j^*, \tau) = \int \text{be}(t \mid \kappa, \tau) \, \text{d}G_j^*(\kappa), \quad G_j^* \sim \text{DP}(\alpha_j, H)$$

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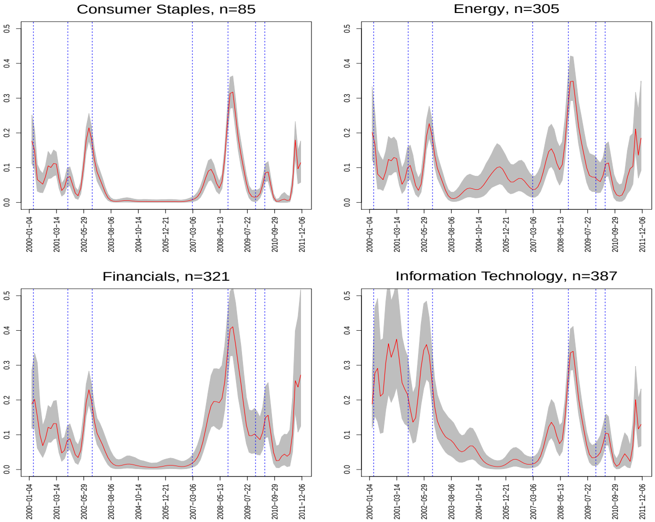
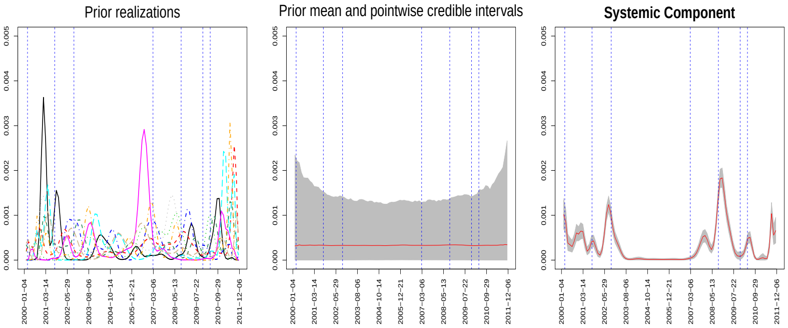
- For the total intensity parameters, $\{\gamma_0^*, \gamma_1^*, \ldots, \gamma_J^*\}$
  - gamma prior for γ_0^*
  - zero-inflated gamma prior for γ_j^*, $j = 1, ..., J$

$$p(\gamma_j^* \mid \pi) = (1 - \pi)\, \delta_0(\gamma_j^*) + \pi\, \text{gamma}(\gamma_j^* \mid a_{\gamma_j^*}, b_{\gamma_j^*})$$

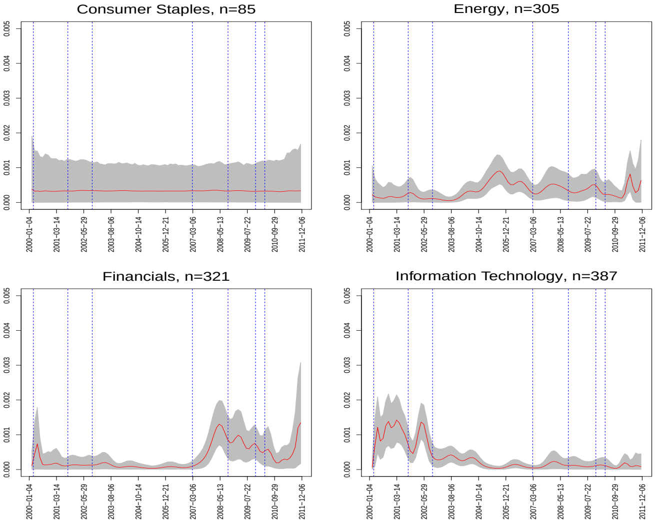
- Enables formal testing for the presence of idiosyncratic risks: $\gamma_j^* = 0$ corresponds to $\varepsilon_j = 1$, i.e., all exceedances in sector j are driven by systematic risks.



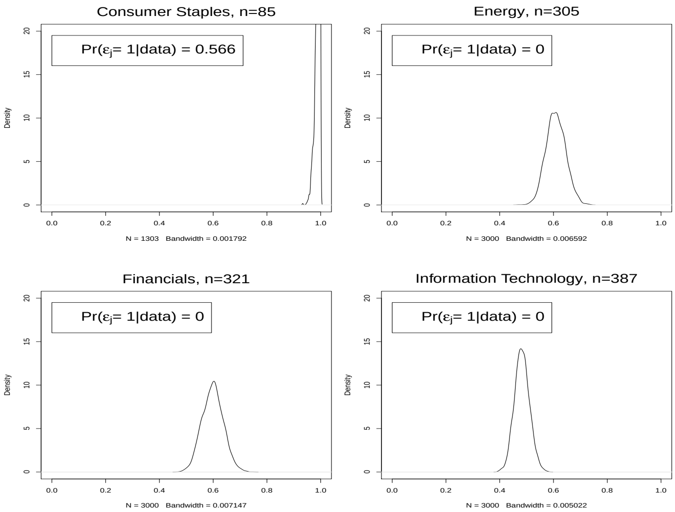
Posterior mean and 95% uncertainty bands for the overall intensity of extreme drops for four sectors of the S&P500 index.



Posterior mean and interval estimates for the density associated with the systematic risk component of the S&P500 index (right panel). The left panel shows realizations from the prior density function, and the middle panel the prior mean density function and 95% prior uncertainty bands.



Posterior mean and 95% uncertainty bands for the idiosyncratic density of extreme drops for four sectors of the S&P500 index.



Posterior densities for the proportion of risk attributable to the systematic component on four sectors of the S&P500 index.

Main results

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- The behavior of the idiosyncratic risk varies drastically with the economic sector, and can be explained by factors that are sector-specific:
  - energy and utilities sectors present increases in idiosyncratic risk during 2005, a period that corresponded to sharp increases in oil prices but that was otherwise relatively calm;
  - idiosyncratic risks of the information technology and telecommunication services sectors are particularly elevated between 2000 and 2002, a period that included the bursting of the so-called dot-com bubble;
  - idiosyncratic risk of the consumer staples sector is almost negligible over the whole period under study – this sector includes companies that produce and trade basic necessities whose consumption might be affected by general economic conditions but is otherwise relatively stable.

Manuscripts

- Kottas, A., Wang, Z. & Rodriguez, A. (2012). "Spatial modeling for risk assessment of extreme values from environmental time series: a Bayesian nonparametric approach." *Environmetrics*, 23, 649-662.
- Wang, Z., Rodriguez, A., & Kottas, A. (2014). "Nonparametric mixture modeling for extreme value analysis." Under review.
- Rodriguez, A., Wang, Z., & Kottas, A. (2014). "Assessing systematic risk in the S&P500 index between 2000 and 2011: A Bayesian nonparametric approach." Under review.

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MANY THANKS !!!