READ ME FIRST
- Don’t spend too much time on any one problem. This exam will take approximately 60 minutes.
- Amount of time spent on a problem is not necessarily proportional to the points.
- Scan through the entire test and do the easy problems first.
- If something is not clear, ASK.
- BE NEAT. We cannot give you points for something that we can’t read.
- Write down your assumptions.
- Don’t just write your answer, show how you got them.
- This is a CLOSED BOOK, CLOSED NOTES exam.

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1. **Quaternions (25 points)**

A stretchable stick initially oriented so that one end is at \( P_A(1,1,1) \) and the other end is at \( P_B(2,2,2) \) is to be rotated (and translated) to a new orientation such that the first end will be at \( P'_A(1,2,3) \) and the other end will be at \( P'_B(3,2,1) \). Ignoring the translation amounts, answer the following questions:

(a) **10 points**
What is the axis of rotation?

(b) **5 points**
What is the angle of rotation?

(c) **10 points**
Represent this rotation using a quaternion.
2. **Behavioral Animation (25 points)**

A boid is moving through a force field. Assume that the boid’s next trajectory is determined according to: 90% based on its current trajectory, and 10% based on external forces. If the boid’s current velocity is: \([10,10,0]\) and the uniform force field is \([0,0,-1]\), what will be the boid’s new trajectory after one time step?
3. **Morphing (25 points)**
Analyze the line-pair using the Beier and Neely algorithm. At time=0, a line goes from (10,10) to (10,70). The corresponding line at time=10 goes from (30,30) to (30,70).

(a) **5 points**
Where is the corresponding line for the intermediate grid at time=5.

(b) **10 points**
Which pixel in the initial (time=0) frame corresponds to the intermediate grid pixel at (30,45)?

(c) **10 points**
Which pixel in the final (time=10) frame corresponds to the intermediate grid pixel at (30,45)?
4. **Inverse Kinematics (25 points)**

In the figure below, joint A is at (0, 0), the end effector is at E (15, 5). The links L1 and L2 have link lengths of $10\sqrt{2}$ and $5\sqrt{2}$ respectively. Currently, $\theta_A$ is 45 and $\theta_B$ is 90. If we want to move the end effector to a new position G at (16, 6), find the instantaneous angular velocities for $\theta_A$ and $\theta_B$. Solve the problem as completely as you can.