

Multiseasonal Management of an Agricultural Pest II: The Economic Optimization Problem¹

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Received August 19, 1982; revised April 1984

The problem of selecting a pesticide application strategy in the face of increasing resistance to the pesticide in the pest population is dealt with. The grower in this situation may do better by sacrificing a portion of the present crop in return for a reduced resistance to future applications. The model presented represents an attempt to forge a compromise between excessive complexity, rendering the model difficult to study, and excessive simplicity, rendering the model useless. The effects of timing of the application of the pesticide within the season are discussed. The principle conclusions are the following: (i) If immigration of pests from refugia is significant then proper timing of the application of pesticide may be used to help alleviate resistance growth. (ii) Resistance growth may best be reduced by spraying earlier than what would otherwise be the best time. (iii) The value of the discount rate (and of the time horizon) has a profound effect on the nature of the optimal policy. © 1985 Academic Press, Inc.

1. INTRODUCTION

The development of resistance to pesticides in agricultural pest populations is common and well known. Each application of a pesticide selects in favor of individuals resistant to that pesticide. Hueth and Regev [11] have pointed out that susceptibility to a pesticide in a pest population may be thought of as an exhaustible natural resource that is "mined" with each pesticide application. In choosing a pest management strategy the agricultural decision maker should take this "mining" effect into account. The intensity and timing of the application of the pesticide should be chosen to balance the reduction in future susceptibility of the pest population with losses in present crop yield due to the pest.

This is the second in a series of papers which addresses some of the theoretical aspects of this pest management problem. We attempt to develop a model which is sufficiently simple that its structure is relatively transparent, and sufficiently realistic that it provides some useful insight into the problem. We study what can only be described as a caricature of the real system; attempts to include further realism lead quickly to overwhelming complexity. Our work is therefore intended primarily as a complement to studies such as those of Gutierrez *et al.* [10] and Shoemaker [14]. We hope our work may provide some intuition for the behavior of their more realistic models.

We do not attempt to link our model too closely to any particular crop-pest system. The model was, however, originally motivated by the cotton-spider mite

¹Partially supported by OPER Grant 4817, University of California, Davis, to R.E.P. and M.M., and by NSF Grants MCS 8121413 to R.E.P. and M.C.S. 8121659 to M.M.

system described by Carey [3]. As such, we make the following basic assumptions. The crop is an annual, harvested at the end of the season. The pests do not overwinter in the field to any extent, rather, they arrive primarily by immigration. Gravid females establish breeding populations that may progress through several generations during the course of the season. The pest population is controlled by application of a chemical pesticide. This pesticide is applied prophylactically. We consider primarily the case of a single pesticide application per year, although we discuss the case of more frequent applications. In many cases, cotton growers are able to limit themselves to a single acaricide application per year (Carey, personal communication), so that this assumption is not entirely unrealistic.

In our previous paper [12] we presented a model for the pest-crop system described above. The intraseasonal dynamics of the model are formulated in continuous time and the interseasonal dynamics are in discrete time. Shoemaker [14] uses a similar approach. The reader may consult our previous paper for a survey of the literature in this field. Our earlier paper presented several variations of the model. The intent was to examine the effect of various assumptions concerning the age structure and genetics of the pest population. The conclusion was that a very simple model gives results which are qualitatively the same and quantitatively close to those obtained using more complex models. Based on our previous results, in this paper we use only the simplest model.

We are concerned with the multiseasonal crop-pest management problem. We assume a fixed time horizon that is sufficiently distant that if the pesticide is applied intensively every season, the population becomes resistant by the time horizon. The basic management problem is to maximize the discounted crop yield over the time horizon of the problem. Pesticide resistance does not enter explicitly into the management problem; rather it enters implicitly by reducing the crop yield in later seasons.

Section 2 contains a brief review of the equations of the model. In Section 3 we begin the study of the multiyear optimization problem. We first formulate and solve a dynamic programming problem that gives an application strategy that approximately maximizes the total net profit given a fixed time horizon. We then check our results by solving the same problem using a nonlinear programming scheme. Our results throughout the paper are entirely numerical; nothing is rigorously proved. There are some questions of mathematical interest associated with our model, particularly regarding the generality of its qualitative conclusions, but we do not pursue them here in any mathematical way.

In Section 4 we briefly study an alternative form of control policy that we call a "myopic" strategy. In this strategy the grower does not plan ahead over the entire time horizon, but rather reacts to events of the previous year. We feel that this strategy may be more realistic in many cases. Section 5 contains a discussion of our results, with particular attention to the robustness of the model.

2. DYNAMIC MODEL OF THE CROP-PEST SYSTEM

In the single season model, we consider events occurring in a single, spatially uniform field during a single season of length T . Since events in one field are obviously influenced by events in another, we are implicitly assuming that either this influence is not strong or that events in neighboring fields are roughly the same as

those in the field under consideration. At the start of the season there are no pests in the field. Immediately after the season begins pests begin to immigrate at a rate $I(t)$, where $0 \leq t \leq T$, from a "pool" into the field. Some of the immigrants are gravid females who lay eggs and establish breeding populations. The descendents of these early immigrants continue to breed and are joined by later immigrants. At time T the crop is harvested and the surviving pests merge with the pool to form the basis for the next season's pool.

In our previous paper [12] we found that a number of simplifying assumptions may be incorporated into the model without seriously changing the form of its solution. In this paper we study essentially the simplest form of the model. After presenting the model below, we discuss some of its more obvious (or glaring) simplifications. It is important to bear in mind that our intent is not to provide a model that realistically simulates the detailed dynamics, but rather one that has a simple form and that simulates the *effect* of the dynamics with reasonable accuracy.

We assume that both the pest and the crop population grow exponentially, that is, that the end of the season is reached before density-dependent population regulation becomes a factor. The pest population is initially zero and grows due to the combined effects of immigration and reproduction. The fundamental measure of the pest level is the population variable $x(t)$. The crop is represented by a variable $c(t)$. We assume that the value of the crop at harvest time is directly related to the value $c(T)$ of this variable. We do not explicitly state what $c(t)$ is, although, for example, Regev *et al.* [13] use an analogous variable that represents leaf mass. In any case, we assume that the pests reduce this variable at a rate proportional to their numbers.

In order to include genetically conferred resistance in the model, the pest population is divided into resistant and susceptible subpopulations. These are denoted by the subscripts R and S, respectively. The basic equations of the single season model are then

$$\begin{aligned} \frac{dx_R}{dt} &= \rho_R(t)x_R + \mu_R I(t), & x_R(0) &= 0, \\ \frac{dx_S}{dt} &= \rho_S(t)x_S + \mu_S I(t), & x_S(0) &= 0, \\ \frac{dc}{dt} &= rc - v(t)(x_R + x_S) & \text{if } c > 0 \\ &= 0 & \text{if } c = 0, & \quad c(0) = c_0. \end{aligned} \tag{2.1}$$

Here $x_R(t)$ and $x_S(t)$ are the growth rates of the resistant and susceptible populations, μ_R and μ_S are the fraction of resistant and susceptible types in the pool, $I(t)$ is the immigration rate from the pool to the field, r is the growth rate of the variable c , and $v(t)$ is the rate of consumption (or damage) of the crop by the pest.

A few comments are in order at this point concerning assumptions. The pest growth rates $\rho_i(t)$ will in general be positive, but will be negative during the period immediately following the application of a pesticide. Our model does not explicitly take natural controls such as predators into account, but these may be implicitly included in the functions $\rho_i(t)$. Similarly, the frequently observed "rebound" effect, in which a pest population grows dramatically after the effects of an insecticide has worn off (due to elimination of predators, hormoligosis, or other effects), may be accounted for in the construction of the $\rho_i(t)$.

In general, a pest population does not economically damage a crop over the entire growing season. This may be taken into account in the formulation of the function $v(t)$. As an alternative, which we shall follow, one may limit the duration of the simulation to the period of susceptibility of the crop to that pest.

In actual agroecosystems, the rate at which events proceed is governed by temperature as well as time. This has led to the introduction of the notion of "physiological time," which is measured in day-degrees and incorporates the effects of temperature (e.g., [16]). Agricultural experiment station advisors are encouraging growers to schedule events by physiological, rather than chronological, time. Since the physiological times for the crop and the pest will, in general, be different, the explicit incorporation of physiological time would introduce an unacceptable level of complexity. We retain a single time variable t , which we regard as incorporating in a general way the effects of temperature.

Finally, the model incorporates no age structure, and only the most simple genetics. In our earlier paper [12], we provided simulation-based evidence that models of this simple form have the same qualitative behavior as more complex, detailed simulation models that include age structure and genetics. Since our objective is to obtain qualitative insight rather than explicit numerical predictions, we feel that the simpler model is more suitable.

Our philosophy of modeling the effect of the process, rather than the process itself, extends to our choice of a function to represent the effect of the pesticide. We model this with the pulse function

$$\begin{aligned} s(t) &= \eta && \text{if } t_s \leq t \leq t_s + \delta \\ &= 0 && \text{otherwise} \end{aligned} \quad (2.2)$$

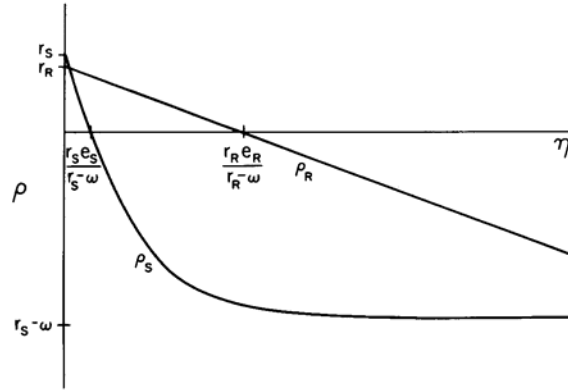
where η represents the intensity of application, t_s is the time of application, and δ is the duration of the effect. The actual effect of an externally applied pesticide would of course wear off gradually. The pulse function, however, gives a more convenient mathematical form to the model and does not change the qualitative behavior of the solutions. We assume that the pesticide is applied prophylactically, and we devote most of our attention to the case of a single application per season. The parameters η and t_s are then the control variables of the model. The basic form for the growth rate functions $\rho_i(t)$ is

$$\rho_i(t) = r_i - \frac{\omega s(t)}{e_i + s(t)}, \quad i = R, S. \quad (2.3)$$

During the period when the pesticide has no effect, we have $\rho_i(t) = r_i$. When the pesticide is active, we have

$$\rho_i(t) = r_i - \frac{\omega \eta}{e_i + \eta}. \quad (2.4)$$

Figure 1 shows sketches of functions of this form. For small values of η , ρ_i decreases in an approximately linear way. The parameters e_i determine the rate of decrease, with $e_S \ll e_R$. As η increases, the effect of the pesticide is assumed to saturate; this saturation level is determined by ω , which is assumed the same for resistants and susceptibles.


 FIG. 1. Sketches of ρ_R and ρ_S as functions of η (cf. Eq. 2.4).

We now turn to a description of the interseasonal dynamics of the model. The simulation is carried out for N seasons. The discrete variable n , with $1 \leq n \leq N$, indexes the season. The control variables t_S and η depend on n , as do the fractions μ_S and μ_R . Indeed, it is only through $\mu_R(n)$ and $\mu_S(n)$ that the dynamics of one season have any effect on those of another. If we let $x_R(t; n)$ denote the value of x_R at time t in season n , and similarly for $x_S(t; n)$, then the discrete dynamics of $\mu_R(n)$ and $\mu_S(n)$ are defined by

$$\mu_R(n) = \frac{x_R(T; n-1)}{x_R(T; n-1) + x_S(T; n-1)}, \quad n = 2, 3, \dots, N, \quad (2.5)$$

$$\mu_R(1) = p_0$$

and $\mu_S(n) = 1 - \mu_R(n)$. This says that the pool is essentially depleted at the end of the season. The survivors of the season are randomly mixed together, and the proportions of the two types in the population at the end of one season determines these proportions for the subsequent season. We assume that these proportions are fixed for the course of a season, and do not change during the interseasonal period. Once again, these assumptions do not alter the qualitative behavior of the model, but make it considerably simpler.

Equations (2.1)–(2.5) completely characterize the dynamics of the model. A considerable economy of representation may be obtained by performing a few simple manipulations. First, we define the variables $y_i(t; n)$ by

$$y_i(t; n) = x_i(t; n) / \mu_i(n), \quad i = R, S. \quad (2.6)$$

Substituting from Eq. (2.6) into (2.1) and noting that the $\mu_i(n)$ do not depend on t yields

$$\frac{dy_i}{dt} = \rho_i(t; n) y_i + I(t). \quad (2.7)$$

This equation is useful, particularly in the dynamic programming solution of Section 3, because it does not contain $\mu_i(n)$. We shall define our optimization problem in terms of $y_i(t; n)$ rather than $x_i(t; n)$.

Next, we define the ratio $F(n)$ by

$$F(n) = \frac{y_R(T; n)}{y_S(T; n)}. \quad (2.8)$$

Substituting from Eq. (2.8) into (2.5) yields

$$\mu_R(n) = \frac{\mu_R(n-1)F(n-1)}{\mu_R(n-1)[F(n-1) - 1] + 1}. \quad (2.9)$$

This relation provides a convenient, explicit form for the dynamics of $\mu_R(n)$. In particular, expanding (2.9) in a Taylor series yields

$$\mu_R(n) = \mu_R(n-1)F(n-1) + 0(\mu_R^2(n-1)), \quad (2.10)$$

which indicates that when the fraction of resistant is small, it grows geometrically. This same effect is seen in the model of Comins [6].

In summary, our model is given by the equations

$$\begin{aligned} \frac{dy_R}{dt} &= \rho_R(t; n)y_R + I(t), & y_R(0; n) &= 0 \\ \frac{dy_S}{dt} &= \rho_S(t; n)y_S + I(t), & y_S(0; n) &= 0 \\ \frac{dc}{dt} &= rc - v(t)[\mu_R(n)(y_R - y_S) + y_S] & \text{if } c(t; n) > 0 \\ &= 0 & \text{if } c(t; n) = 0, \\ c(0; n) &= c_0 \\ \mu_R(n) &= \frac{\mu_R(n-1)F(n-1)}{\mu_R(n-1)[F(n-1) - 1] + 1}, & \mu_R(1) &= p_0 \\ 0 \leq t \leq T, & & 1 \leq n \leq N, & \end{aligned} \quad (2.11)$$

together with Eqs. (2.2), (2.3), and (2.8). In the next section we consider the economic optimization problem associated with this model.

3. THE MULTISEASON ECONOMIC OPTIMIZATION PROBLEM

The problem is to maximize profit from the crop harvest over N seasons, subject to discounting. We assume that the decision maker expects a new pest control technology to be available at the time horizon so that resistance to the pesticide does not enter explicitly into the profit function, which is

$$J = \sum_{n=1}^N \alpha^{n-1} [c(T; n) - c_p \eta(n)]. \quad (3.1)$$

In this equation $c(T; n)$ is the value of the crop variable at time T in year n , $\alpha = (1 + \gamma)^{-1}$ where γ is the discount rate, and c_p is the relative cost per unit of pesticide.

The multiseason economic optimization problem is to choose a strategy $\{\eta(n), t_s(n)\}$, $n = 1, \dots, N$, to maximize the profit function J given in Eq. (3.1), subject to the dynamics of Eqs. (2.11). Our problem is so simple that it can be solved numerically using standard nonlinear programming subroutines. However, most real problems of this type are too complicated for this method of solution, and must instead be solved using dynamic programming, often with simplifying assumptions to make the problem tractable. Shoemaker [14] presents an example of the ingenious use of dynamic programming to solve a very complicated problem of this sort. Since we are interested in relating our problem to the more complicated variety, we focus on the dynamic programming solutions, and use nonlinear programming solutions as a check. Our nonlinear programming code uses the IMSL package ZXMIN, and is based on a similar program written by Dr. S. Yakowitz. We begin by making a further simplification, which may, however, more closely represent actual practice.

In our earlier paper [12] we concluded that the pest population in a given year could be controlled with pesticide intensity and that the buildup of resistance could be controlled by proper timing of the pesticide application. An increase in pesticide intensity leads to an increase in crop yield that is uniform over the first several seasons. In general, pesticide intensity has a much lower effect on the buildup of resistance than does timing of the pesticide application.

Therefore, in the dynamic programming solution we fix η at a value independent of n and use $t_s(n)$ as the sole control variable. To select a proper value for η we consider the single year optimization problem of maximizing over η the quantity $c(T) - c_p\eta$, with t_s held fixed. The solution η^* of this problem depends on t_s and μ_R . Mangel and Plant [12] find that the value of η^* is virtually independent of μ_R and t_s over a wide range of values of μ_R and t_s . We therefore adopt the following strategy. Before the start of the dynamic programming problem, t_s and μ_R are fixed at moderate values and the optimal η^* maximizing $c(T) - c_p\eta$ is determined. The parameter η is then fixed at this value η^* for the solution of the dynamic programming problem. In the nonlinear programming solution, we treat both η and t_s as free parameters. Our solution to the dynamic programming problem is as follows. Let $\Gamma(t_s(n), \mu_R(n)) = c(T; n) - c_p\eta^*$. Then the simplified multiseason problem is to find a $t_s(n)$ to

$$\text{maximize } J = \sum_{n=1}^N \alpha^{n-1} \Gamma(t_s(n), \mu_R(n)) \quad (3.2)$$

subject to the dynamics described in Eqs. (2.11). Let $J_n^*(\mu)$ denote the return from stages n through N , assuming that the optimal policy is followed and that the current value of $\mu_R(n)$ is μ . Then by the principle of optimality the appropriate functional equation is

$$J_n^*(\mu) = \max_{0 \leq t_s \leq T-\delta} \{ \Gamma(t_s, \mu) + \alpha J_{n+1}^*(G(n, \mu; t_s)) \} \quad (3.3)$$

where $G(n, \mu; t_s)$ is defined by the right-hand side of Eq. (2.9). The solution proceeds by finding $J_N^*(\mu)$ using

$$J_N^*(\mu) = \max_{0 \leq t_s \leq T-\delta} \Gamma(t_s, \mu) \quad (3.4)$$

and then solving Eq. (3.3) recursively. The optimal profit is then

$$J^* = J_1^*(p_0). \quad (3.5)$$

The solutions to the dynamic programming problem were obtained as follows. The analytical solutions of Eqs. (2.11) were first written out. These solutions are simple to obtain but long to state, and are not reproduced here. Following the usual procedure of Bellman and Dreyfus [1], the continuous problem of Eq. (3.3) was replaced by one in which the control variable t_s assumed discrete values. For simplicity we restricted ourselves to integer values; the nonlinear programming solution removed this restriction.

Table I shows the values of the parameters chosen for our calculations. These are the same as those used in our earlier paper [12]. The choice of these parameters is somewhat arbitrary. As will be pointed out in the discussion, the qualitative behavior of the model is highly insensitive to the values of the parameters.

A convenient indication of the single season yield is the ratio f defined by

$$f = c(T)/c_0 e^{rT}. \quad (3.6)$$

Figure 2 shows a plot of f vs t_s for the parameter values of Table I. The value of μ_R is p_0 . The optimal value of t_s is $t_s^* = 6$. We shall refer to this colloquially as the *single season optimal* value. Note that, in fact, the optimal value t_s^* depends on μ_R . We use the term "single season optimal" to mean the optimal value if resistance is negligible. In fact, for $\mu_R = 1$, t_s^* is also approximately 6.

Also shown in Fig. 2 is a plot of the ratio $\mu_R(2)/p_0$ versus t_s . Unlike the crop yield, this ratio increases with increasing t_s . For any multiseasonal strategy that conserves susceptible pests, the value of t_s may not be the single season optimal during some years. It is evident from Fig. 2 that it will never be useful to set t_s greater than t_s^* since this only decreases the fraction of susceptibles. This discussion should make easier the interpretation of the multiseason optimization results, which we now present. Figure 3 shows plots of the optimal timing $t_s(n)$ with zero discount rate ($\alpha = 1$). The figure shows $t_s(n)$ plots for several different time horizons.

For a sufficiently short time horizon, the multiseasonal optimal strategy is simply to spray at the single season optimum in each season. Moreover, for any value of the time horizon it is optimal to spray at this value of t_s when n is nearly equal to N , since at this point the cost of future resistance is small. As the time horizon

TABLE I

Parameter	Value	Parameter	Value
I_0	1	δ	2
I_c	1	ω	20
r	0.1	p_0	10^{-4}
v	0.005	c_0	1
T	20	c_p	0.5
e_R	35		
e_S	5		
r_R	0.2		
r_S	0.2	η^*	2

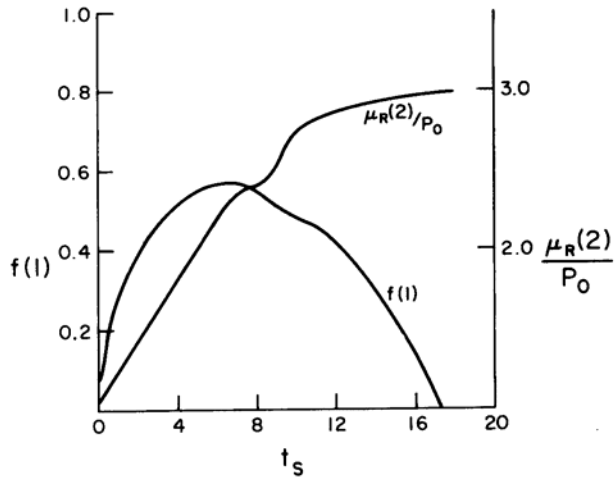


FIG. 2. The effect of spraying time t_s on relative yield and resistance buildup. Parameters are set to their first year values. The curve $f(1)$ gives the relative yield, and the curve $\mu_R(2)/p_0$ gives the ratio of μ_R in the second year to that of the first.

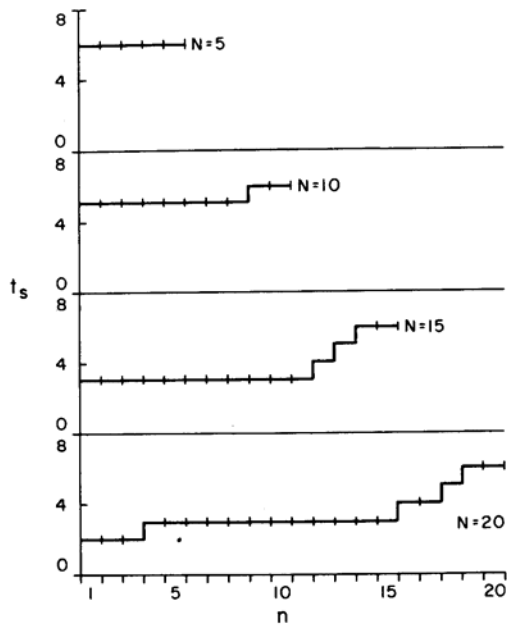


FIG. 3. Optimal spraying time $t_s(n)$ for four different time horizons: $N = 5$, $N = 10$, $N = 15$, and $N = 20$.

lengthens, it becomes cost effective to spray early during the early seasons. Based on Fig. 3, we can say that by spraying early the grower sacrifices a portion of his present crop to delay the buildup of resistance, thereby gaining a greater crop in the future. Note that under our assumption of zero discount rate, each season's crop has equal value.

Figure 4 presents the results of Fig. 3 in another, more compact form. The abscissa is still n , but the ordinate is a logarithmic plot of $\mu_R(n)$. The dots show values of $\mu_R(n)$ assuming the optimal policy is followed. These dots are connected by thin lines to aid the eye. The heavy lines show intervals of a given optimal value of t_s as a function of the season in a 20-year problem, and of the current value of μ_R . Once again, the thin lines serve only to aid the eye. The optimal control policy for a given time horizon $N \leq 20$ and a given present value of μ_R may be obtained by starting at $n = 20 - N + 1$ and following the appropriate trajectory. Since t_s is actually a continuous control variable, the figure may be thought of as giving an approximation to the level curves on the optimal t_s surface; we shall see that this is a good approximation of these level curves.

For small values of n , the $\mu_R(n)$ trajectories are approximately linear, indicating the approximately geometric growth mentioned in Section 2. Also, for small n the $\mu_R(n)$ trajectories are approximately parallel to the "level curves" of t_s . This indicates that the optimal policy during the early seasons is to spray at about the same fixed time in each season, and is due to the negligible value of μ_R in the early seasons.

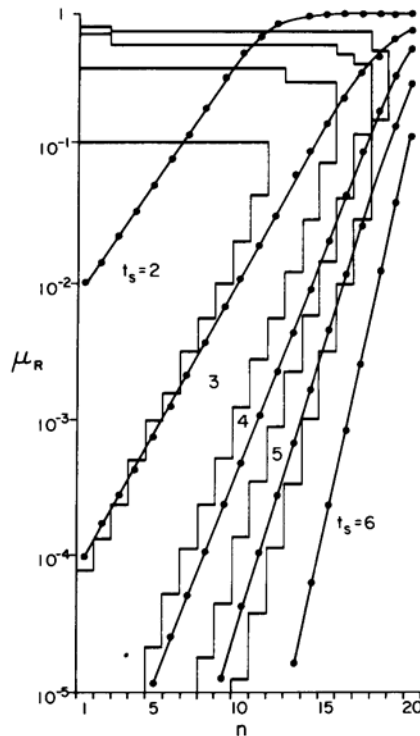


FIG. 4. Optimal strategies as functions of μ_R and n . The dots show values of $\mu_R(n)$ over a 20-season horizon, assuming the optimal policy is followed. The heavy lines separate intervals of optimal t_s for each n . The thin lines are only an aid to the eye.

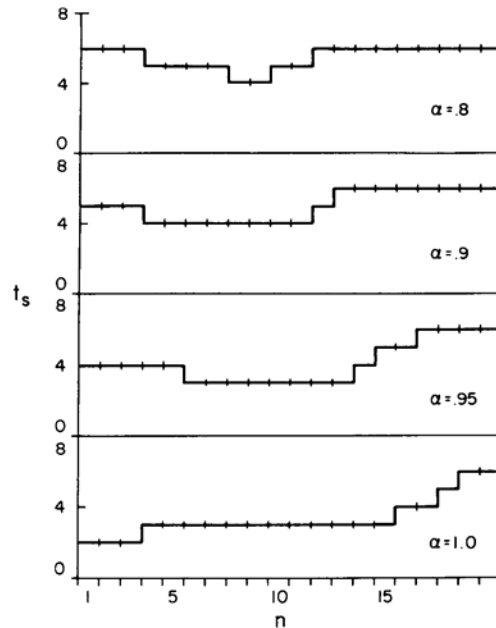


FIG. 5. Optimal spraying time $t_s(n)$ for $N = 20$, with four values of α .

Figure 5 shows how the discount rate influences the optimal policy for a 20-year time horizon. There are two principle effects. The first is to make the policy nonmonotonic; the second is to move the policy toward the single season optimal in each season. The reason for the former effect is that the early season crop, which does not suffer from the effects of a highly resistant pest population, becomes much more valuable due to the discounting of future crops. The reason for the second effect may be explained as follows.

Figure 6 shows a plot similar to that of Fig. 4, but for the case $\alpha = 0.9$. This figure indicates that it is never optimal to spray earlier than $t_s = 4$. The figure also shows that it is, roughly speaking, a good policy to spray early during those seasons when μ_R , the fraction of resistant, has an intermediate value. During the early seasons the high value of present crops due to discounting motivates spraying near the single season optimal. After the resistant type dominates the population there is nothing to be gained by spraying early, so the optimal policy returns to the single season optimal.

The simplifications made in implementing the dynamic programming solution are the assumption of constant η and the restriction of t_s to integer values. We implemented a nonlinear programming solution to test the effect of these simplifications. Because of the size of the problem we were forced to restrict the time horizon in the nonlinear programming solution to $N = 10$, so our comparisons are made based on this value. The discount rate was set at zero (i.e., $\alpha = 1$) and all other parameter values are as in Table I. Table II shows the values of t_s and η for three solution methods: (i) dynamic programming with η fixed and t_s held to integer values; (ii) nonlinear programming with no restriction on t_s and η held fixed; and (iii) nonlinear programming with no restriction on either parameter. The correspondence between these is very good. A similar good correspondence was found for $\alpha = 0.9$.

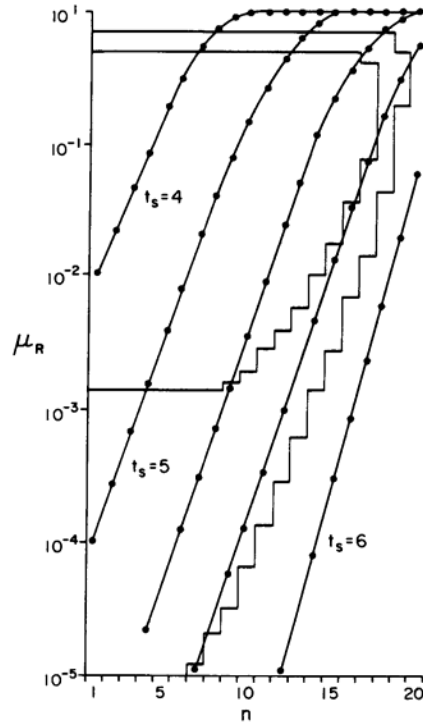


FIG. 6. Optimal strategies as in Fig. 4, but with a discount term $\alpha = 0.9$.

TABLE II
Optimal Values of t_s and η Using Three Different Assumptions^a

n	(i)		(ii)		(iii)	
	t_s	η	t_s	η	t_s	η
1	5	2	4.7	2	4.8	2.0
2	5	2	4.7	2	4.8	2.0
3	5	2	4.7	2	4.8	2.0
4	5	2	4.7	2	4.8	2.0
5	5	2	4.8	2	4.8	2.0
6	5	2	4.8	2	4.8	2.0
7	5	2	4.9	2	4.9	2.0
8	5	2	5.0	2	5.0	2.1
9	6	2	5.5	2	5.5	2.3
10	6	2	6.5	2	6.5	2.7

^a(i) Dynamic programming, integral t_s , fixed; (ii) nonlinear programming, η fixed; (iii) nonlinear programming, η free.

In studying the economic optimization model in the form of Eq. (3.1), we implicitly assume that the grower follows a procedure of actively planning ahead over the entire time horizon. An alternative procedure is to simply react to perceived changes in the system. We refer to this as a myopic policy and study it in the next section.

4. MYOPIC STRATEGIES

In motivating this section, two points are especially worth noting. The first is that the time horizon of the problem, which is generally unknown to the manager, may play a significant role in determining the optimal strategy. In particular, if the time horizon is relatively short, then the strategy of spraying at the single season optimal time in each season may provide a multiseason optimum. The second point is that the multiseason optimum strategy is to spray at or near the single season optimum time except during those years when resistance is at an intermediate level, in which case the spraying should be done earlier.

These observations motivate the consideration of a model in which the strategy is decided "myopically," i.e., without explicit regard for the future, but in which the present state of the system is measured and used to influence future decisions. The strategy is to spray at time t_s^* under ordinary circumstances. If during a given year an appreciable increase of resistance to the pesticide is detected, then in the next year the pesticide is applied earlier. In practice the grower might implement such a strategy by simply using subjective judgment. In order to study such a strategy in the context of the model, however, we must provide specific decision rules. Our rule is to infer the increase in resistance by observing a reduction in yield. This is possible in our model since the only factor affecting yield is resistance. It might not be possible in reality since other factors may mask the effect of increased resistance. In any case, the increase in resistance is measured in the following way. The yield $c(T; 1)$ during the first season is established as the baseline. The decline $\Delta c(n)$ in yield is measured as

$$\Delta c(n) = [c(T; n) - c(T; n - 1)]/c(T; 1). \quad (4.1)$$

The decision rule is: if $\Delta c(n) < \theta$ then $t_s(n + 1) = t_s^* - \Delta t_s$. Computational experimentation with this simple strategy indicates that for the proper choice of the parameters θ and Δt_s , the myopic strategy may significantly increase the value of the profit function J [Eq. (3.1)], although the increase is not as great as that obtained through the dynamic programming strategy. The values of J for each of these strategies are highly dependent on the parameters of the model, so we do not present them.

We conclude this section with a brief discussion in which we drop the restriction of a single pesticide application per season. We consider only the myopic strategy of spraying early in response to decreased yield in the previous season.

The problem is studied numerically in a way that tries to simulate actual practice. As in the single dose case the first dose is applied prophylactically at time t_s . At time $t_s + \delta$ the population of pests x is estimated. If x is larger than some threshold level x_T , a second dose is applied. This procedure is repeated a maximum of M times, and is terminated in less than M times if at some $t_s + m\delta$, where $m < M$, the relation $x < x_T$ is satisfied.

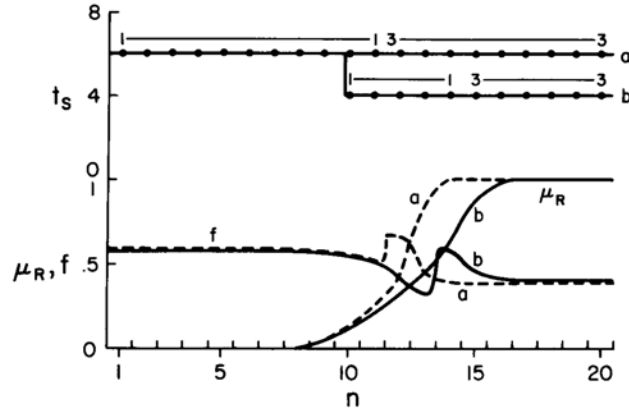


FIG. 7. A comparison for the case of multiple applications of the optimal strategy with that of spraying at the single season optimal time in each season. The upper curves show application times $t_s(n)$ and the number of applications. Curve (a) correspond to the single season optimal strategy, and curve (b) corresponds to the myopic feedback strategy. The lower curves show values of $\mu_R(n)$ and $f(n)$. To aid the eye these are plotted as continuous curves. The solid curve corresponds to the myopic feedback strategy; the dashed curve corresponds to the single season optimal strategy.

Figure 7 shows the results of numerical solution using this strategy. Curves *a* correspond to the case in which the pesticide is first applied at t_s^* in each season. The number of applications in each season is shown above the t_s curve. The maximum number allowed is 3. A single application is used until season 12; three applications are required from this season on. The yield is high in season 12, but resistance builds very rapidly and the yield declines quickly. Curve *b* shows the results when the myopic feedback strategy of Section 3 is used. The value of x_T is set slightly lower in this case to partially compensate for the earlier initiation time t_s . The strategy is modified from that described earlier in that the value of t_s is maintained at $t_s^* - \Delta t$, once it has been set there. The feedback strategy causes the multiple dose application of pesticides to be delayed until season 15 and significantly delays the buildup of resistance, as well as reducing the total amount of pesticide applied over the time horizon.

5. DISCUSSION

Our paper is concerned with the economic trade-off faced by a grower who must balance short-term gains achieved through the application of a pesticide with long-term losses due to the buildup of resistance to the pesticide in the pest population. We study this problem by means of a model that we hope is realistic enough to have some usefulness, but simple enough to permit a detailed examination and understanding of its behavior. Our conclusions are not intended to be prescriptive, but rather simply to provide some insight, as does the Schaeffer model in the study of fisheries [4].

One result of our study is that the time horizon of the problem and the discount rate have a substantial effect on the optimal policy. This effect may best be interpreted by considering pesticide susceptibility as a resource that is mined with each pesticide application [11]. As with any other resource, the optimal rate of

exploitation of susceptibility increases with increasing discount rate and with decreasing time horizon. We may expect this result to be quite robust with regard to the choice of model for the system.

A second result is depicted in Fig. 2. This figure shows that the curve representing yield as a function of time of application of pesticide has a maximum. If the pesticide is applied too early then the pest population is rebuilt by immigration; if the pesticide is applied too late then the crop is destroyed before the application. This result must be interpreted in the context that the susceptibility of the crop to pest attack is assumed uniform over time in our model. Within this context the result appears reasonable. In our model, the optimal spraying time is virtually independent of the value of μ_R .

The second curve in Fig. 2 shows that resistance increase in a given year is a monotonically increasing function of time of application of pesticide. This effect occurs because if the pesticide is applied later, the immigrating population, which is not subjected to the pesticide, has less time to dilute its effect. Once again this result agrees with intuition, provided that immigration takes place over the course of the growing season. The principal conclusion obtained by combining the curves in Fig. 2 is that, if the grower is going to try to delay the onset of resistance by spraying at a time other than the optimal one, then he should spray earlier than optimal, rather than later.

In our earlier paper [12] we noted that while the intensity of application η has a significant effect on the yield in a given year, over a broad range it does not have much effect on the rate of growth of resistance. This is reflected in Table II, in which the optimal intensity is almost constant. This result holds for a wide range of parameter values in our model, but we are not prepared to say that this conclusion holds for all possible models. We can, however, say that this phenomenon in our model permits (or restricts) us to focus on the effects of timing of application on resistance growth, which, in our model, are substantial.

We examined our model over a wide range of parameter values and found that its conclusions were generally insensitive to parameter value provided that the growth rate of the pest population is substantially higher than that of the crop. We feel that this is a reasonable restriction. If the rate of decline I_c of the immigration function is reduced, then the optimal time t_s is delayed, but the qualitative results are identical. The same thing happens if the intrinsic growth rates r_i are increased after the pesticide is applied, to reflect the destruction of natural enemies.

The dynamics of interseasonal resistance buildup reflect three assumptions: the pool is depleted at the end of the year, there are no pests overwintering in the field, and the ratios of susceptibles and resistants do not change in the overwintering period. The effect of modifying these assumptions is exactly what one would expect. Resistance growth is slowed if the pool is not depleted, speeded up if there is overwintering in the field, and may have either effect depending on which type has the higher growth rate during the overwintering period. The qualitative results are not changed.

Since our model is deterministic, there is no difference between the open loop optimal strategy presented in Section 3 and the corresponding closed loop optimal strategy. The implementation of such a strategy would require knowledge of the time horizon and future discount rates (in general, knowledge of one of these implies knowledge of the other). This motivated the consideration of a "myopic" strategy in Section 4. This strategy for some parameter values resulted in a substantial increase

in total yield over simply applying the pesticide at the single season optimal in each season.

Although we briefly considered the case of multiple pesticide applications in this section, we restricted ourselves primarily to the consideration of a single application. Many crops require multiple applications of the same pesticide, indeed, this is the rule rather than the exception. The effect of multiple applications could easily be incorporated into our model by making the value of δ , the duration of pesticide intensity, larger and the value of η , the intensity smaller, to reflect a sort of "average" level of pesticide over several applications. In the alternative, several applications could be built in by making the spray function $s(t; n)$ in Eq. (2.2) consist of several pulses. As with other such changes, these would increase the complexity of the model without substantially changing its qualitative results.

The robustness of our model to detailed changes is both its strength and its weakness. On the one hand, we feel that we can have some confidence in the qualitative nature of the results, and that these results may be useful in understanding the behavior of the solutions of more complex models. On the other hand, we certainly do not expect that our model will ever have any predictive value. A more thorough consideration of the specific properties of the individual agroecosystem, and a more detailed model reflecting the complexities of this system, are required before one can make realistic predictions.

ACKNOWLEDGMENTS

We are grateful to James R. Carey, Richard E. Howitt, and Sidney J. Yakowitz for their helpful discussions. We are also grateful to an anonymous referee for a very thorough and insightful critique of an earlier draft.

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