Amazing! Regardless of how many tapes, I can simulate in almost the same time.

Consider old T solution

Gap between *'s is arbitrary
(as large as T)
Cannot control spacing. What can it be?
(R) Constant
(G) As large as T
(B) \log T
(C) T

Thus a single move requires \( \Omega(T) \) simulation time, and \( k \) moves can require \( \Omega(kT) \) simulation time.

Instead keep all the head locations in one place, and simulate move \( \theta \) by shifting entire tape.

Use new symbol \( \triangleright \) to create buffers. This acts like \( \triangleright \) but has different meaning.
Use a new "copy" to aid efficient copying.

Create zones in tape:

\[ L_3 \quad L_2 \quad L_1 \quad R_1 \quad R_2 \quad R_3 \]

\( L_i, R_i \) have \( 2^i \) cells.
Invariants:
(1) Cell $0$ has current head location
(2) Each $L_i, R_i$ has either all non-$\sigma$ symbols (full), no non-$\sigma$ all $\sigma$ (empty) or exactly $2^{-i} \sigma$ (half-full).
(3) Total number of $\sigma$s in $L_i \cup R_i$ is $2^i$.

A move to the right
Q: What is cost of initialization?

(R) $O(T)$
(G) $O(1)$
(B) $O(\log T)$
(C) Tape is already initialized, no cost.

Shifting: Consider head moving right, so tape shifts left.

(1) Find first non-empty zone $R_i$.

What does $L_i$ have?

(R) Arbitrary
(G) Full
(B) Half-full
(C) Empty

$\sigma$ needs to be moved to midpoint of tape.
$\sigma$ stored in register.

(2) $R_i$ has $2^{i-1}$ or $2^{i-1}$ non-$\sigma$ symbols.
$\sigma$ moved to midpoint. $2^{i-1} - 1$ symbols moved into remaining zone.

\[
\sum_{i=1}^{n} 2^{i-1} = 2^n - 1
\]
Q. How many symbols need to be moved into $R_i - \ldots, R_{i-1}$ to make them half-full?

R) $2^{i_0}$
B) $2^{i+1}$
A) $2^{i_0 - 1}$
O) $2^{i_0} - 1$

Q. $R_{i_0}$ is now

A) Empty
B) Half-empty
O) Depends

Q. What needs to be done to maintain invariant?

R) Nothing
B) Make $L_i - \ldots, L_i$ empty
A) Make $L_i - \ldots, L_i$ half-empty
O) Make $L_i - \ldots, L_i$ full

Q. How do we bound the cost of movement?

Q. How many moves before $L_{i_0}, R_{i_0}$ accessed again?

R) $2^{i_0} O(1)$
B) $10$
A) Cannot be determined
O) $2^{i_0}$

Thus, a $O(2^i)$ move happens only after $O(2^i)$ steps.

Total cost $= O\left(\sum_{i=1}^{\log T} \frac{T_i \times 2^i}{2^i}\right) = O(T \log T)$