A Lattice-Based Approach to Deterministic Parallelism with Shared State

Lindsey Kuper and Ryan R. Newton
Indiana University

University of California, Berkeley
August 16, 2012

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\text{let } \_ = \text{put } l\ 3 \text{ in} \\
\quad \text{let par } v = \text{get } l \\
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What do we want?

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- A **deterministic** program is one that always produces the same observable result on multiple runs.
- A **deterministic-by-construction** programming model is one that only allows deterministic programs to be written.
  - Examples: Kahn process networks, Intel Concurrent Collections, Haskell’s monad-par, ...

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Serialize?

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Disallow shared state?

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A few single-assignment languages

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  - Compel (Tesler and Enea, 1968)

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    - Specifically, *Featherweight CnC*

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- Today:
  - Intel Concurrent Collections (Budimlić *et al.*, 2010)
    - Specifically, *Featherweight CnC*
  - monad-par for Haskell (Marlow *et al.*, 2011)

Disallow multiple assignment?

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Deterministic programs that single-assignment forbids

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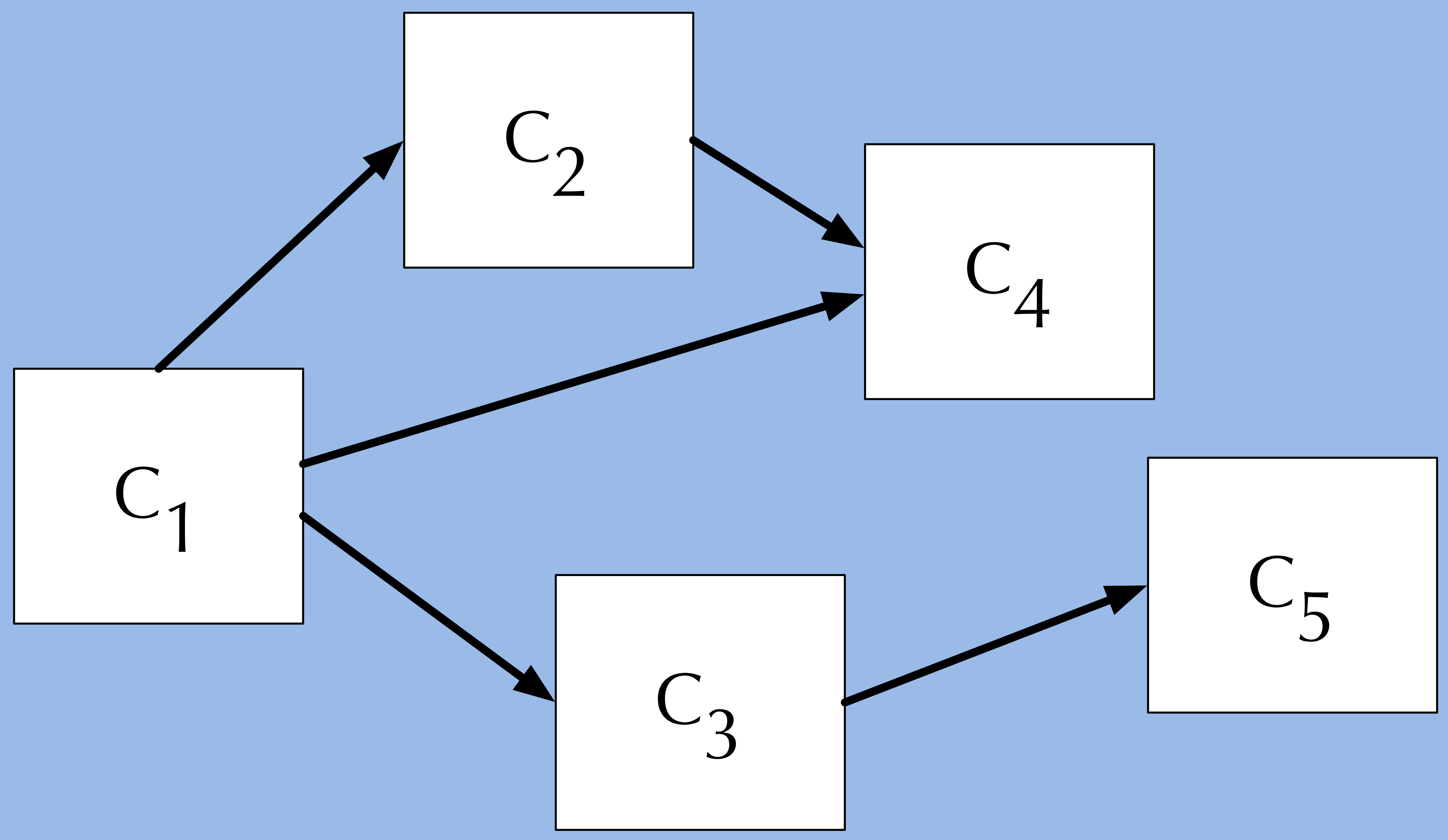
$$

$$
\begin{array}{l}
\text{let par } \_ = \text{put } l\ (4, \bot) \\
\qquad\quad\ \ \_ = \text{put } l\ (\bot, 3) \\
\quad \text{in let } v = \text{get } l \text{ in } v
\end{array}
$$

Kahn process networks (Kahn, 1974)



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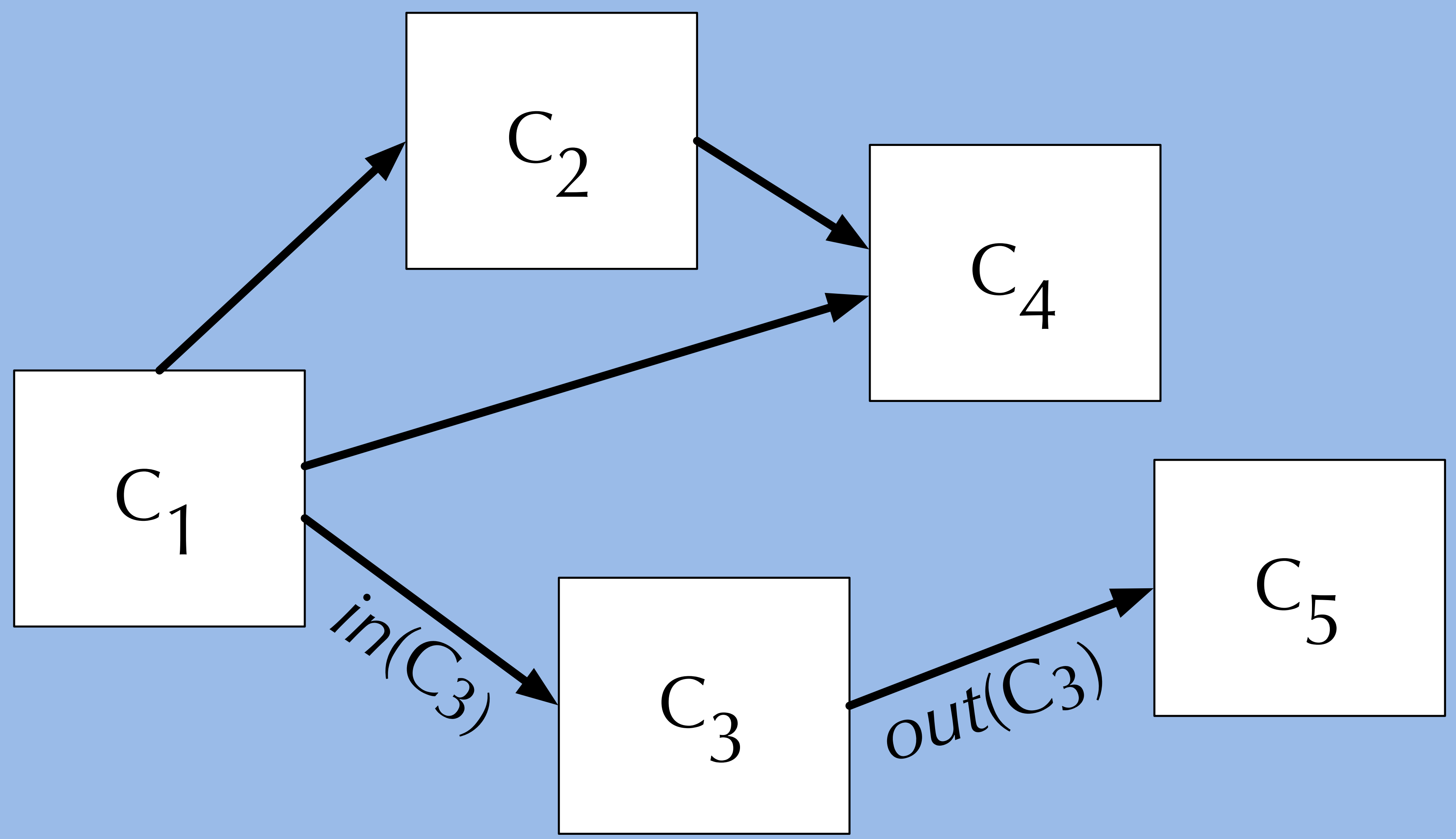

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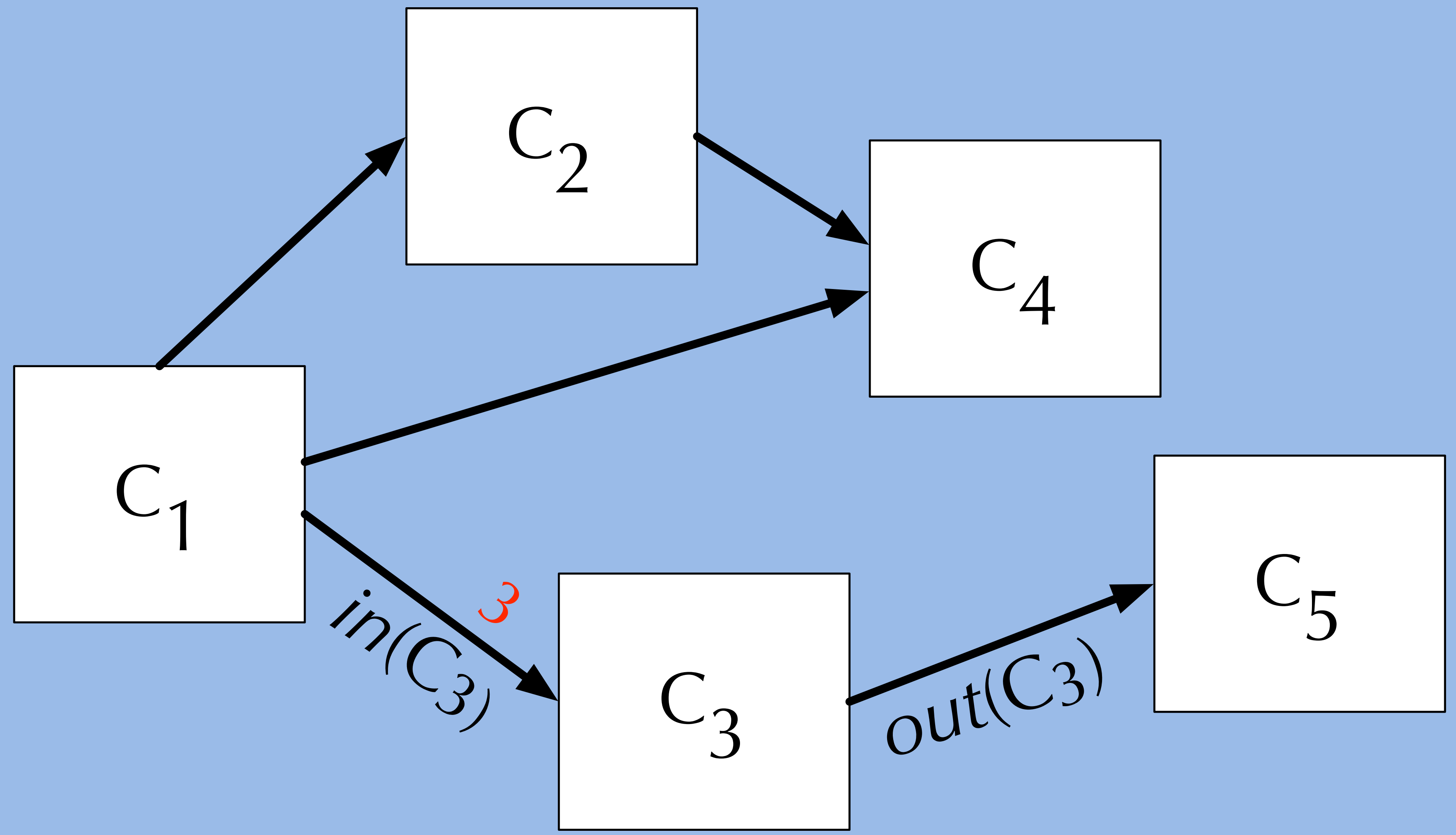

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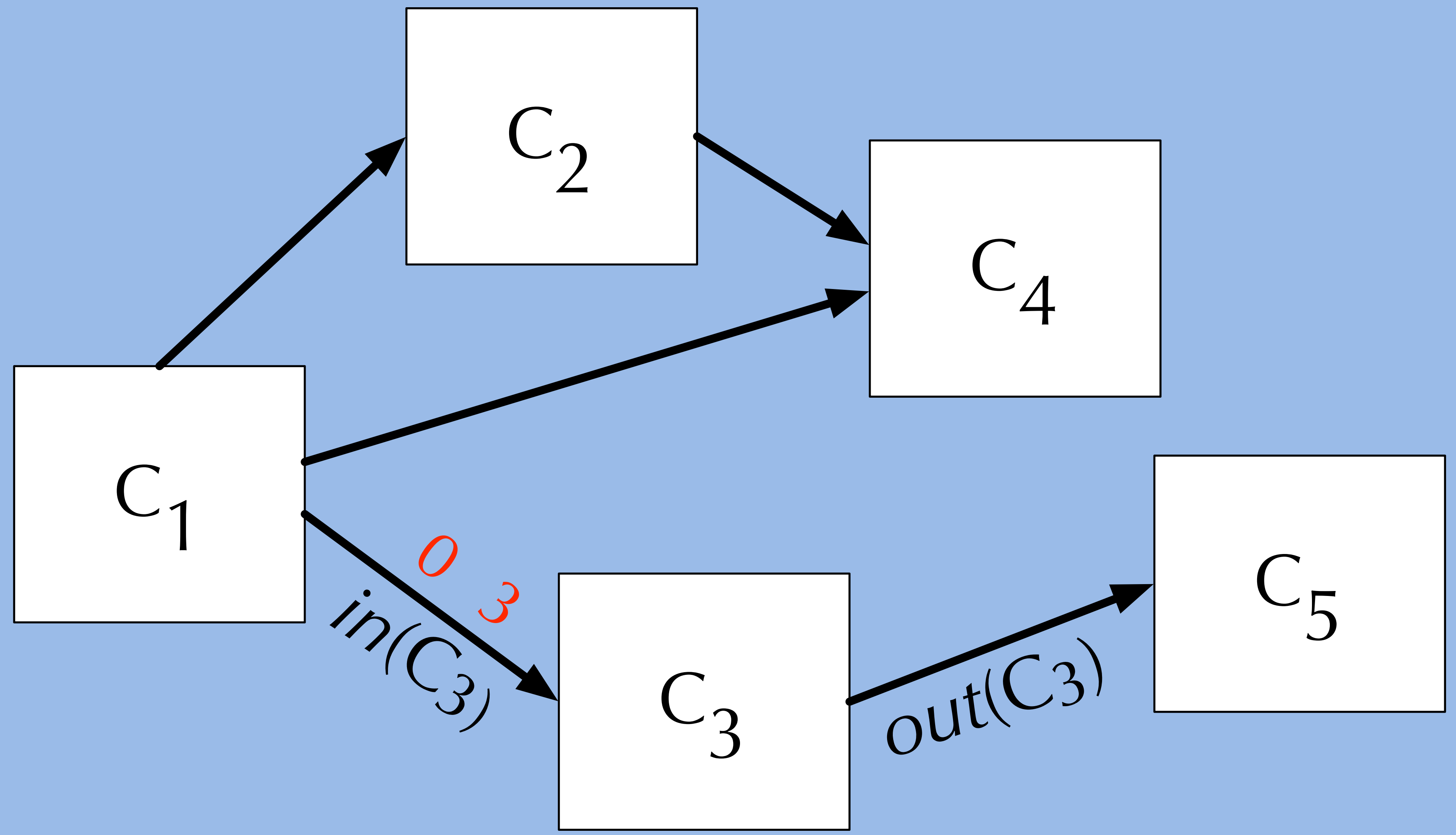

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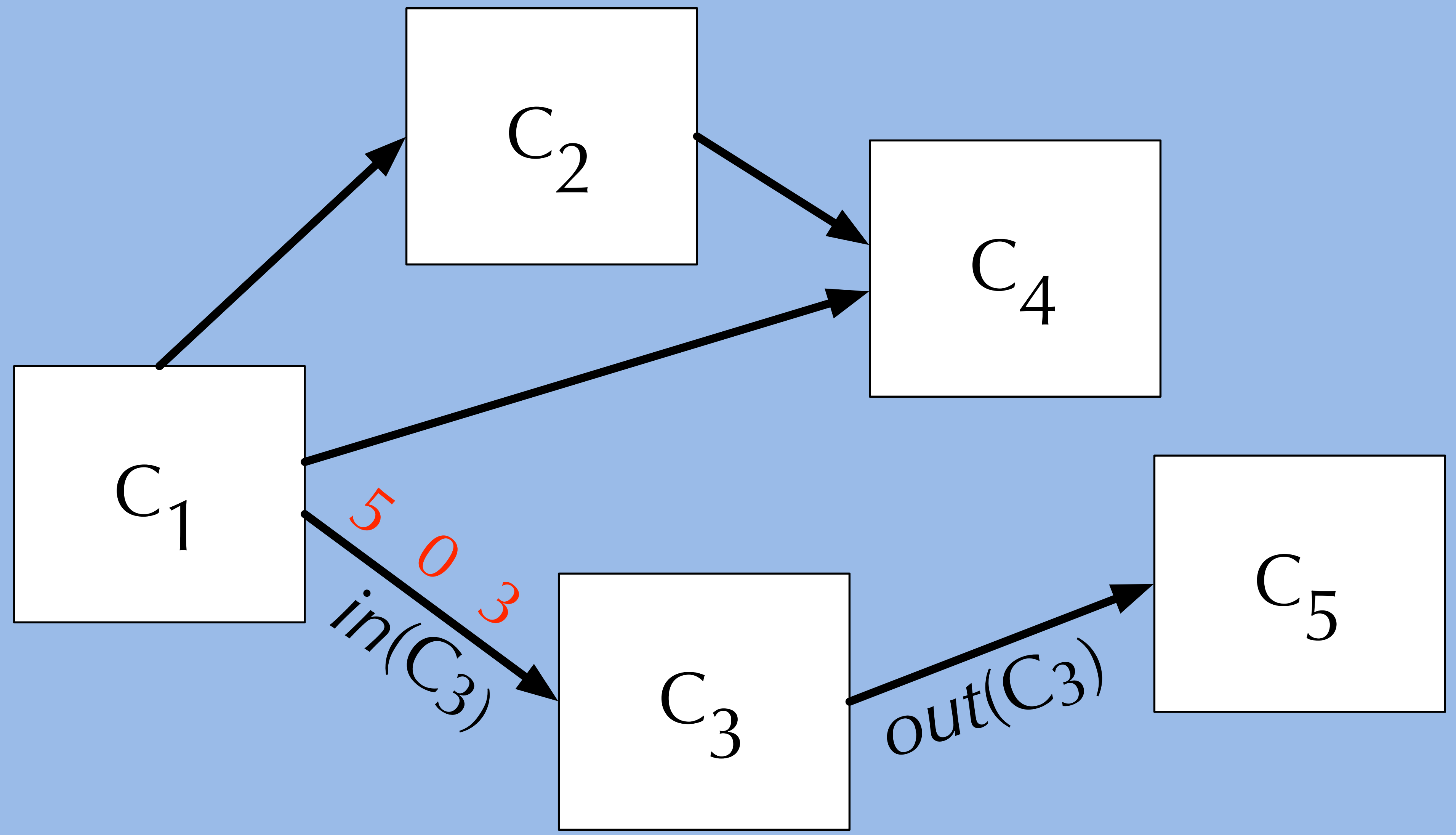

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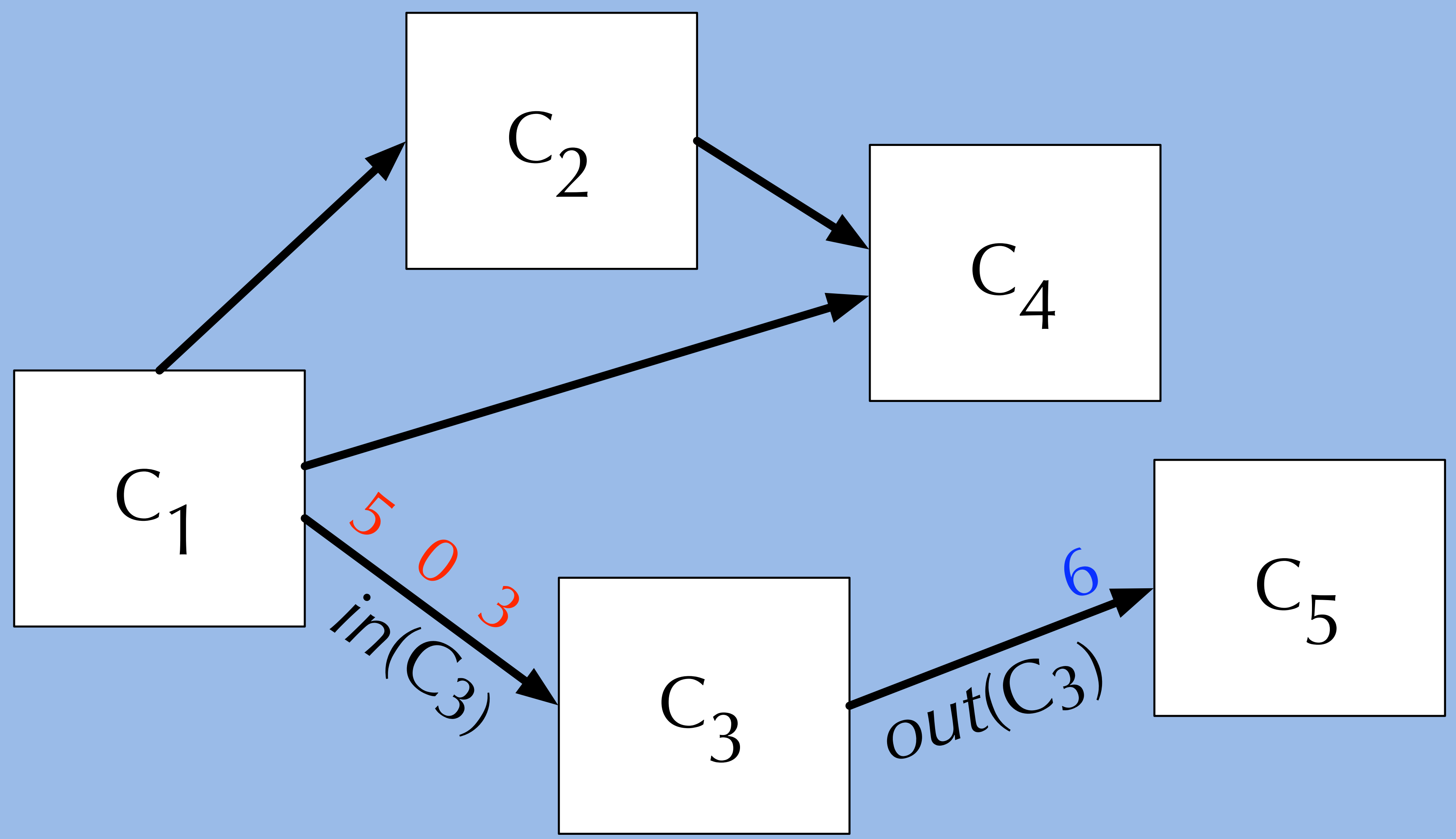

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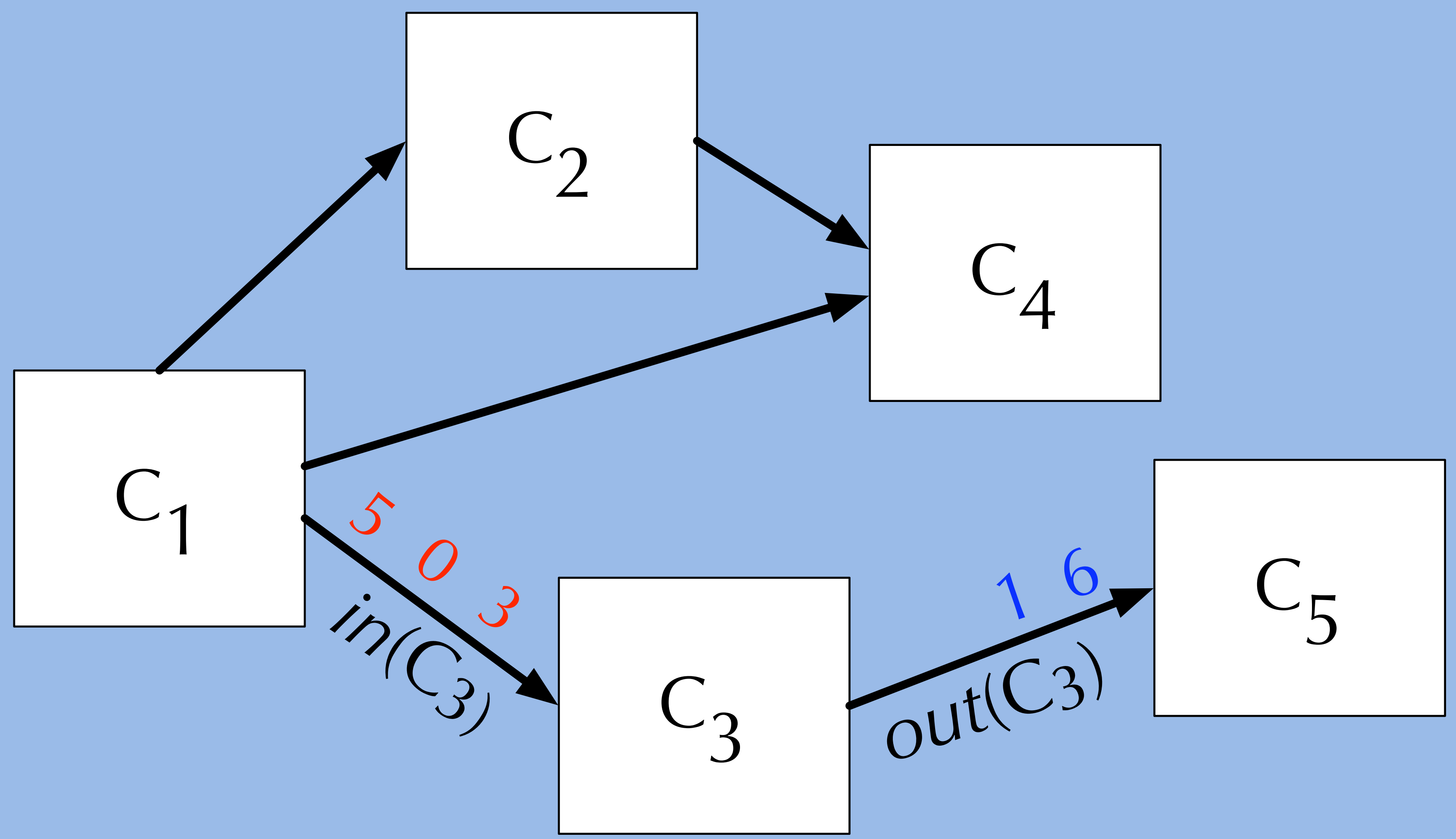

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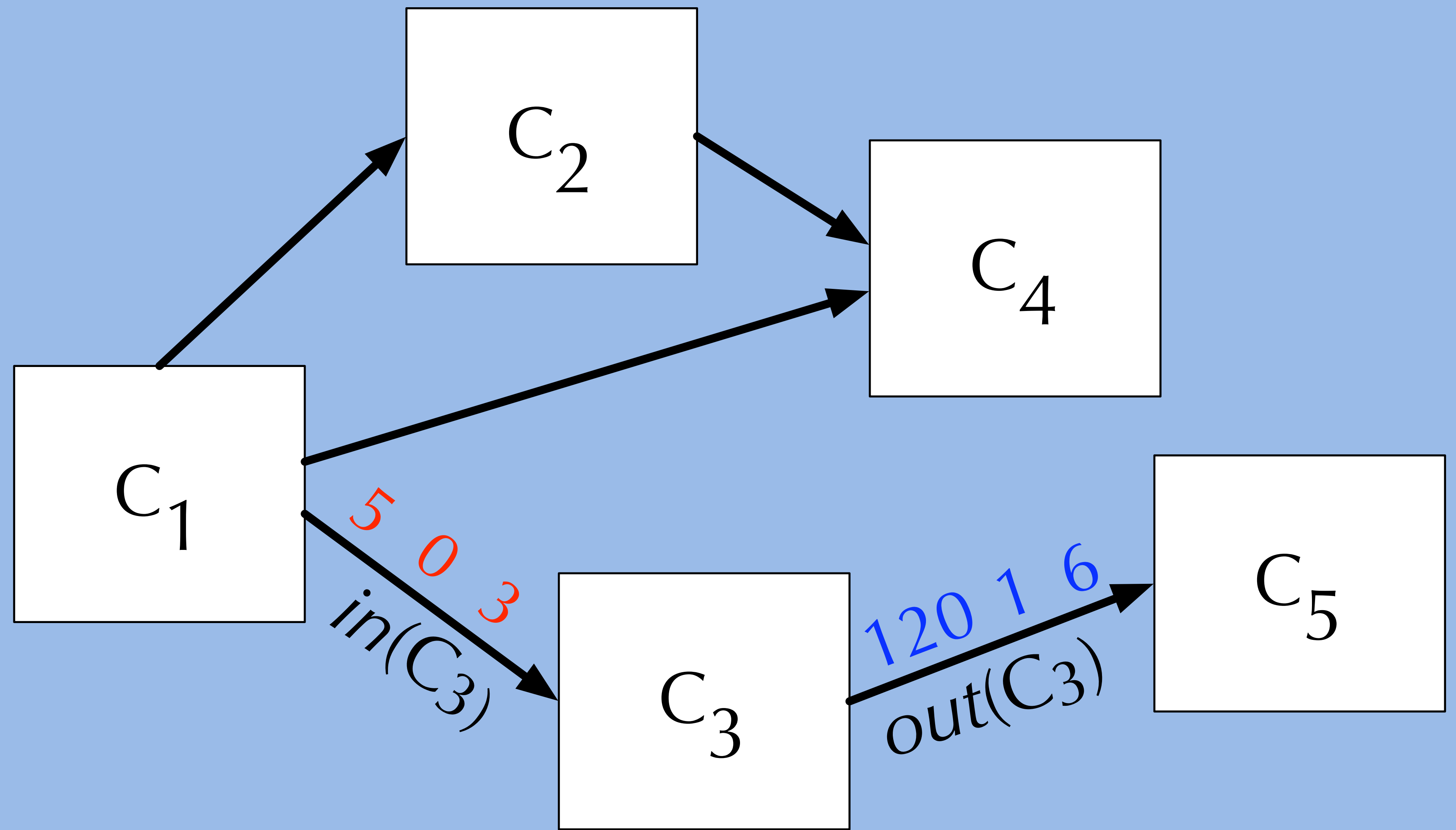

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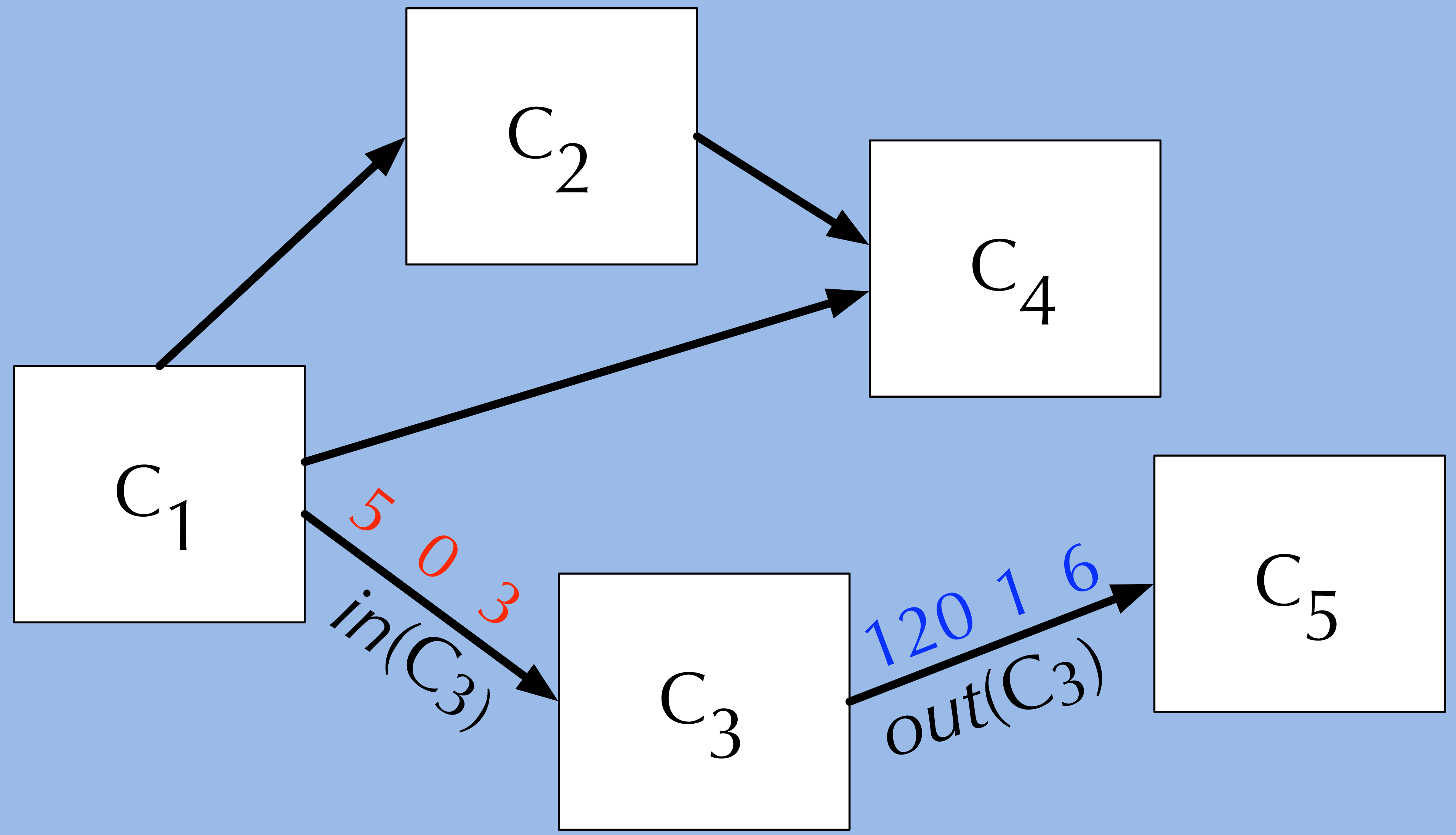
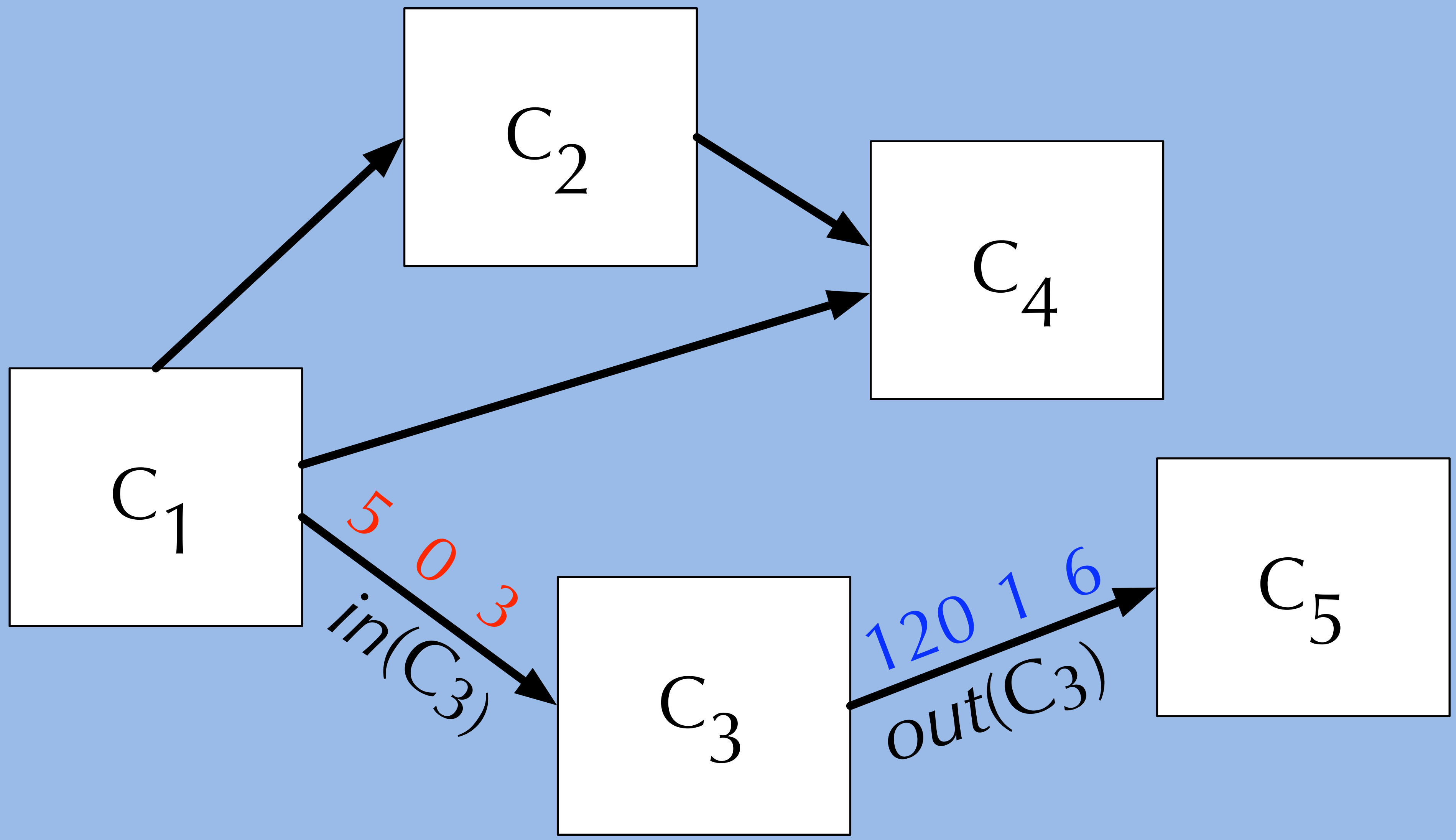


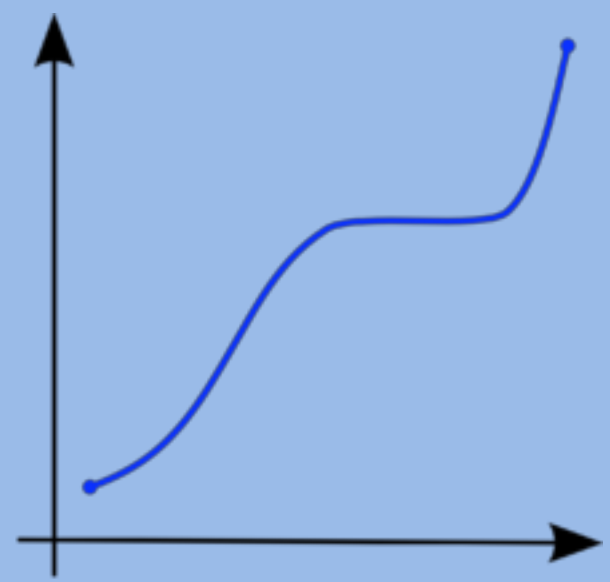
hist(*in*(C$_3$)): [3, 0, 5, ...]

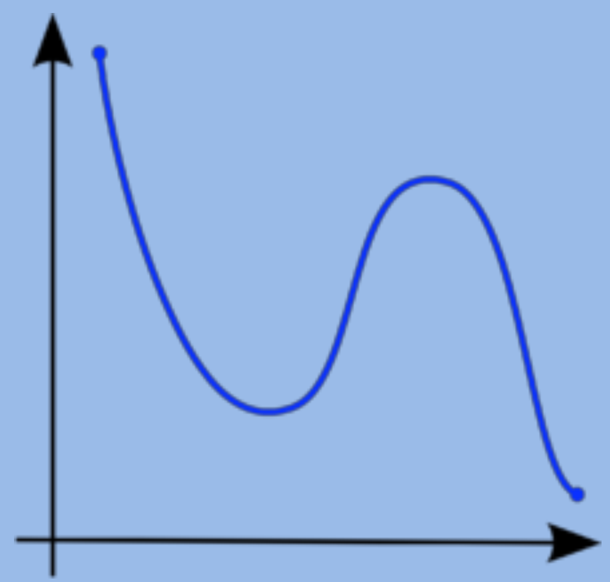
Kahn process networks (Kahn, 1974)



hist(*in*(C3)): [3, 0, 5, ...] *hist*(*out*(C3)): [6, 1, 120, ...]

Monotonicity





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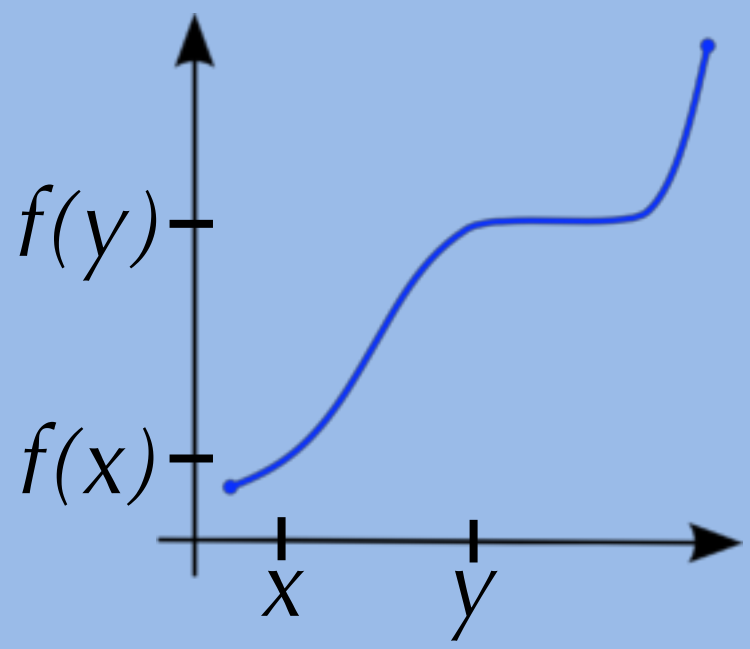

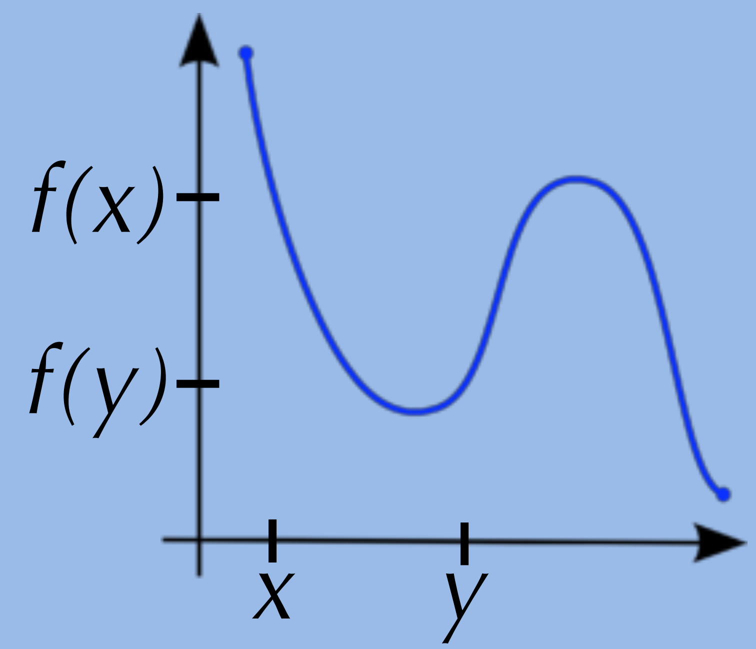

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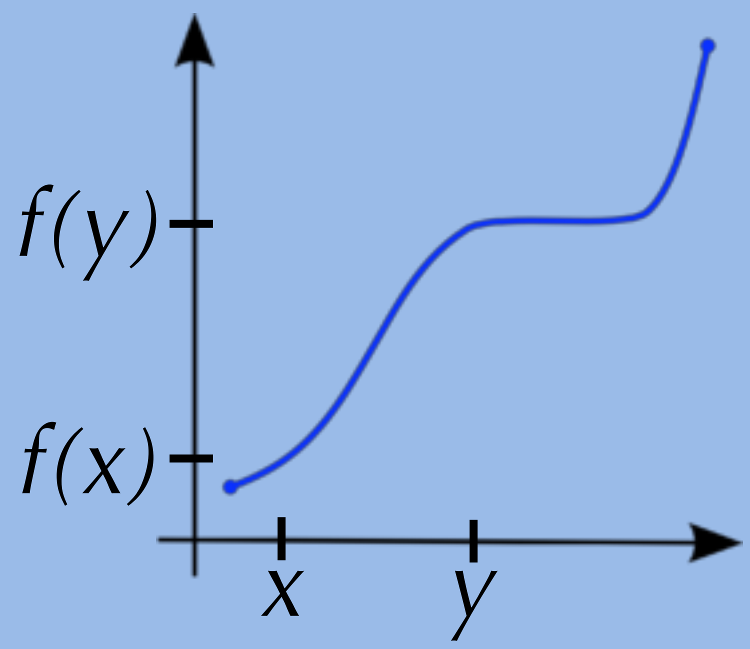
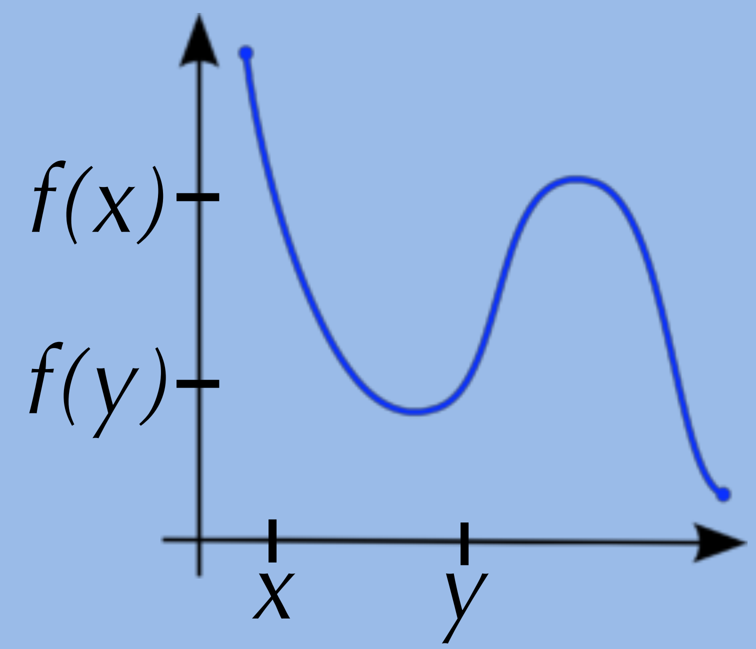
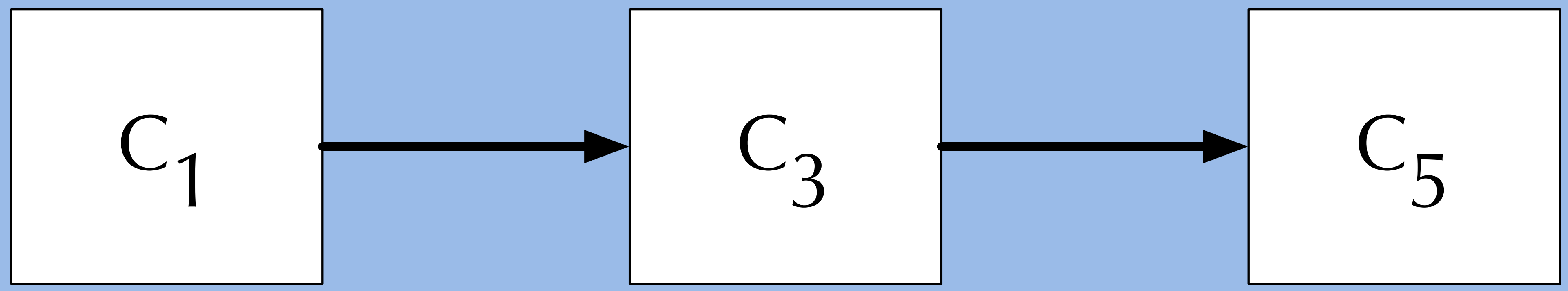




f is monotonic iff $x \leq y \implies f(x) \leq f(y)$

Monotonicity in KPNs

$$f \text{ is monotonic iff } x \leq y \implies f(x) \leq f(y)$$



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$$\boxed{C_1} \xrightarrow[\mathit{in}(C_3)]{} \boxed{C_3} \xrightarrow[\mathit{out}(C_3)]{} \boxed{C_5}$$

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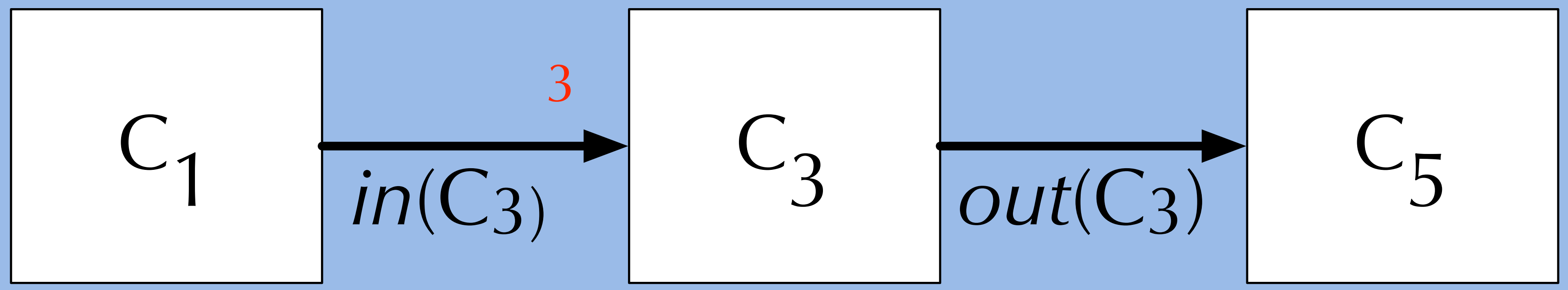

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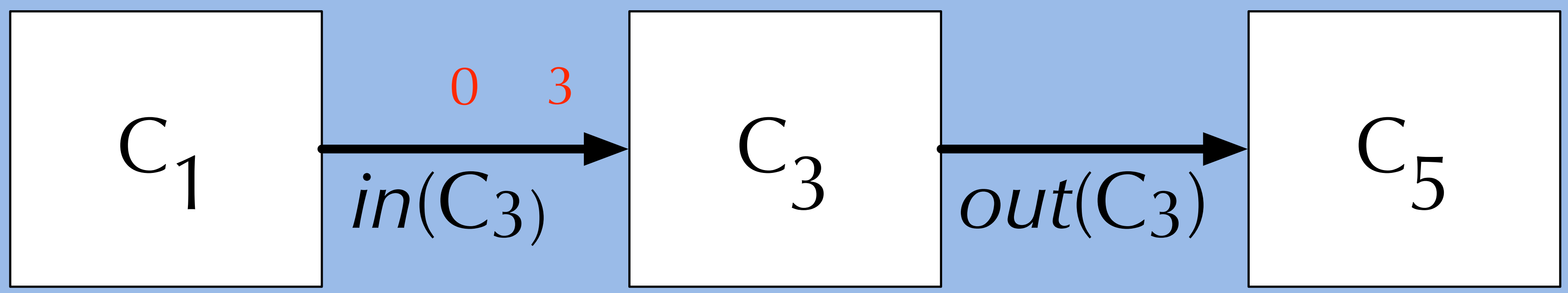

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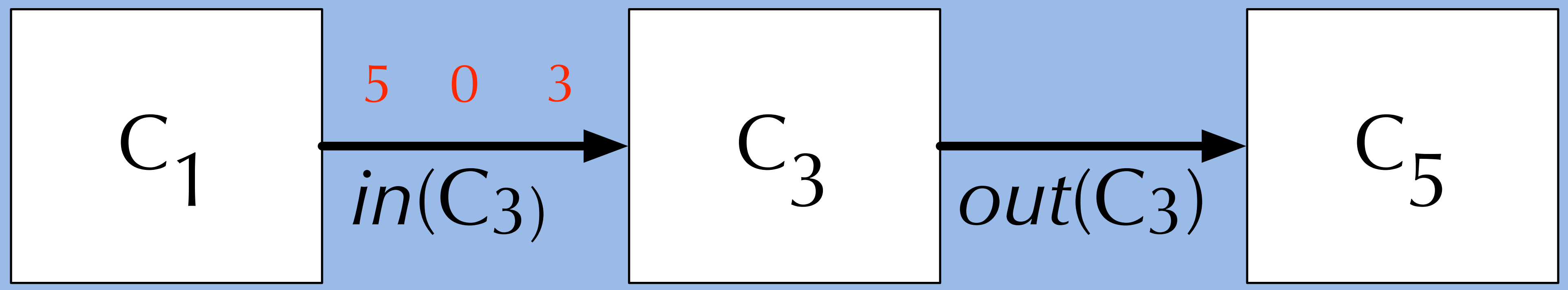

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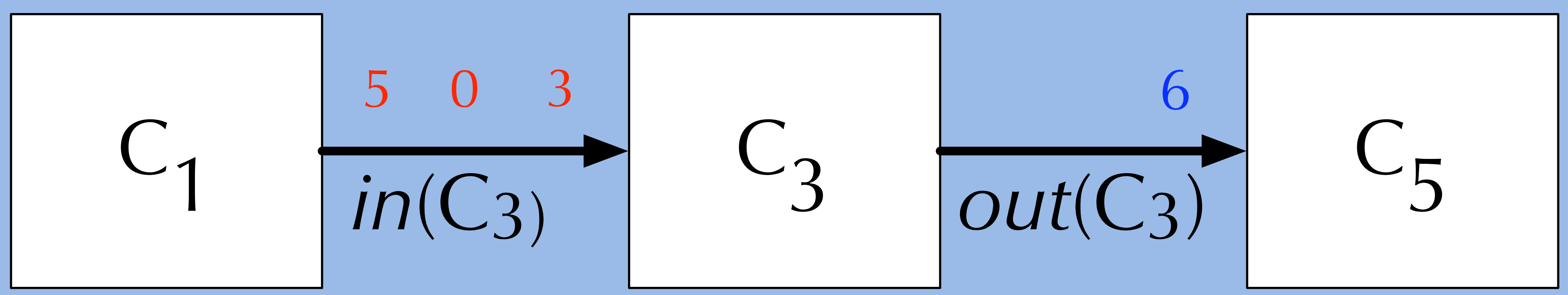

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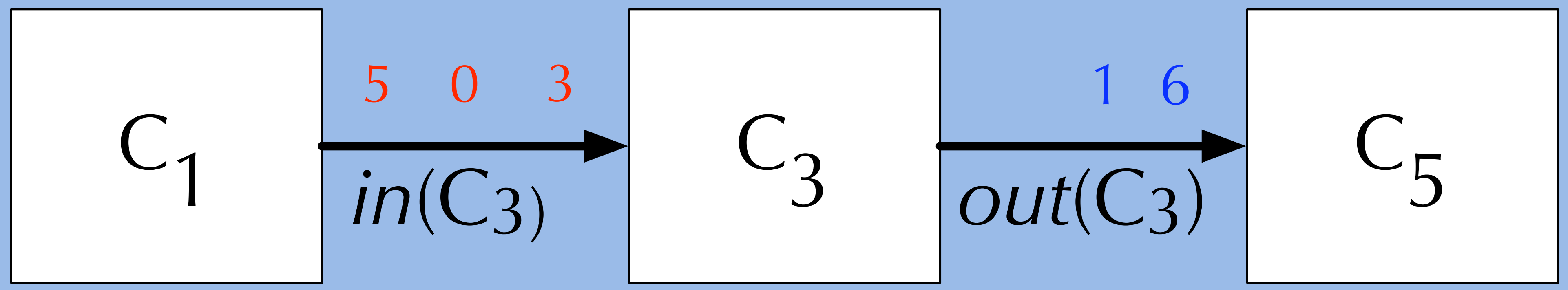

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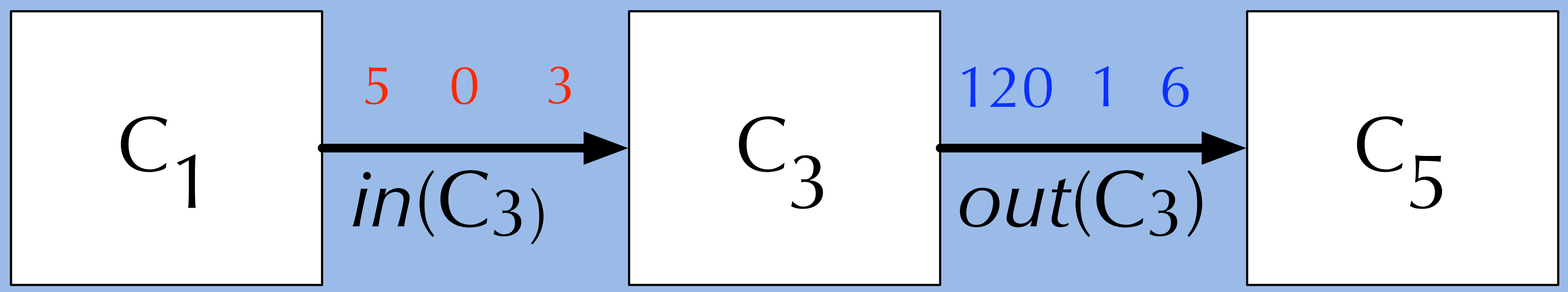

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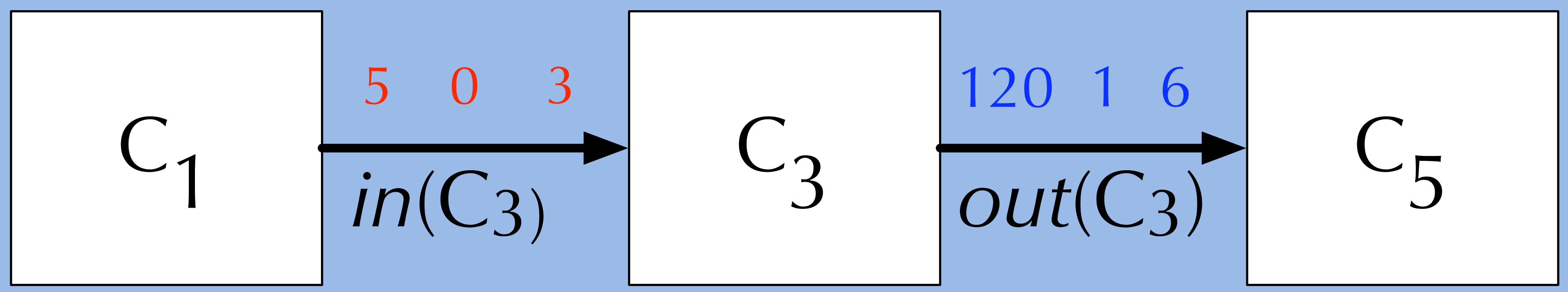

For KPNs, the $\leq$ relation is just *prefix-of*:

[3] *prefix-of* [3, 0] $\implies$ [6] *prefix-of* [6, 1]

[3, 0] *prefix-of* [3, 0, 5] $\implies$ [6, 1] *prefix-of* [6, 1, 120]

...

Monotonicity causes deterministic parallelism!

Back to single-assignment languages

$$
\begin{array}{l}
\text{let } \_ = \text{put } l_1 \; 4 \text{ in} \\
\quad \text{let } \_ = \text{put } l_2 \; 3 \text{ in} \\
\quad\quad \text{let par } \_ = \text{put } l_4 \; 3 \\
\quad\quad\quad\quad\quad\;\; \_ = \text{put } l_3 \; 5 \\
\quad\quad\quad \text{in get } l_4
\end{array}
$$

Store:

Back to single-assignment languages

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Store:

l_1	4

Back to single-assignment languages

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Store:

l_1	4
l_2	3

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Store:

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l_2	3
l_3	5

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$$

Store:

l_1	4
l_2	3
l_3	5
l_4	3

For stores, the $\leq$ relation is $\subseteq$:

$$\{l_1 \rightarrow 4,\ l_2 \rightarrow 3\} \subseteq \{l_1 \rightarrow 4,\ l_2 \rightarrow 3,\ l_3 \rightarrow 5\} \Longrightarrow$$

$$\{l_1 \rightarrow 4,\ l_2 \rightarrow 3,\ l_4 \rightarrow 3\} \subseteq \{l_1 \rightarrow 4,\ l_2 \rightarrow 3,\ l_3 \rightarrow 5,\ l_4 \rightarrow 3\}$$

Generalizing our notion of monotonicity

For stores, the ≤ relation is ⊆:

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- Given stores S and S', we say that $S \leq S'$ iff:
  - $\mathrm{dom}(S) \subseteq \mathrm{dom}(S')$, and
  - for all locations l in $\mathrm{dom}(S)$, $S(l) = S'(l)$

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Idea: restrict reads

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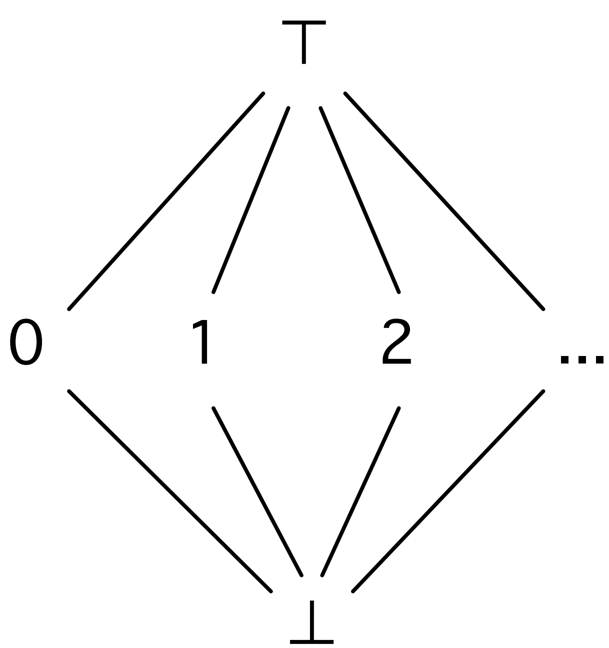
\end{aligned}$$

return 4

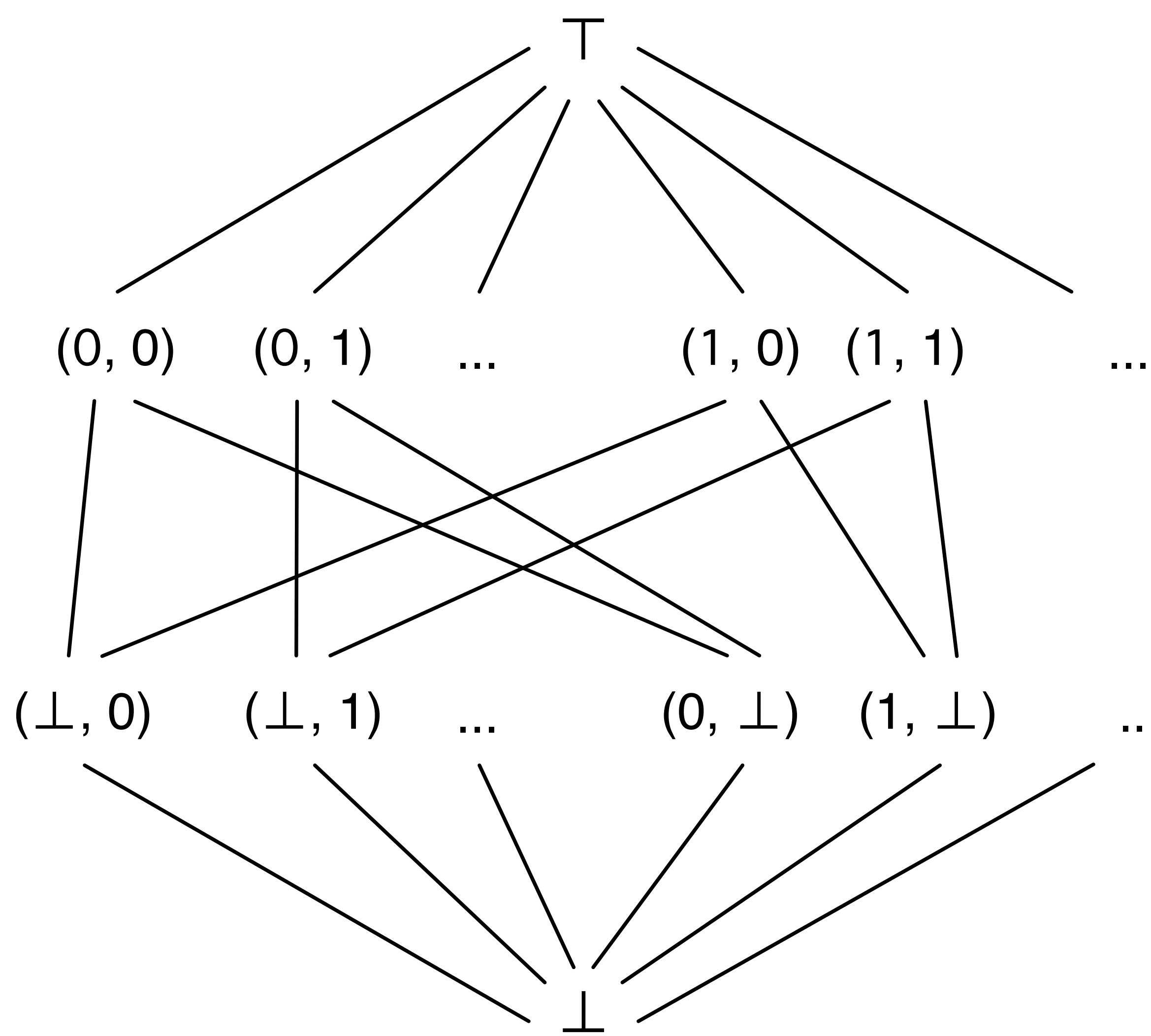
$$\begin{aligned}
&\text{let } _ = \text{put } l\ 3 \text{ in}\\
&\quad \text{let par } v = \text{get } l\ 4\\
&\quad\quad\quad\quad\ \ _ = \text{put } l\ 4\\
&\quad\quad\quad\quad\ \ _ = \text{put } l\ 5\\
&\quad\ \ \text{in } v
\end{aligned}$$

Monotonically increasing writes
+ restricted reads
= deterministic-by-construction parallelism

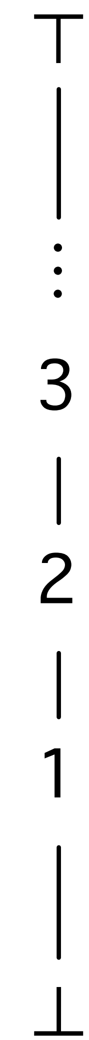
Parameterizing our language: “LVars”



IVar

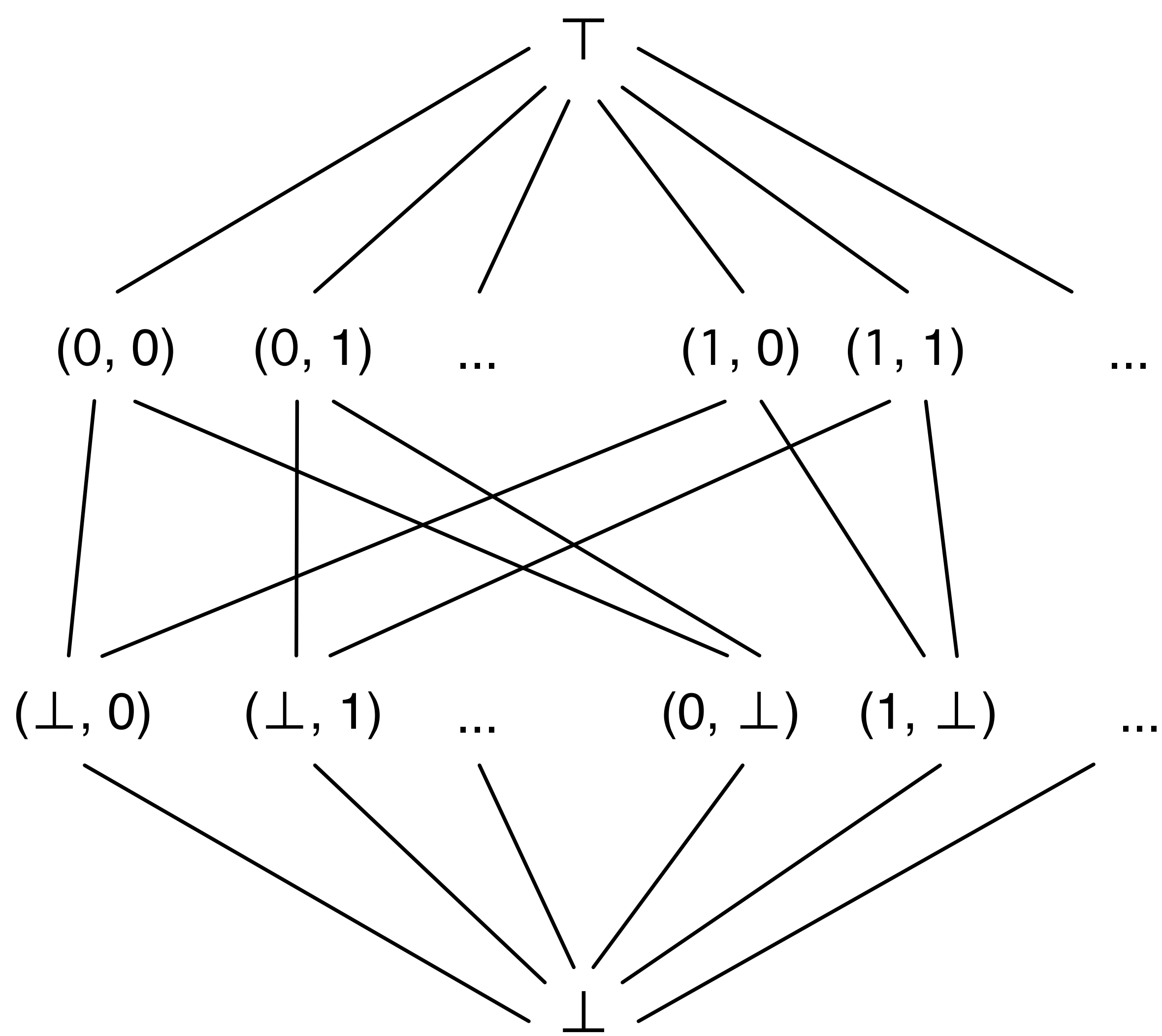


Pair of IVars



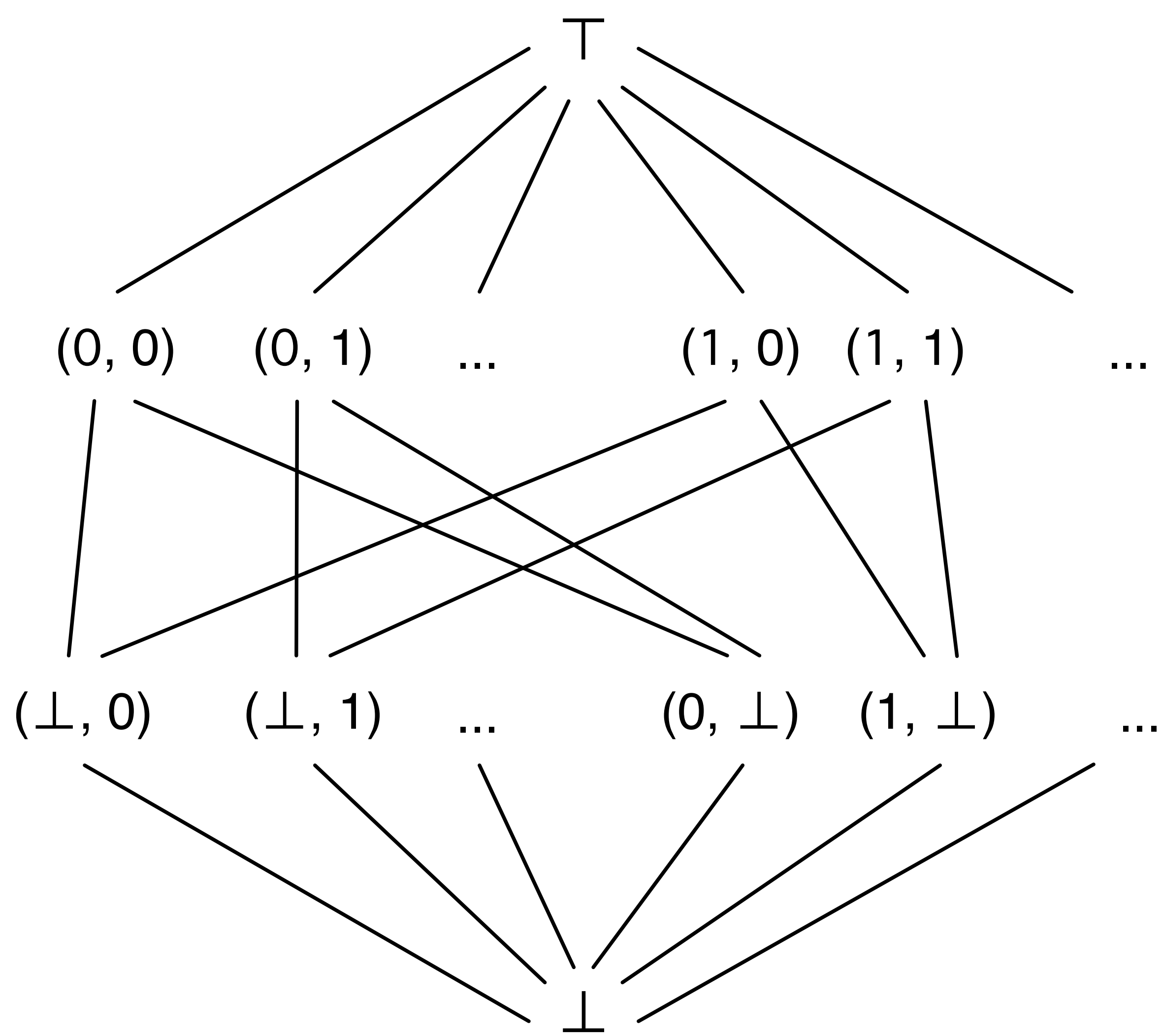
Counter

Parameterizing our language: “LVars”



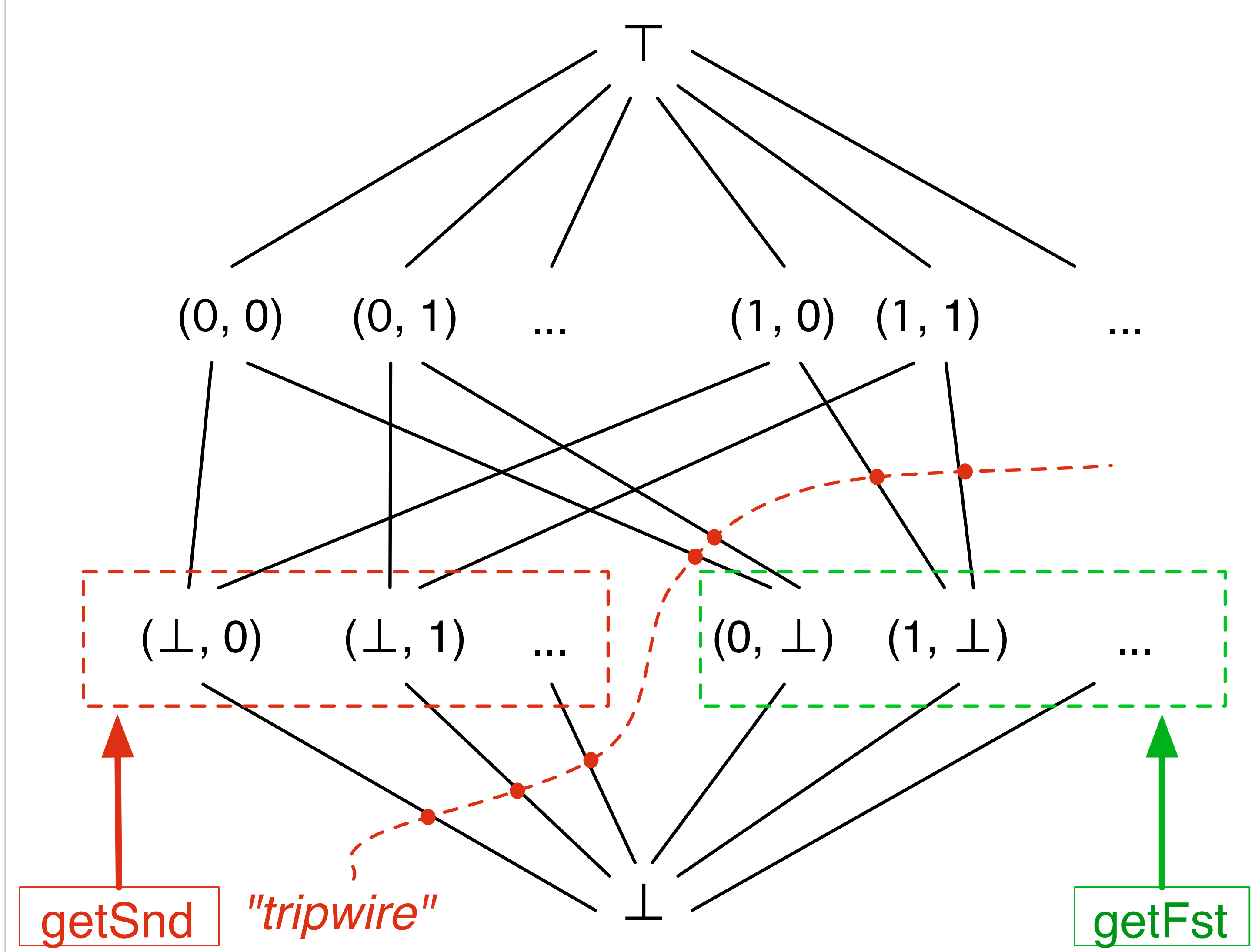
Pair of IVars

Parameterizing our language: "LVars"



Pair of IVars

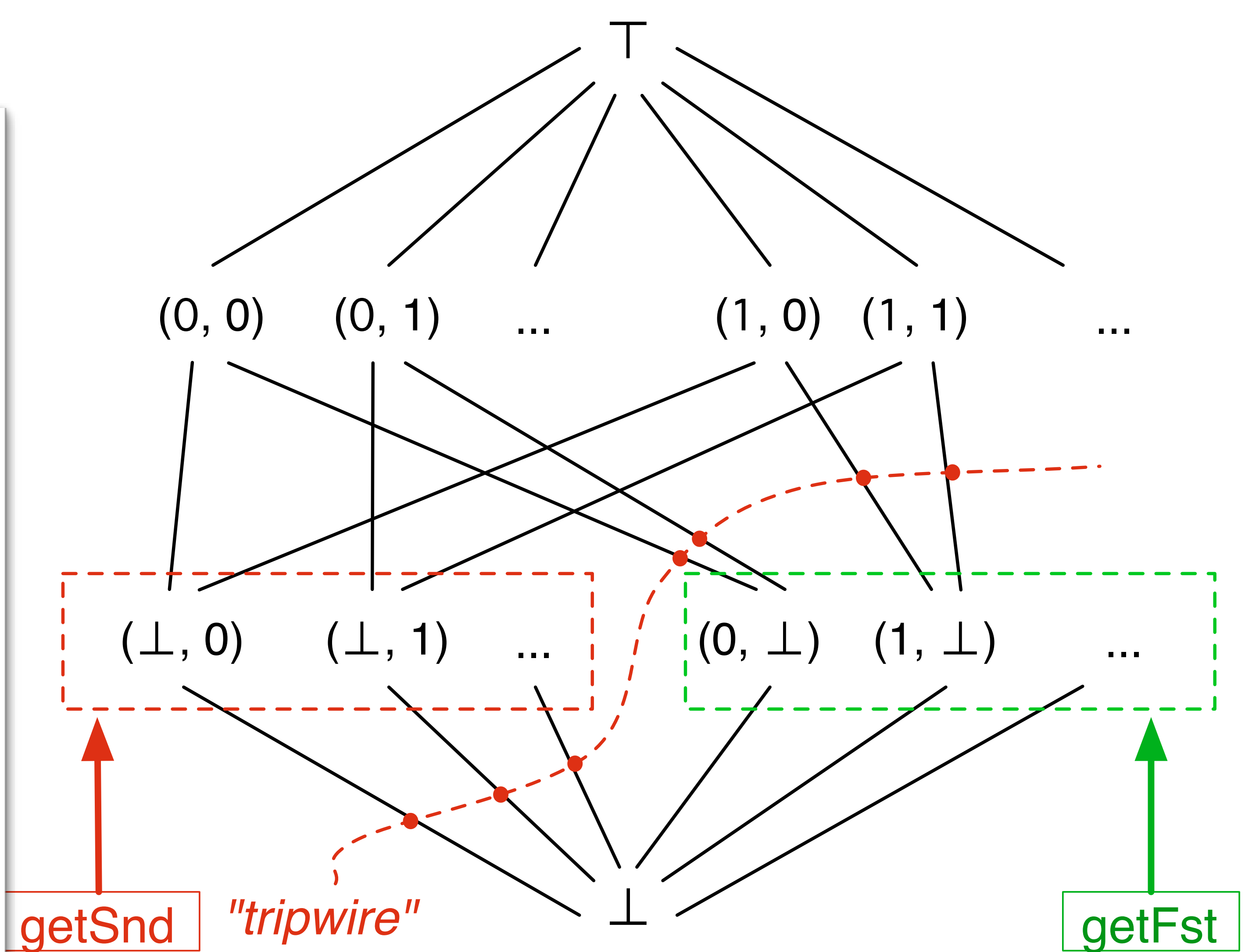
Parameterizing our language: “LVars”



Pair of IVars

Parameterizing our language: "LVars"

$$
\begin{aligned}
&\text{let } p = \text{new in} \\
&\quad \text{let } \_ = \text{put } p \ \{(\bot, 4)\} \text{ in} \\
&\qquad \text{let par } v_1 = \text{getFst } p \\
&\qquad\qquad\quad \_ = \text{put } p \ \{(3, 4)\} \\
&\qquad \text{in } \ldots v_1 \ldots
\end{aligned}
$$



Pair of IVars

More in our paper draft and TR

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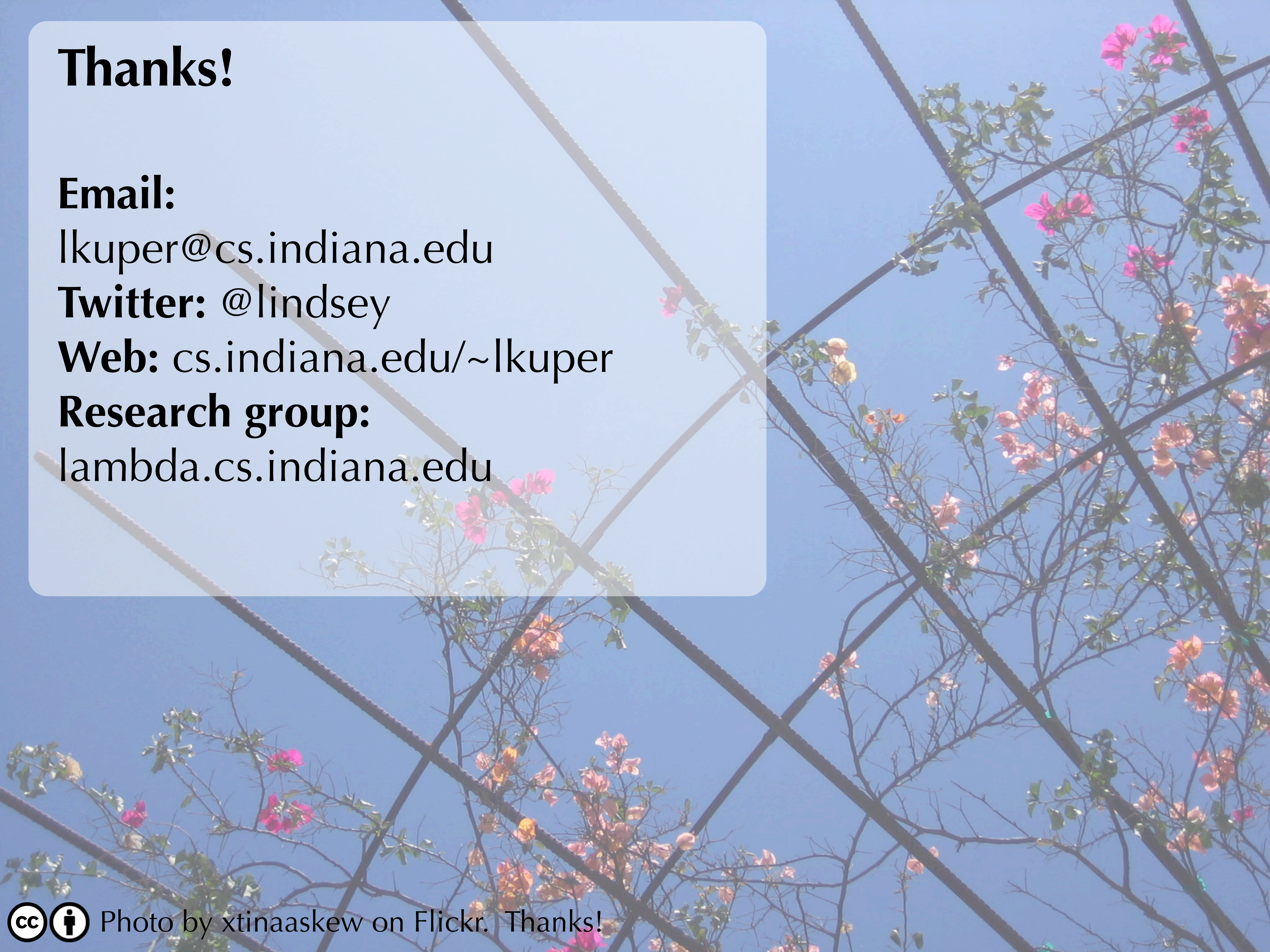
  - “probation” state



Thanks!

Email:
lkuper@cs.indiana.edu
Twitter: @lindsey
Web: cs.indiana.edu/~lkuper
Research group:
lambda.cs.indiana.edu