IP = language of feasible linear 0/1 programs

Thm: 3SAT \leq_p IP

Pf: Given \Phi: for each clause \( x_i \lor \bar{x_j} \lor \bar{x_k} \)
introduce constraint \( x_i + (1-x_j) + \bar{x_k} \geq 1 \)

QP = language of feasible quadratic programs

Thm: CLIQUE \leq_p QP

Pf: Given \((G, k)\), write program

\[
\begin{align*}
\sum x_i &= k \\
\forall i \quad x_i^2 &= x_i \\
\forall (u,v) \in E \quad x_u x_v &= 0
\end{align*}
\]

DHAMPATH \leq_p \langle G \rangle \quad G \text{ is directed and has Hamiltonian paths}

Thm: 3SAT \leq_p DHAMPATH

Pf: DHAMPATH \in NP

Variable gadget: \( \Phi \) has \( n \) variables and \( m \) clauses
Each variable is

\[
\begin{array}{c}
\text{variable}\text{-}\text{gadget} \\
\text{3m+2 nodes}
\end{array}
\]
Each clause is a vertex $C_j$

If $C_j = \{i \vee j \vee k\}$

Directions of paths in graph's variable gadget determines assignment:

True
False

If $\phi \in SAT$, we can construct path that visits all vertices exactly once.

If there is Hamiltonian path, it must enter clauses from a variable and go back to it.
We can also HAMCYCLE is NP-complete with above construction by connecting last vertex to first vertex.

Thus: HAMPATH is NP-complete.

PF: DHAMPATH \ \not\ \leq \ \text{HAMPATH}.

We do vertex splitting.

Paths in these graphs have a direct correspondence to each other.

Thus: SUBSET-SUM is NP-complete.

PF: 3SAT \ \leq \ \text{SUBSET-SUM}

Let \ \phi \ \text{has} \ n \ \text{variables and} \ m \ \text{clauses.}