Learning Rotations

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On-line learning

- Regret: on-line loss minus best off-line loss
- Parameter is typically a weight vector in
  - probability simplex (as in expert setting)
  - L1 simplex or L2 ball

Matrix parameter

- Symmetric positive definite matrices of trace one [TRW05]
  - Generalization of expert framework, where experts are dyads $\mathbf{u}\mathbf{u}^T$
  - Based on eigen decomposition
  - Regularization with quantum relative entropy
- $n \times m$ dimensional matrices
  - Based on SVD decomposition [W07]
  - $EG^{\pm}$ trick
  - Regularization w. two-sided version of quantum relative entropy

Why rotation matrices?

$SO(n)$ group is

- Curved, compact manifold
- Can't use linear combinations of rotations
- With suitable embeddings
  - All Euclidean transformations [WCL05]
  - $O(n)$ has universal representation for all classical Lie groups [DHSA93]

Fact: Rotation matrices are exponentiated skew-symmetric matrices

A concrete problem

Learning proceeds on trials

- After receiving a unit vector $\mathbf{x}_t$
- Algorithm predicts with unit vector $\hat{\mathbf{y}}_t$
- Receives unit vector $\mathbf{y}_t$
- Incurs loss $||\hat{\mathbf{y}}_t - \mathbf{y}_t||^2$

Find an online algorithm with bounded regret wrt best rotation $\mathbf{R}$

$$\sum_{t=1}^{T} ||\hat{\mathbf{y}}_t - \mathbf{y}_t||^2 - \inf_{\mathbf{R}} \sum_{t=1}^{T} ||\mathbf{R}\mathbf{x}_t - \mathbf{y}_t||^2$$

Off-line problem solved and known as Wahba's problem [Wah65]

What needs to be explored

- On-line algorithms that exploit Lie group structure the rotations
- Divergences that lead to suitable updates
- Alternative loss functions that better exploit spherical geometry
- Upper and lower bounds on the regret

A teaser problem

On-line learning of rotations in the plane (points on the unit circle)

- How to deal with wrap around?

RESULTS

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- Your job :-)