

TP-351

PROPAGATION MODE ESTIMATION: A PREREQUISITE FOR OTH RADAR FUSION

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Abstract - Over-the-Horizon (OTH) radars operate in the High Frequency (HF) band of the radar spectrum and utilize ionospheric reflections to detect and track targets in regions up to 2000 nautical miles from the radar site. OTH radars thus have the capability to serve as cost-effective sensors for detecting and tracking aircraft over large surveillance areas. In this paper we describe an algorithm for fusing data from multiple OTH radars. We show that estimation of the radar signal propagation mode is tightly coupled to the tracking and correlation functions in the fusion algorithm. Results using test data collected from two operational OTH radars show that fusion improves both track continuity and track accuracy.

1. INTRODUCTION

Over-the-Horizon (OTH) radars operate in the High Frequency (HF) band of the electromagnetic spectrum. At these frequencies, energy directed by an OTH radar toward the ionosphere is refracted and ultimately reflected back to the surface of the earth several hundred miles down range from the radar. This enables the OTH radar to detect and track targets in regions far beyond the ranges of conventional radars which operate in UHF and higher frequency bands and which detect and track targets which are within line-of-sight from the radar.

Surveillance assets which are currently used for detecting and tracking drug carrying aircraft originating in South America and entering the U.S. and Canada include a chain of tethered aerostats, ground radars, Navy ships, and airborne surveillance systems. Because of the limited coverage provided by these surveillance assets, drug smugglers can plan and follow routes that do not fall within the coverage region of these surveillance assets and thereby avoid being detected. OTH radars provide three features which complement the capabilities of the existing surveillance assets for detecting and tracking drug carrying aircraft. First, as pointed out earlier, OTH radars have relatively large coverage areas. These large coverage areas provided by OTH radars will prevent the presence of gaps in the surveillance

coverage area through which drug smugglers can fly without being detected. Second, with a suitable choice of OTH radar location, they can be made to look at airspace deeper into South America. This is the region where the drug traffic originates and, because the commercial air traffic density is lower in these countrysides, it is easier to detect, track, and identify them in this region. Third, in contrast to the look-up capability of conventional ground radars, OTH radars provide a look-down capability. This feature enables the OTH radar to detect drug-carrying aircraft that follow flight paths through valleys in mountainous regions. Such flight paths would be camouflaged to ground radars.

Use of OTH radar for the Counter-Drug mission also raises several technical issues. The primary issue arises from the fact that the OTH radar relies on the ionosphere to propagate the radar signal to and from the target. For each detected target, the OTH measures the slant range, the slant azimuth, and the slant range-rate of the target. Target tracks are typically required in ground coordinates (e.g., latitude, longitude or x, y position in a Cartesian frame with origin at the radar receiver). Transformation of the slant measurements to ground coordinates requires knowledge of the path followed by the radar signal. Such a path is also referred to as the propagation mode and it specifies the reflecting ionospheric layers and the number of bounces the radar signal undergoes on its way to the target and back to the OTH receiver. Several such propagation modes are feasible, and for each mode there is a corresponding transformation of the slant measurements to ground tracks. It is difficult to predict the path followed by the radar signal because the ionospheric conditions depend on many variables (e.g., the frequency of radar operation, the geographical area of operation, the time of the day, the Sun Spot activity, etc.) and they constantly vary with time [1]. If the mode is not selected correctly, then the ground track will be inaccurate and it will be difficult to correlate the ground tracks of one OTH radar with another. The errors in the ground tracks due to incorrect mode selection will thus represent registration errors.

The second issue associated with the use of OTH radars is that range and cross-range measurement errors for an OTH radar are much larger than those of microwave radars. Consequently, OTH radars will have difficulty resolving

targets which are separated by a few nautical miles. The poor resolution is compounded by the errors in transforming the slant radar measurements. The third issue associated with the use of OTH radars is clutter. Two factors which cause the clutter problem to be serious are the large foot-print of the OTH radar and the long propagation path. The first factor causes ground clutter over the entire footprint to be integrated. The second factor causes any irregularity in the ionosphere (e.g., equatorial ionospheric bulge) to be picked up by the OTH receiver as clutter. Ground clutter can be mitigated by Doppler processing; Doppler processing, however, makes it difficult to detect and track targets which fly paths which are cross-radial to the radar.

Fusing data from multiple OTH radars can mitigate the three issues mentioned above. Because a fusion algorithm attempts to combine target information from multiple sensors, it will be able to detect registration errors and also be able to correct for them. A fusion algorithm will improve the accuracy of tracked targets for the following two reasons. First, it has target data from more than one sensor and so it can average out the noise more effectively. Second, if the sensors provide geometrically diverse measurements (e.g., if the range component of one sensor is orthogonal to the other sensor and the measurements have different in-range and cross-range measurement uncertainties), then the fusion algorithm will provide tracking uncertainties which will be the intersection of the uncertainties of the individual sensors. Finally, a fusion algorithm can reduce the clutter in regions where the two sensors have overlapping coverage because tracks generated from clutter will not correlate across multiple sensors. Hence, by fusing data from multiple OTH radars, it will be possible to identify tracks generated by clutter reports and then drop them. Multiple viewing geometries provided by two or more OTH radars can also minimize the blind speeds introduced by the Doppler based clutter mitigation procedure.

A fusion algorithm will provide three other benefits. First, a fusion algorithm will have the capability to correlate duplicate tracks from multiple sensors, combine them, and present an integrated air picture to the surveillance operator. Note that when the number of such duplicate tracks is more than three or four, it will be very difficult for an operator to correlate them. Second, the fusion algorithm will be capable of integrating information from other sensors (e.g., microwave radars, aerostats) with minimal new design effort. Third, knowledge of the coverage areas and measurement capabilities of all the sensors available to the fusion algorithm will enable it perform sensor management functions such as sensor cueing and sensor energy management much more effectively.

In this paper we will describe an algorithm for fusing OTH radar data and demonstrate some of the fusion benefits listed above with test data. In Section 2, we provide a summary of parameters and tables which characterize OTH radars for the fusion algorithm. In Section 3, we provide a description of the OTH radar fusion algorithm. In Section 4, we describe the OTH fusion design testbed developed during this effort. In Section 5, we provide results which demonstrate the benefits provided by the OTH radar fusion algorithm.

2. OTH RADAR CHARACTERIZATION

The ionosphere is a thick shell of free electrons which envelops the Earth at altitudes between 90 and 600 km. The typical daytime electron density tends to peak around four layers at different altitudes. The first peak occurs at altitudes ranging from 50 to 90 km and is known as the *D layer*. The second peak occurs at altitudes ranging from 100 to 110 km and is known as the *E layer*. The third peak occurs at altitudes from 150 to 200 km and is known as the *F1 layer*. The fourth peak occurs at altitudes from 270 to 300 km and is known as the *F2 layer*. Electromagnetic waves traversing the ionosphere get refracted because of the changes in the electron density.

An OTH radar typically operates in the high frequency (HF) band (5-28 MHz) of the electromagnetic spectrum. When a radar signal operating in the HF band is directed toward the ionosphere, it is refracted and eventually reflected by it. Figure 1 shows the typical path followed by an OTH radar signal. The radar signal emitted by the OTH radar transmitter at an elevation angle Φ travels a distance $R_S/2$ before striking the ionosphere at the point P where it is reflected back towards the ground. In reality, the signal is not reflected at P but it is continuously refracted towards the ground as it passes through the ionosphere. However, for practical purposes, we can model the signal as being reflected at P. Near the ground, the radar signal backscatters off a target (e.g., an aircraft) A. Some of the backscattered energy retraces the transmitted signal path back to the OTH radar receiver R located in the vicinity of the transmitter. The path of the signal, $T \rightarrow P \rightarrow A \rightarrow P \rightarrow R$, is called a *raypath*. The distance between the target and the OTH radar along the raypath is called the *slant range* (R_S), and the distance between the target and the OTH radar along the ground is called the *ground range* (R_G). The ionospheric layer at the point of reflection P is at a height h above the midpoint between the radar and the target.

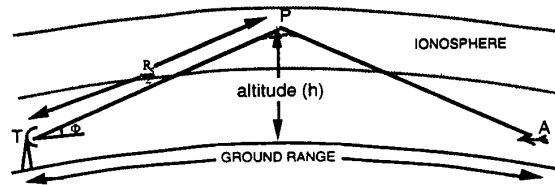


Fig. 1 Typical Path Followed by OTH Radar Signal

In practice, the OTH radar signal can reach a target by more than one raypath. Figure 2 shows two rays originating from the transmitter at two different elevation angles and reaching the same target after reflecting from two different ionospheric layers. In addition to rays reaching the target after a single reflection from the ionosphere (i.e., a single hop), other rays might reach the target after two reflections from the ionosphere with an intermediate reflection from the ground (i.e., two hops). Furthermore, the reflected ray may

follow a path different from the transmitted ray. This shows that OTH radar signals may follow many different paths to and from the target. Each such raypath for a target report is referred to as a *propagation mode*.

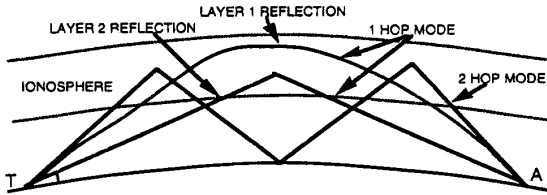


Fig. 2 Multiple Paths Followed by OTH Radar Signal

The OTH radar measures the slant range, the slant azimuth, and the slant range-rate for each target. Because target track information is generally required in an absolute coordinate system, the data processing algorithms in the OTH radar transform these slant space measurements to target latitude and longitude. The transformation is done in two steps: First the slant space measurements are converted to ground space measurements (i.e., ground range and ground azimuth). This is a difficult step because it requires knowledge of ionospheric conditions. In the second step the ground space measurements are converted to target latitude and longitude.

OTH radars are equipped with auxiliary systems which constantly monitor and update the ionospheric conditions over the regions of interest [2]. An OTH radar uses information from these auxiliary systems to construct what are called as coordinate registration (CR) tables. These tables contain information for transforming both slant range and slant azimuth to ground range and ground azimuth for each of the feasible propagation modes. The data processing algorithms in the OTH radar system transform the target slant space measurement to the most likely ground space measurements.

The OTH radar parameters and data that are required by a fusion algorithm are:

- Coverage area of the radar;
- Probability of target detection and the probability of false alarm;
- Uncertainties in the slant space measurements for each target report; and
- Data for converting the slant space measurements to ground space measurements.

3. OTH FUSION ALGORITHM

A generic surveillance system represents a feedback system whose objective is to detect, track, and identify targets within a surveillance volume. It consists of seven modules shown in Fig. 3. The *Sensor Suite* module represents

all the sensors available to the surveillance system. Sensors in the sensor suite may or may not be collocated and they may or may not be homogenous. Raw sensor data is passed to the *Signal Processing* module which generates report data for objects detected by the sensors. Each report contains measurements such as range, azimuth for the detected object. Biases in the report data for each sensor are first removed by the *Sensor Registration* module. These sensor biases are generally computed off-line using a sensor calibration procedure. Report data is then passed to the collection of three modules labeled Report Correlation, Target Tracking, and Target Identification. The function of the *Report Correlation* module is to correlate the reports for each target across sensors and across time. The function of the *Target Tracking* module is to take the set of correlated reports for each target and estimate the kinematic state (typically position and velocity) for the target. The kinematic state is generally referred to as the target track. The function of the *Target Identification* module is to take the set of correlated reports for each target and identify the type of target. The Report Correlation, the Target Tracking, and the Target Identification modules are tightly coupled and they collectively represent the fusion module [3]. The fusion module generates the track and ID for each of the targets in the surveillance volume. Based on the estimated track and ID of targets, the *Sensor Management* module controls the parameters for each sensor to achieve the mission of the surveillance system. For example, if the mission requires that a target originating in a specified region be tracked with a specified accuracy and the fusion algorithm has detected the presence of a target in that region, the Sensor Management module can determine what sensor resources need to be utilized (e.g., dispatching a sensor to the region, switching on a sensor to examine the region, etc.) to achieve the specified tracking accuracy. The directives of the Sensor Management module to the Sensor Suite module closes the feedback loop in the surveillance system.

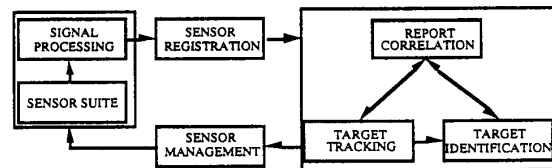


Fig. 3 Components of a Generic Surveillance System

A counter-drug surveillance system will require each of modules in the generic surveillance system. The mission of the counter-drug surveillance system will be to detect and identify drug carrying aircraft, and track them with an accuracy of a few nautical miles. The inclusion of OTH radars in the counter-drug surveillance system requires that the fusion algorithm address the issues associated with OTH radars. The primary issue is the error associated with transforming the slant space measurements to ground space measurements. Because this transformation relies on the selection of the correct propagation mode for the sensor report, we will refer to this as the propagation mode estimation error. The function of compensating for the propagation mode estimation error falls within the scope of the sensor registration module. Propagation modes supported by the ionosphere change with time and consequently the

propagation mode estimation error will also change with time. Because of this, it will not be possible for the sensor registration module to calibrate the propagation mode estimation error off line. Rather, the sensor registration module will have to perform the propagation mode error estimation on line.

The tracking algorithm will be able to provide information for estimating the propagation mode because it has access to target information from multiple sensors and from multiple time samples. Because the sensor registration module will use information from the tracking algorithm, these two modules will be tightly coupled. For this reason we will treat the sensor registration module as an integral part of the fusion algorithm in the counter-drug surveillance system.

During the Piper Aztec experiment (see Section 4), truth trajectory for only the test aircraft was recorded with on-board GPS equipment. To demonstrate the benefits of fusion for the test target, we developed a prototype fusion algorithm that is capable of tracking this test target. The prototype fusion algorithm comprises a target tracking module, a sensor registration module, and a single target report correlation module. We have used a Kalman Filter (KF) to implement the target tracking module. The KF computes the linear minimum-mean-square-error estimate of the target kinematic state. The KF also computes the likelihood of how well the measurements in the sensor reports match those predicted for the target. We have used a multiple hypothesis approach to implement the sensor registration module. The basic idea here is to postulate all feasible propagation modes for the radar signal, and transform the slant measurements to ground tracks for each of these modes. Probabilities for each of these modes are computed and the most probable mode is chosen after sufficient information is available to make that decision. The probability for each mode is computed based on the likelihood generated by the KF. Finally, we have used a multiple hypothesis approach to implement the correlation algorithm. The basic idea here is to determine whether the target of interest was detected or not detected at every reporting interval. At each reporting interval, the correlation algorithm postulates both a detection track and a missed detection track. Probabilities for both these tracks are computed and the more probable one is selected after sufficient information is available to make that decision. The probability for the detection track is computed based on the KF likelihoods and the model for OTH radar detection characteristics. The probability of the missed detection branch is computed from the model for the OTH radar detection characteristics.

The fusion algorithm processes slant reports from the OTH radars sequentially. At any point in time, the fusion algorithm maintains a set of feasible track hypotheses for the target. These feasible track hypotheses, which we will denote as old track hypotheses, are sequentially updated with the slant reports from the OTH radars. Measurements in a slant report are first fed to the Sensor Registration module which constructs the feasible ground tracks for that slant report. Each of the old track hypotheses is associated with each of these ground tracks to form a new set of track hypotheses. The Report Correlation algorithm adds a track for each of the old track hypotheses to account for the fact that the target may not have been detected by the OTH radar. Each new track

hypothesis thus contains an old track hypothesis and either a ground track or no ground track to update it. All new track hypotheses are next updated by the Target Tracking module. The Target Tracking module also computes the probabilities for all of the new track hypotheses. All new track hypotheses are then predicted to the time of the next slant report, at which point they become the old track hypotheses. After all the slant reports have been processed, the most likely track represents the target track estimated by the fusion algorithm.

Figure 4 shows the flow of execution in the prototype fusion algorithm. A preprocessing program selects from the set of all slant reports from the OTH radars those which could have potentially originated from the test target. These slant reports are sorted by time and processed sequentially by the prototype fusion algorithm. As indicated in Fig. 4, each slant report is converted to multiple ground tracks using different propagation modes provided in the coordinate registration (CR) tables.

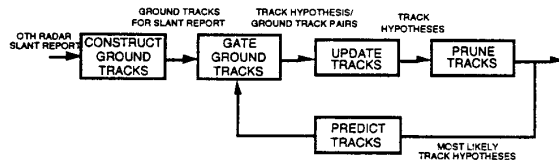


Fig. 4 Components of the Prototype Fusion Algorithm.

We will denote the time of the slant reports by the discrete integers 1, 2, ..., k, ..., K. The time interval (in seconds) between successive slant reports at time index k and (k+1) is denoted by ΔT_k . Assume that at time k after processing the kth slant report, the fusion algorithm postulates a set of t track hypotheses for the target. Each track is represented by the triplet $\{X_t, S_t, L_t\}$ where

- X_t represents the estimated kinematic state of the target; i.e. position and velocity
- S_t represents the covariance matrix for the estimated state; and
- L_t represents the likelihood that the track represents the true target.

Each of the track hypotheses is propagated to the time of the (k+1)th report by the Predict Tracks module. The four-dimensional state vector X_t consists of the target position in latitude and longitude (measured in degrees), and the target velocity in latitude and longitudinal directions (measured in nautical miles per hour). Hence, the state transition matrix $\Phi_t(k)$ is give by:

$$\Phi_t(k) = \begin{bmatrix} 1 & 0 & \Delta T_k & 0 \\ 0 & 1 & 0 & \Delta T_k \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The process (target state equation) is hence described by:

$$X_t(k+1) = \Phi_t(k) X_t(k) + Bw(k)$$

where $w(k)$ is a zero-mean white Gaussian noise process of unit intensity, and the noise shaping matrix B is estimated from the given GPS data for any particular experiment and held fixed throughout the experiment. When running the fusion algorithm, the value of the matrix B can be scaled by multiplying by a scalar q to study the effect of the varying process noise.

The measurements $Z_t(k)$ of the target position are recorded in degrees of latitude and longitude. Hence, the measurement equation becomes:

$$Z_t(k) = HX_t(k) + Dv(k) \text{ with } H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

where $v(k)$ is a zero-mean white Gaussian noise process of unit intensity, and the matrix D is set based on OTH radar characteristics. Prediction and update function are implemented using KF equations. Equations for the gating and likelihood calculations, are provided in the Appendix.

4. OTH RADAR FUSION TESTBED

The Piper Aztec Data Collection program was conducted by Rome Laboratory in May, July, and September of 1993. During the experiments a Piper Aztec was flown in the vicinity of Puerto Rico. The Piper Aztec is a twin-engine airplane whose characteristics are similar to those of a typical of drug-carrying aircraft. The Piper Aztec was equipped with a GPS receiver which recorded time-tagged GPS position, altitude, heading, and velocity. Data collected simultaneously from the Maine Over-the-Horizon Backscatter (OTH-B) radar and the Virginia Relocatable Over-the-Horizon Radar (ROTHR) was provided to ALPHATECH.

Figure 5 shows the three major components of the OTH Fusion Testbed [4]. The first component is a set of data preprocessing programs which reduce the content in the sensor data to that necessary for demonstrating the prototype OTH fusion algorithm. The second component is a set of programs which include the prototype fusion algorithm program, a program which computes simple performance measures, and two display programs. The third component of the testbed is a graphical user interface which enables the user to control the processing performed by the fusion algorithm, and the data displayed to the user.

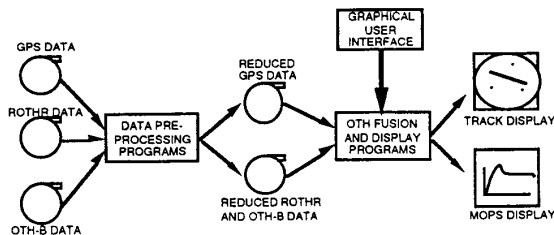


Fig. 5 Components of OTH Fusion Algorithm Testbed.

The motivation for including a sensor data preprocessor in the OTH fusion testbed is that a large amount of data was collected during the Piper Aztec experiments and using it to test the different components of the prototype fusion algorithm is not a simple task. For example, the ROTHR data for each flight test contains three files: the Ground Track File, the Slant Track History File, and the Coordinate Registration Table file. Each file contains a time ordered sequence of records, and each record is partitioned into several fields. Each field contains a specific data item. The ROTHR Ground Track File, for example, contains fields such as the report time, the track ID, the Dwell Illumination Region (DIR), etc.. Because each experiment could be as long as six hours, and the recorded data contains many items for each sampled time, the size of each of these files is fairly large. The function of the sensor data preprocessor is to extract from the flight test data only that which is necessary to readily test and demonstrate different components of the prototype fusion algorithm.

The sensor data preprocessor contains four programs. The first program, Sensor Data Reducer, processes each of the sensor files to eliminate unnecessary fields in each record. The second program, Track Data Selector, selects track data of interest for each sensor. It processes the sensor Ground Track File and the GPS file, to screen out all but the set of ground tracks which are either close to the test target or have a specific track ID. Note that retaining only those ground tracks which are likely to have originated from the test target has enabled us to confine attention to a single target fusion algorithm. The third program, Multiple Ground Track Constructor, constructs multiple ground tracks for each slant track reported by the sensor. It processes the sensor Ground Track file, the sensor Slant Track History file, and the sensor Coordinate Registration Table file to create a new ground track file for the sensor containing multiple ground tracks for each slant track reported by the sensor. Note that forming multiple ground tracks (corresponding to different propagation modes for each slant track) and making them accessible to the fusion algorithm enabled ALPHATECH to test the Sensor Registration algorithm. The fourth program, Sensor Track Merger, merges the new ground track files (formed for each sensor) into a single file containing ground tracks from both sensors. The sensor ground track data in this file will be ordered sequentially in time. This file contains ground track data which can be easily processed by the prototype fusion algorithm.

5 . RESULTS

Figure 6 shows the location and coverage areas of the Maine OTH-B radar and the Virginia ROTHR. Figure 7 shows the flight path of the Piper Aztec for a particular day. The dotted line in the figure represents the outline of Puerto Rico, and the circles represent the GPS recorded data. We have processed data from several segments of the flight data [5]. Results from an approximately 30 minute sample segment of data will be discussed here. This segment represents the final leg of the flight where the Piper Aztec test target follows a South-to-North path before it turns East and heads toward the airport.

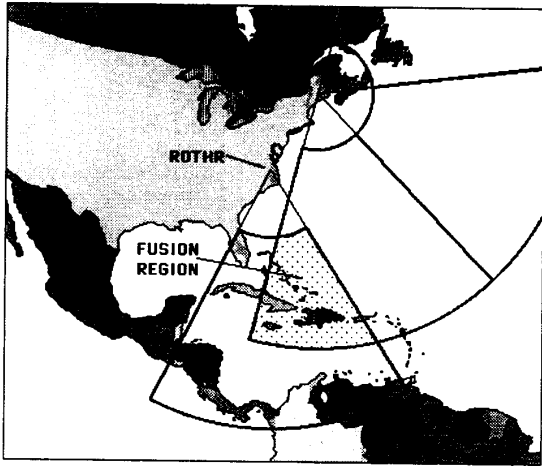


Fig. 6 Coverage Regions of OTH-B radar and ROTHR.

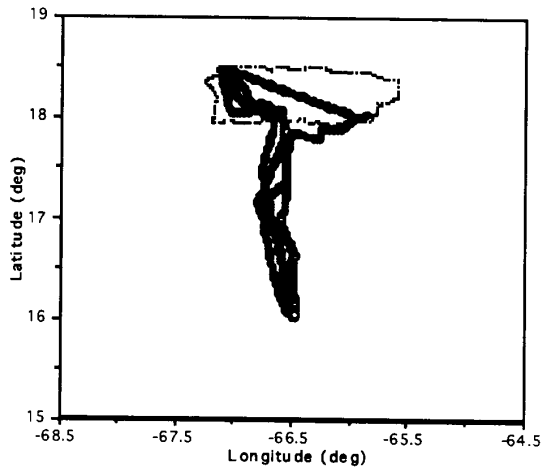


Fig. 7 Piper Aztec Flight Path.

A. Fusion Without Propagation Mode Estimation

Figure 8 shows the ground tracks reported by ROTHR and the OTH-B radar. Note that we have changed the scale on the x axis to show the tracks in more detail. Figure 9 shows the results when only the ROTHR reported track and the OTH-B radar reported are processed through the prototype fusion algorithm. The figure also shows the GPS track to provide a qualitative measure of the fusion performance. Because time is not shown in the figure, however, tracks which may appear

to be close together in the figure may not be close together at the same point in time.

The results show that the fusion track is initialized based on the average of the OTH-B and the ROTHR tracks. After that, the fusion algorithm relies exclusively on the OTH-B data. This is because the OTH-B tracks and the ROTHR tracks are far apart. Once the fusion algorithm updates the fused track with the OTH-B track, the gate sizes are small enough to prevent the ROTHR tracks from associating with the fused track at subsequent times. Note that the fused track accuracy is only slightly better than the OTH-B track accuracy, and both the fused track and OTH-B track accuracies are worse (27% poorer) than the ROTHR track accuracy.

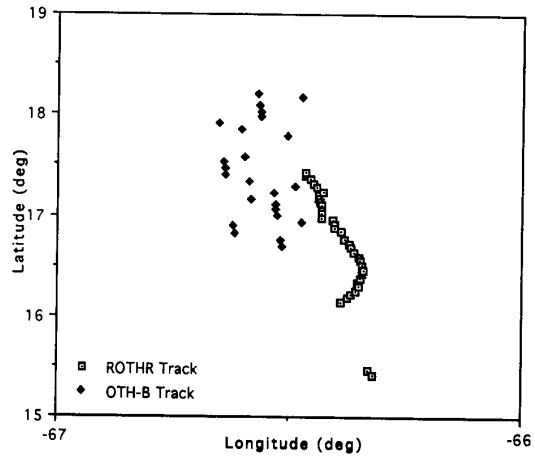


Fig. 8 ROTHR and OTH-B Reported Tracks.

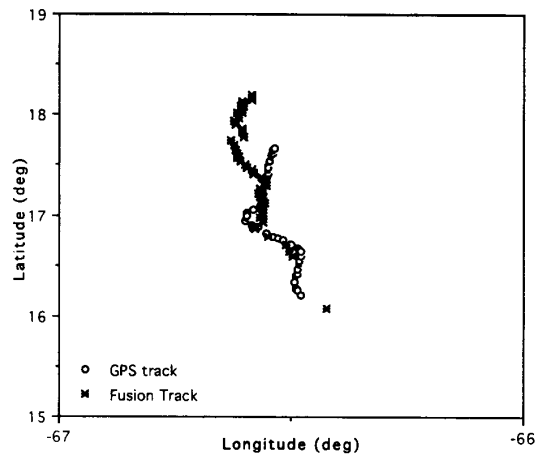


Fig. 9 Fused Track and GPS Track.

B. Fusion With Propagation Mode Estimation

Next we provided the fusion algorithm with not only the ROTHr reported track, but also the ground tracks for two additional modes constructed by the preprocessing programs. The ground tracks for these additional modes are shown in Fig. 10. Mode 1 represents the highest power mode. Modes 2 and 3 represent the second highest power modes at different update times.

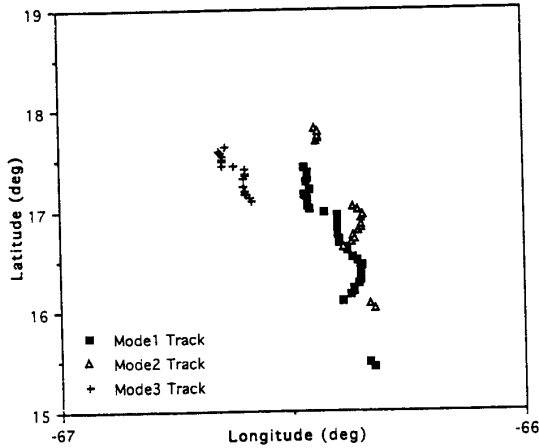


Fig. 10 Multiple Mode Tracks Constructed for ROTHr

Figure 11 shows the fused track which uses the additional propagation modes from ROTHr. The fusion track accuracy is now better than the both the ROTHr track accuracy (20% better) and the OTH-B track accuracy (38% better). Fig. 12 shows the input tracks used by the fusion algorithm at each update time. Aside from a single update time at scan 29, the fusion algorithm relies on mode 2 and mode 3 ground tracks from the ROTHr.

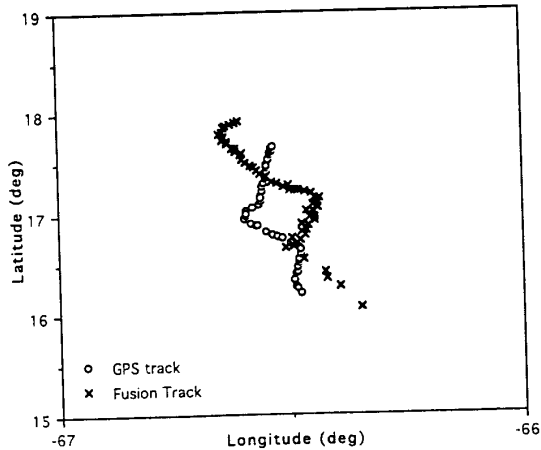


Fig. 11 Fused Track Based on Multiple Modes.

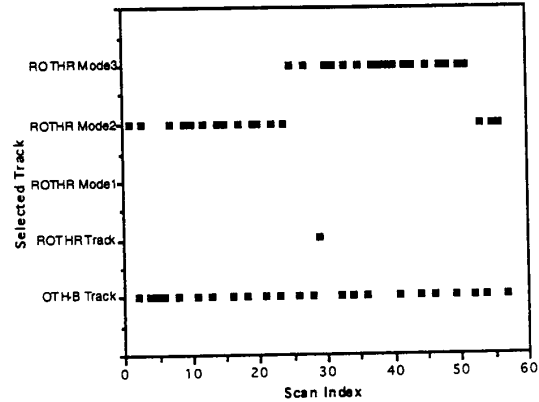


Fig. 12 Input Tracks Selected by the Fusion Algorithm.

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APPENDIX

GATING AND LIKELIHOOD CALCULATIONS

At every scan, measurements $Z_t(k)$ in every sensor report are gated with the predicted measurements $Z_t^{(i)}(k)$ to form branches of the hypothesis tree. The superscript i denotes the i^{th} track hypothesis. The KF innovations, meanwhile, are used to compute the incremental likelihoods of the branches being created. The equations for these are as follows.

Gating Measurements.

$$\begin{array}{l} \text{Pass} \\ (Z_i(k) - \hat{Z}_i(k))^\top G^{-1}(k) (Z_i(k) - \hat{Z}_i(k)) < a^2 \\ > \\ \text{Fail} \end{array} \quad (\text{A-9})$$

where

G denotes the covariance of the innovations process; and

a denotes a parameter, typically chosen between 1 and 3, which controls the gate size.

Likelihood calculations.

When a report passes the gating test, the negative log likelihood is given by:

$$L_i^{(i)}(k) = \frac{1}{2} \log(\det(G(k))) + \frac{1}{2} (Z_i(k) - \hat{Z}_i(k))^\top G^{-1}(k) (Z_i(k) - \hat{Z}_i(k)) + \log(2\pi) - \log(P_d) \quad (\text{A-10})$$

Otherwise the negative log likelihood is given by:

$$L_i^{(i)}(k) = -\log(1 - P_d). \quad (\text{A-11})$$

where P_d denotes the probability of detection.