tape

Multi-headed to standard TM

(1) Interleave tapes
(2) Mark head using new symbol * on top of symbol

Always shift head back to leftmost or rightmost *

Program in "assembly" \[\Rightarrow\] 2-dim TM with multiple types
\[\Downarrow\]
TM with multiple 1-D types
\[\Downarrow\]
Standard TM

All equivalent

TM's are robust to specifications.

Read up to section 1.4 in Arora-Barak.

Q. Consider a TM that has k-bit register. On transition, TM overwrites content of register. TM can transition depending on register contents.

This
R) TM might be more powerful.
B) Same power can be proven by changing states.
A) Same power can be proven by adding tape.
D) Same power, proven by updating alphabet

Q. Consider TM that has register, that stores natural number. On transition, TM increments/decrements register. TM transitions depending on register contents.
Q. TM has register with natural number. TM can transition depending on whether register has value 1.

Q. TM has register with natural number. TM can transition depending on whether register has prime number.

Q. The main tape of TM stores alphabet of natural numbers. TM can increment/decrement current symbol. TM's transitions function depends on whether symbol is 1.

Q. Same as above, but TM transition depends on whether symbol is prime.

Q. Same as above, TM transition depends on symbol.

Defn: Let \( f: \{0,1\}^* \rightarrow \{0,1\}^* \) and \( T: \mathbb{N} \rightarrow \mathbb{N} \).

- \( M \) computes \( f \) if \( M(x) \) is output, and \( M(x) = f(x) \).
- \( M \) computes \( f \) in \( T(n) \)-time if \( M(x) \) is output in \( T(1^n) \)-time.

Claim: If \( f \) is computable in time \( T(n) \) by \( M \) with alphabet \( \Sigma \), then it is computable in time \( 4^1/T(n) \) using \( \Sigma' = \{1,0,\} \).

Pf: Use unary encoding. \( M' \) reads symbol in \( \Sigma' \) steps.

(1) \( M' \) reads symbol in \( \Sigma' \) steps.
(2) \( M' \) "stores" symbol in its register/state.
(3) \( M' \) transitions appropriately.
(4) \( M' \) writes symbol on tape.
Q: Suppose M' has alphabet $\Sigma'$. Then M can be simulated in:

$O\left( \frac{\log |\Sigma|}{\log |\Sigma'|} T \right)$

A) $O\left( \frac{\log |\Sigma|}{\log |\Sigma'|} T \right)$

B) $O\left( \frac{\log |\Sigma|}{\log |\Sigma'|} T \right)$

C) $O\left( \log |\Sigma| \log |\Sigma'| T \right)$

B) $O\left( \log |\Sigma| T \right)$

Claim: If $f$ is computable in time $T(n)$ by $k$-tape TM, $f$ is computable in time $5kT(n^2)$ by single tape TM.

Proof:
Represent $k$-tapes by 1-tape

\[1 \ 2 \ 3 \ 4 \ 2 \ 3 \ 4 \ 1 \]

M' uses dotted symbols to indicate where head is.

1) M' first computes $T(1x1)$.
2) M' sweeps $T(1x1)$ steps, and records each head symbol.
3) At the end, M' transitions into state, and records what needs to be written, and direction of head.
4) M' sweeps back and make updates.
We can modify $M'$.

4) On sweeping back, $M'$ shifts all heads that moved left.
5) $M'$ sweeps forward, shifts all heads that moved right.
6) $M'$ moves back to start.

Q. At any intermediate time
Q. How many steps does a single move of $M$ take?

A) Depends on $f(n)$, but at most $O(T(n))$
B) A constant number
C) $T(n)$
D) Exactly $T(n)$

Q. At any intermediate step $t$, where is the head?

A) Depends on $f(n)$, not possible to give closed form.
B) Periodic
C) Corresponds to head position in original $M$.

Thus, machine is oblivious. Head position only depends on $f(n)$ and $t$.

Q. How can a TM be represented?

A) All TMs can be represented by finite string.
B) Every TM is necessarily represented by an infinite string.
C) Some TM is represented by an infinite string.