On-line learning

- Regret: on-line loss minus best off-line loss
- Parameter is typically a weight vector in
  - probability simplex (as in expert setting)
  - L1 simplex or L2 ball
Matrix parameter

- Symmetric positive definite matrices of trace one \[\text{[TRW05]}\]
  - Generalization of expert framework, where experts are dyads \(uu^T\)
  - Based on eigen decomposition
  - Regularization with quantum relative entropy
- \(n \times m\) dimensional matrices
  - Based on SVD decomposition \[\text{[W07]}\]
  - \(EG^\pm\) trick
  - Regularization w. two-sided version of quantum relative entropy
Why rotation matrices?

$SO(n)$ group is
- Curved, compact manifold
- Can’t use linear combinations of rotations
- With suitable embeddings
  - All Euclidean transformations
  - $O(n)$ has universal representation for all classical Lie groups

Fact: Rotation matrices are exponentiated skew-symmetric matrices
A concrete problem

Learning proceeds on trials

- After receiving a unit vector $x_t$
- Algorithm predicts with unit vector $\hat{y}_t$
- Receives unit vector $y_t$
- Incurs loss $||\hat{y}_t - y_t||^2$

Find an online algorithm with bounded regret w.r.t best rotation $R$

$$\sum_{t=1}^{T} ||\hat{y}_t - y_t||^2 - \inf_R \sum_{t=1}^{T} ||Rx_t - y_t||^2$$

Off-line problem solved and known as Wahba’s problem [Wah65]
What needs to be explored

- On-line algorithms that exploit Lie group structure the rotations
- Divergences that lead to suitable updates
- Alternative loss functions that better exploit spherical geometry
- Upper and lower bounds on the regret
A teaser problem

On-line learning of rotations in the plane (points on the unit circle)

- How to deal with wrap around?
RESULTS

- Your job :-}