A PROJECT SUMMARY

Game theory has emerged as the key tool for understanding and designing complex multiagent environments such as the Internet, systems of autonomous agents, and electronic communities or economies. To support these relatively recent uses of game theory — as well as for more ambitious modeling in some of the traditional application areas — there is a growing need for a computational theory that is largely absent from classical game theory research. Such a computational theory needs to provide a rich and flexible collection of models and representations for complex game-theoretic problems; powerful and efficient algorithms for manipulating and learning these models; and a deep understanding of the algorithmic and resource issues arising in all aspects of game theory. The overarching goal of the proposed work is to “scale up” the applicability of game theory, in much the same way that Bayesian networks and associated advances made complex, high-dimensional probabilistic modeling possible in a wide set of applications in computer science and beyond.

Two of the most important topics that have materialized to date — and the primary emphases of the current proposal — are the representation and efficient manipulation of large and complex games, and new approaches to learning in game-theoretic settings. On the topic of representation, the proposal includes the development of methods to model structured interaction in large-population games; the intersection of social network theory and game theory; new representations in repeated games; and representational issues for a variety of equilibria types. On the topic of learning, it includes the development of online multiplicative update methods for large and structured games; modeling cooperation in learning; applications of game theory to the analysis of machine learning methods; learning for games that change over time; and the relationship between game theory and reinforcement learning. It also covers many interesting topics in the intersection of representational and learning issues.

The work proposed here will have a number of broader impacts. It will make it possible to study interactions in social and ecological systems in drastically larger systems using the new game-theoretic representations and algorithms, allowing researchers in other scientific disciplines to apply game theory to a broader array of problems. Through highly visible symposia, workshops, and tutorials at international conferences and interdisciplinary classes on computational game theory, the new discoveries will be disseminated to interested researchers and students. In addition, new programs in game theory will be created for two established outreach programs based at Penn’s Institute for Research in Cognitive Science. One of these is a summer school for advanced undergraduates, while the other is for minority and female high school students in the Philadelphia area.
## Contents

### A PROJECT SUMMARY

- A–1

### B TABLE OF CONTENTS

- B–1

### C PROJECT DESCRIPTION

- C–1
  - C.1 Introduction and Overview ........................................ C–1
  - C.1.1 New Representations for Complex Games ................... C–1
  - C.1.2 New Learning Methods for Complex Games .................. C–2
  - C.2 Representation in Computational Game Theory ........... C–3
    - C.2.1 Topics in Graphical Games ............................... C–3
    - C.2.2 Representation and Computation for Repeated Games ... C–5
  - C.3 Learning in Computational Game Theory .................... C–6
    - C.3.1 General Methods for Deriving Updates for Games ..... C–6
    - C.3.2 Shifting Games ........................................... C–7
    - C.3.3 Game Theory and Analysis of Learning Algorithms ... C–8
    - C.3.4 Stochastic Games and Reinforcement Learning .......... C–10
  - C.4 Applications and Tools ........................................ C–11
  - C.5 Broader Impact ................................................ C–11
  - C.6 Results from Prior NSF Support ............................. C–13
  - C.7 Budget Discussion and Management Plan .................... C–14

### D References

- D–1
C PROJECT DESCRIPTION

C.1 Introduction and Overview

Game theory is a topic with an illustrious history, and has been applied as a modeling tool across a staggering variety of disciplines, including geopolitical conflict, evolutionary biology, neuroscience, and ecology. Most recently, and particularly within artificial intelligence (AI) and related disciplines, there has also been burgeoning interest in game theory as a tool for understanding and designing complex environments such as the Internet, systems of autonomous agents, and electronic communities or economies.

For these more recent uses of game theory — as well as for more ambitious modeling in traditional application areas — there is a growing need for a computational theory that is largely absent from classical game theory research. By a computational theory we mean a rich and flexible collection of models and representations for complex game-theoretic problems; powerful and efficient algorithms for learning in and reasoning about these models; and a deep understanding of the algorithmic and resource issues arising in all aspects of game theory. The overarching goal is to “scale up” the applicability of game theory in much the same way that Bayesian networks and associated advances made complex, high-dimensional probabilistic modeling possible in a wide set of applications in computer science and beyond (see Section C.4). The term strategic reasoning is sometimes used to draw the parallel between a computational theory of game-theoretic interaction and the statistical interactions modeled in probabilistic reasoning.

The discipline that is emerging from such concerns is broadly called Computational Game Theory (CGT), and is populated by ideas from a number of related fields, including artificial intelligence, machine learning, computational learning theory, economics, and theoretical computer science. The research so far has established detailed technical connections to many methods from these areas, including Bayesian networks and algorithms for probabilistic inference, reinforcement learning and Markov decision processes, and boosting algorithms and multiplicative updates. While important foundations have been established, CGT remains in a state of relative infancy.

Two of the most important topics that have materialized to date — and the primary emphases of the current proposal — are the representation and efficient manipulation of large and complex games, and new approaches to learning in game-theoretic settings. We now discuss each of these briefly, and then elaborate in the main body of the proposal.

C.1.1 New Representations for Complex Games

When attempting to apply the methods of classical game theory to large or complex systems, one often quickly encounters problems of representation — finding a way of succinctly describing a system precisely for subsequent computational processing. Perhaps the most striking example of this phenomenon arises in settings with many players (which are becoming increasingly prevalent in applications of game theory to multiagent systems, electronic markets, and the like). The classical matrix or normal form for one-shot games inherently scales exponentially with the number of players, and thus has been applied primarily to very small problems. While large-population formalisms have been developed for certain special areas (such as evolutionary game theory, and potential and congestion games), even these tend to make strong and unwarranted assumptions of symmetry between players for the sake of analytical tractability.

Over the last several years, inspired by an analogy with graphical models for probabilistic inference, a number of AI researchers (including proposed PIs Kearns and Littman) have been directly tackling such modeling issues and the algorithmic challenges that accompany them. The basic insight is that in many potential applications of game theory, there is significant structure to the problem, and one must develop new game representations that allow the parsimonious modeling and exploitation of this structure. Perhaps the most natural example comes from multiparty game theory: in many settings, each player does not interact directly with the entire population, but only with a much smaller collection of immediate “neighbors”, who in turn interact with others, and so on, to form a network of strategic or competitive interaction. This invites a graph-theoretic or network representation, which has been developed considerably over the last few years.

We note that another major research topic in recent years has been the study of algorithmic mechanism design. While not directly a focus of this proposal, we believe that many of the ideas proposed here will have important generalizations to mechanism design as well.
years. Similarly, one can envision settings in which a large action space “decomposes” into collections of more atomic actions, thus yielding a factorization of the game matrices.

The primary agenda of this line of work is to

- Identify natural and powerful sources of structure in complex, multiplayer games;
- Develop new representations that permit the flexible and parsimonious description of such structure;
- Develop algorithms for all manner of game-theoretic computations (including the computation of all varieties of equilibria, such as Nash, correlated, and other well-studied concepts from game theory) that operate directly on the new representations, as efficiently as possible;
- Grow the set of operations supported by the new representations.

By this last point, we mean that CGT should not be limited to mere equilibrium computation, but should support a wide range of predictive and hypothetical queries about the system of strategic interaction captured by the models (again, analogous to graphical models for probabilistic inference). Examples of richer queries might include the ability to compute equilibria conditioned on exogenous events (such as an external force, such as the government, restricting the range of actions available to certain players), or the computation of the conditional independences between player actions that hold in the correlated equilibria of a game.

While significant advances have been made in the general arena of game representation recently, we feel that it remains underdeveloped, and that the most important territory is yet to be explored. Thus new models for complex games, and algorithms for manipulating them, form one of the two main emphases of our proposal. We detail our proposed research in this area in Section C.2.

C.1.2 New Learning Methods for Complex Games

Learning is one of the most central and important topics in game theory, both “classically” and in CGT. Without some account of how (or partially) “rational” players might arrive at some global state or states, theories of equilibrium are at best incomplete and at worst irrelevant. While there is a vast literature on learning and dynamics in traditional game theory, it again tends to focus on small problems and largely sidesteps computational issues.

Within CGT, there have been significant advances in understanding learning in games, and the development of new learning methods that scale well to certain complex settings. Many of these contributions have come from the computational learning theory (COLT) and machine learning communities, and in particular from the study of models of online learning.

When one looks to the field of machine learning for guidance in the study of learning in games, one is immediately struck by the irrelevance of the dominant statistical paradigms of most theoretical and experimental learning work. Typical machine learning methods explicitly or implicitly assume that the learning environment is statistical, and either static or at least Markovian. Thus the overwhelming majority of work in supervised learning assumes that there is a fixed distribution on inputs to a fixed target function that is to be learned or modeled; clustering or unsupervised methods assume a static data distribution; and reinforcement learning examines learning in state-based models of Markovian change. All of these assumptions (as well as variants designed to accommodate slow “drift”) are rather severely violated in almost any natural study of learning in games, where the dynamics are continually changing due to the choices of the other agents, but not in accordance with any statistical model.

At the other extreme, COLT research in online learning aims to develop algorithms whose performance can be guaranteed on absolutely any sequence of “trials”, including those obeying no underlying statistical pattern whatsoever. The performance guarantees are usually expressed in the form of the total regret over the entire sequence (a comparison to the best possible behavior), as in the online or amortized analysis of algorithms in theoretical computer science more generally. Furthermore, these algorithms often enjoy a very gradual degradation in performance as the number of actions increases, and thus are promising candidates for large-scale applications.

Because of their lack of assumptions on the underlying process generating the data or trials, results in online learning are immediately applicable to the game-theoretic setting. A number of fundamental connections between online learning and game theory have recently been established (many of them by PIs Schapire and
Warmuth), and the computational concerns of such COLT work have direct benefits in the game-theoretic setting. A compelling example is the recent development of an algorithm for learning in zero-sum games that converges to a Nash equilibrium, but with a regret bound that grows only logarithmically with the number of actions (Freund and Schapire 1997a). The algorithms developed in the on-line setting often employ what are known as multiplicative updates (Littlestone 1988, Cesa-Bianchi et al. 1997), and tend to look quite different from classical learning methods in games (such as fictitious play or Bayesian approaches).

It can be argued that the on-line learning models go too far in their lack of assumptions, and that even deeper connections might be found by specializing the approach to assumptions broadly applicable in game theory, such as some minimal notion of rationality among the individual players. Fortunately, much of the on-line learning work has been developed in great generality, permitting the examination of such specialized objectives or constraints on the players.

Again, we feel that while cornerstones for this area have been laid, important issues lie unexplored. Thus the development of on-line learning generally, and of new learning methods in games specifically, forms the second of our two emphases. We elaborate on this proposed line of work in Section C.3.

Finally, we emphasize that there are many compelling open questions regarding the intersection of our proposed research that we will highlight throughout the exposition.

C.2 Representation in Computational Game Theory

As discussed in Section C.1, the first of our two major emphases is the development of new computational models in game theory. In this section we shall assume familiarity with the basics of classical game theory, including the notions of Nash and correlated equilibria.

C.2.1 Topics in Graphical Games

As discussed briefly in Section C.1, graphical games are a recently introduced model that permit the succinct representation of certain large-population games. They rely on the insight that in many natural game-theoretic settings, the payoffs of each player may be a function of only a relatively small number of others, even though the global population is considerably larger. To capture such situations, a network or graph-theoretic representation is adopted in which each player is a node whose payoff is a function only of its own action and those of its neighbors in the graph (Kearns et al. 2001, Koller and Milch 2001).

In many cases, this model may permit an exponential reduction in the size of the representation compared to standard normal form. A series of results over the past several years has also established significant computational benefits to the models. In particular, there are now provably efficient algorithms for computing Nash equilibria in graphical games with special topological structure such as trees (Kearns et al. 2001, Littman et al. 2002), as well as local message-passing heuristics for general graphs whose early empirical performance is quite encouraging (Ortiz and Kearns 2003, Vickrey and Koller 2002).

Proposed PIs Kearns and Littman have been central figures in the introduction and algorithmic development of graphical games. Here we propose extending this line of work in three main directions we feel are especially promising. Throughout the proposal, we discuss other open problems related to graphical games that we would also investigate; several of these have to do with the application of multiplicative updates to graphical games (Section C.3.1).

Graphical Games and Social Network Theory.

Game theory has a long history of application to the social sciences and to problems of human, organizational, and military conflict. While the few theoretical and experimental results examining special network structure in graphical games to date have focused on topologies common to computer scientists (such as trees, grids, cycles, random graphs, etc.), the applicability of game theory to the social and economic sciences presents an even more interesting opportunity.

The last several years have seen an explosion of research in what has come to be called social network theory. This line of work has empirically quantified a variety of naturally occurring, real-world networks — social, organization, political, corporate, cultural, and technological. Even more fascinating, it has documented the existence of a number of “universals” that seem to perpetually appear in such networks, such as so-called small world phenomena, hubs and authorities, and power laws of neighbor set decay. A mathematical and generative theory has also been developed alongside (Kleinberg 2000, Watts and Strogatz 1998). However, much of the work in social network theory so far has been descriptive (identifying the common structures

C-3
of natural networks) rather than prescriptive or predictive (identifying the behavioral implications of such structures). Some of the questions we find particularly intriguing are the following:

- What are the strategic and competitive implications of the universal structures identified by social network theory?
- Are there algorithmic aspects of such structures that can be exploited in the game-theoretic context?
- Is there a game-theoretic basis for why networks of interacting agents tend to take on particular forms?

We thus propose to extensively study, both theoretically and experimentally, the intersection of graphical games and social network theory. More specifically, we will examine the equilibria (both Nash and other varieties, such as correlated) and computational properties of graphical games whose topologies match those identified by social network theory. Areas to be examined include:

- **Equilibria computation in natural networks.** Here we would examine several proposed generative models for naturally occurring networks (such as the Watts-Strogatz model and its generalizations) as graphical games, and see if they permit efficient algorithms for equilibria computation (possibly in conjunction with special payoff functions; see subsequent topic on parametric payoffs).

- **Semantics of correlated equilibria in natural networks.** For the same kinds of topologies, we would like to examine the distributional implications for correlated equilibria (see later discussion), including which dependencies and independences of actions are entailed by common universals.

**Parametric Payoffs in Graphical Games**

One of the primary potential benefits of the graphical game representation lies in providing a flexible language in which one can succinctly express exactly those interactions that are important in a large-population game, omit those that are not, and pay a computational price that depends only on the complexity of the former. However, there remain a number of relatively unexplored avenues for introducing even further representational and computational power into this basic formalism.

One of the most important such directions is the introduction of parametric modeling of the local payoff functions in a graphical game. In a standard graphical game, the payoff for a given player is a function only of its neighbors in the graph, but this function remains in a tabular form of size exponential in the neighborhood size. Much as in Bayesian networks (where sigmoidal, noisy-OR and other parametric conditional probability functions are introduced for succinct modeling), we propose to study the effects of introducing natural parametric representations for the payoff functions to allow the succinct modeling of graphs with large neighborhood sizes. Among the questions to be addressed:

- **Generalized algorithms.** Which of the existing algorithms for equilibrium computation (such as the NashProp algorithm and junction-tree approaches for Nash equilibria, and the method for computing correlated equilibria described in Section C.2.1) can be generalized to parametric payoff functions in a computationally efficient manner?

- **Choice of parameters.** From an application perspective, which parametric representations are most important? Congestion and summarization games (Kearns and Mansour 2002) are parameterized payoff games that have been considered when there is a complete graph; we plan to examine how other graphical structure can be exploited to allow for more realistic modeling.

- **Large action spaces.** The topic of parametric payoffs is closely related to the succinct representation of very large action spaces (even for 2-player games), where there is a history of proposals for how a continuous action (such as setting a price) may impact strategic considerations. We also plan to investigate compact models for games with many actions, both in the graphical game context and elsewhere.

**Graphical Games and Alternative Equilibria**

The ability to model structured interaction in graphical games permits one to ask a host of natural questions about the strategic interaction so described. These can go well beyond the basic notion of finding Nash equilibria. We again use Bayesian networks as a helpful analogy: such models support not only the
generation of the joint distribution, but allow diagnostic and inferential queries regarding conditional dependencies. Similarly, one might ask in what ways the interactions in a graphical game influence the correlations, collusions, coalitions and other strategic structures one might find in broader notions of equilibrium, learning and so on.

Initial steps towards a more general theory of strategic reasoning were made recently in regards to *correlated equilibria* (Kakade et al. 2003). There it was shown that associated with any graphical game (which is a representation of entirely game-theoretic interaction), there is a unique Markov network (which represents only probabilistic interactions) sufficient to represent all correlated equilibria of the game up to expected payoff equivalence. The Markov network retains the parsimony of the graphical game, and in some cases supports the efficient computation of the correlated equilibria. This Markov network allows one to immediately “read off” exactly those conditional dependences required to achieve the correlated equilibria implied by the game structure.

There are a number of compelling reasons to favor correlated over Nash equilibria, including the fact that correlated equilibria are the natural description of the convergence point for many of the regret-based learning algorithms discussed in Section C.3.1, and are often much easier to compute as well. We propose to grow this avenue of inquiry in several directions:

- **Structure and learning.** How does the structure of a graphical game and its associated Markov network influence the correlated equilibria found by regret-minimizing algorithms like those described in Section C.3.1?

- **Cooperative equilibria.** What are the implications for other types of equilibria? For example, how does the structure of a graphical game determine the coalitions and stable sets that can form in the standard notions of cooperative game theory?

- **Evolutionary game theory.** Can we develop a graphical formalism appropriate for *evolutionary stable strategies*, the equilibrium notion for evolutionary game theory? Here the graph might naturally represent the geographic dispersal of a species.

- **Behavioral game theory.** Work in behavioral game theory has found that it is extremely common for human subjects to exhibit non-equilibrium behavior when placed in strategic situations (Camerer 2003). Experimental data reveals a very different set of rules governing human decision making and we plan to examine the computational implications of these rules. For example, can we develop algorithms that compute behavioral equilibria efficiently in graphical games?

### C.2.2 Representation and Computation for Repeated Games

Much of the CGT work to date, including the graphical games work discussed above, has focused on representation and equilibrium computation for a single engagement of a matrix game in normal form. However, the very notion of equilibrium presupposes a process of learning, usually via repeated engagements, to respond appropriately to other players. This leads naturally to the idea of a repeated game, in which players are faced with the same strategic situation over and over again, with performance measured by, say, average payoff. A set of (now history-dependent) strategies constitutes a Nash equilibrium in a repeated game if each strategy is a best response to the others over the set of all possible (history-dependent) policies.

Unlike strategies for one-shot games, which can be represented simply and succinctly by probability distributions over actions for each player, history-dependent strategies can be arbitrarily complex. Consider the tit-for-tat strategy for repeated Prisoner’s Dilemma (Axelrod 1984), in which a player begins by cooperating, then responds to any defection by defecting once in turn before returning to a cooperating strategy. A pair of tit-for-tat strategies constitutes a Nash equilibrium for the repeated game because it is in each player’s long-term best interest to cooperate if the other player responds to defection with defection. Tit-for-tat can be expressed as a 2-state finite-state transducer. A recent result (Littman and Stone 2003) builds on the well-known “folk theorem” of game theory to show that any two-player repeated game has a Nash equilibrium that can be represented by a polynomial-size finite-state transducer augmented with “counting nodes”, and that such an equilibrium can be found in polynomial time. This is an example of a representational result that has important computational repercussions.

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2There is some evidence that human decision makers create similar strategies when stakes are high (Weber 1997).
We propose to further study core algorithmic issues for equilibria in repeated games. Specific areas for investigation include:

- **Multi-player repeated games.** The efficient repeated game Nash equilibrium algorithm cited above is limited to two-player games represented as matrix games. We seek to extend the algorithm to the general multi-player case; we believe this is possible as there are corresponding folk theorems for multi-player games. The chief difficulty is in finding the appropriate “threat” (as in tit-for-tat) that stabilizes cooperation when appropriate.

- **Repeated games in large populations and action spaces.** Given the importance of studying large population games, we think it is critical to extend multi-player repeated game algorithms to large, succinctly represented games, including graphical games. In addition, we also plan to study equilibrium computations in games in which the action spaces are very large but compactly represented, and to generally examine equilibrium computation (including alternate equilibrium notions) in large-scale repeated games.

### C.3 Learning in Computational Game Theory

We now turn to the second primary emphasis of our proposal, the problem of learning in CGT. As discussed in Section C.1, the COLT research in on-line learning is perhaps the most promising direction for learning in CGT, due both to the lack of assumptions made on the generation of data, and to the significant connections already discovered between the two topics. Moreover, in keeping with the goal of providing methods that have the potential to scale to large games, many of the on-line learning algorithms for games discovered to date posses remarkably mild dependencies on the number of actions or pure strategies available to the players.

In the simplest game theory setting, an on-line learning algorithm plays a repeated game with an unknown opponent. The learner maintains a mixed strategy or distribution over the actions available to it. Each round’s payoff may cause the learning algorithm to update its mixed strategy. In the following sections, we discuss methods for updating strategies, ways of handling games that change over time, the use of game-theoretic analysis in studying learning algorithms in general, and reinforcement learning in games.

#### C.3.1 General Methods for Deriving Updates for Games

In the COLT community, a family of on-line learning algorithms broadly known as *multiplicative update methods* have been the subject of intense study (Littlestone and Warmuth 1994, Freund and Schapire 1997a). These methods dictate a particular way of updating a mixed strategy in response to the payoff at each trial, typically scaling the weight given to an action by a multiplicative factor depending on how good or poor its payoff is. For our purposes, there are a number of striking properties held by multiplicative update methods in the game-theoretic setting:

- It is possible to bound the payoff regret of such algorithms over any possible sequence of trials, with no need to assume rationality of opponents or statistical processes generating the trials.

- The regret bounds obtained typically have only a logarithmic dependence on the number of pure strategies available to the player, and thus have the potential to scale well to games with many actions.

- The algorithms are simple and intuitive, and can be seen as implementing a particular form of rational behavior. When two “copies” of such algorithms are played against each other in the context of a zero-sum game, their strategies converge to a Nash equilibrium (Freund and Schapire 1997a).

Multiplicative update methods can be derived using an important general technique developed by Kivinen and Warmuth (1997), in which the updated mixed strategy minimizes the mixture loss on the last trial plus an inertia term that prevents the updates from being too drastic. This inertia term is based on a *divergence function* measuring the difference between the new and the old mixed strategy. Using relative entropy as a divergence function gives standard multiplicative update rules.

This general design principle can be used to derive a great number of on-line learning algorithms (Kivinen and Warmuth 2001, Kivinen and Warmuth 1999a). One of the goals of this research is to understand these and other design principles, particularly as applied to problems of a game-theoretic flavor.
The proposed areas of study include:

- **General-sum setting.** We saw above that multiplicative update algorithms converge to Nash equilibria in the zero-sum case. What happens in general-sum games (including special subclasses of interest)? Can such algorithms be used to efficiently compute equilibria for a broader class of games? This would continue a line of work started by Hart and Mas-Colell (2000), who showed that such algorithms often converge to correlated equilibrium (see Section C.2.1) and could provide direct methods for computing equilibria in a wide range of computational scenarios.

- **Multiplicative updates in graphical games.** What is the behavior of multiplicative updates in multi-player graphical games, or other models of structured interaction? For instance, can we show that the convergence is faster, or that the loss bounds are more favorable, or that the algorithms become more tractable? If so, this would lend support to the idea that the graphical representation is natural and appropriate for this class of problems.

- **Pricing games.** Can multiplicative updates be used to solve multi-player pricing games where each player has a different linear utility function and the total utility over all players needs to be minimized (Devanur et al. 2002)? This is a class of games of particular interest and seem to be a natural fit for multiplicative update algorithms.

- **Divergence functions for game theory.** What are the divergence functions that are most appropriate for multi-player games? In other words, what is the right way to derive useful algorithms for learning to play games when there are many players? Do existing divergence functions apply, or do we need to develop novel divergence functions that produce theoretically grounded algorithms for large populations of players?

- **Modeling cooperation.** The on-line learning setting assumes that Nature is highly adversarial. But in game-theoretic settings, better long-term performance can often be achieved through cooperation (for instance, in Prisoner’s Dilemma). How can we modify our algorithms to take advantage of opportunities for cooperation? As a first step, how can we even explicitly modify the definition of regret so that both players are enticed to cooperate?

- **Last-step Minimax Algorithm:** This powerful new algorithm determines its predictions for finite trial on-line learning problems based on a modified single trial problem (Takimoto and Warmuth 2000a). Surprisingly, its regret bound is often provably close to the optimum, thus raising the following fundamental question: Which finite horizon repeated games can be well approximated by similarly modified one-round games?

Work on learning to play repeated games in the sense that we have described goes back to Hannan (1957) and Blackwell (1956), and includes more recent work by Foster and Vohra (1993, 1997, 1999), Fudenberg and Levine (1995), and Hart and Mas-Colell (2001, 2000). Some of this work is based on the approachability theory of Blackwell (1956), a beautiful generalization of games to vector payoffs, and one that has proved quite powerful. Hart and Mas-Colell (2001) have made some interesting connections between approachability theory, fictitious play and multiplicative updates used in on-line learning. These connections seem worthy of further study, since often understanding the relationship between methods used for different purposes can lead to the discovery of entirely new and practical algorithms.

### C.3.2 Shifting Games

Game theory is traditionally concerned with defining various forms of equilibria, while learning in games often asks: How fast does the learner reach what kind of equilibria? In a sense, the traditional focus is on the final result of playing a static game, largely ignoring the dynamics of the game.

However, consider a shifting game, which consists of repeated plays in which the payoff matrix and the degree of cooperativeness between the players is changing with time. No equilibria might ever be reached in such a situation. Indeed, in a non-stationary game, it is hard to define precisely what the learning process ought to converge to. During learning, the decision maker will be faced with many conflicting goals. How
much should the learner hold onto previous parameters versus trust the more recent data? How can we incorporate long-term performance information about different strategies?

Many of these issues have been considered by the on-line learning community using divergence-based methods (Herbster and Warmuth 1998, Auer and Warmuth 1998, Herbster and Warmuth 2001, Bousquet and Warmuth 2002), and an exciting research topic is to adapt these techniques to playing shifting games:

- **Switching modes.** When environments change during repeated play, they often exhibit a small number of different “modes” (like rush-hour mode versus midday traffic, sleep mode versus active usage, aggressive opponents versus conservative opponents). The responses given by Nature mainly switch back and forth between these modes (Choi et al. 2000). Assume we have a pool of base strategies that is large enough to contain at least one strategy suitable for each mode. A master algorithm is used to maintain a mixture over the base strategies. We plan to apply a technique developed in on-line learning (Bousquet and Warmuth 2002) that uses a divergence function to anchor the current mixture vector of the master to the average of all mixtures used in the past. This helps the recovery when the request stream switches to a mode that was seen before; it could be a successful method for helping an agent adapt quickly to changes in its strategic situation given sufficient experience.

- **Applications.** On-line master algorithms that combine simpler strategies have been used in a number of applications with bursty, time-changing data that does not seem to be amenable to statistical approaches, such as predicting the idle time or disk (Helmbold et al. 2000), choosing a good caching policy (Gramacy et al. 2003, Blum et al. 1999) and load-balancing (Blum and Burch 1997). We plan to greatly expand these promising applications to multiagent domains including dynamic routing and congestion protocols.

- **Game theory and Kalman filters.** The Kalman filter is the main tracking algorithm used in many scientific and engineering disciplines (for example, Brown and Hwang 1992). It is derived and analyzed based on Gaussian noise assumptions. Building on recent work (Kivinen et al. 2003), we plan to do a game-theoretic analysis of the Kalman filter, proving regret bounds that hold for arbitrary sequences of inputs and noise.

- **Exponentially large strategy spaces:** Since the regret bounds for multiplicative updates grow logarithmically with the number of base strategies, we hope to adapt techniques from machine learning (for example, Helmbold and Schapire 1997, Helmbold et al. 2002, Takimoto and Warmuth 2002b, and Takimoto and Warmuth 2002a) to efficiently explore exponentially large strategy spaces. This would greatly expand the applicability of our methods.

**C.3.3 Game Theory and Analysis of Learning Algorithms**

While the work proposed in this section has emphasized what COLT can provide to CGT, it is also the case that the close relationship between the topics can provide new insights into fundamental topics in machine learning as well. An example is the method known as **boosting**, first invented by proposed PI Schapire.

Boosting is based on the observation that finding many rough prediction rules can be a lot easier than finding a single, highly accurate rule. To apply the boosting approach, we start with a method or algorithm for finding such prediction rules. The boosting algorithm executes this “weak” or “base” learning algorithm repeatedly, each time feeding it a different distribution or weighting over the training examples — this weighting encodes the importance of each example on that round. Each time it is called, the weak learner generates a new weak classifier (prediction rule), and after many rounds, the boosting algorithm must combine these weak rules into a single prediction rule with the aim of making it much more accurate than any one of the weak rules. The key to the effectiveness of any boosting algorithm is in the choice of the distribution over training examples that is chosen on each round. The AdaBoost algorithm of Freund and Schapire (1997b) was the first practical boosting algorithm and it is still the most widely studied (see Schapire 2002 for an overview of research on AdaBoost).

This interaction between the boosting algorithm and the weak learner can be viewed as a two-player zero-sum game, a connection that was spelled out later by Freund and Schapire (1996, 1999). The booster’s pure strategies are the training examples themselves, while the weak learner’s pure strategies are the weak classifiers at its disposal. On each round, the booster chooses a distribution over the training examples: a
mixed strategy. In response, the weak learner chooses one of its pure strategies: one of the weak classifiers. By choosing the payoff matrix appropriately, the weak learner is motivated to choose a weak classifier with minimum error for the booster’s chosen distribution. In these terms, AdaBoost is nothing more than an application of the algorithms for repeated game play described above. Moreover, by general properties of these algorithms, one can show that the empirical mixture of pure strategies chosen by the weak learner will be an approximate Nash equilibrium. Returning to the boosting interpretation, this turns out to mean that the combined classifier output by AdaBoost will classify all of the training examples perfectly correctly.

In early experiments (Breiman 1998, Drucker and Cortes 1996, Quinlan 1996), AdaBoost performed better than predicted by its initial analysis (Freund and Schapire 1997b). In particular, AdaBoost showed resistance to overfitting, a pernicious phenomenon exhibited by many machine learning methods in which performance on test data degrades even as performance on the training set is improving. This tendency not to overfit was later explained by Schapire et al. (1998) who argued that AdaBoost tends to increase the margins of all of the training examples, where the margin can be viewed as a measure of confidence of a prediction made by the combined classifier. Moreover, it was proved that larger margins on the training set imply a better bound on performance on separate test data.

The notion of margin, which is fundamental to our best explanation of how AdaBoost works, is also essentially a game-theoretic concept. In particular, the best possible minimum margin is exactly the value of the zero-sum game that is played between the booster and the weak learner. The results of Schapire et al. (1998) show that AdaBoost approximately maximizes the minimum margin, and thus, that it approximately solves the boosting game. Although this game-theoretic view has helped us to understand boosting, there are many unanswered questions:

- **Convergence of AdaBoost.** Most importantly, although AdaBoost is widely studied and widely used in practice, we still do not have a complete characterization of what it converges to. Can game theory help? Can we show that, with a “best case” weak learner (say, one that always returns the best available weak classifier for the given distribution), AdaBoost converges to a minimax solution for the appropriate game (rather than merely an approximation of this solution)?

- **Improved boosting.** Rätsch and Warmuth (2002) very recently introduced a version of AdaBoost called AdaBoost* which provably solves the boosting game. How well does this new algorithm perform in practice? Theory suggests that it should perform even better than AdaBoost. Does it? Also, as mentioned above, AdaBoost is a special case of a more general algorithm for playing repeated games. How can AdaBoost* be modified so that it too can be applied to other games?

- **Margins.** Although, in our view, the theory of margins gives the best explanation for the effectiveness of boosting, Breiman (1999) demonstrated a nasty case in which better margins did not improve performance. His example clearly deserves more study. Perhaps it will point to a more refined and accurate theory. The theoretical bounds that have been proved to date give guarantees that suggest how the algorithm will perform qualitatively. However, as Breiman’s example emphasizes, the quantitative predictions made by existing theory are rather poor. Significant improvements in this regard have been made recently by Koltchinskii and Panchenko (2002), Koltchinskii et al. (2001a, 2001b), and Panchenko (2001). It would be of considerable practical value if we could derive better posterior estimates on generalization error of boosting in terms of margins, since they would let us reasonably predict how well the algorithm’s combined classifier will perform based on the margins achieved on the training data.

- **Boosting and repeated games.** AdaBoost can be viewed as implementing one particular algorithm for playing repeated games. By now, however, there are a substantial number of methods that are used for this purpose. What happens if other game-playing algorithms are used to derive new boosting algorithms? Would they have interesting properties very different from AdaBoost?

- **Game theory and support vector machines.** Finally, there is another methodology for maximizing the margins of training examples, namely, using support-vector machines (SVMs). Here, one explicitly maximizes the minimum margin by solving a quadratic program. The notion of margin that is used here, however, is slightly different from the one used in boosting and is more geometric, but less clearly related to games. One can ask then if SVMs also can be understood in terms of games, and if this new interpretation leads to new insights about this very popular learning method.
C.3.4 Stochastic Games and Reinforcement Learning

An alternative formulation of the problem of learning in games is provided by reinforcement learning (RL). RL (Kaelbling et al. 1996, Sutton and Barto 1998) is a learning paradigm for agents in complex, multi-state environments. Although commonly studied from the perspective of a single agent learning to maximize its reward in an initially unfamiliar environment, an RL approach is also quite appropriate for multi-state strategic interactions. And, whereas Markov decision processes serve as a formal foundation for single-agent RL, the stochastic game or Markov game framework (van der Wal 1981, Shapley 1953, Owen 1982) is a formalization of multiagent interaction that is a good match for existing RL theory.

Repeated games can be viewed as a very basic kind of stochastic game consisting of a single state. More generally, however, players' action choices influence not only their immediate payoffs, but their future payoffs as well via discrete changes of the environmental state. A series of papers has developed the theory of RL in games. The earliest paper (Littman 1994) introduced a Q-learning algorithm called minimax-Q for zero-sum two-player games. Later work (Littman and Szepesvári 1996) showed that minimax-Q converges to the game-theoretic optimal value. A separate line of work (Hu and Wellman 1998) described an extension to minimax-Q, now called Nash-Q, that attacks general-sum games by using a Nash equilibrium computation in the learning rule.

While Nash-Q is highly general, the assumptions that are known to be sufficient to guarantee its convergence are quite restrictive; no general-sum game has been shown to satisfy them. The friend-or-foe Q-learning (FFQ) algorithm (Littman 2001) provides an approach for guaranteeing convergence to equilibrium in games with coordination or adversarial equilibria, but a complete treatment of general-sum games is still lacking.

We feel the critical areas to address next are:

- **On-line methods in RL.** We have been very successful at applying divergence-based methods from on-line learning to derive and analyze updates for temporal-difference learning (a class of algorithms for single-agent RL, Forster and Warmuth 2001, Schapire and Warmuth 1996). The resulting regret bounds hold for arbitrary sequences of instances and reinforcements and go beyond the usual results, which need to make probabilistic assumptions. We believe that these methods can be applied directly, or nearly so, to richer settings of RL such as stochastic games, to produce algorithms with strong theoretical properties.

- **Rationality assumptions.** Existing RL algorithms for stochastic games are either distribution dependent, treating opponents as part of the environment; or distribution free, and attempt to learn an equilibrium strategy ignoring the observed behavior of other players. In parallel to our proposed work in on-line learning, we will examine learning approaches that are sensitive to the rationality of other players while at the same time being responsive to their observed behavior. This will allow players to learn a kind of "negotiation" procedure that is critically important in general-sum strategic interaction. We will show how to apply existing and emerging results in on-line learning to produce more effective learning algorithms for stochastic games.

- **Repeated stochastic games.** RL has been applied to negotiation in repeated games (Littman and Stone 2001). It is straightforward to conceive of repeated stochastic games, in which the same stochastic game is played over and over again — in fact, this is the typical learning scenario RL researchers have implicitly used. Given that these games are repeated, it is natural, and perhaps quite helpful, to apply folk-theorem-like ideas to stochastic games and enable a richer strategic space for learning. We plan to develop this connection and examine the computational and learning implications of repeating stochastic games.

- **Generalization.** While a key concept underlying much work in computational learning theory, generalization has not played a role in multiagent learning to date. Nevertheless, researchers creating multiagent systems have remarked upon the importance of generalizing across interactions as a way of developing “reputations”, which are very important for establishing and enforcing stability of learning and behavioral norms. We plan to study generalization in a variation of the repeated-game setting in which the game to play on each round is repeatedly drawn from a distribution (a PAC-like setting). We would like to derive formal results linking restrictions on the set of strategies to the effectiveness of generalizing between game instances. First steps in this direction may be provided by recent work relating the VC dimension to the RL setting (Kearns et al. 1999a, Kearns et al. 1999b).
C.4 Applications and Tools

While the emphasis of the current proposal is clearly on fundamental representational and algorithmic issues in CGT, we believe the proposed research will have significant impact on a number of emerging applications, including some with the direct participation of the PIs.

As we have suggested elsewhere, there are many important connections between CGT and graphical models for probabilistic inference, and we believe the potential applicability of CGT as a modeling tool rivals that of the latter field. In the same way that tools such as Bayesian networks (and their attendant inference algorithms) made large-scale probabilistic modeling feasible or more widespread in disciplines such as medical diagnosis, information retrieval, computational biology, and many others, CGT may provide the same power in application areas where reasoning about strategic interactions is an important component.

A recent series of papers has proposed game-theoretic formulations of a number of complex, real-world problems with large populations. These include game-theoretic or utility-theoretic views of the problems of vaccination against infectious disease (Bozzette et al. 2003, Kunreuther and Heal 2002, Kaplan et al. 2002), airline security, network management and security (Kunreuther and Heal 2003), and several others. In each of these proposals, the number of players ranges from the hundreds to the millions, and in the interests of analytical tractability, unreasonable assumptions of symmetry are often made (such as the assumption that any two individuals in a population are equally likely to come into infectious contact).

With colleagues in risk management at the Wharton School and epidemiology in the Penn medical school, proposed PI Kearns has begun a project to apply graphical games to the existing game-theoretic models of vaccination in infectious disease. The basic idea is to use sources of epidemiological and travel data to build a model that (even coarsely) accounts for the presence of metropolitan concentrations of population, relatively isolated rural areas, traffic between regions, and so on. The result will be a more realistic model of infectious disease spread that accounts for the empirical geographic distribution of population in the U.S., and therefore a more realistic version of the game-theoretic vaccination models proposed thus far or in the future. This line of work is perfectly suited to the graphical games formalism.

This work is not a topic of the current proposal; it will be the subject of a separate funding proposal to the Social, Behavioral and Economic Sciences directorate of the NSF, and will be primarily devoted to data collection and management, model construction, and subsequent computation. However, many of the proposed algorithmic tools discussed here will have direct benefits for this project and similar ones in the future. The participation of PI Kearns on both lines of work will maximize the chances for beneficial influences between the two projects.

For the current proposal, we do plan to develop software tools to support applications such as those discussed above. A promising candidate is a graphical games toolkit, which would provide simple GUI functionality for creating game models, and a suite of algorithms for equilibria and other computations based on the work proposed here. While earlier research has implemented some of the methods discussed (such as the NashProp algorithm for graphical games, Ortiz and Kearns 2003) for illustrative purposes, here we are imagining software that a third party could use effectively. It is possible that our software efforts may build on the GAMBIT system created at CalTech.

C.5 Broader Impact

Historically, game theory is perhaps the branch of mathematical modeling that has enjoyed the greatest interdisciplinary interest, having been employed now as a tool in social sciences, economics, evolutionary biology, ecology, medicine, engineering, and countless other fields. As such, the development of powerful computational tools to support game-theoretic modeling has a tremendous opportunity for broad impact.

The proposed PIs firmly believe in the scientific and educational benefits of game theory’s diverse applications, and have shown consistent commitment to encouraging and growing this diversity. In addition, as part of the current proposal we would exploit a number of established and successful educational outreach programs at Penn specifically for younger and minority students. We detail each of these points in turn below.

Graduate classes on CGT: Kearns is currently teaching a joint course between Penn’s Computer and Information Science department and the Wharton School on Computational Game Theory (www.cis.upenn.edu/~mkearns/teaching/cgt). This course is explicitly designed to bring together students and faculty in the Wharton School, whose interests run more towards risk management, social science and economics, with
computer scientists, to study algorithmic and modeling issues in game theory. The course features a number
of guest lectures from the represented fields, including Dean Foster and Howard Kunreuther from Wharton,
proposed PI Littman, and COLT researcher Yoav Freund. The course is new this year and will continue for
many subsequent years, and would be a prime venue for the research proposed here.

Warmuth jointly taught a course at U.C. Santa Cruz on Evolutionary Game Theory together with
economist Dan Friedman and biologist Barry Sinervo (www.cse.ucsc.edu/classes/cmps272/Winter02).
A large number of students were exposed to computational issues in game theory and there is a growing
interest in this type of interdisciplinary class at UCSC. The new algorithms will enable them to model and
simulate much larger games in biology and economy than what was previously possible.

Littman is creating a new yearly AI graduate course at Rutgers on Planning and Acting, which will
feature a substantial game theory segment.

Tutorials at international conferences: The PIs are also quite active in giving game theory tutorial pre-
sentations to interdisciplinary audiences. Kearns gave a 2-hour invited tutorial on CGT at the recent Neural
Information Processing Systems (NIPS 2002) conference, a premier AI and machine learning conference with
heavy attendance by neuroscientists, physicists, and cognitive psychologists. Kearns will give a similar in-
vited 3-hour tutorial at the upcoming ACM Conference on Electronic Commerce (EC 2003). Kearns will give
an invited talk on his CGT work at this summer’s conference on Theoretical Aspects of Reasoning about
Knowledge (TARK 2003), and an invited talk at the International Conference on Machine Learning (ICML
2003). Kearns is also due to participate in a conference on Neuroeconomics in May organized by Terry
Sejnowski.

Littman co-organized a NIPS 2002 workshop on multiagent learning and will be giving an invited tutorial
on multiagent systems and game theory at the International Joint Conference on AI (IJCAI 2003).

Warmuth has given numerous invited tutorials on on-line algorithms and the connections to game theory:
Neural Information Processing (NIPS 99), in December 1999 at Denver; Conference on Computation Learning
Theory (COLT 00), in July 2000 at Stanford; workshop on “Inference Principles and Model Selection”, in
July 2001 at Dagstuhl, Germany; and Machine Learning Summer School, in February 2001 at ANU, Calgary,
Australia.

Conferences: Warmuth is program chair of the 2003 Conference on Computational Learning Theory (COLT
2003), where he is deliberately growing the scope of the conference to include game theory; Kearns has been
enlisted as a program committee member expressly for this purpose. The program will include an invited
tutorial on game theory.

Symposium and workshops: As part of the proposed project, we plan to organize several symposia and
workshops. In the second year of the project, we will request a AAAI Spring Symposium on Computational
Game Theory, to which we will invite a multidisciplinary group of distinguished researchers. The focus of
the symposium will be on identifying the most promising algorithmic advances and to help “marry” them
with emerging applications in other disciplines. We will also run more traditional workshops at NIPS and
ICML during the course of the project.

Educational outreach activities: Kearns is the co-director of Penn’s Institute for Research in Cognitive Science
(IRCS), an interdisciplinary research institute at Penn that includes faculty from computer science, linguistics,
psychology, neuroscience, cognitive science, and many other departments. IRCS was established in 1990
and soon received an NSF Science and Technology Center (NSF-STC) grant, and continues to thrive under
subsequent funding sources. One of the great successes of IRCS has been the establishment of several ongoing
educational programs, which we propose participating in by providing game theory courses or projects as
a way of introducing students to the topic. We initially propose teaching game theory as an introduction to
social, economic and evolutionary modeling in two existing IRCS programs: the Summer School for Cognitive
Science, and the PENNlines program.

Now in its sixth year, the IRCS Summer School in Cognitive Science (www.ircs.upenn.edu/summer2002)
is a 3-week program taught by Penn and other faculty affiliated with IRCS. The program is for undergrad-
uates, typically in the summer after their junior year, and is an intense and broad introduction to topics in
psychology, neuroscience, computer science (typically AI and related areas), language and linguistics. Ad-
misions are competitive, with typically 30 students out of 125 applicants accepted. Kearns already taught
a full-day course on AI in the 2002 program.

As part of this program, we plan to give a 2- or 3-day course and project in game theory each summer
during the duration of the current proposal. The emphasis would be on game theory as a model of social

C–12
and biological interaction and evolution, but would also provide the students with a broader computational perspective on game theory. Planned course activities would include interactive demonstrations of behavioral game theory (having participants engage in game-theoretic exercises), automated game-playing tournaments such as those developed by (Axelrod 1984), and studies and simulation of evolutionary game theory.

Whereas the IRCS Cognitive Science Summer School is targeted at advanced undergraduates who may be contemplating graduate study in one of the aforementioned areas, we also plan to develop a second, more elementary version of our proposed short course for K–12 students. Within IRCS, such courses are coordinated through the PENNlincs group, which specializes in applications of cognitive science to education. PENNlincs has developed extensive contacts and partnerships focused on science, math and technology learning in local public schools and institutions such as the Philadelphia Zoo and the Franklin Institute Science Museum. We have already discussed the incorporation of our course into the program with Pennlincs director Dr. Christine Massey, who is extremely enthusiastic; we include her letter of support as a supplementary document.

The proposed short course on game theory is a particularly good fit with an ongoing PENNlincs program, the Agents for Change: Robotics for Girls project (http://www.ircs.upenn.edu/pennlincs/index.html), funded through NSF’s Program for Gender Equity (PGE/LCP 9976527). The Agents for Change project activities are specifically designed to engage and support those students-especially girls and minority students-who continue to be drastically underrepresented in education programs and career tracks in the physical sciences, engineering, and technology (http://www.nsf.gov/sbe/srs/nsf00327/start.htm). While this program has a primary focus on robotics, it emphasizes the larger themes of artificial intelligence and “smart technology,” and endeavors to introduce students and teachers to related contemporary interdisciplinary science and engineering domains in ways that are age-appropriate and motivating. Few public schools have the internal capacity to incorporate these new sciences into their curriculum, yet students are often intrigued by them.

C.6 Results from Prior NSF Support


The grant funded research into algorithmic techniques for sequential decision making and their mathematical analysis. A major contribution of the research was the development of novel planning algorithms for large-scale sequential decision problems (Majercik and Littman 1998, Majercik and Littman 2002), and its connection to stochastic satisfiability (Littman 1999, Littman et al. 2001) and computational complexity theory (Littman et al. 1998). Progress was also made in fundamental and applied research on reinforcement learning (Szepesvári and Littman 1999, Singh et al. 2000), including the development of reinforcement-learning algorithms for two problems in adaptive algorithm design (Lagoudakis and Littman 2000, Lagoudakis et al. 2002).

The research produced new exact algorithms for planning in partially observable Markov decision processes (POMDPs) (Cassandra et al. 1997, Kaelbling et al. 1998), which have been used in a large number of comparative studies since. Novel elements were studied in combination with sequential decision making, including worst-case influences (Kao and Littman 1997), time constraints (Boyan and Littman 2001), and simultaneous objectives (Fulkerson et al. 1997). The funding also helped to support the organization of the first symposium on POMDPs and to train two graduate students (Steve Majercik, now a professor at Bowdoin College, and Michail Lagoudakis, completing his Ph.D. at Duke University with Ron Parr) and was results were disseminated via numerous conference talks, invited talks, a new graduate course, a specialized workshop, a AAAI tutorial, and outreach programs in Durham, NC.


We developed a framework for deriving and analyzing on-line learning algorithm using Bregman divergences (Helmbold et al. 1999, Kivinen and Warmuth 2001, Gentile and Warmuth 1998). We also made important connections to previous work on exponential families of distributions in statistics (Azoury and Warmuth 2001) and explored alternate game-theoretic frameworks for deriving on-line updates (Takimoto and Warmuth 2000b, Takimoto and Warmuth 2000a).
We made great advances in our work on on-line algorithms for non-stationary data (Herbster and Warmuth 1998, Auer and Warmuth 1998). We were the first to use projections with respect to Bregman divergences for making on-line algorithms robust against changes (Herbster and Warmuth 2001). We also introduced a long-term memory effect that help the on-line algorithms to switch back to previously good states (Bousquet and Warmuth 2002). These algorithms were applied as an adaptive disk spin-down technique for PCs (Helmbold et al. 2000) and led to a 50% power saving over the standard method.

Boosting techniques create a good classifier from weak learning methods. We showed how these techniques can be viewed as entropy projection methods (Kivinen and Warmuth 1999b). Our work also gives a partial characterization of when a boosting method has the PAC boosting property (Duffy and Helmbold 1999). In recent work, we have extended the boosting methodology to regression problems (Duffy and Helmbold 2000b, Duffy and Helmbold 1999, Duffy and Helmbold 2000a).

C.7 Budget Discussion and Management Plan

The funds will be divided approximately equally between the four PIs, and the proposed usage is fairly straightforward: it will be used primarily to fund doctoral students in the proposed research areas; for support of the workshop and educational outreach activities; for summer salary for the PIs; and for other miscellaneous expenditures. The four PIs have collaborated in the past in various combinations (all \binom{4}{2} pairs except Littman–Warmuth) and we expect that coordination of the research will be feasible even at a distance. However, the institutions of three of the PIs (Kearns, Littman and Schapire) are all within an hour of each other, and as listed below, we plan to physically meet for collaboration and coordination regularly.

The following activities will be supported by the proposed funding:

- Several interdisciplinary workshops on CGT at the PIs institutions and major conferences;
- Development of the educational outreach programs for PENNlines and the IRCS Cognitive Science Summer School;
- A gathering of all four PIs at one of the host institutions on at least an annual basis;
- Travel funding for doctoral students on the grant to conferences and to the other PI institutions.


D References

References


