Computational Complexity Classes

Def: Let \( t(n) \) be a fn. \( \mathbb{N} \rightarrow \mathbb{N} \). A language \( L \in \text{DTIME}(t(n)) \) if \( \exists \) a TM that runs in time \( c \cdot t(n) \) for constant \( c > 0 \) and decides \( L \).

Q. Let us define \( \text{DTIME}_{0,1}(t(n)) \) w.r.t alphabet \( \{0,1\} \) and define \( \text{DTIME}_{0,1,2,3}(t(n)) \) w.r.t alphabet \( \{0,1,2,3\} \)

R) \( \text{DTIME}_{0,1}(t(n)) \subseteq \text{DTIME}_{0,1,2,3}(t(n)) \)
S) \( \ldots \ldots \ldots \ldots \ldots \ldots \)
B) \( \text{DTIME}_{0,1}(t(n)) = \text{DTIME}_{0,1,2,3}(t(n)) \)
O) None of the above

(Suppose \( \text{DTIME}(t(n)) \) was defined without the constant \( c \). Then?)

Q. Define \( \text{DTIME}_k(t(n)) \) w.r.t \( k \)-tape TMs. Let \( k < k' \)

R) \( \text{DTIME}_{k}(t(n)) \subseteq \text{DTIME}_{k'}(t(n)) \)
S) \( \ldots \ldots \ldots \ldots \ldots \ldots \)
B) \( \text{DTIME}_k(t(n)) \supseteq \text{DTIME}_k(t(n)) \)
O) None of the above

Q. Consider situation above.

R) \( \text{DTIME}_k(t(n)) \subseteq \text{DTIME}_{k'}(t(n) \log t(n)) \)
S) \( \ldots \ldots \ldots \ldots \ldots \ldots \)
B) \( \ldots \subseteq \text{DTIME}_{k'}(t(n)^2) \)
O) \( \ldots \ldots \ldots \ldots \ldots \ldots \)

O above when \( k \geq 2 \).
Def: \( P = \bigcup_{n \geq 0} \text{DTIME}(n^k) \)

Ask previous questions not P.

**Church-Turing Thesis:** Every physically realizable computational device can be simulated by a TM.

**Strong CT Thesis:** Every physically realizable computational device can be simulated by a TM with polynomial overhead.

1) P contains what we consider efficient: "Cethanisthesis" 
2) P contains "closure" of efficient algorithms. An algorithm that invokes a poly-time algorithm poly-times is also in P. 
3) For many problems/languages of interest, existence in P implies structure, implies a non-trivial algorithm strongly avoiding brute force

[Edmonds 64, Gödel 52, Trakhtenbrot 50s, Cethan 62]

Def: \( \text{LGNP} \) if \( L \) is "poly-time" verifiable.

\[ \exists \text{ polynomial } p \text{ and } q \text{ s.t. } \forall x \in \Sigma^* \exists L \exists y \text{ s.t. } M(x,y) \text{ accepts and terminates in } q(\text{poly}(x,y)) \]

\[ \forall x \in L \forall y \text{ s.t. } M(x,y) \text{ rejects and terminates } \]
Q. Hilbert's 10th problem: Given a set $S$ of diophantine equations, does it have a solution (integer)?

Q. $Ax=b$. Is this in $\text{NP}$?

Q. Is Clique $\text{NP}$? Is bipartite matching $\text{NP}$?

What complement?

Thm: $\text{P} \text{E} \text{NP}$

Q. What is certificate?

(A) Everything
(B) Anything
(C) Nothing
(D) Anything can be shown as certification

Def: A non-deterministic TM has a non-deterministic transition function. The running time is a bound on all possible computational paths.

Thm: $\text{NP} = \bigcup \text{NTIME}(n^c)$

**Reductions**

Def: A reduction is a computable mapping $f: L \rightarrow L'$

st. if $x \in L$ then $f(x) \in L'$

$x \notin L$ then $f(x) \notin L'$