SDN-based Privacy Preserving Cross Domain Routing

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Abstract—Today’s large-scale enterprise networks, data center networks, and wide area networks can be decomposed into multiple administrative or geographical domains. Domains may be owned by different administrative units or organizations. Hence protecting domain information is an important concern. Existing general-purpose Secure Multi-Party Computation (SMPC) methods that preserves privacy for domains are extremely slow for cross-domain routing problems. In this paper we present PYCRO, a cryptographic protocol specifically designed for privacy-preserving cross-domain routing optimization in Software Defined Networking (SDN) environments. PYCRO provides two fundamental routing functions, policy-compliant shortest path computing and bandwidth allocation, while ensuring strong protection for the private information of domains. We rigorously prove the privacy guarantee of our protocol. To improve time efficiency we design the QuIck Pathing (QIP) technique. QIP only requires one-time offline preprocessing and very fast online computation. We have implemented a prototype system that runs PYCRO and QIP on servers in a campus network. Experimental results using real ISP network topologies show that PYCRO and QIP are very efficient in computation and communication costs.

Index Terms—Privacy, Secure Multi-Party Computation, Software Defined Networking, Routing.

1 INTRODUCTION

Large-scale enterprise networks, data center networks, and wide area networks (WANs) may be decomposed into multiple administrative or geographical domains [4], [31], [17], [15], [32], [42], [28], [1]. Multi-domain networks are deployed to interconnect community networks, data centers, corporation sites, and university campuses. In a multi-domain network such as a WAN, different domains may belong to different administrative units with an organization or different organizations [31], [17], [15], [28], [24], [1]. For example, a number of organizations may own their own sub-networks, and those subnetworks are mutually interconnected to form a multi-domain network [1]. Hence individual domain may have security and privacy concerns regarding revealing its domain information to other domains. This paper focuses on Enterprise-scale and Metropolitan-scale multi-domain WANs that consist of a few to less than 20 domains. Internet-scale privacy-preserving routing requires future study.

Routing optimization, such as finding policy-compliant paths that have least routing cost or satisfy bandwidth demands, plays a critical role of network management. Recent advances of Software Defined Networking (SDN) has brought tremendous convenience to routing optimization by separating the control plane from routers and allowing a central controller to make routing decisions. Using centralized optimization, the controller can efficiently and effectively find a desired routing path for each flow and install forwarding rules on corresponding switches. In this paper we refer to all network forwarding units as “switches” for consistency to SDN terminology. Although SDN simplifies routing optimization in a single domain, privacy-preserving cross-domain routing optimization is still a challenging problem. Suppose each domain has a centralized controller. The state-of-the-art approach to route a cross-domain flow is using local optimization for intra-domain path selection and BGP for inter-domain routing, such as the design in Google’s SDN B4 [17] and DISCO [28]. This approach protects the autonomy and privacy of domains. However, BGP heavily relies on the local execution model. It only allows each domain to compute the “best route” according to their local policies and pass the results to other domains. More importantly, BGP cannot incorporate application requirements. For example, delivering voice over IP and video streaming require low-latency paths while file transfer requires high-throughput paths. These requirements are completely unaware of by BGP. The cross-domain paths computed by BGP plus local policies may result in sub-optimal decisions: it cannot guarantee even the shortest paths in terms of hop-count. Hence BGP routing optimization is unpredictable, and may exhibit slow convergence [19] and persistent oscillation [38] [12]. Similarly, hierarchical routing cannot perform global optimization on routing paths based on any metric. Another solution is to allow every controller to broadcast its domain information to the entire network and maintains a network-wide map, similar to a controller-level OSPF protocol. This approach causes privacy and security concerns because every domain has to expose its private information such as network topology, link latencies, bandwidth, and routing policies. In fact, there is no practical and privacy-preserving solution to the

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most fundamental routing problem, i.e., computing shortest paths, for multi-domain networks. Existing secure BGP protocols (S-BGP [18], soBGP [25], and RPKI [20]) cannot satisfy the privacy preserving requirements because they are used to provide authentication, integrity, and validity of routing updates rather than domain privacy.

Privacy-preserving cross-domain network problems can be modeled as secure multi-party computation (SMPC) [41], [2], [4], [16], [22], [26]. However, general-purpose SMPC solutions such as SEPIA [4] are extremely slow and may take days to complete [9] [13]. Therefore, customized algorithms are needed for the privacy-preserving cross-domain routing problems.

In this paper, we present the first work for privacy-preserving cross-domain routing optimization that has reasonable efficiency in practical networks. We design and implement a protocol named PYCRO (PrivAcY-preserving Cross-domain Routing Optimization) and its extensions to provide two fundamental routing functions, namely policy-compliant shortest path computing and bandwidth allocation, while protecting the private information of domains. PYCRO is executed on SDN controllers in a distributed manner and does not rely on any trusted third party. PYCRO is developed based on a novel cryptographic tool called Secure-If operations.

The properties of PYCRO can be summarized as follows.

- PYCRO can compute policy-compliant cross-domain shortest paths and allocate bandwidth for flows while protecting private information of domains. The privacy guarantee of PYCRO is cryptographically strong. (Please see Section 7 for formal analysis of privacy.)
- PYCRO also preserves the autonomy and local policies of domains. A domain can independently determine whether and how to forward different flows and these preferences are unknown to other domains.
- PYCRO is efficient in both computation and communication costs.
- Compared to BGP, PYCRO is able to perform network-wide optimization and incorporate fine-grained application-specific metrics, such as latency and bandwidth, by setting proper link weights for both intra-domain and inter-domain links [39]. In addition, PYCRO includes a multi-path routing protocol.

PYCRO is the first work of privacy-preserving cross-domain routing optimization in SDN environments. We have implemented a prototype system that runs PYCRO on machines in a campus network. Experimental results using real ISP network topologies show that PYCRO has reasonably good efficiency. It spends < 30 seconds and < 700 KB messages in computing a shortest path tree for networks consisting of thousands of switches and links.

In the section, we discuss possible extensions of our protocols in Section 9. Finally, we conclude our paper in Section 10.

2 RELATED WORK

Privacy-preserving cross-domain routing can be modeled as a secure multi-party computation (SMPC) problem. Yao’s seminal work [41] introduces the first algorithm, called Yao’s garbled circuits, to allow two parties to compute an arbitrary function with their inputs without revealing private information. Since then, many studies about SMPC have been conducted [2], [4], [16], [22], [26]. In [23], a secure two-party computation system called Fairplay is introduced and the system implements generic secure function evaluation. FairplayMP proposed in [2] supplements the Fairplay system. FairplayMP is a generic system for secure multi-party computation while Fairplay only supports secure two-party computation. SEPIA [4] is a recently proposed SMPC system for general inter-domain network applications. A common limitation of these SMPC solutions is that the computation time can be way too long for practical applications. For example, [9] shows that it takes thousands of days to track cross-domain connectivity of a few domains using SEPIA [4]. An SMPC-based routing algorithm proposed to replace BGP also experiences long execution time [13] which makes the existing SMPC methods impractical for inter-domain routing.

Recent researchers have proposed custom privacy-preserving algorithms for different network applications. Chen et al. [6] use Bloom filters to combine access control lists of multiple domains and determine network reachability in a privacy-preserving manner. Djatmiko et al. [9] propose to apply counting Bloom filters for privacy-preserving multi-domain connectivity tracking. STRIP [14] is a privacy-preserving inter-domain routing protocol to replace BGP and achieve fast convergence. To our knowledge, no existing work in this category studies the privacy-preserving cross-domain routing optimization problem. Route Bazaar [5] is a contractual system that provides ASes with automatic means to form, establish, and verify end-to-end connectivity agreements. Compared to this work, Route Bazaar focuses on Internet accountability which is very different from the objective of our work, because Route Bazaar does not compute and optimize routing paths.

3 PROBLEM OVERVIEW AND BACKGROUND

In the section, we formalize the problem in this paper and then introduce a novel cryptographic tool we will use to solve the problem.
3.1 Problem Formulation

We formalize the problem to be solve in this paper as follows.

Consider a large network that consists of $N$ domains: $D_1$, $D_2$, ..., $D_N$, where each domain $D_i$ has a domain controller $C_i$ that makes routing decision and updates the forwarding tables of switches in the domain. A domain controller can access any information of its domain, including the network topology, access control policies, link bandwidth, and authenticated hosts. A domain controller can add, delete, and update forwarding entries of switches in its domain. It communicates with controllers in other domains via pre-established secure channels.

For any two switches $v, v' \in D_i$ ($v \neq v'$), we use $v \sim v'$ to denote that there is a link between $v$ and $v'$ and we denote its link cost by $c(vv')$. Clearly, each $C_i$ should know the topology of $D_i$, and should also know all the link costs within this domain: $\{c(vv') | v, v' \in D_i, v \neq v'\}$. The privacy is defined such that the intra-domain topology and the intra-domain link costs are confidential information of $C_i$. That is, any other domain controller should not know anything about this topology or these link costs. We assume different domains are managed by different parties, such as ISPs, organizations, or departments of a corporation. Parties do not share domain information. We also assume no two domain may collude. If a party owns multiple physical domains or multiple domains agree to share information with each other, all these domains can be considered a single logical domain and our protocol works correctly under this scenario.

There are some inter-domain links that connect switches from different domains. We assume that information about an inter-domain link is available of the two end domains, and domains can share it with other domains. That is, for any inter-domain link $v \sim v'$ (where $v \in D_i$, $v' \in D_j$ and $D_i \neq D_j$), all domain controllers could know the two endpoints $v$ and $v'$, and also $D_i$ and $D_j$—the domains they belong to. A switch that is connected to switches in other domains is called a gateway switch. Clearly, information about the connectivity between these gateway nodes can hardly be kept confidential. Hence we assume gateway switches can be publicly known.

Suppose that there are a source switch $v_s$, which belongs to a domain $D_s$, and a destination switch $v_t$, which belongs to another domain $D_t$. Our objective is to design a private-preserving optimized routing solution. Specifically, we need to design a protocol that allows each domain controller $C_i$ to find the forwarding table $T(v)$ for all $v \in D_i$, where each entry $T(v)[v_s, v_t]$ is the next-hop switch of $v$ on the optimal routing path from the source $v_s$ to destination $v_t$.

We uses the SDN model for routing optimization mainly due to the following reasons. First, the cross-domain routing protocol includes cryptographic operations such as homomorphic encryption/decryption and Secure-If operations, which incurs relatively complex computation for routers or switches. Hence we need general-purpose servers, i.e., controllers in our model, to perform these operations and compute optimized routes. Second, SDN provides routing and forwarding flexibility for flows with different application requirements. For example, if flow 1 requires a shortest path in latency and flow 2 requires a path with high bandwidth, the corresponding paths can be easily computed separately in the controllers and installed to the forwarding tables of switches. In traditional IP networks, such forwarding decisions are difficult to implement.

In this work, PYCRO focuses on two major routing optimization problems.

1) Policy-compliant shortest path routing. Each link has an associated routing cost (also known as link weight), representing a performance metric such as hop count, latency, or traffic load [39], [30]. The routing object is to find a path from the source to the destination that has the minimum sum of link cost without violating policies of domains.

2) Bandwidth allocation. Bandwidth allocation has been applied to practical traffic engineering solutions such as B4 [17]. Each flow has a bandwidth demand and link bandwidth is allocated to different flows. When flows are competing for bandwidth, a single flow may need multiple paths to satisfy its bandwidth demand. The routing object is to find one or more paths for a flow such that the flow bandwidth demand can be satisfied. At this stage, we do not consider fairness among flows [17], [21].
other domains’ privacy.

### 3.2 Cryptographic Tool

Here we introduce the cryptographic tool we will use in this work, namely the Secure-If operation.

**Secure-If operation.** Our protocol depends on a cryptographic technique developed by us, which we call the Secure-If operation. This operation allows the protocol to choose between two options \(Y\) and \(Z\), based on whether a particular condition \(X\) is satisfied. Denote by \(\text{SecIf}(X, Y, Z)\) the Secure-If operation, and then we have

\[
\text{SecIf}(X, Y, Z) = \begin{cases} Y, & X \text{ is satisfied;} \\ Z, & \text{otherwise.} \end{cases}
\]

(1)

Note that this operation is privacy preserving. It is infeasible for anybody to decide whether the condition is satisfied or not, i.e., which of the two options is actually chosen. For example, suppose that \(X, Y, Z\) are ciphertexts; consider a condition that “\(X\) is an encrypted 1”. This operation can return a rerandomization of \(Y\) when the plaintext of \(X\) is indeed 1, and return a rerandomization of \(Z\) otherwise. However, nobody can learn whether the returned value is a rerandomization of \(Y\) or a rerandomization of \(Z\) unless the result is decrypted. A re-randomization operation (also called re-encryption) [29] of an encryption \(E()\) is denoted by \(R()\).

The re-randomization of a ciphertext \(c = E(m)\) of plaintext \(m\), is a new ciphertext \(c' = R(c)\) such that both ciphertexts decrypt to the same plaintext, i.e., \(D(c) = D(c') = m\). The ElGamal encryption system [36] used in our implementation supports rerandomization operations. In general, the privacy guarantee is that no knowledge about any plaintext(s) involved is leaked to any party.

The involved conditions may be complicated and thus this technique itself can depend on other cryptographic building blocks. For instance, we may need to use the building block of partial decryption. Suppose that the private key for a ciphertext is shared among a number of parties using a \((N, 2)\)-secret sharing scheme [33]. We formally define the \((N, 2)\) partial decryption as follows.

**Definition.** Let \(E()\) and \(D()\) be a public key encryption/decryption system. The encryption of plaintext \(m\) using public key \(k^+\) is a ciphertext \(c = E(m)_{k^+}\). Each of \(N\) parties holds a share of the private key and the let the share of party \(i\) be \(k_i^-\). Then the partial decryption of the \((N, 2)\)-secret sharing allows \(D(D(c)_{k_i^+})_{k_j^-} = m\), for \(i \neq j\).

The above definition can be extended to \((N, k)\)-secret sharing for \(k > 2\). Partial decryption allows a party with a share of the private key to partially decrypt a ciphertext. The partially decrypted ciphertext does not leak any knowledge about the plaintext. However, when a threshold number of parties apply partial decryption one by one, the plaintext will finally be revealed. As an example, the ElGamal encryption system [36] we use in our implementation supports partial decryption. Detailed implementation of Secure-If operations are custom-built and depend on different algorithms. We leave the description of our implementation of Secure-If operations for routing optimization to Section 4.4.

Also notice that we will use a few variants of Secure-If operations in this paper. Each of these variants is constructed in a distinct way. Please see Section 4.4 for the detailed constructions.

### 4 Design of the PYCRO Protocol

In this section, we present the PYCRO protocol with three steps: equivalent cost graph construction, privacy-preserving shortest path tree protocol and path establishment. In the PYCRO protocol, we need two homomorphic encryption systems \(E()\) and \(E'()\), both of which must be semantically secure. The difference between \(E()\) and \(E'()\) is that \(E()\) must be additively homomorphic, while \(E'()\) must be multiplicative homomorphic. Specifically, for two messages \(x\) and \(y\),

\[
E(x) + E(y) = E(x + y)
\]

\[
E'(x) \cdot E'(y) = E'(xy)
\]

Each of the \(E()\) and \(E'()\) encryption operations in this paper use a public key whose corresponding private key is shared among the domain controllers using \((N, 2)\)-secret sharing. Every domain controller holds a share of the private key. However any single domain is not able to decrypt a ciphertext \(E(x)\). The only approach to know the plaintext \(x\) is to let at least two domain controllers to apply partial decryption, according to the property of \((N, 2)\)-secret sharing. There exist cryptosystems [10], [37], [34] that are both additively and multiplicatively homomorphic. However, we do not use them due to efficiency considerations. We denote by \(D()\) and \(D'()\) the corresponding decryption operations, respectively. In addition, we allow both of them supports re-randomization operations, denoted by \(R()\) and \(R'()\).

As mentioned earlier, another main cryptographic tool we use in the PYCRO protocol is the Secure-If operation.

#### 4.1 Equivalent Cost Graph Construction

In this subsection, we show how to construct the equivalent cost graph of a multi-domain network, as defined as follows.

As for nodes, we define a switch as a significant node if it is the source switch or a gateway switch and the nodes of the equivalent cost graph are the significant nodes in the entire network. We denote by \(S_i\) the significant node set of domain \(D_i\) and we also denote by \(S\) the significant node set of all domains.

As for links, for any two significant nodes \(v\) and \(v'\) \((v \neq v')\), we distinguish two cases:

**Case 1:** If \(v, v' \in S_i\), then link \(v \sim v'\) is in the equivalent cost graph. In this case, the link is called intra-domain link since two nodes are in the same domain. Note that an intra-domain link does not necessarily correspond to a physical link, and could be a multi-hop path between two switches. The path from \(v\) to \(v'\) is selected by \(D_i\) in the best effort based on \(D_i\)'s local policies and is not necessarily the shortest path. If a domain does not wish to forward the packets from the source domain, it sets the path length as infinity or the pre-defined path length upper limit. We use \(d(vv')\) to denote the path length assigned by \(D_i\).

**Case 2:** If \(v \in S_i \land v' \in S_j \land S_i \neq S_j \land v \sim v'\) then link \(v \sim v'\) is in the equivalent cost graph. In this case, the link is called inter-domain link since two nodes are in different domains. We use \(c(vv')\) to denote the length of link \(v \sim v'\).
As an example, Figure 2(a) shows the equivalent cost graph of a network consisting of four domains, in the view of the controller $C_s$ of the source domain $D_s$. The nodes of the graph are the source switch $v_s$ and all gateway switches $v_1$–$v_7$.

Clearly, $C_s$, the controller of the source domain, knows the connectivity information of the equivalent cost graph. Furthermore, for links in Case 2 above, $C_s$ also knows the link costs in the equivalent cost graph. For links in Case 1 above that are not in $D_s$, $C_s$ does not know the link costs in the equivalent cost graph, which are private information of different domains.

4.2 Privacy-preserving Shortest Path Tree Protocol

This subsection describes how the source controller computes a Privacy-preserving Shortest Path Tree (PSPT) on the equivalent cost graph rooted at $v_s$ while providing strong protection for the private information of other domains. We use $c_{\text{max}}$ to denote the maximum link cost and assume the length of cryptographic keys in use is much greater than the length of $c_{\text{max}}$.

Each domain controller $C_i$, except the source domain controller $C_s$, encrypts all its link costs in Case 1 of the equivalent cost graph, and sends them to $C_s$. Specifically, for any two switches $v$ and $v'$ in $D_i$, if $C_i$ computes $e(vv') = E(d(vv'))$ and sends it to $C_s$. The source domain controller $C_s$ needs to encrypts all its link costs in Case 1 of the equivalent cost graph. $C_s$ is also responsible for encrypting the link costs in Case 2 of the equivalent cost graph. Specifically, for any $v$ in $D_1$ and $v'$ in $D_2$, if there is an inter-domain link between these two nodes, then $C_i$ computes $e(vv') = E(e(vv'))$. For any $v$, $v' \in D_2$ ($v \neq v'$), if there is an intra-domain link between these two nodes, then $C_i$ computes $e(vv') = E(e(vv'))$.

For each node $v$ in the equivalent cost graph, except the source node itself, $C_s$ computes three indicators: $f(v) = E'(2^{-1})$, $g(v) = E(0)$, and $h(v) = E'(\phi)$. Here $f(v)$ is an encrypted indicator for node $v$, indicating whether it has been added to the shortest path tree. We use an encrypted 2 to represent “No”, and an encrypted $2^{-1}$ to represent “Yes”. The plaintext of $g(v)$ will be used for the length of the shortest path from the source node to $v$, once $v$ is added to the shortest path tree. The plaintext of $h(v)$ will be used to store the information of the parent node of $v$ in the shortest path tree, once $v$ is added to the shortest path tree. All these indicators are essential in the computation of the shortest path tree.

For the source node, $C_s$ computes the three indicators: $f(v) = E'(2^{-1})$, $g(v) = E(0)$, and $h(v) = E'(\phi)$, because it is the root of the tree. Then the source controller repeats the two steps below for $|S| - 1$ iterations, where $S$ is the set of nodes in the equivalent cost graph. At each iteration, a node with the minimum distance to the root among the remaining nodes is added to the tree.

Step 1. For each link $vv'$ in the equivalent cost graph, $C_s$ uses a Secure-If operation (denoted as $\text{SecIf}$) to compute $\alpha(vv')$. The condition here is that the plaintext of $f(v)$ is equal to the plaintext of $f(v')$. If this condition is satisfied, $\alpha(vv') = E(c_{\text{max}}|S| + 1)$; otherwise, $\alpha(vv') = R(g(v) + g(v') + e(vv'))$. If the condition is satisfied, it means either $v'$ and $v$ are both in the tree or neither in the tree. We just let $\alpha(vv')$ be a maximum value and do not consider it. If the condition is not satisfied, one of $v'$ and $v$ is in the tree and the other is not. Then the plaintext of $\alpha(vv')$ is the distance from the node not in the tree to the root.

Step 2. For each link $vv'$ in the equivalent cost graph, $C_s$ uses a Secure-If operation (denoted as $\text{SecIf}$) to re-compute $f(v)$, $g(v)$, $g(v')$, $h(v)$, $h(v')$. The condition is that the plaintext of $\alpha(vv')$ is the smallest among the $\alpha$ values of all links in the equivalent cost graph. The node not in the tree that corresponds to the smallest $\alpha$ should be added to the tree and its three indicators should be updated.

If the condition in Step 2 is satisfied, then we use another Secure-If operation (denoted as $\text{SecIf}$) to decide how to update the indicators. The condition of this new Secure-If operation is that the plaintext of $f(v)$ is equal to 2, i.e., whether the node $v$ is not in the tree.

- When the condition is satisfied ($v$ is not in the tree), $f(v) = E'(2^{-1})$, $g(v) = R(\alpha(vv'))$, $h(v) = E'(v')$. The indicators of $v'$ are re-randomized.
- Otherwise, $v'$ is not in the tree, hence $f(v') = E'(2^{-1})$, $g(v') = R(\alpha(vv'))$, $h(v') = E'(v)$. The indicators of $v$ are re-randomized.

If the condition in Step 2 is not satisfied, all indicators $f(v)$, $f(v')$, $g(v)$, $g(v')$, $h(v)$, $h(v')$ are just re-randomized based on the original values.

We show an example of the above iteration in Figure 2(b). $v_s$, $v_1$, and $v_2$ are already in the tree. Since $v_3$ is not in the tree, we compute $\alpha(v_2v_3) = R(g(v_2) + g(v_3) + e(v_2v_3))) = R(E(1) + E(0) + E(2)) = R(E(3))$. Suppose the plaintext of $\alpha(v_2v_3)$, i.e., 3, is the smallest $\alpha$ value. Then $v_3$ should be added to the tree. The indicators of $v_3$ are updated as follows: $f(v_3) = E'(2^{-1})$, $g(v_3) = R(E(3))$, $h(v) = E'(v_2)$. The indicators of $v_2$ are all re-randomized.
Once the algorithm is completed, for each $v$ in the equivalent cost graph, $C_s$ actually obtains the ciphertexts of $g(v)$, the shortest path length from $v_s$ to $v$, and $h(v)$, the parent of $v$ on the PSPT. Figure 2(c) shows the constructed PSPT of the network. With all the $g(v)$ and $h(v)$, we can construct the path from $v_s$ to $v_t$ using the method proposed in the next section (Section 4.3). Note that we use three types of Secure-If operations ($SecIF_0$, $SecIF_1$, and $SecIF_2$). We will describe how they are implemented in Section 4.4.

### 4.3 Path Establishment

After running the PSPT construction protocol, each domain controller knows all its significant nodes’ values of $g$ and $h$ from $C_s$. Using the values, we can construct the path $P$ from $v_t$ back to $v_s$ step by step (e.g., first $v_t$, and then the parent of $v_t$, and then the parent of the parent of $v_t$, until the source $v_s$).

After finishing computing the shortest path tree, $C_s$ then partially decrypts each $g(v)$ and each $h(v)$, and sends the partial decrypted ciphertexts to the domain controller of node $v$. The domain controller of $v$ also applies partial decryption, and thus obtains the plaintexts of $g(v)$ and $h(v)$, i.e., $dg(v)$ and $dh(v)$. Since $E()$ uses $(N,2)$-secret sharing, the encrypted indicators can be decrypted by partial decryption of two domains.

For any destination $v_t$, the shortest path and corresponding forwarding table entries are constructed using the path establishment protocol with all plaintext indicators $dg()$ and $dh()$.

If $v_t$ is not a significant node, the domain controller $C_t$ of $v_t$ compares all the significant nodes in its domain, for the sums of their distances from $v_s$ and to $v_t$. Suppose the significant node with the smallest distance sum is $v$. Then the intra-domain path from $v$ to $v_t$ is chosen as part of the shortest path from $v_s$ to $v_t$, and the forwarding table entries for destination $v_t$ in this part of path are computed and installed accordingly. The forwarding table entries in the other parts of the path are computed in a way similar to $v_t$ being a significant node presented below.

If $v_t$ is a significant node, the domain controller of $v_t$ decides what to do based on the type of link between $v_t$’s parent $dh(v_t)$ on the shortest path tree and $v_t$ in the equivalent cost graph.

- If the link represents an intra-domain path, i.e., $dh(v_t)$ is another significant node in the destination domain, the intra-domain path between $dh(v_t)$ and $v_t$ is picked as part of the shortest path from $v_s$ to $v_t$. The forwarding table entries for destination $v_t$ in the destination domain are installed by $C_t$ accordingly.
- If the link is an inter-domain link, the link is added directly as part of the shortest path from $v_s$ to $v_t$. $C_t$ then sends a message to the domain controller of $dh(v_t)$ and asks it to install a corresponding forwarding table entry at switch $dh(v_t)$.

Next, the domain controller of the predecessor of the destination domain on the selected shortest path computes the forwarding table entries similarly. This process is repeated until the source switch is reached and all forwarding table entries for destination $v_t$ have been computed.

### Algorithm 1 Secure-If Operation Sketch

**Input:**

- $x$: value when condition is satisfied;
- $(t_0,t_1,t_2)$: three parameters.

**1:** $C_s$ randomly choose a domain controller $C_i$.

**2:** $C_s$ computes $PD(t_0)$ and sends $(PD(t_0),t_1,t_2)$ to $C_i$. ($PD()$ is partial decryption operation)

**3:** Upon receiving $(PD(t_0),t_1,t_2)$, $C_i$ do partial decryption on $PD(t_0)$ and gets the plaintext $dt_0$ of $t_0$.

**4:** $C_i$ sends $R(t_1)$ if $dt_0 = x$ $R(t_2)$ otherwise back to $C_s$.

**5:** $C_s$ gets the result.

For the example of Figure 2(c), the destination controller $C_t$ selects $v_d$ as part of the optimal path from $v_s$ to $v_t$. It then installs forwarding entries on switches between $v_t$ and $v_d$ and also notifies $C_t$ to install a forwarding table entry at $v_d$, specifying that packets from $v_d$ to $v_t$ should be forwarded to $v_t$ by $v_d$. The routing path can be established by repeating this process.

Note that there is no need to perform the above process for every source and destination pair. When a shortest path tree is established, the shortest paths from the source to all possible destinations can be obtained.

### 4.4 Implementation of Secure-If Operations

In this section, we will introduce the implementation of the three Secure-If operations used in the PSPT construction protocol.

First, we present a sketch of the Secure-If operation (See Algorithm 1). Each Secure-If operation needs to construct three parameters $(t_0,t_1,t_2)$ and a condition satisfied value $x$ as input. $t_0$ is a condition while $t_1$ and $t_2$ are two options. The output of Secure-If is $t_1$ when condition is satisfied ($t_0 = x$); otherwise, the output is $t_2$. Such operation is achieved by an interactive process between two controllers, say $C_s$ and $C_i$. $C_s$ first applies partial decryption to $t_0$ and sends the result $PD(t_0)$, together with $t_1$ and $t_2$, to any other domain controller $C_i$. Then $C_i$ can fully decrypt $t_0$ and get the plaintext $dt_0$ as the threshold of secret sharing is 2. $C_i$ verifies whether $dt_0$ is equal to $x$ and replies one of $t_1$ and $t_2$ (with re-randomization) to $C_s$. With the Secure-If operation sketch, we need to show the construction of $(t_0,t_1,t_2)$ and $x$ when we introduce a Secure-If operation.

**Security property.** The security property of the Secure-If operations is that it is infeasible for any single party to decide whether the condition is satisfied or not, i.e., which of the two options is actually chosen.

The PSPT construction uses three Secure-If operations ($SecIF_0$, $SecIF_1$, and $SecIF_2$). As the Secure-If operation $SecIF_2$ is used in $SecIF_1$, our decryption is in the order of $SecIF_0$, $SecIF_2$, and $SecIF_1$.

**Construction of $SecIF_0$**

$x$ in $SecIF_0$ is 1 and $(t_0,t_1,t_2)$ are constructed as the following two cases, each may occur in probability $\frac{1}{2}$.

**Case 1.** With probability $\frac{1}{2}$, $C_s$ computes $t_0 = \left(\frac{f(v)}{f(v')}\right)^x$.
where \( r \) is a randomly picked exponent \( 1; t_1 = E(c_{\text{max}}|S| + 1); t_2 = R(g(v) + g(v') + e(vv')). 

**Case 2.** With the remaining probability \( \frac{1}{2} \), \( C_s \) computes 
\[
t_0 = \frac{1}{(v_0 f(v'))^2}, 
\]
where \( r \) is a randomly picked exponent; 
\[
t_1 = R(g(v) + g(v') + e(vv')); t_2 = E(c_{\text{max}}|S| + 1).
\]

**Proof of Secure-If property.** In Case 1, if \( f(v) \) is equal to \( f(v') \), \( t_0 = 1 = x \) and \( \alpha(vv') = E(c_{\text{max}}|S| + 1). \)
Otherwise, \( \alpha(vv') = R(g(v) + g(v') + e(vv')). \)

Hence the function of SecIf0 can be achieved. In Case 2, if \( f(v) \) is not equal to \( f(v') \), \( t_0 = \frac{1}{(vv)^2} = 1 = x \) and \( \alpha(vv') = R(g(v) + g(v') + e(vv')). \)

Hence the function of SecIf0 can be achieved.

The reason for that we use an uncertain calculation is to protect privacy. If we only apply the first case, any attacker that decypts \( t_0 \) and finds \( t_0 = x \) can determine that \( f(v) = f(v') \). However, in the current implementation, even if an attacker knows \( t_0 = x \), it cannot guess whether \( f(v) = f(v') \) as \( f(v) = f(v') \) and \( f(v) \neq f(v') \) have equal probability.

**Construction of SecIf2** 
\( x \) in SecIf2 is 2 and \((t_0, t_1, t_2)\) are constructed as follow.

**Case 1.** With probability \( \frac{1}{2} \), \( C_s \) computes 
\[
t_0 = R(f(v)). 
\]
Let \( \langle t_1, t_2 \rangle = \langle E'(2^{-1}), R'(f(v)) \rangle \) for the SecIf2 of \( f(v); \langle R(\alpha(vv')), R(g) \rangle \) for the SecIf2 of \( g(v); \langle E'(v'), R'(b') \rangle \) for the SecIf2 of \( h(v); \langle R'(f(v')), E'(2^{-1}) \rangle \) for the SecIf2 of \( f(v); \langle R(g(v')), R(\alpha(vv')) \rangle \) for the SecIf2 of \( g(v); \langle R'(h(v')), E'(v') \rangle \) for the SecIf2 of \( h(v) \).

**Case 2.** With the remaining probability \( \frac{1}{2} \), \( t_0 = R(\frac{1}{(vv)}) \) and the above values of \( t_1 \) and \( t_2 \) are swapped, i.e., \( (t_1, t_2) \) be \( R'(f(v)), E'(2^{-1}) \) for \( f(v) \) and so on.

**Proof of Secure-If property.** In Case 1 if \( t_0 = f(v) = x = 2 \), i.e., \( v \) has not been added to the shortest path tree, then 
\[
f(v) = E'(2^{-1}), g(v) = R(\alpha(vv')), h(v) = E'(v').\]
Otherwise, \( f(v') = E'(2^{-1}), g(v') = R(\alpha(vv')), h(v') = E'(v').\)

In Case 2, if \( t_0 = f(v) = x = 2 \), i.e., \( v' \) has not been added to the shortest path tree, then 
\[
f(v') = E'(2^{-1}), g(v') = R(\alpha(vv')), h(v') = E'(v').\]
Otherwise \( f(v') = E'(2^{-1}), g(v') = R(\alpha(vv')), h(v') = E'(v').\)

Hence the function of SecIf2 can be achieved.

**Construction of SecIf1**
To show the construction of \( x \) and \((t_0, t_1, t_2)\) in SecIf1, we first introduce two comparison protocols called \( sc \) and \( osc \) which are necessary in SecIf1.

The comparison protocol \( osc \) is designed by us based on the secure comparison protocol \( sc \) which is necessary in SecIf1.

The protocol \( sc(a, b) \) in [27] takes two ciphertexts of \( E(\cdot) \), denoted as \( a \) and \( b \), as input, and outputs another ciphertext of \( E(\cdot) \). The output is \( E(1) \) if \( a \)’s plaintext is greater than or equal to \( b \)’s; otherwise, the output is \( E(-1) \).

Based on this comparison protocol, we design a new comparison protocol \( osc(a, a_{idx}, b, b_{idx}) \) which can distinguish not only two edges with \( \alpha \) values (the two \( \alpha \) values are denoted by \( a \) and \( b \)) but also two edges with the same \( \alpha \) by comparing their indexes (denoted by \( a_{idx} \) and \( b_{idx} \)).

If \( a > b \) or \( a = b \) and \( a_{idx} < b_{idx} \), \( osc(a, a_{idx}, b, b_{idx}) \) returns \( E(1) \). Otherwise it returns \( E(-1) \).

We show how to construct \( osc(a, a_{idx}, b, b_{idx}) \) as follows.

1. Assume the plaintext space and the ciphertext space are both the same cyclic group. The value of \( r \) needs to be picked uniformly at random from between 0 and the order of the group minus 1, including the two endpoints.

2. Using DGK, we make a small sacrifice of privacy for efficiency. However, it’s worth since only a little information is revealed.
path length of candidate node \( v_i \). Once the DGK protocol finishes, \( C_i \) tells \( C_s \) the result of the comparison. According to the result, if the plaintext of \( g(u) \) is less than that of \( g(v_i) \), \( C_i \) then updates \( u \leftarrow v_i \).

Step 3. After the two steps above, the controller \( C_s \) get the shortest-distance node \( u \). Next, \( C_s \) requests the controller of \( u \)'s domain for the information of \( g(u) \) and \( h(u) \) and add \( u \) into the shortest path tree under its parent. \( C_s \) broadcasts the new shortest path tree with encrypted distance information to the other domains.

After \(|S| - 1\) iterations of the above loop, \( C_s \) finishes the computing of the shortest path tree.

5 Quick Pathing Protocol

It is very common that a domain \( D \) will apply the same access and routing policy to many flows transmitted from another domain \( D_s \). Based on this fact, we propose a protocol named QIP (Quick Pathing) to improve the computation efficiency of PYCRO. The main idea of QIP is to perform a one-time offline preprocessing at first. Specifically, the preprocessing is to run PYCRO for some gateway switches offline. Due to the preprocessing, the online computation and communication cost is very low.

We define an equal-flow group \( G \) as a group of flows from domain \( D_s \) such that for all flows in \( G \), any other domain \( D \) will apply the same access and routing policy. \( D \) will provide same paths between any two gateways of \( D \). Therefore all flows in \( G \) can use a number of shared shortest path trees. A source domain with \( k \) gateway switches maintains \( k \) shared shortest path trees for each equal-flow group. Each of the trees is rooted at a gateway switch. To compute each shared shortest path tree, the nodes of the equivalent cost graph include gateway switches (significant nodes) of all domains.

Correspondingly, when constructing links in the equivalent cost graph, for any two significant nodes \( v \) and \( v' \) \((v \neq v')\), we distinguish two cases:

- Case 1: \( v \) and \( v' \) belong to the same domain, then there is a link in the equivalent cost graph between those two nodes, and the cost of this link is \( d(v,v') \), which is only known to the domain of \( v \) and \( v' \).
- Case 2: \( v \) and \( v' \) belong to different domains. If there is an inter-domain link between \( v \) and \( v' \), then there is a link in the equivalent cost graph between those two nodes, and the cost of this link is \( c(v,v') \). If there is no such inter-domain link then there is no link in the equivalent cost graph between these two nodes.

The algorithm in Section 4.2 can be run to build each shared shortest path tree.

When the source controller receives a flow query from the source \( v_s \) to destination \( v_t \), for each gateway switch \( v_i \) in the source domain, the source controller \( C_s \) computes the distance from \( v_s \) to \( v_i \) plus the distance from \( v_i \) to a gateway \( w \) in the destination domain on the shared tree rooted at \( v_t \). Thus for any \( w \), there are \( k \) potential paths from \( v_s \) to \( w \) and \( C_s \) chooses the shortest path to \( w \) in them. Suppose the destination domain has \( k' \) gateways, then there are \( k' \) paths chosen by \( C_s \) and each chosen path stands for the shortest path from \( v_s \) to a gateway switch in the destination domain.

Then \( C_s \) simply sends all \( k' \) path lengths to the destination controller \( C_t \). \( C_t \) can determine the shortest path from \( v_s \) to \( v_t \) and install forwarding entries using the method similar to that in Section 4.3.

For example, in Figure 2(a), both \( D_s \) and \( D_t \) have two gateways. Hence for a group of flows from \( D_s \), \( D_s \) can maintains two PSIPTs rooted at \( v_1 \) and \( v_2 \). The shortest path from \( v_s \) to \( v_0/v_7 \) is \( v_s \rightarrow v_2 \rightarrow \cdots \rightarrow v_0/v_7 \). There are two chosen paths between \( D_s \) and \( D_t \), and \( D_t \) can select one of them for each flow and tell \( D_s \) the paths it selects.

6 Bandwidth Allocation

Bandwidth allocation has been applied to practical traffic engineering solutions such as B4 [17]. We solves a relatively simple version of the bandwidth allocation problem. Before we define the problem, we introduce some preliminaries.

Besides the link cost, every link \((v,v')\) also has a bandwidth \( b(v,v') \). \( b(v,v') \) represents the maximum bandwidth that link \((v,v')\) can provide. And the definition of the cost of a flow on a path is:

**Definition 1.** Given a path \( p \) from node \( v \) to node \( v' \) whose length is \( l_p \), if a flow \( f \) consumes bandwidth \( b_f \) on \( p \), then the cost of \( f \) on \( p \) is \( l_p \).

A flow \( f \) has a bandwidth demand \( q_f \). However as link bandwidth is limited, it may need multiple paths to satisfy a flow’s bandwidth demand [17]. We assume a flow can be split to multiple subflows to be transmitted on different paths. And the cost of \( f \) is defined as:

**Definition 2.** The cost of \( f \) for bandwidth allocation is the sum of path cost \( \Sigma b_f \cdot l_P \) for \( p \in P \) where \( P \) is the set of paths that \( f \) is split on.

Given Definition 1 and Definition 2, we define the Bandwidth Allocation problem as follows:

**Definition 3.** Bandwidth Allocation: for any flow \( f \) with bandwidth demand \( q_f \), we should find \( k \) paths such that the sum of allocated bandwidth of these paths to \( f \) is no less than the bandwidth demand \( q_f \) and the routing cost of \( f \) should be minimized.

We design a bandwidth allocation protocol of PYCRO, named PYCRO-BA, works in the following steps:

Step 1. During the construction of the equivalent cost graph, each domain controller assigns an available bandwidth \((v,v')\) amount between two significant nodes \( v \) and \( v' \), which is also encrypted by a homomorphic encryption system.

Step 2. The source controller creates the shortest path tree and finds the shortest path \( p \) from the source \( v_s \) to destination \( v_t \) using the protocol presented earlier.

Step 3. The source controller determines the available bandwidth \( b_p \) on the shortest path, which is the minimum value of \((v,v')\) for all links \((v,v')\) on the paths. This process is similar to the previous protocol to determine the minimum cost candidate. We skip the protocol details here and have implemented them in the experiments.

Step 4. If \( b_p \) is smaller than the bandwidth demand \( q \), \( C_s \) computes a residual demand \( q - b_p \) and find another path to satisfy the demand.
Step 5. $C_s$ deletes all links of $p$ from the equivalent cost graph, and repeats Steps 2-4 to find more paths until the bandwidth demand is satisfied.

The above bandwidth allocation protocol requires multiple calls of the shortest path tree protocol. To improve its efficiency, $C_s$ may find multiple disjoint paths to different gateways of the destination domain and suggest these paths to the destination controller $C_i$. If $C_i$ can also find multiple disjoint paths from different gateways to $v_t$, multiple paths can be established by a single call of the shortest path tree protocol. We plan to develop more sophisticated protocol to optimize this process in future work.

7 PRIVACY ANALYSIS OF PYCRO

We analyze the privacy-preserving property of PYCRO in a standard cryptographic model, the semihonest model [11], which is widely used in the literature (e.g., [40] and [3]). In this model, all involved parties are assumed to follow the protocol faithfully, although they may attempt to violate privacy using the information they obtain. Note that such an assumption is acceptable in our scenario of cross-domain routing, because domain controllers usually have long-term relationship with each other. Despite their curiosity about others’ private information, it is uncommon for them to deviate from the protocol just in order to violate others’ privacy.

The main result we get as shown in Proposition 4 below, is that PYCRO only leaks to each domain controller its significant nodes’ distances from the source node and parent nodes in the shortest path tree. We stress that this leaked distance information is about a small number of pairs of nodes only. Any other information, including distances between other pairs of nodes, are protected by PYCRO. Furthermore, our protection is cryptographically strong, i.e., no partial knowledge about the protected information is leaked by PYCRO. In contrast, the performance cost we pay for the privacy protection is very reasonable. The execution time varies among different topologies, from seconds to tens of seconds (please see Section 8 for details). We show that PYCRO is weakly privacy-preserving, which means each domain controller learns the distances from the source node and parent nodes to its significant nodes, in the shortest path tree, which is necessary information in computing a shortest path tree. The domain controller knows nothing about other nodes in another domains topology.

Proposition 4. PYCRO is weakly privacy preserving in the semihonest model, in the sense that it reveals to each domain controller no more than its significant nodes’ distances from the source node and parent nodes in the shortest path tree.

Proof: Our proof is established by demonstrating a probabilistic polynomial-time simulator according to the definition and proof methodologies of cryptographic protocols discussed in a standard textbook [11]. Our proof is based on the zero-knowledge proof, for which one party (called the prover) can prove to another party (called the verifier) that a given statement (shortest path tree) is true, without revealing private information (topology and intra-domain link cost).

Zero-knowledge requires that if the statement is true, no verifier learns anything other than the fact that the statement is true. This is formalized by showing that every verifier has some simulator that, given only the statement to be proved (and no access to the prover), can produce a transcript that “looks like” an interaction between the honest prover and the cheating verifier.

For each domain controller $C_i$, we construct a simulator for its view, which takes as input its significant nodes’ distances from the source node and parent nodes in the shortest path tree. All coin flips in the view can be easily simulated, and thus we focus on generating simulated messages below.

If $C_i \neq C_s$, the simulator simulates the messages received from $C_s$ for each of its significant node, using two ciphertexts. The first ciphertext is an encrypted distance of the significant node from the source node, where the cryptosystem used is $E()$ and the key used is $C_i$’s own public key. The second ciphertext is an encrypted identity of the significant node’s parent node in the shortest path tree, where the cryptosystem used is $E'(())$ and the key used is still $C_i$’s public key.

For $C_1$, we add the following simulated messages. In the Secure-If operation $SecIf_0$, the messages from $C_s$ is simulated using three ciphertexts. The first of these three is under $E(0)$, with the plaintext being 1 with probability $\frac{1}{3}$, or a uniformly random number with probability $\frac{2}{3}$. The remaining two are encryptions of random plaintexts under $E()$. The public key used for encryption of all these three are $C_1$’s own public key.

For the Secure-If operation $SecIf_1$ and $SecIf_2$, the simulator goes as follows. For $SecIf_1$, the messages from $C_s$ are simulated using 8 random ciphertexts under $E'(())$ and 4 random ciphertexts under $E()$, and also another ciphertext under $E'(())$ with the plaintext being 2 or $\frac{1}{2}$ each with probability $\frac{1}{2}$, where the public key used for encryption is $C_1$’s own public key. For $SecIf_1$, in addition to simulating the received messages in the executions of secure comparison, the simulator simulates the earlier round of message from $C_s$ using three ciphertexts under $E()$, with the first being an encrypted 0 or encrypted $-1$, each with probability $\frac{1}{2}$, where the public key used for encryption is $C_1$’s own public key. The remaining two ciphertexts are randomly generated. The simulator simulates the later round of message from $C_s$ using 8 random ciphertexts under $E'(())$ and 4 random ciphertexts under $E()$, and also another ciphertext under $E'(())$ being an encrypted 1 or encrypted $-1$, each with probability $\frac{1}{2}$, where the public key used for encryption is $C_1$’s own public key.

For $C_s$, the simulator goes as follows. First, it simulates the first round messages from other domain controllers using random ciphertexts. For each pair of significant nodes in any other domain, there should be a random ciphertext under the cryptosystem $E()$. Then the simulator proceeds to simulate the message received from $C_1$ in the Secure-If operation $SecIf_0$. This should again be a random ciphertext under the cryptosystem $E()$.

For $SecIf_1$, the messages from $C_1$ can be simulated by using 4 random ciphertexts under cryptosystem $E'(())$ and 2 random ciphertexts under cryptosystem $E()$. For $SecIf_2$, in addition to simulating the received messages in the executions of secure comparison, the simulator simulates...
8 PERFORMANCE EVALUATION

The most significant concern of a privacy-preserving protocol is its computation and communication efficiency. In this section, we conduct experiments to evaluate the efficiency of PYCRO, bandwidth allocation protocol and QIP.

We have implemented a prototype system on seven Dell PowerEdge R720 servers with Linux operation systems and Intel(R) Xeon(R) CPU E5-2650 0 @ 2.00GHz. All servers are connected via a campus network. Each machine runs a program to emulate a controller. If the controller number is larger than seven, we may run multiple threads on a single machine. We configure the controller placement such that two neighboring controllers are in different machines. In all experiments, cryptographical operations are implemented using the Crypto++ library [7]. We choose ElGamal as the multiplicative homomorphic encryption system $E'$ and an variant of ElGamal as the additively homomorphic encryption system $E$. The key length in $E'$, $E$, and the secure-comparison protocols are all in 512-bit.

We use the router-level topologies of seven real ISP networks collected by the Rocketfuel project [35]. The detailed information of the seven networks can be found in Table 1 and networks are identified as I to VII. Based on topology analysis, we set a number of routers as gateways. Based on the seven networks, we construct 30 multi-domain topologies in six groups as shown in Table 2. For example, topologies 1 to 5 are constructed using the same domains I and II, but have different number of gateways and inter-domain links.

### Table 1 Information of the seven Rocketfuel topologies

<table>
<thead>
<tr>
<th>Network ID</th>
<th>Network name</th>
<th># routers</th>
<th># links</th>
<th># gateways</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>AS 1221</td>
<td>318</td>
<td>758</td>
<td>231</td>
</tr>
<tr>
<td>II</td>
<td>AS 1239</td>
<td>604</td>
<td>2208</td>
<td>242</td>
</tr>
<tr>
<td>III</td>
<td>AS 1755</td>
<td>172</td>
<td>381</td>
<td>61</td>
</tr>
<tr>
<td>IV</td>
<td>AS 2914</td>
<td>960</td>
<td>2821</td>
<td>507</td>
</tr>
<tr>
<td>V</td>
<td>AS 3257</td>
<td>240</td>
<td>404</td>
<td>89</td>
</tr>
<tr>
<td>VI</td>
<td>AS 3967</td>
<td>201</td>
<td>434</td>
<td>110</td>
</tr>
<tr>
<td>VII</td>
<td>AS 7018</td>
<td>631</td>
<td>2078</td>
<td>246</td>
</tr>
</tbody>
</table>

The earlier message from $C_1$ using a random ciphertext, being $E(1)$ with probability $\frac{1}{2}$ and $E(-1)$ with probability $\frac{1}{7}$. The final messages from $C_1$ are simulated using 4 random ciphertexts under $E'$ and 2 random ciphertexts under $E$.

We demonstrate that we are able to construct the simulators such that the distributions produced by the simulators and the real proof protocol are close.

### 8.1 PYCRO and bandwidth allocation protocol

#### Computation cost.
We first conduct experiments to construct shortest path trees on every topology. For each topology, we randomly select 20 nodes and construct a shortest path tree for each of them. By computing time, we mean the average execution time of the protocol for one shortest path tree. We find that the computing times for different nodes in a same topology vary very little. It is because the execution time mainly depends on the number of domains, number of inter-domain links, and number of gateways. Figure 3 shows the average execution time of PYCRO on different topologies. The deviations are too small to be shown in the figure. We find that, for topologies consisting of the same domains (e.g., topologies 1-5), the execution time increases linearly with the number of inter-domain links and number of gateways. By comparing topologies of different domains, the execution time also increases linearly with the number of domains. In general PYCRO is very efficient: it takes a short time to compute a shortest path tree on a topology with thousands of switches and links in a privacy-preserving manner. Since a shortest path tree can be shared with multiple paths and the response to a path query takes much less time. Specially, if we have got a shortest path tree rooted at $v_s$, the paths that start from $v_s$ to any destination can be constructed easily and quickly using the the path establishment protocol in Section 4.3.

We then conduct experiments to evaluate the execution time of the bandwidth allocation protocol. We assign every link a random capacity from 1 to 5. In each experiment, we set the bandwidth demand as 20 and find multiple paths between the sender and destination to satisfy the bandwidth demand. This bandwidth demand can be considered as the aggregated demand of all flows in the sender switch. For each topology we perform 20 runs and compute the average.

The results are shown in Figure 4. We find that there is no strict linear dependency of the execution time and number of inter-domain links, because more inter-domain links also make it easier to find multiple disjoint paths at a shortest path tree.

#### Communication cost.
We then show the communication cost of PYCRO in the average size of all messages per domain and plot the results in Figures 5 and 6. We observe that the communication cost also increases with the number of domains, number of inter-domain links, and number of gateways. Each domain spends less than 700 KB to compute a shortest path tree and less than 1 MB to allocate bandwidth for the largest topology. For other topologies the communication cost is much less. In general, PYCRO is communication efficient.

### 8.2 Evaluation of QIP

We then evaluate the performance improvement from QIP. For each test case, we do a one-time offline preprocessing and 100 times online computations.

#### Computation cost.
First we evaluate the execution time of QIP. Figure 8 shows the execution time of the one-time offline preprocessing. For the largest topology, it takes 647.7 seconds to perform preprocessing. We know that the preprocessing time increases linearly with the number of
.gateway switches in source domain. The average online computation time is shown in Figure 7. We can see that the longest execution time is less than 20 milliseconds, bringing significant improvement to time efficiency.

Communication cost. Figure 10 shows the communication cost of QIP’s one-time offline preprocessing in the size of all message payload. It takes about 20 MB to complete the largest topology, which is tolerable. For online computation, the size of all messages in each test case is shown in Figure 9 and the communication costs of all the test cases are all less than 1 KB. Hence we conclude that the QIP online computation is communication-efficient.

8.3 Comparison with Other Solutions

It is hard to find an existing work achieving the same objectives as PYCRO. It is non-trivial to apply existing secure multi-party computation such as Fairplay [23] and SEPIA [4] to the problems of this paper, because they were not designed for the same objectives of PYCRO.

A cross-domain privacy-preserving protocol for quantifying network reachability is proposed in [6]. From their experimental results, we find that about 400 or 550 seconds offline computation cost, about 5 or 25 seconds online computation cost and about 450 or 2100 KB communication cost are needed for every party on average in their synthetic data.

In [23] [2], a full-fledged system called Fairplay that implements generic secure function evaluation is introduced. Their experimental results show that it takes 1.41 seconds to make a comparison. In our optimized protocol, \((|S| - 1)(n - 1)\) comparison operations are needed in total, where \(|S|\) is the significant node number (from tens to hundreds) and \(n\) is the domain number(from 2 to 7). Hence, if we apply Fairplay to our protocol, the average comparison operation time of each domain is 1.41\((|S| - 1)(n - 1)\) seconds. For a case \(|S| = 185\) and \(n = 7\), the average comparison time of each domain is 222.38 seconds while the average time that PYCRO consumes is only 32.3 seconds and the online computation cost of QIP is less than 20 ms.

Note that the computation platform we used is no better than that in [2].

In summary, PYCRO and QIP can improve the time and bandwidth efficiency by an order of magnitude for cross-domain routing optimization, compared to existing solutions.

9 Discussion and Extended Design

9.1 Dealing with the Dynamic Change of Topology

In practice, a topology may change for various reasons (e.g. node failure, link failure, new nodes added to the network, etc). We discuss how PYCRO and QIP would deal with topology dynamics. We distinguish four cases:

Case 1: Link \(v \sim v'\) cost increases. Note that link cost can be infinity, and it means that the link is broken. The basic idea is to find all the shortest path trees which contain link \(v \sim v'\) and then update them. The update method is to run PYCRO in Section 4. When all affected shortest path trees are updated, the online computation of QIP can work properly. However when updates are still running, the online computation of QIP may not get the shortest routing path. Hence in the online computation of QIP, we do not consider the affected shortest path trees during their updating process. The protocol works in the following way. The domain controller that realizes a link cost increase will notify the domain controller of the parent node of this link. Such information will propagate to the source node. Then if the shortest path between the source to the destination includes the changed link, the domain controller of the source node will temporarily ignore this shortest path tree and select a shortest path from the remaining trees. As a result, we may not get the shortest routing path in some cases, but the path is comparatively short. More specifically, it is the shortest among the unaffected shortest path trees. When the convergence is reached, all trees can be used.

Case 2: Link \(v \sim v'\) cost decreases. Note that \(v \sim v'\) is a new link if its cost is infinity before being decreased. The basic idea is similar to that in Case 1. First we need to
update the affected shortest path trees and then the online computation of QIP can work properly. However during the updating process, the affected shortest path trees are also considered as all affected paths become shorter. So we can see that a comparatively good path can be found during the updating process.

Case 3: A gateway switch $v$ crashes. It is equivalent to increasing the cost of all $v$’s links to infinity. So we can solve this case with the method of Case 1.

Case 4: A new gateway switch $v$ is added. In this case, all shortest path trees should be updated. During the updates, we should use the updated trees as many as possible when we run the online computation of QIP. It guarantees that the result is the best one if we do not consider the new node $v$.

The above discussion shows that our QIP protocol can get a comparatively good result with a little modification when topology changes. Note when the offline computation is finished and the convergence is reached, PYCRO always achieves the shortest paths. The convergence time is close to the results shown in Section 8.2. Hence the critical question is that, whether the above methods in four cases during the protocol execution (before convergence) can still achieve relatively good results. Hence we implement this extended protocol and show the results in Figures 11 and 12. The metric shown is the routing stretch, defined as the length of the alternative path found by PYCRO using the above methods during convergence, over the actual shortest path.

We randomly select 10,000 source-destination pairs and compute the average, for each topology. Figure 11 shows the routing stretch when link cost increases and Figure 12 shows the routing stretch when link cost decreases. We find that the routing stretch is always maintaining low. For most topologies in Figure 11, the routing stretch is under 1.05. Only two small topologies may cause higher values ($\alpha$). In Figure 11, the routing stretch is always lower than 1.01. The results demonstrate that the protocol to deal with the topology changes is very effective.

9.2 Dealing with A Large Number of Domains

Though the PYCRO protocol works well in many scenarios, when it is difficult to work for Internet-scale multi-domain networks. Based on the analysis, the computation bottleneck of PYCRO is the step to select the smallest $\alpha(vv')$ among the $\alpha$ values of all links in the equivalent cost graph, as we have discussed in Section 4.5. We present the candidate recommendation technique to reduce the calls of the secure comparison operation. Its main idea is that each domain recommends the best one of it as a candidate. However when the number of domains is very large, the number of recommended candidates is big too. As a result, PYCRO is still inefficient. To deal with this situation, we may find the smallest $\alpha(vv')$ with parallelism. Suppose there are $N$ domains and the number of candidates is also $N$. For the case $N = 2^k (k = 0, 1, 2, \ldots)$, the process could be described by a full binary tree as shown in Figure 13. First, at height $k$, we have $N$ domains/candidates and we divide them into $N/2$ pairs. Each pair selects a smaller $\alpha$ value in parallel. So we have $N/2$ candidates remaining at height $k - 1$. Similarly, we get $N/(2^i)$ candidates remaining at height $k - i$. Finally at height 0, we can get the only 1 candidate, $t_{0,1}$. And $t_{0,1}$ is the smallest $\alpha$ value. It is obvious that the algorithm only performs $k$ rounds of comparison since the comparisons at the same height are performed in parallel. For the case $N \neq 2^k (k = 0, 1, 2, \ldots)$, we can find a $k$ s.t. $2^{k-1} < N < 2^k (k = 1, 2, \ldots)$ and we add $2^k - N$ dummy candidates with infinity value in them.

10 Conclusion

In this paper we present PYCRO, the first privacy-preserving cross-domain routing optimization protocol in multi-domain SDN environments. We develop a new cryptographic tool named the Secure-If operation and apply it with homomorphic encryption to compute the shortest cross-domain paths without revealing private information. PYCRO also provides bandwidth allocation, a fundamental traffic engineering solution. Experimental results show that PYCRO can improve the time and bandwidth efficiency by an order of magnitude compared to general-purpose solutions. In future we will design more complex routing optimization functions based on PYCRO. We believe our study may lead to useful discussion of the same problem for the Internet.

References
