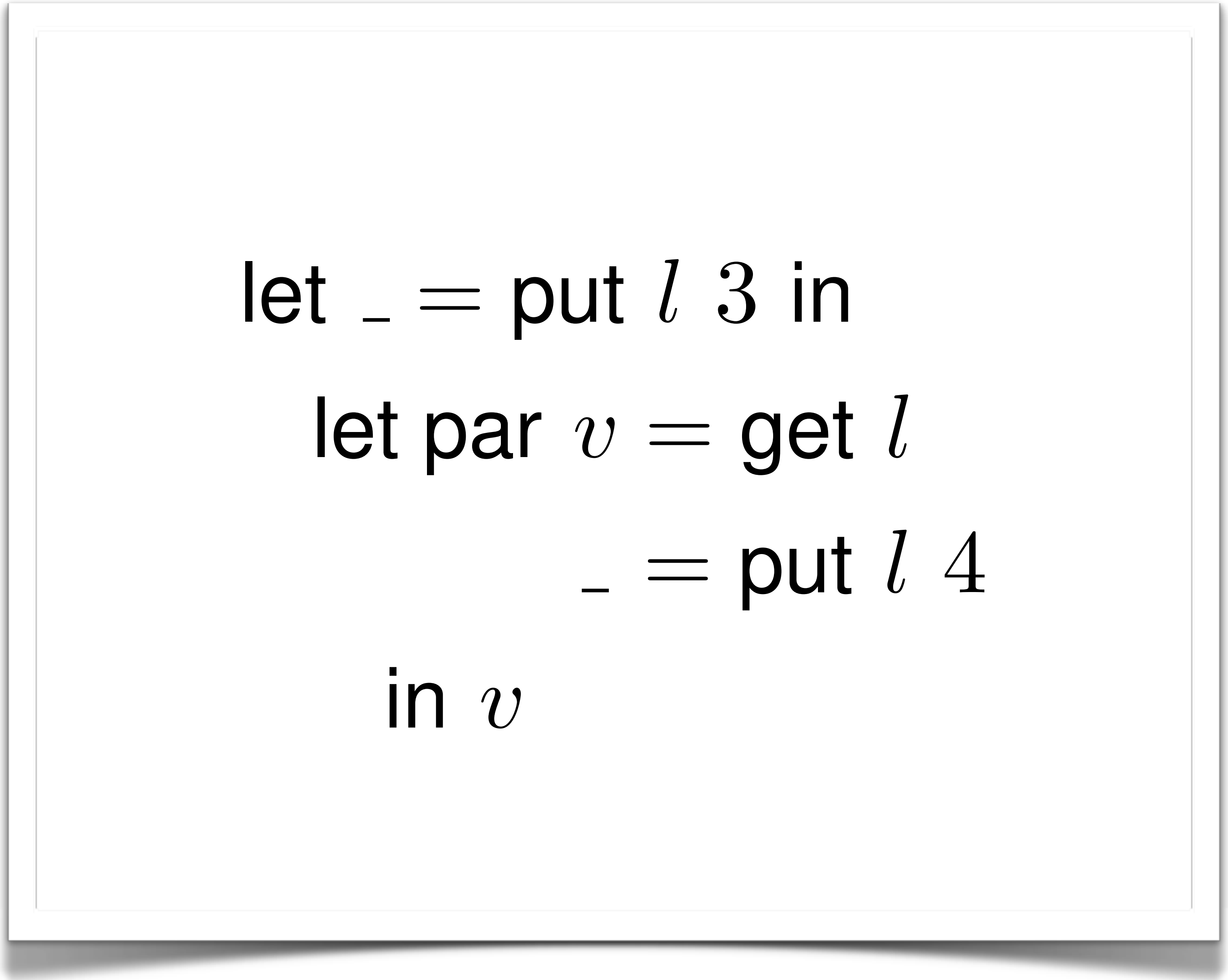
# A Lattice-Based Approach to Deterministic Parallelism with Shared State

**Lindsey Kuper** and Ryan R. Newton
Indiana University
Bloomington, Indiana, USA

Aarhus University
14 September 2012



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let _ = put $l$ $3$ in
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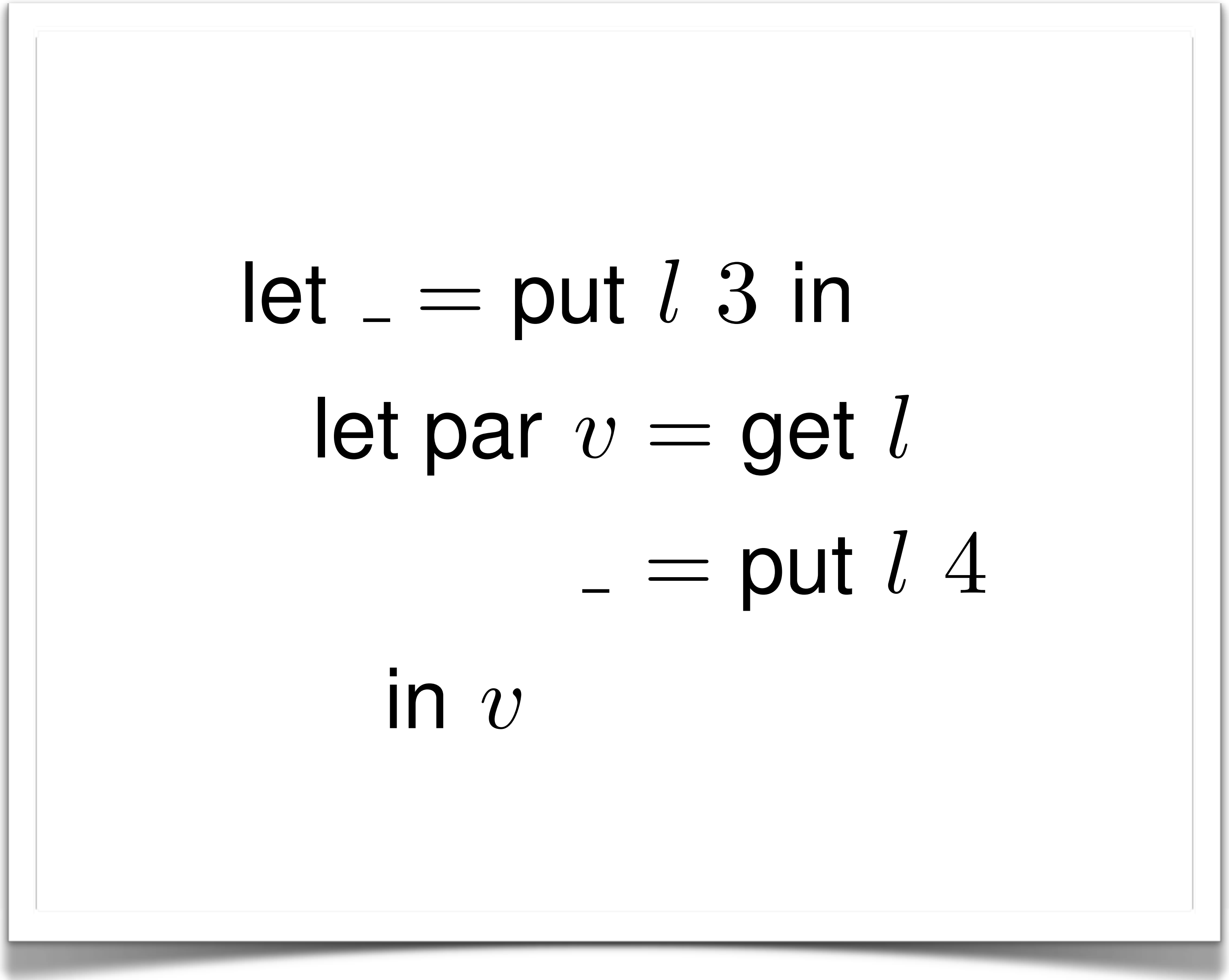
- A **deterministic** program is one that always produces the same observable result on multiple runs.
- A **deterministic-by-construction** programming model is one that only allows deterministic programs to be written.
  - Examples: Kahn process networks, Intel Concurrent Collections, Haskell's monad-par, ...



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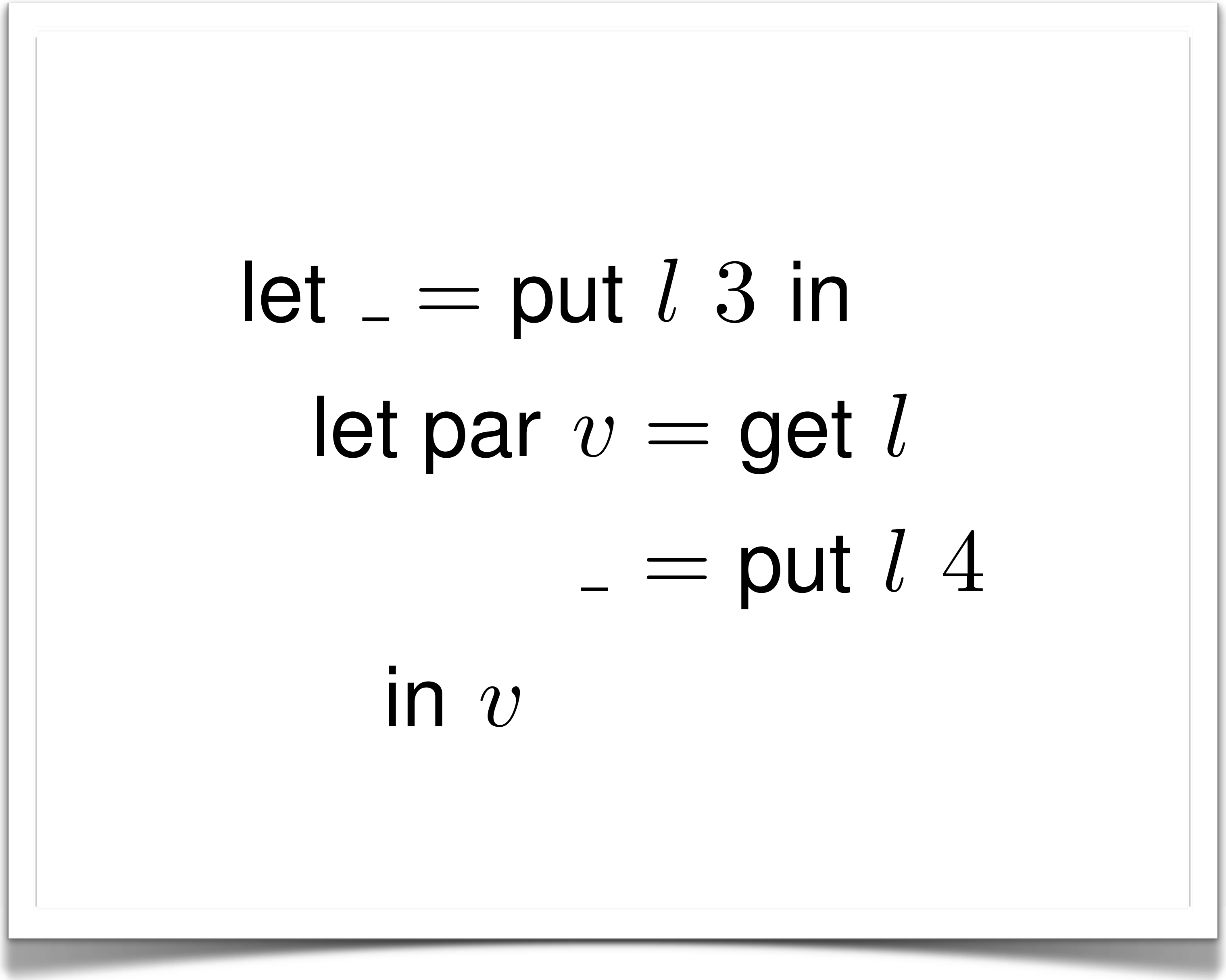
# Serialize?



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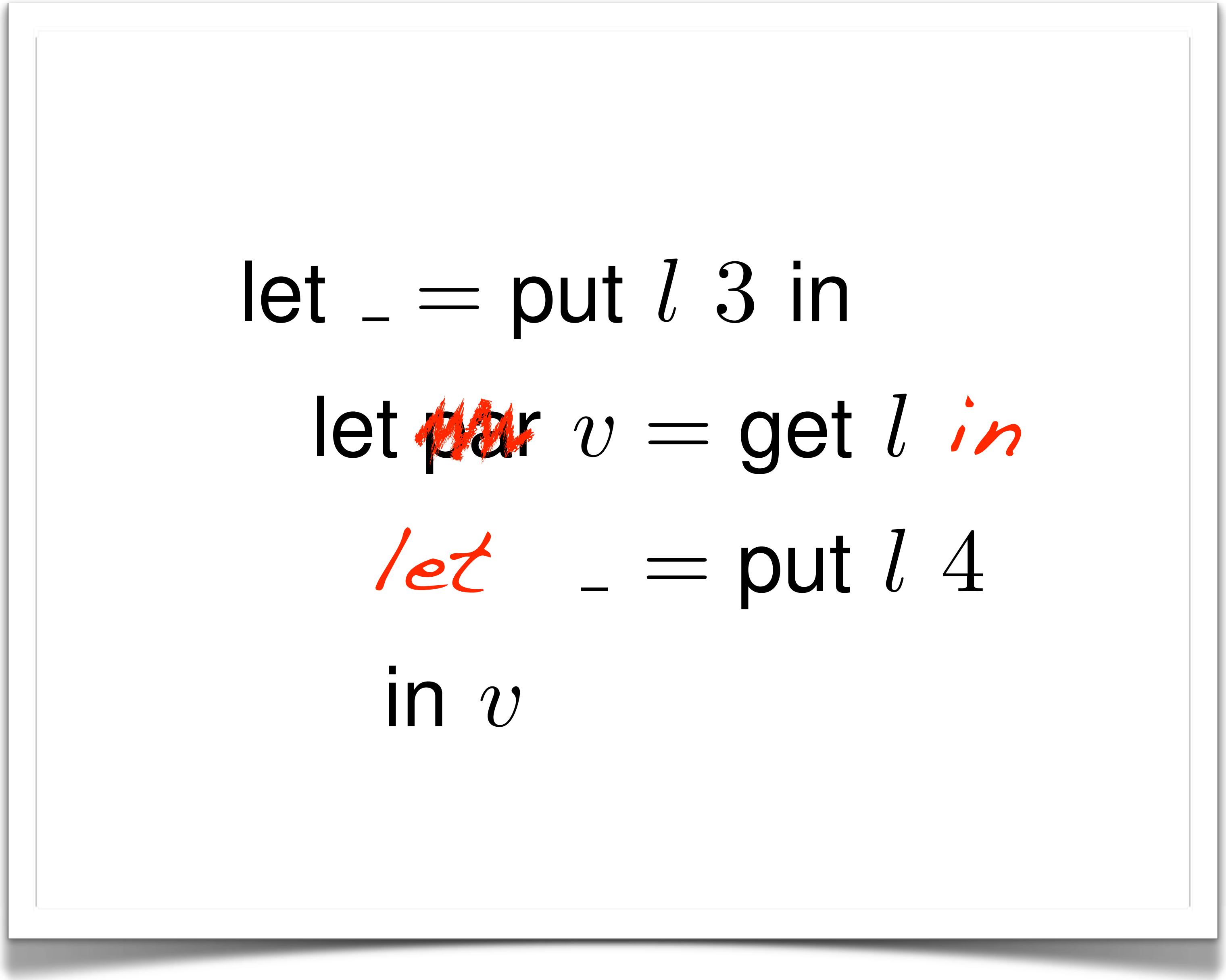

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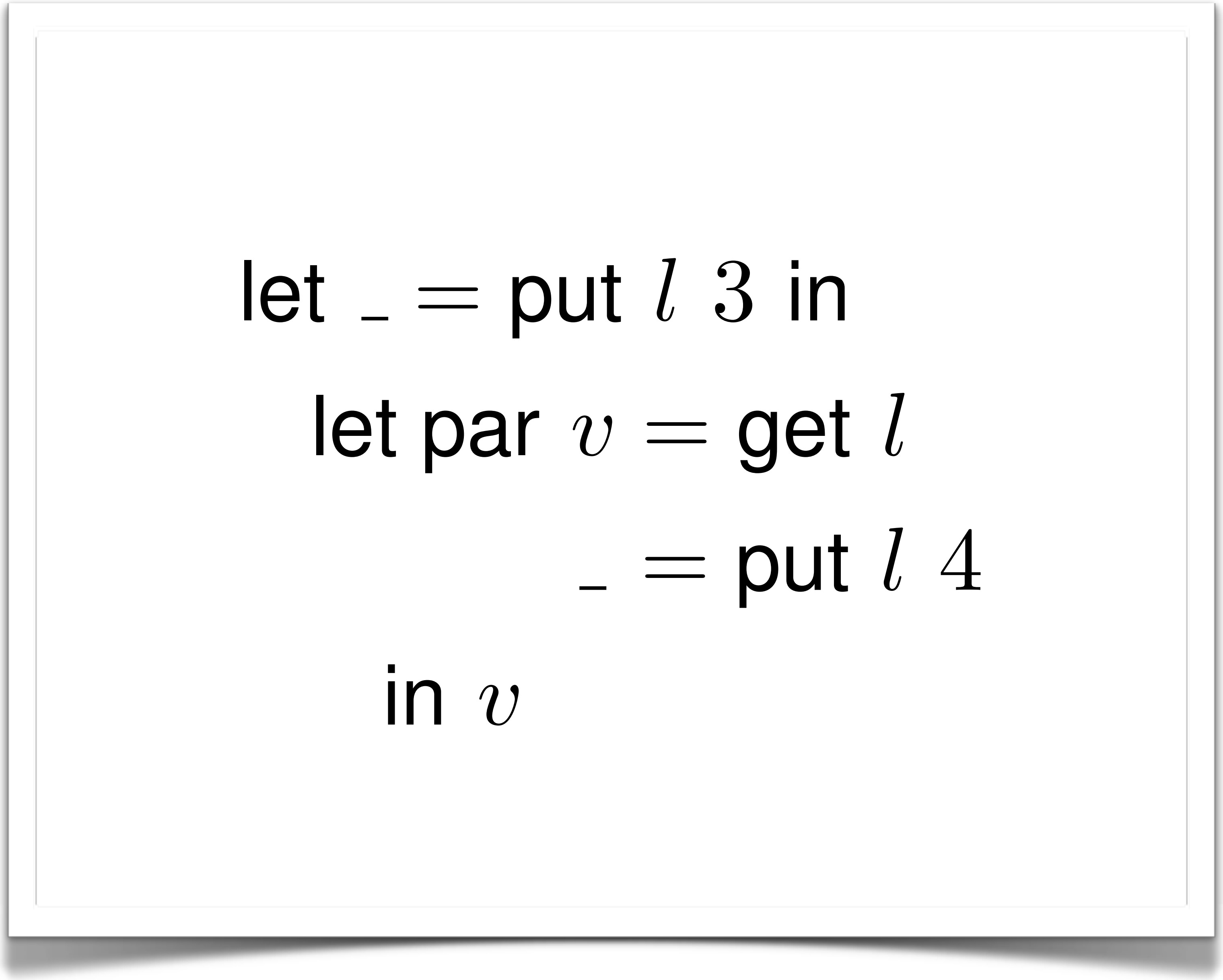

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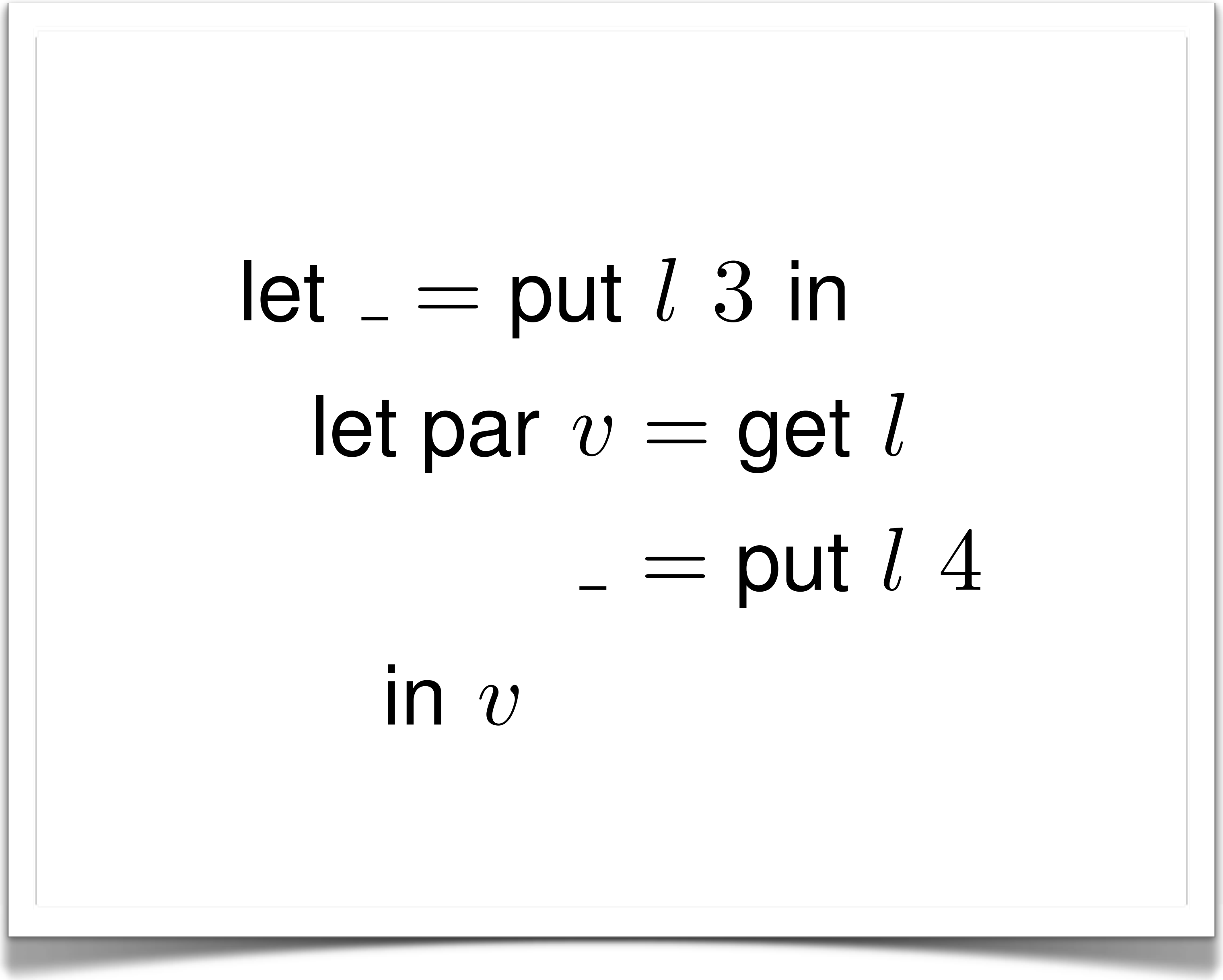

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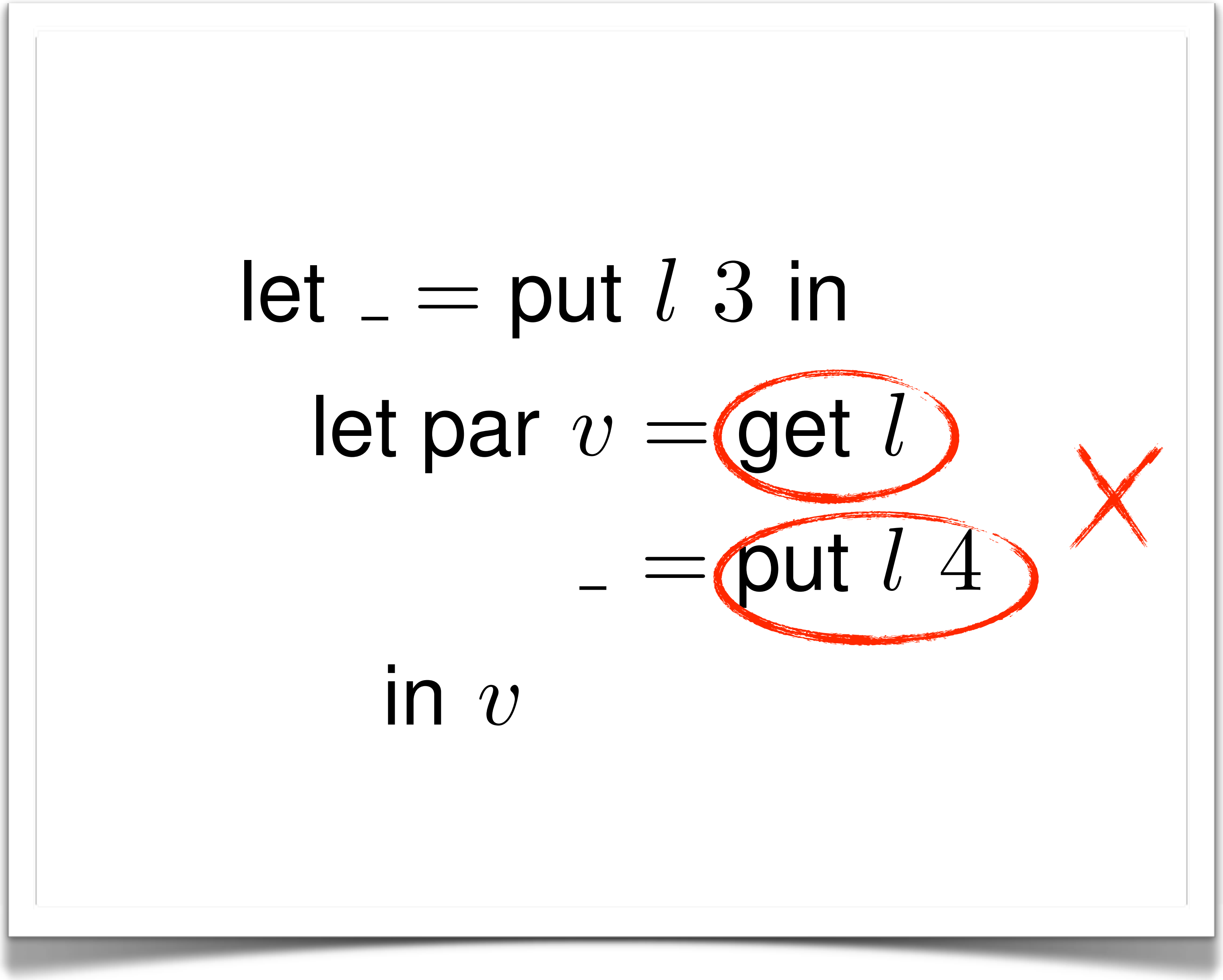

$$
\begin{aligned}
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&\quad \text{let par } v = \text{get } l \\
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\end{aligned}
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✗

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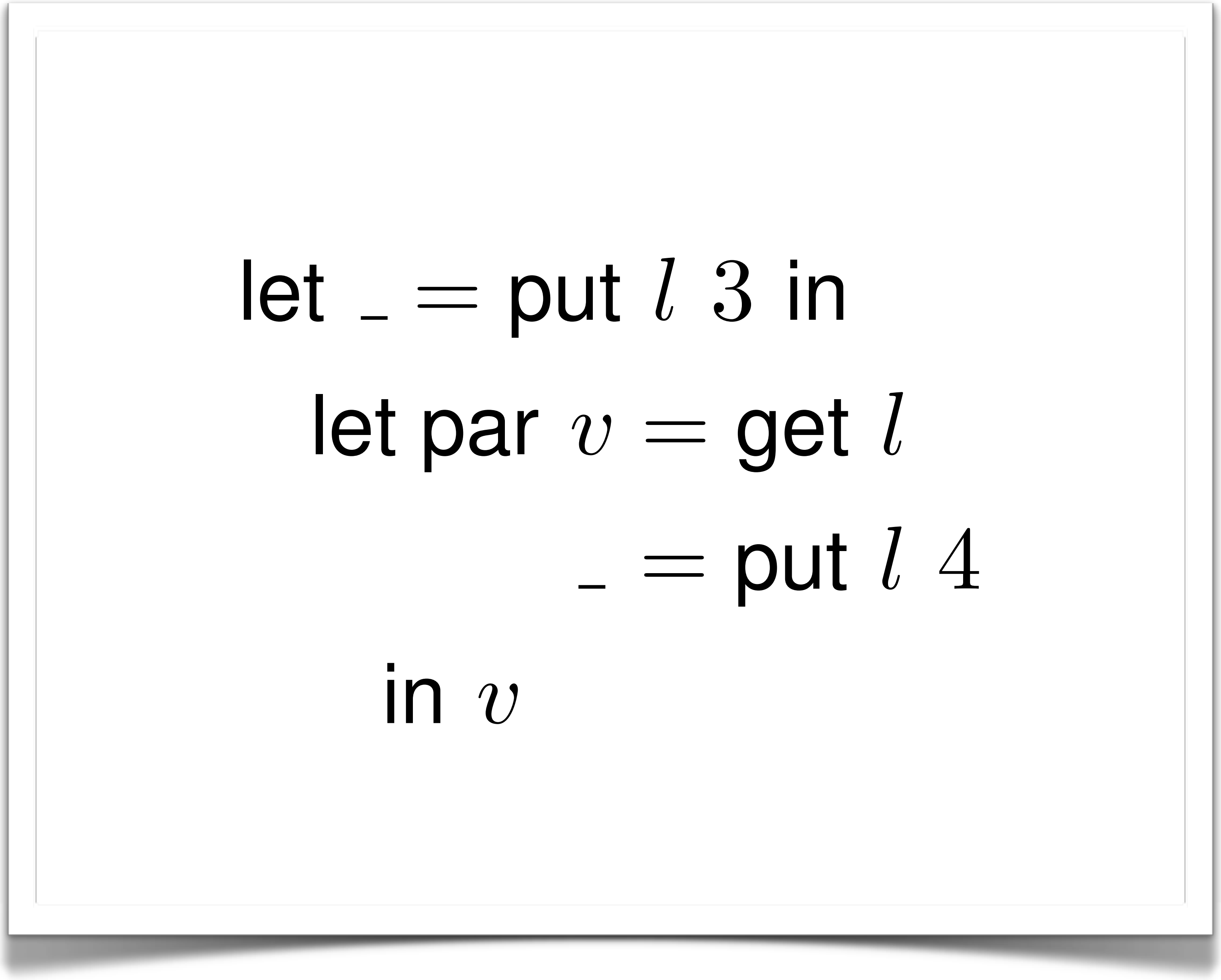

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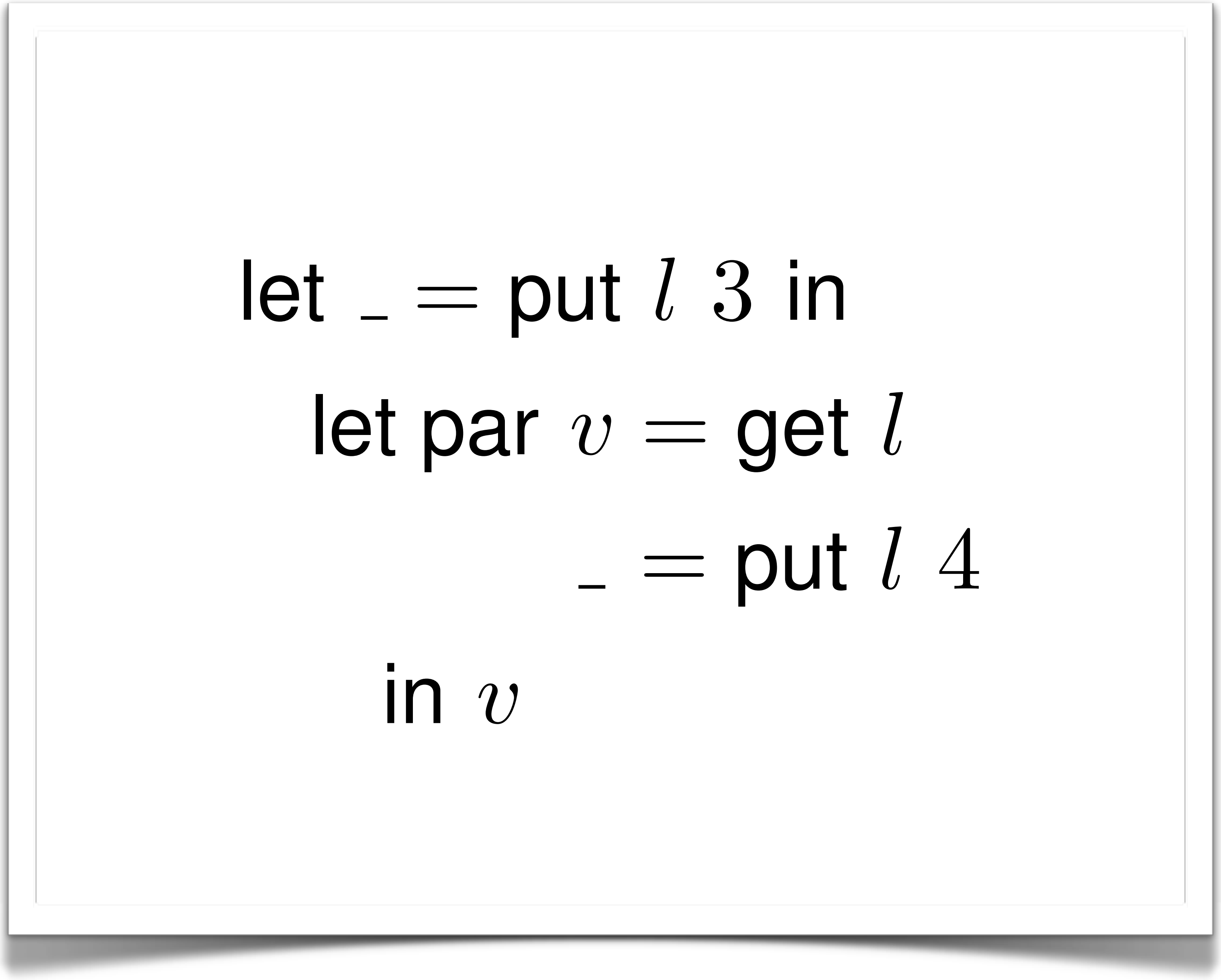

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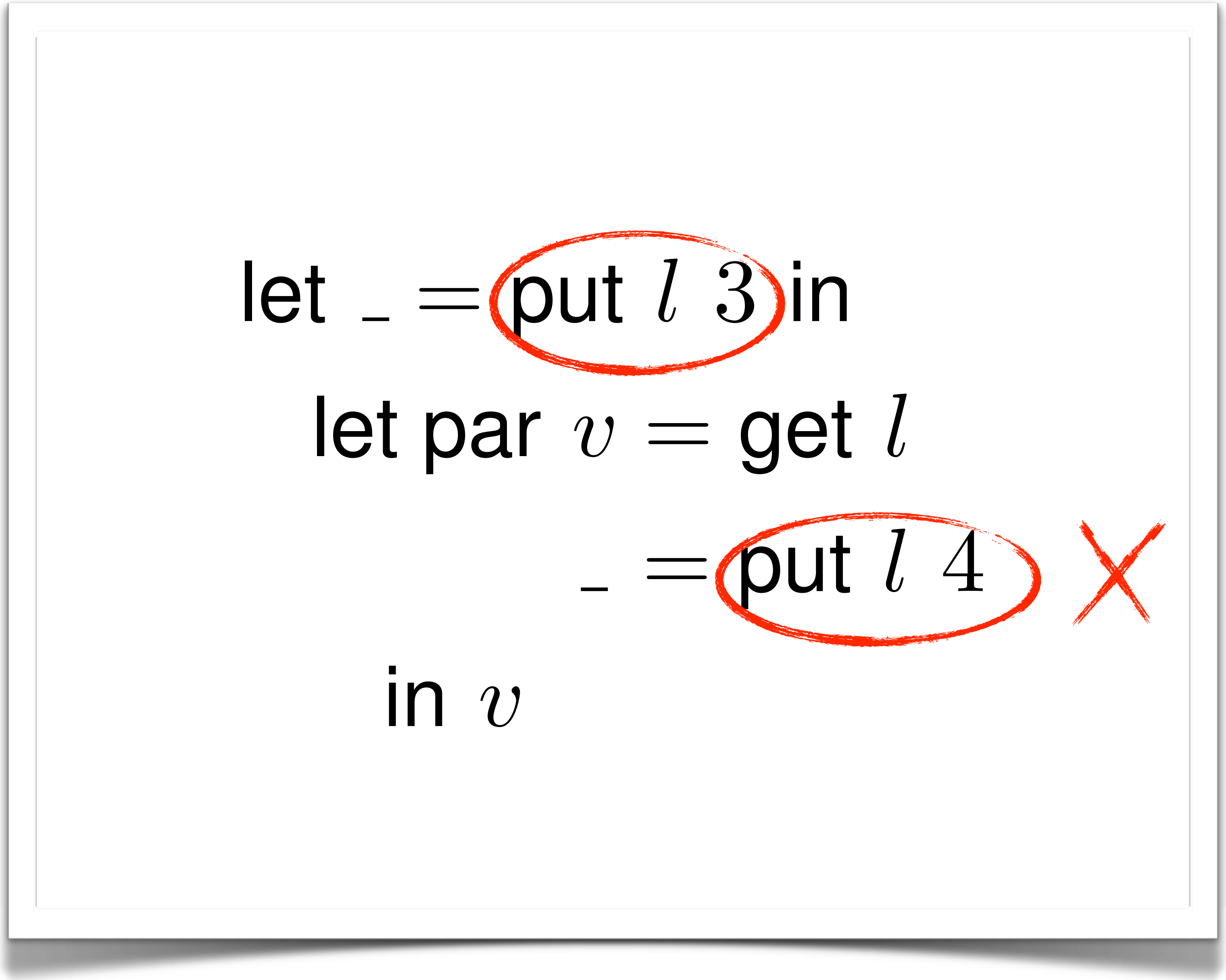

$$\begin{aligned} &\text{let } \_ = \boxed{\text{put } l\ 3} \text{ in}\\ &\quad \text{let par } v = \text{get } l\\ &\qquad\qquad\qquad \_ = \boxed{\text{put } l\ 4} \quad \times\\ &\quad \text{in } v \end{aligned}$$

# A few single-assignment languages

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  - Compel (Tesler and Enea, 1968)

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- Today:
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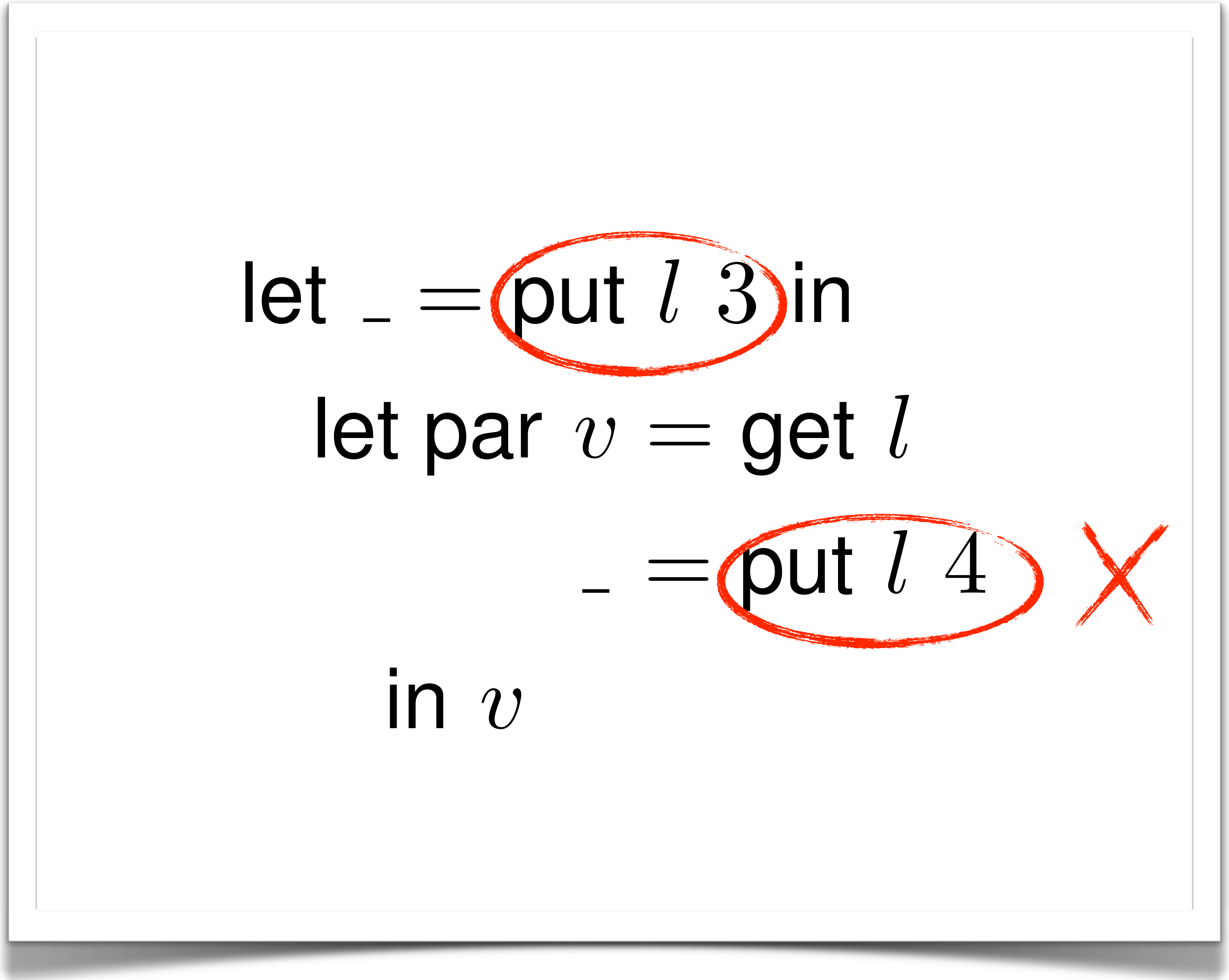
  - monad-par for Haskell (Marlow *et al.*, 2011)

# Disallow multiple assignment?



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          _ = put l 4   ✗
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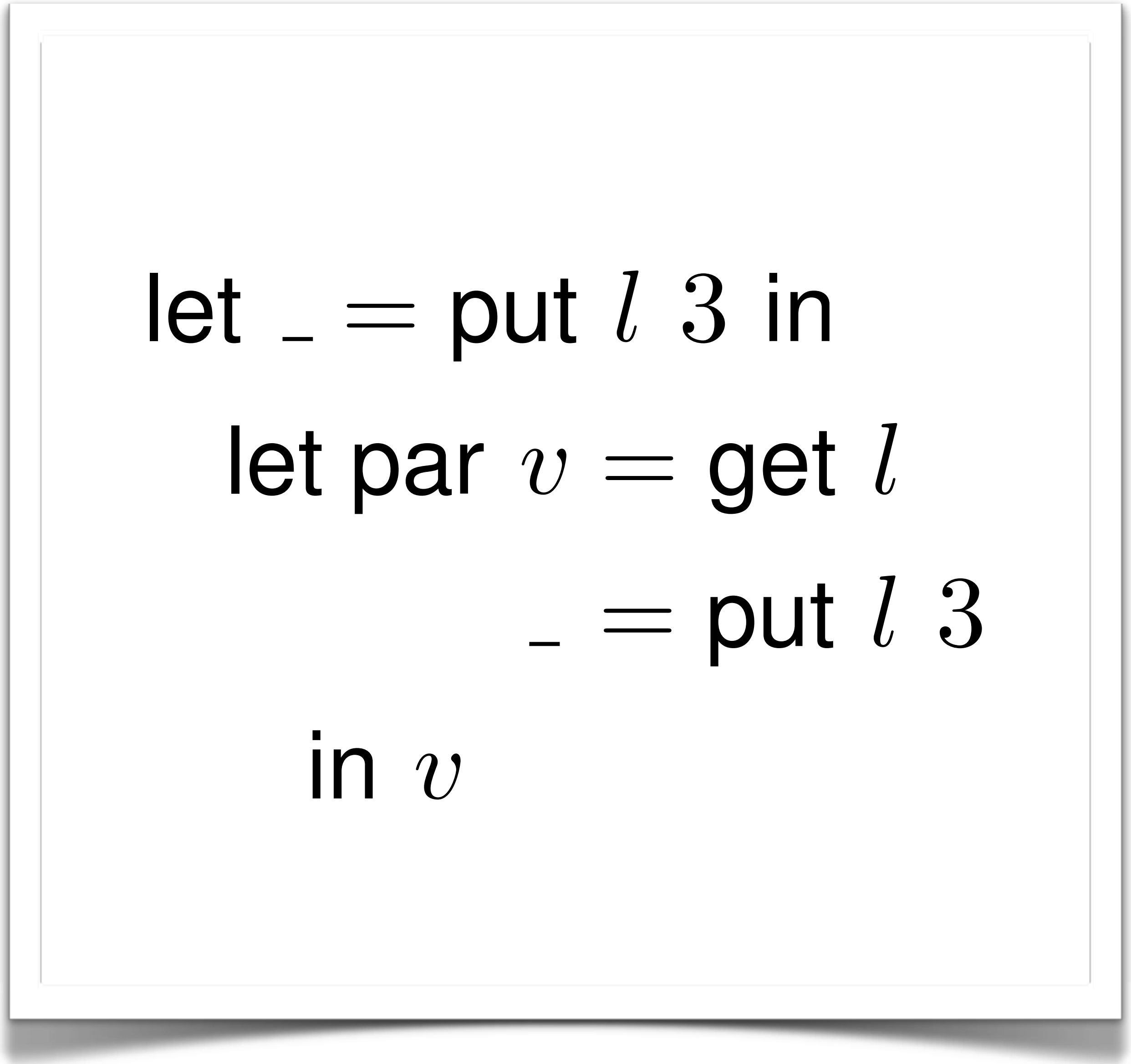
# Deterministic programs that single-assignment forbids



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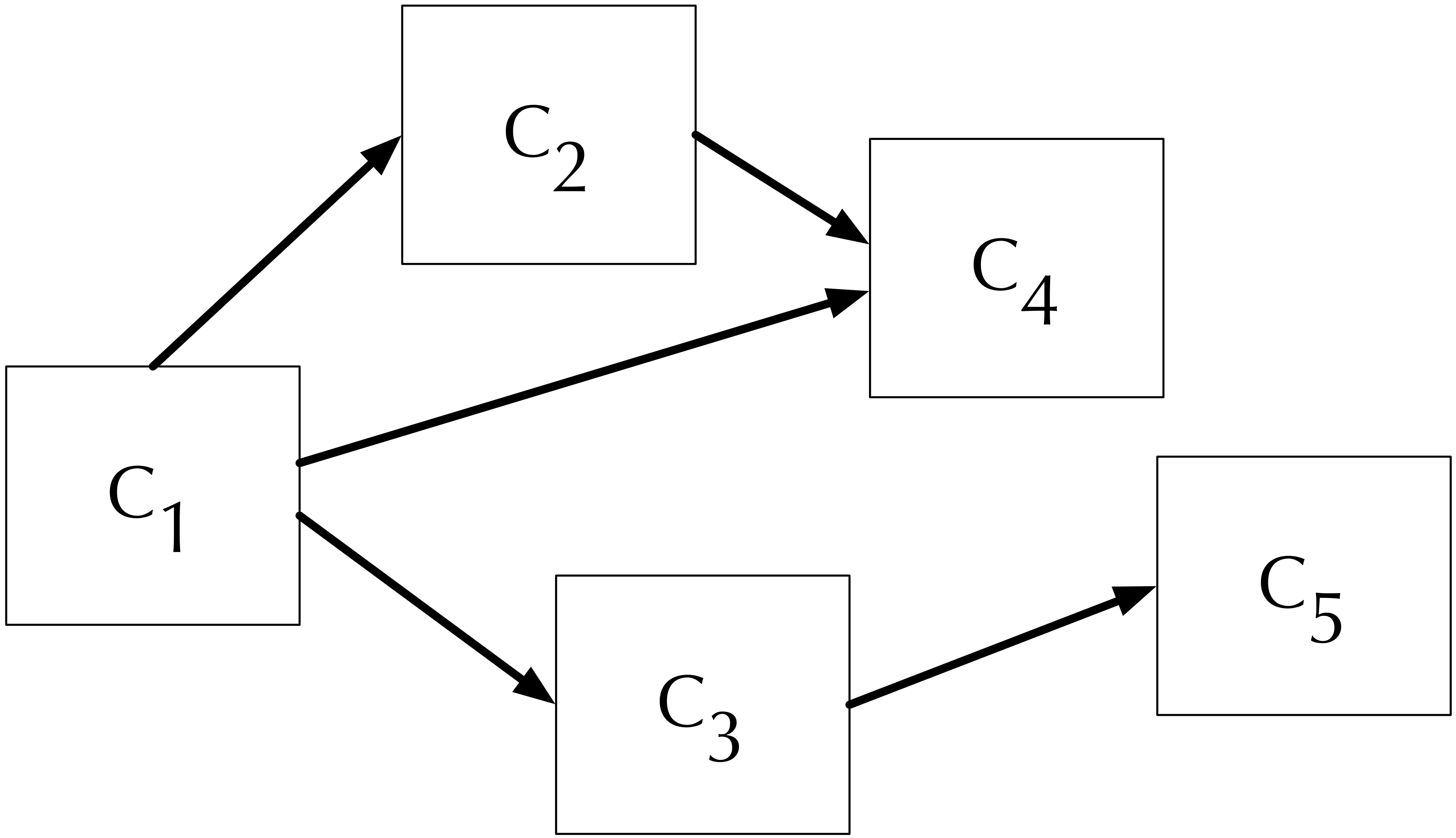
```
let par $\_ =$ put $l\ (4, \bot)$
        $\_ =$ put $l\ (\bot, 3)$
  in let $v =$ get $l$ in $v$
```

# Kahn process networks (Kahn, 1974)



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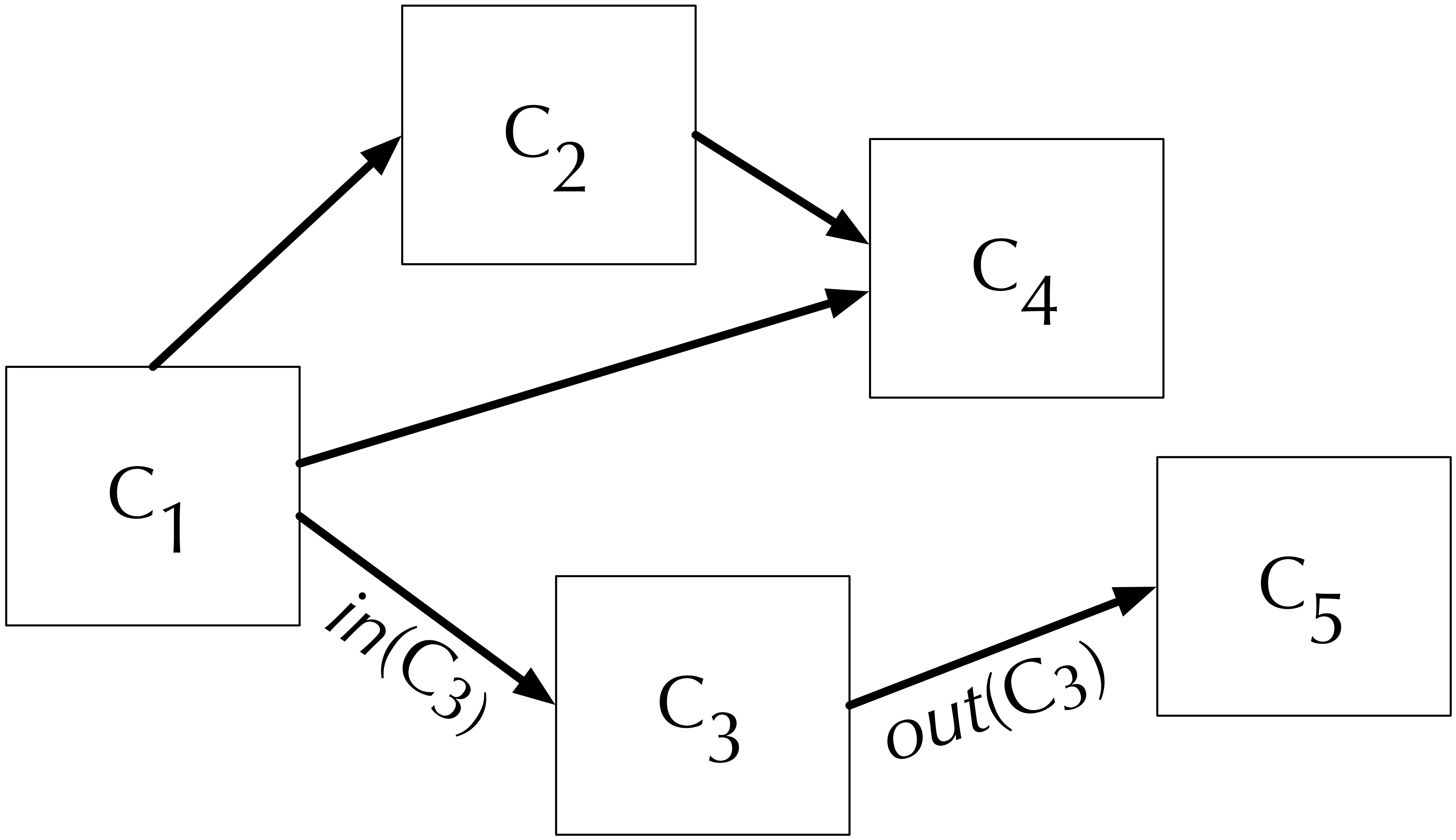

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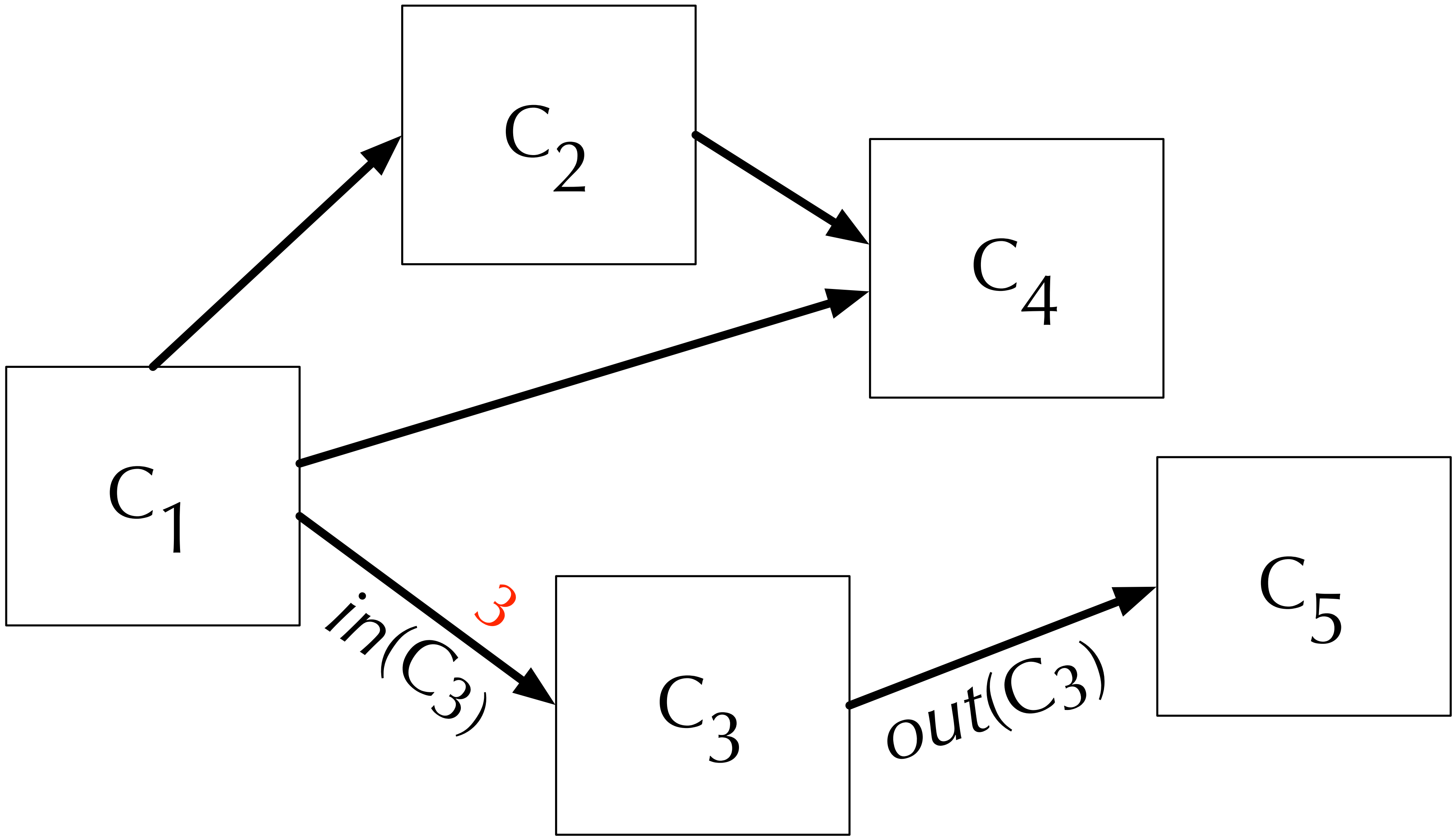

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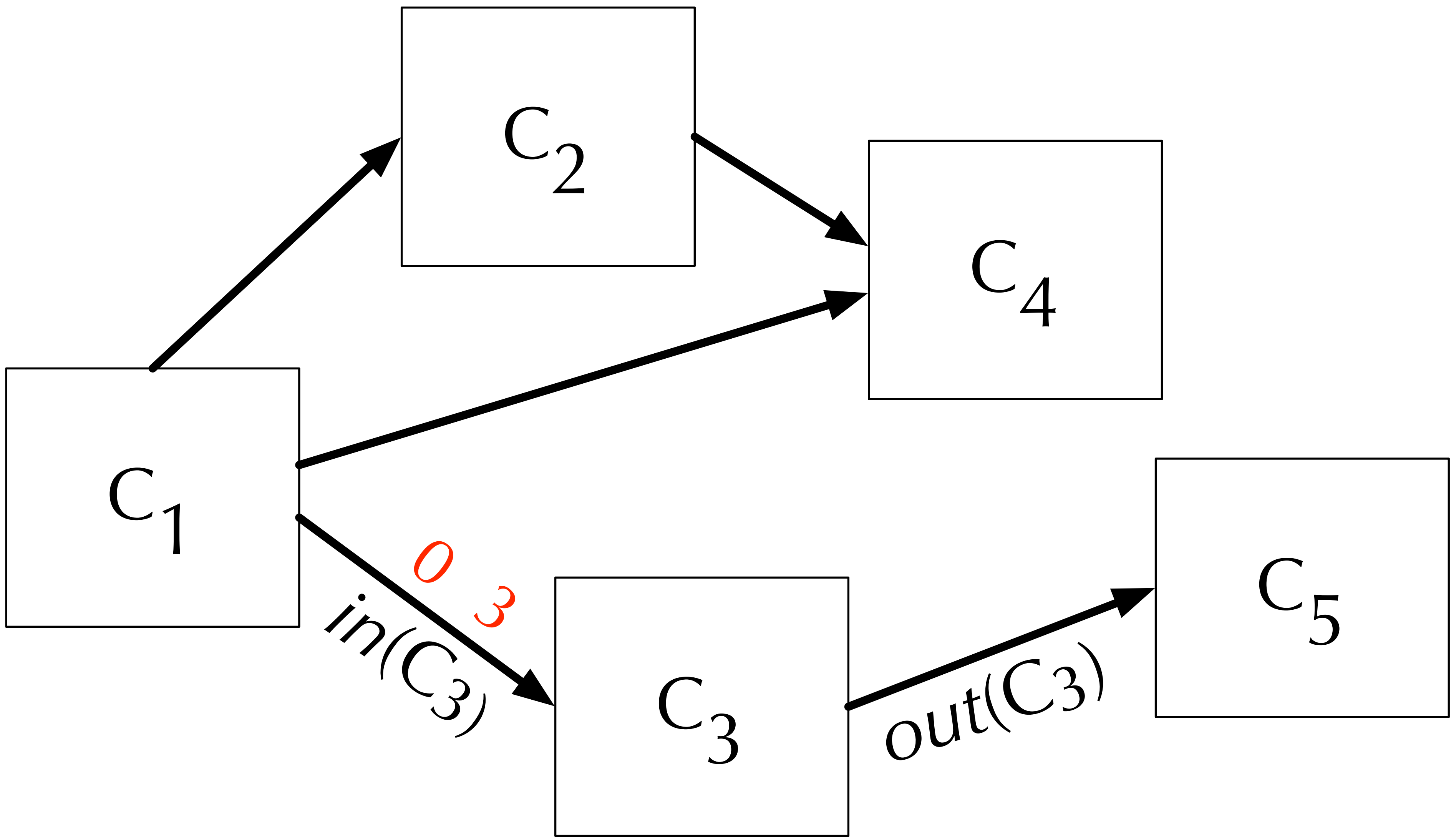

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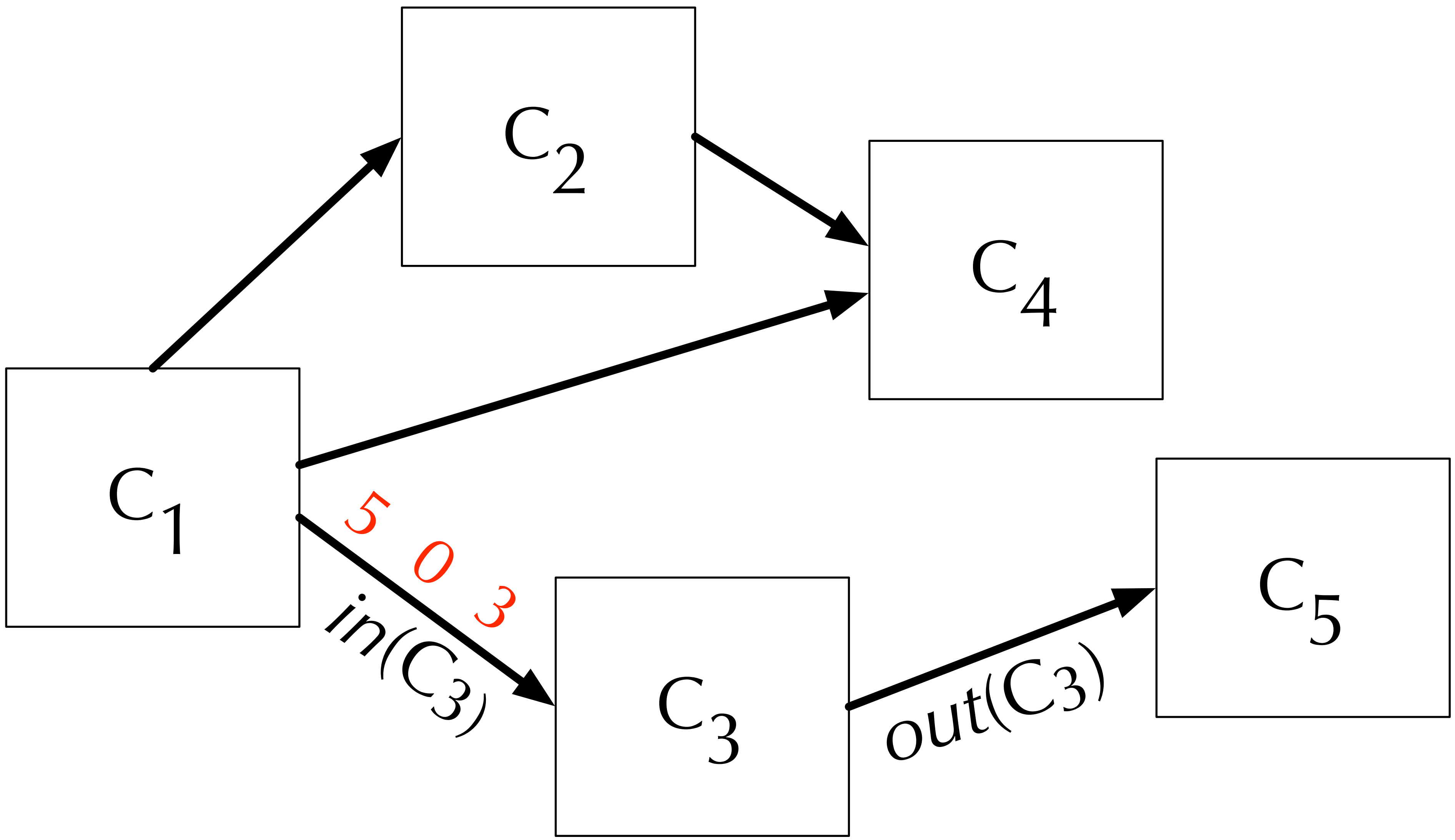

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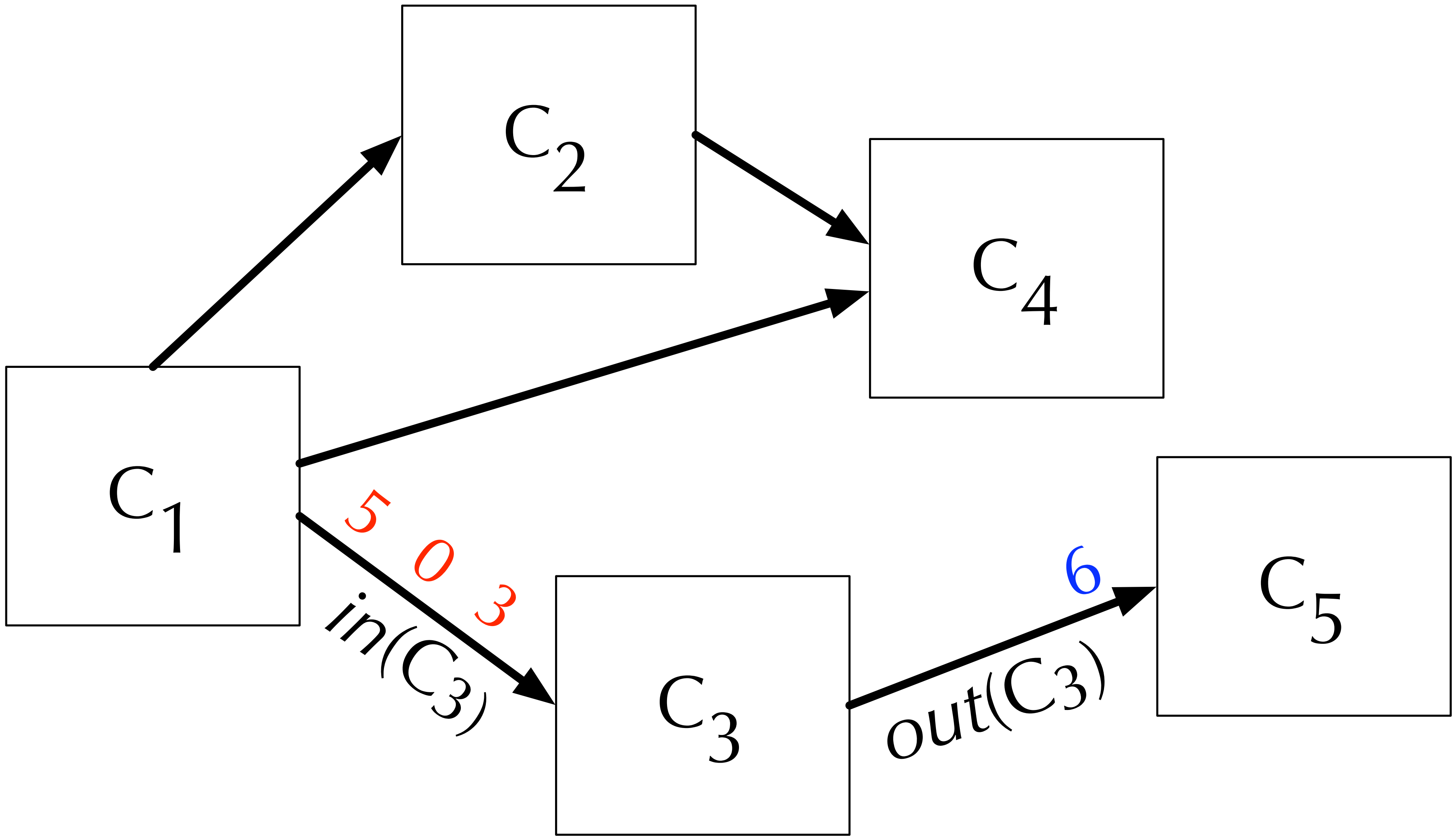

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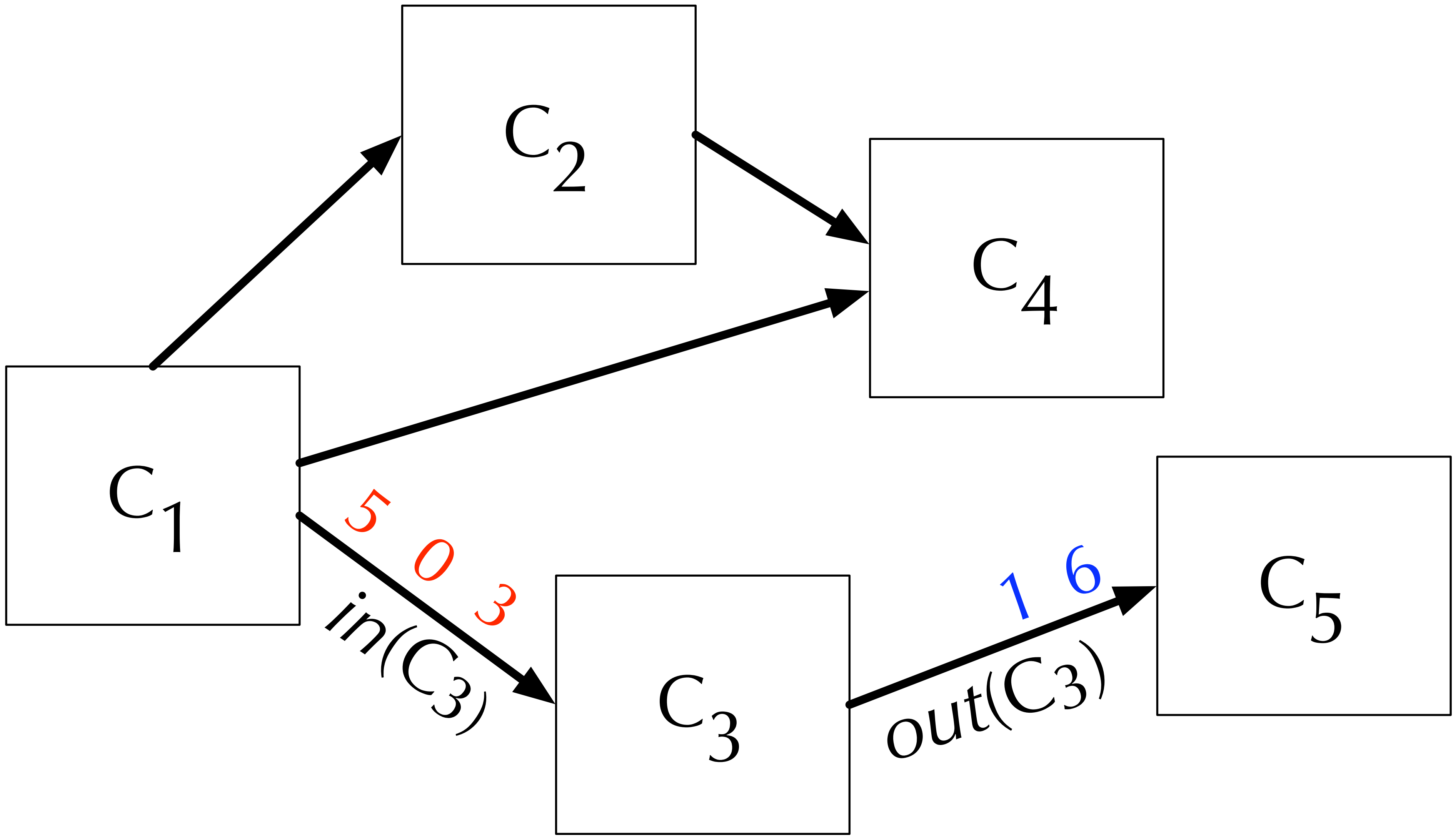

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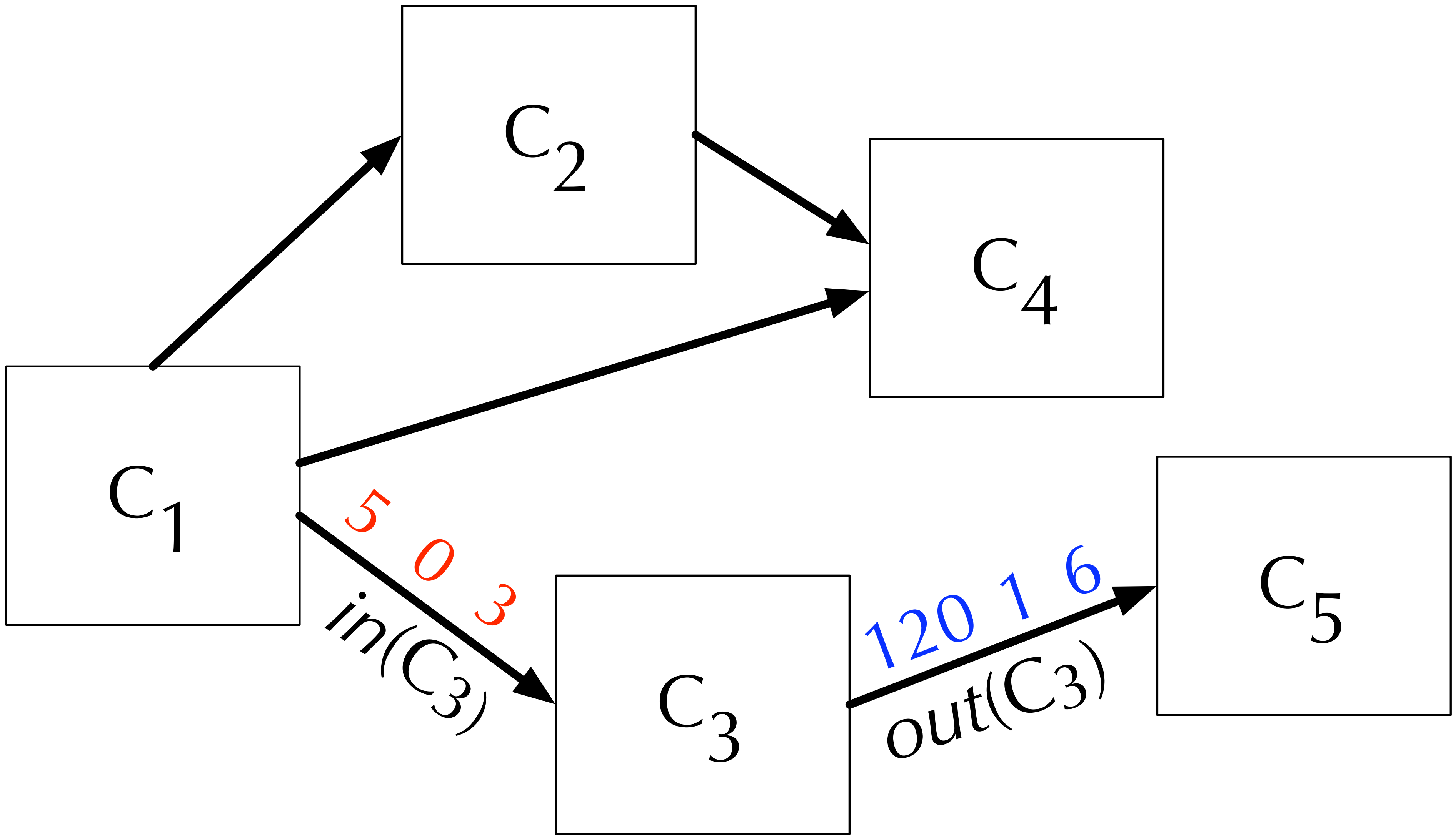

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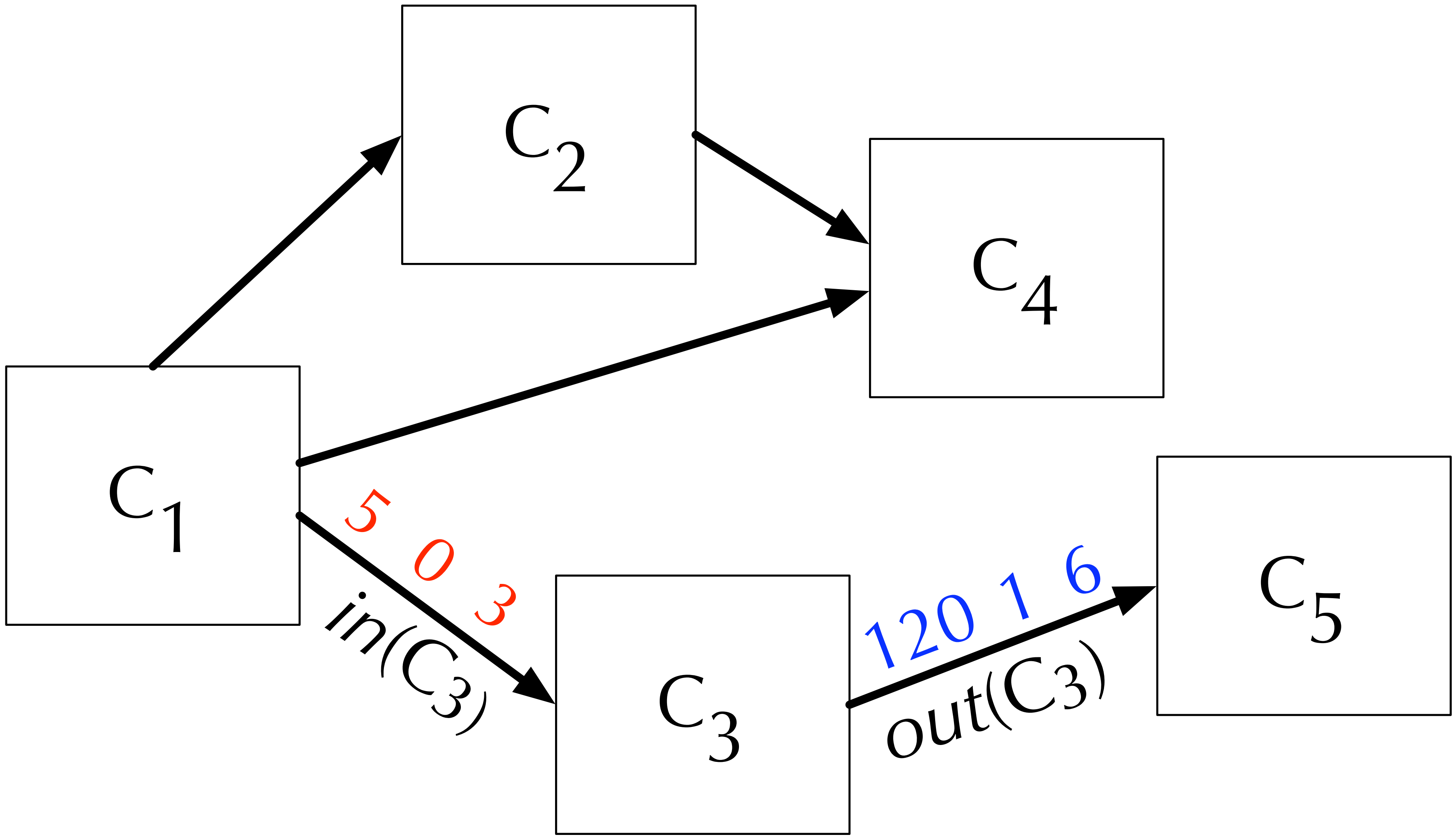

*hist*(*in*(C₃)): [3, 0, 5, ...]

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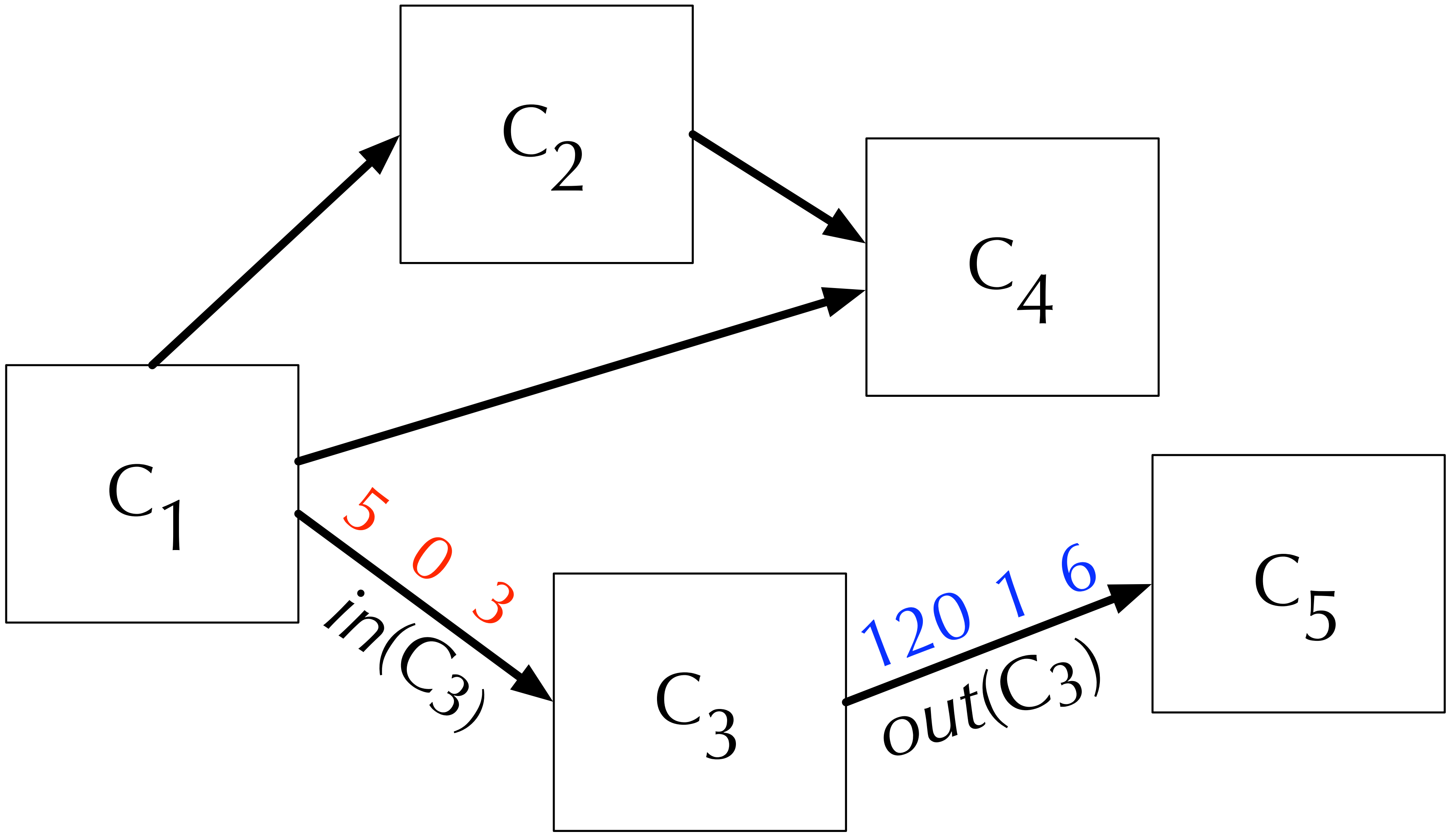
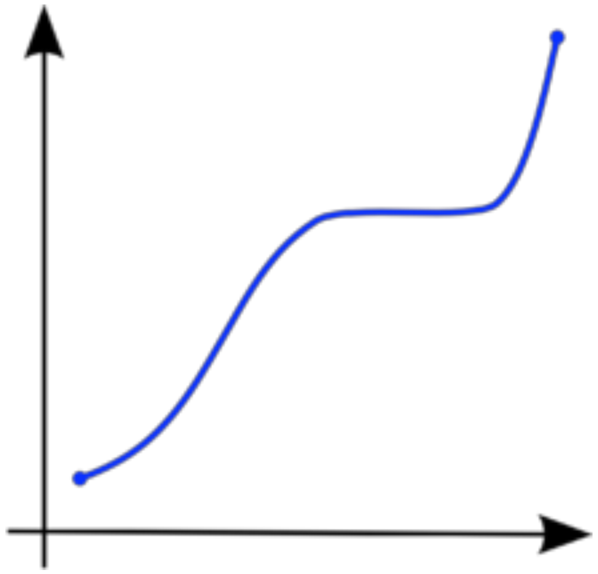
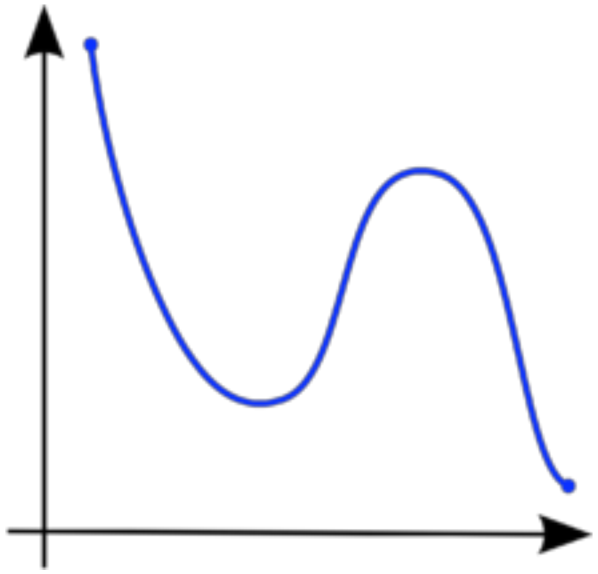


*hist*(*in*(C3)): [3, 0, 5, ...]  *hist*(*out*(C3)): [6, 1, 120, ...]

# Monotonicity





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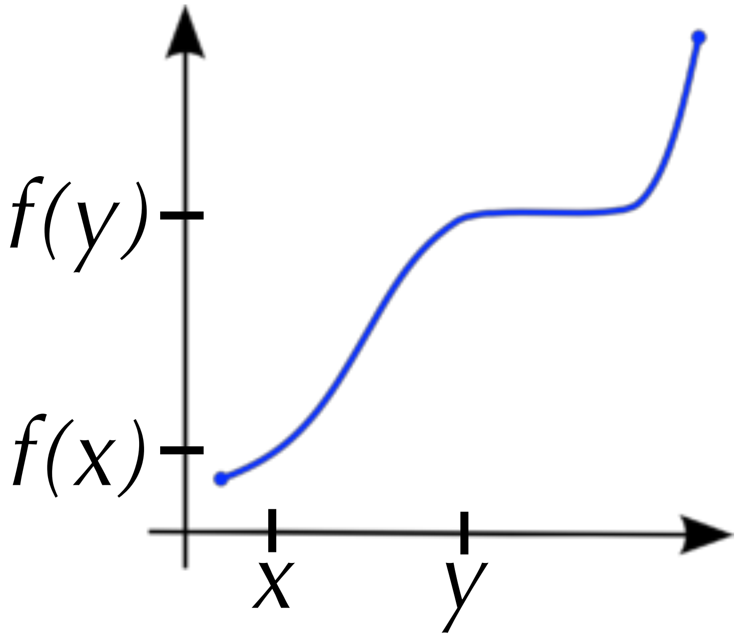

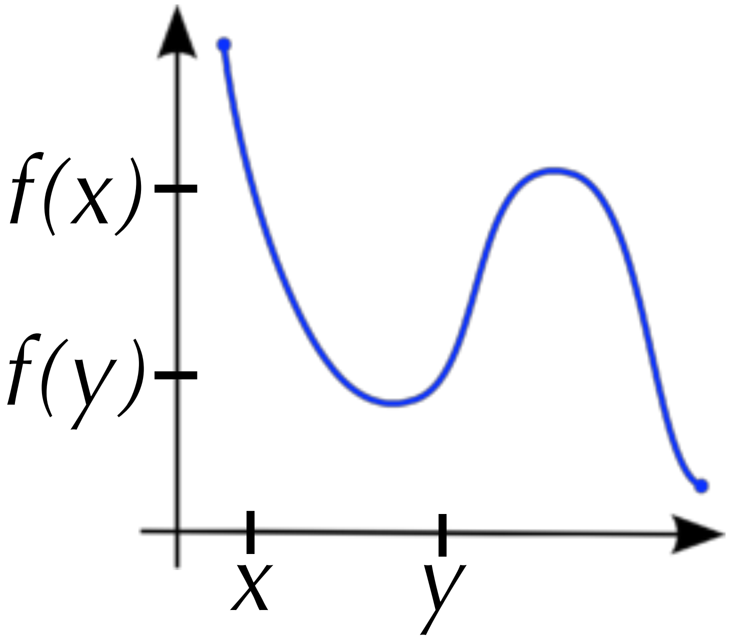

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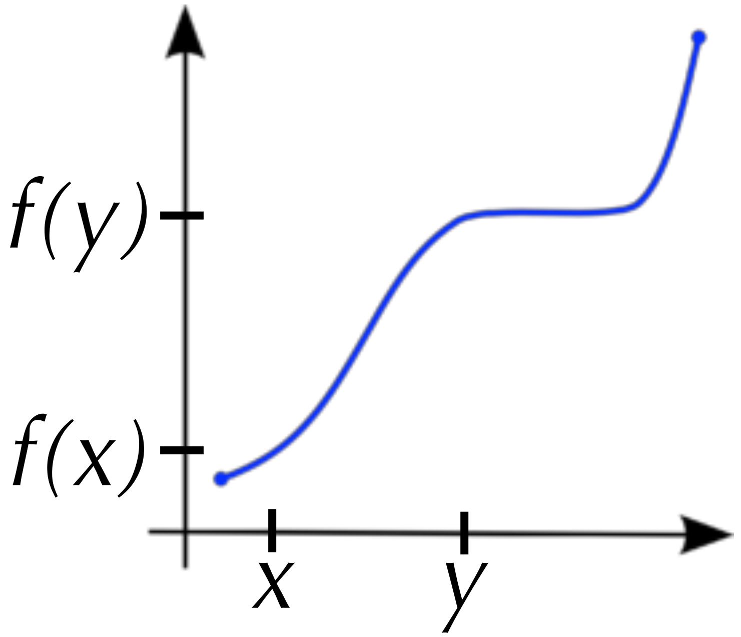

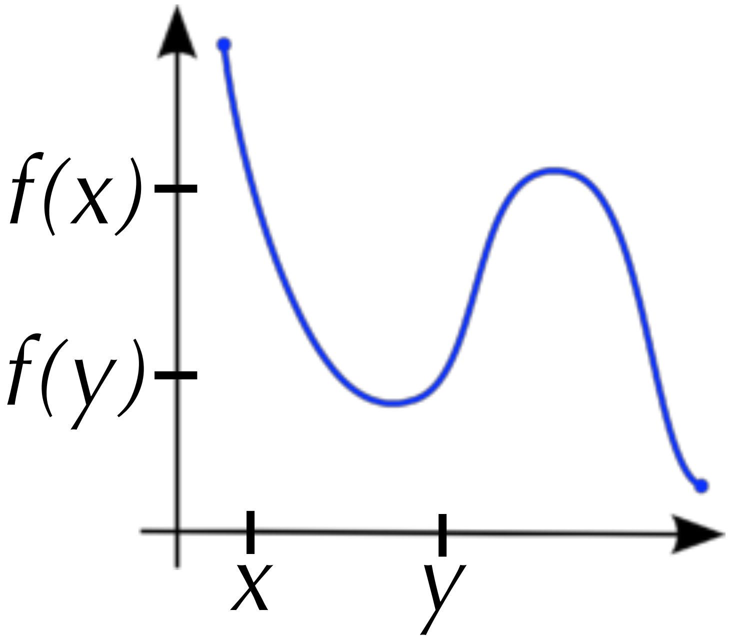

$f$ is monotonic iff $x \leq y \implies f(x) \leq f(y)$

# Monotonicity in KPNs

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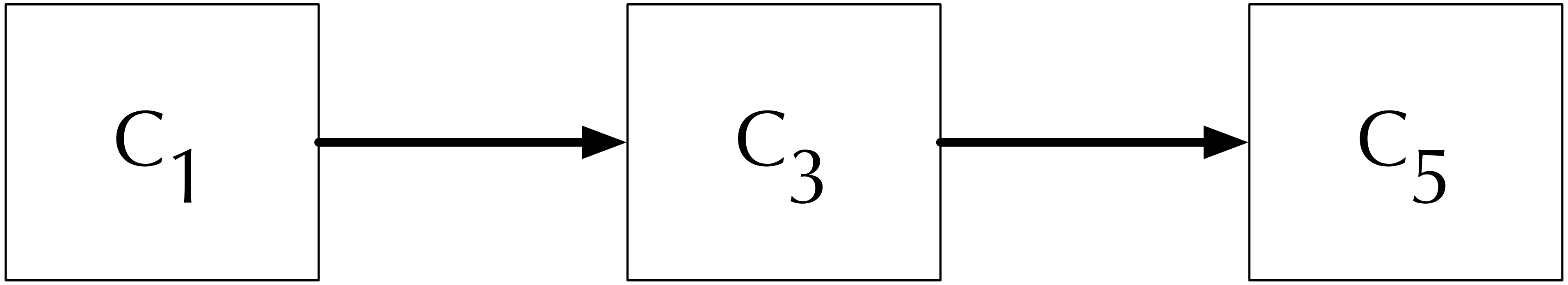

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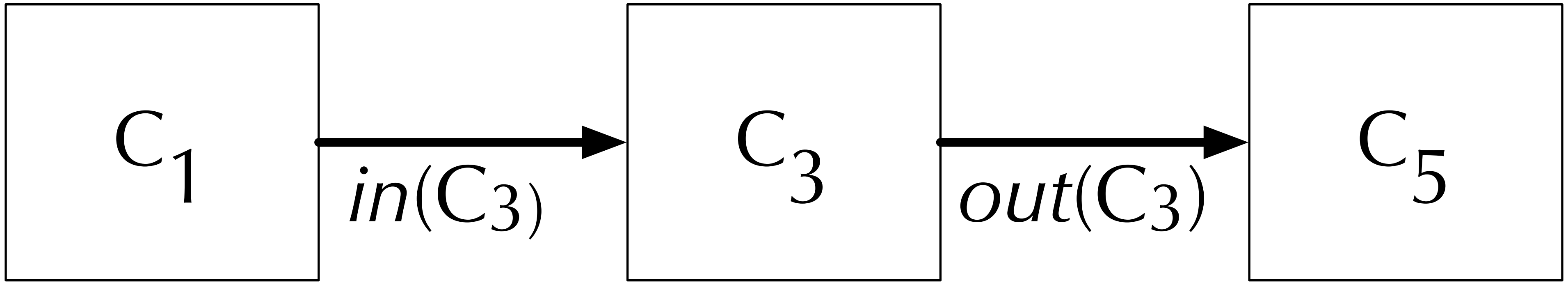

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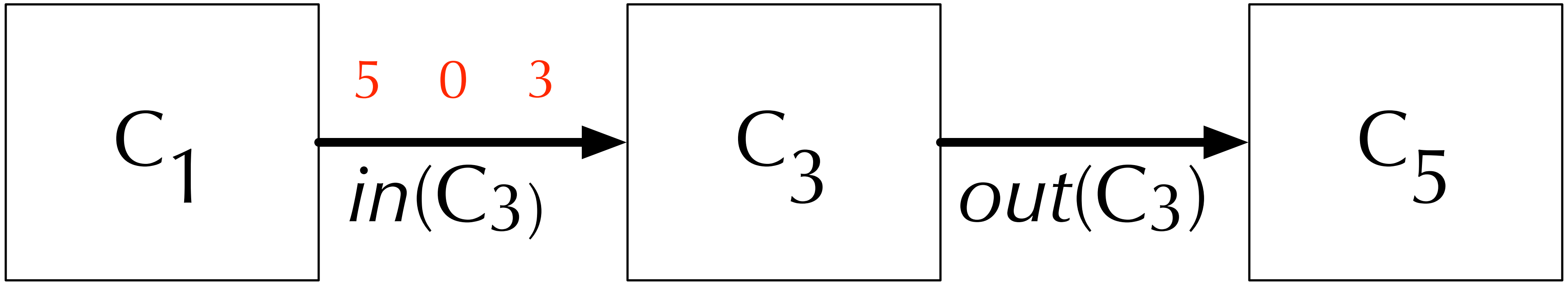

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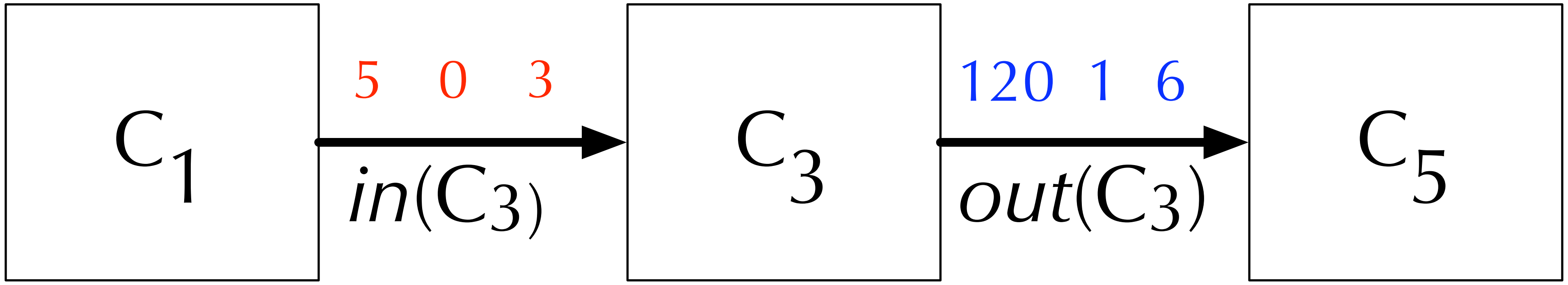

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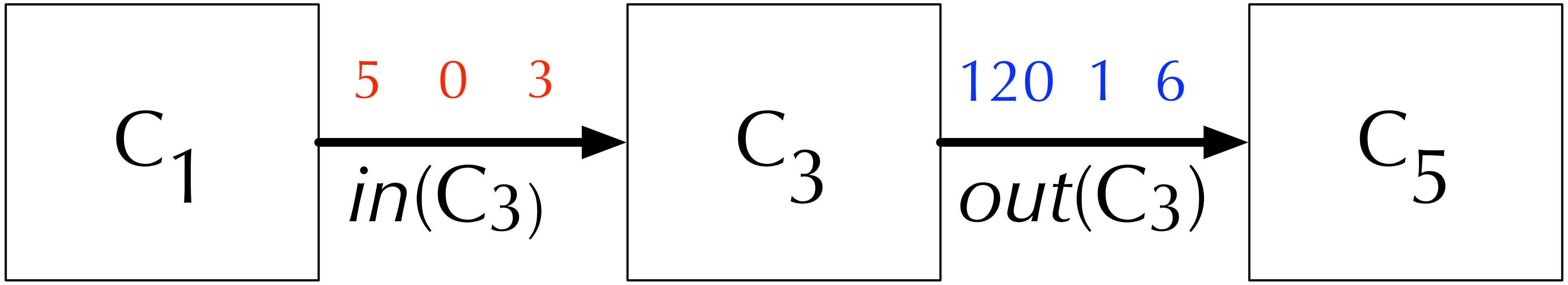


For KPNs, the ≤ relation is just *prefix-of*:

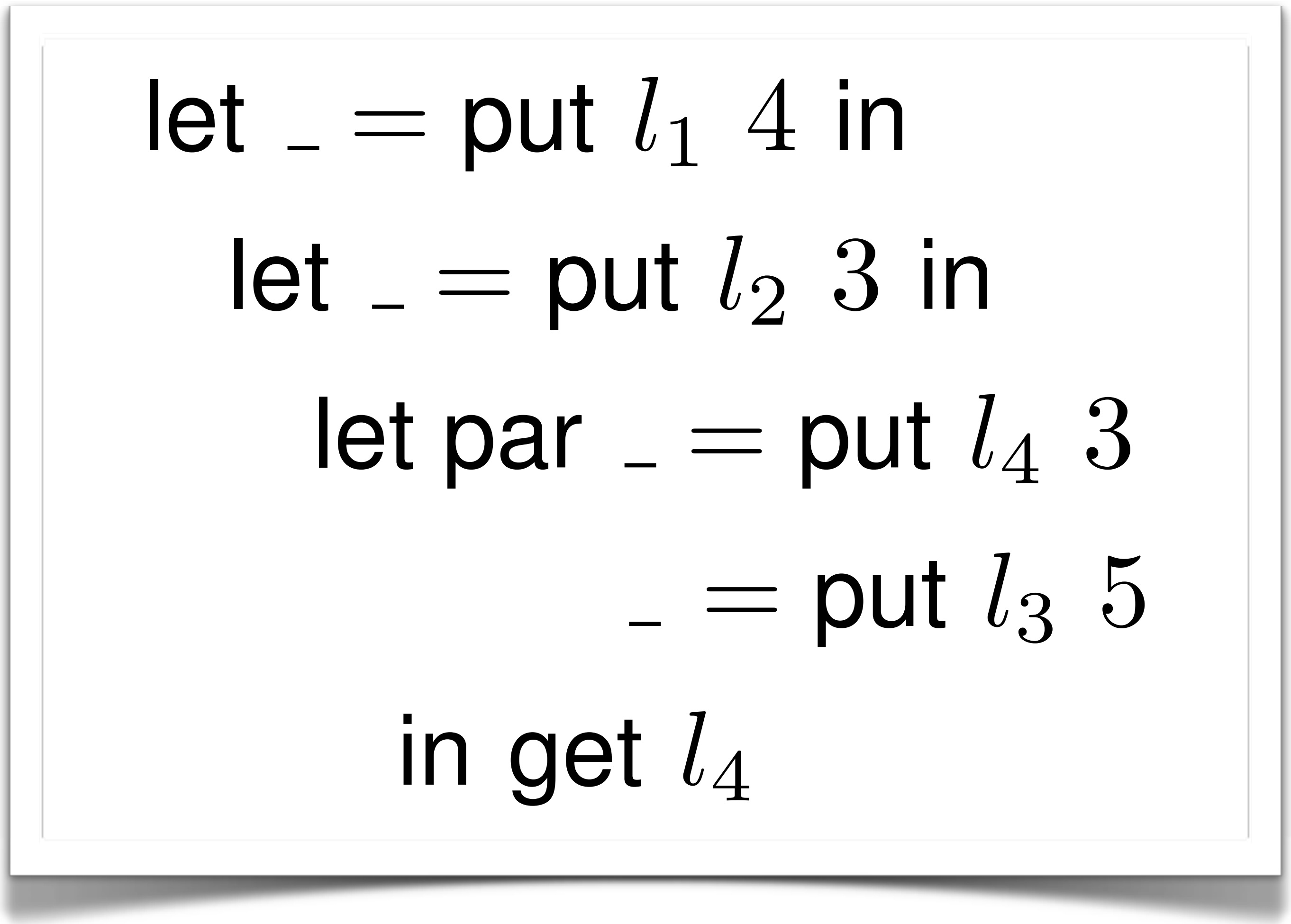
[3] *prefix-of* [3, 0] $\implies$ [6] *prefix-of* [6, 1]

[3, 0] *prefix-of* [3, 0, 5] $\implies$ [6, 1] *prefix-of* [6, 1, 120]

…

Monotonicity causes deterministic parallelism!

# Back to single-assignment languages



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   let _ = put $l_2$ $3$ in
      let par _ = put $l_4$ $3$
                 _ = put $l_3$ $5$
         in get $l_4$
```

Store:

# Back to single-assignment languages

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Store:

| | |
|---|---|
| $l_1$ | $4$ |

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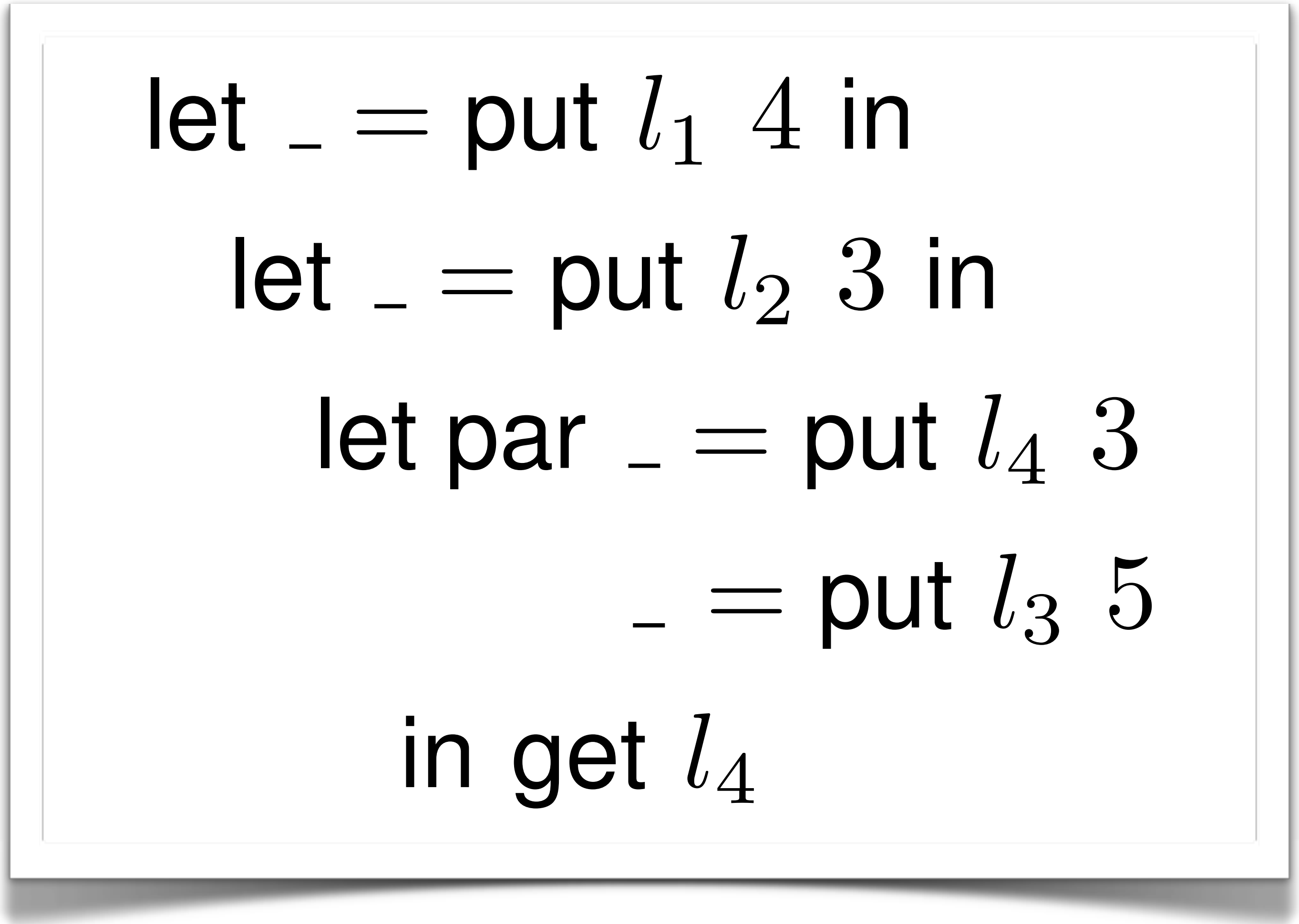

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            in get $l_4$
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Store:

| | |
|---|---|
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| $l_2$ | 3 |
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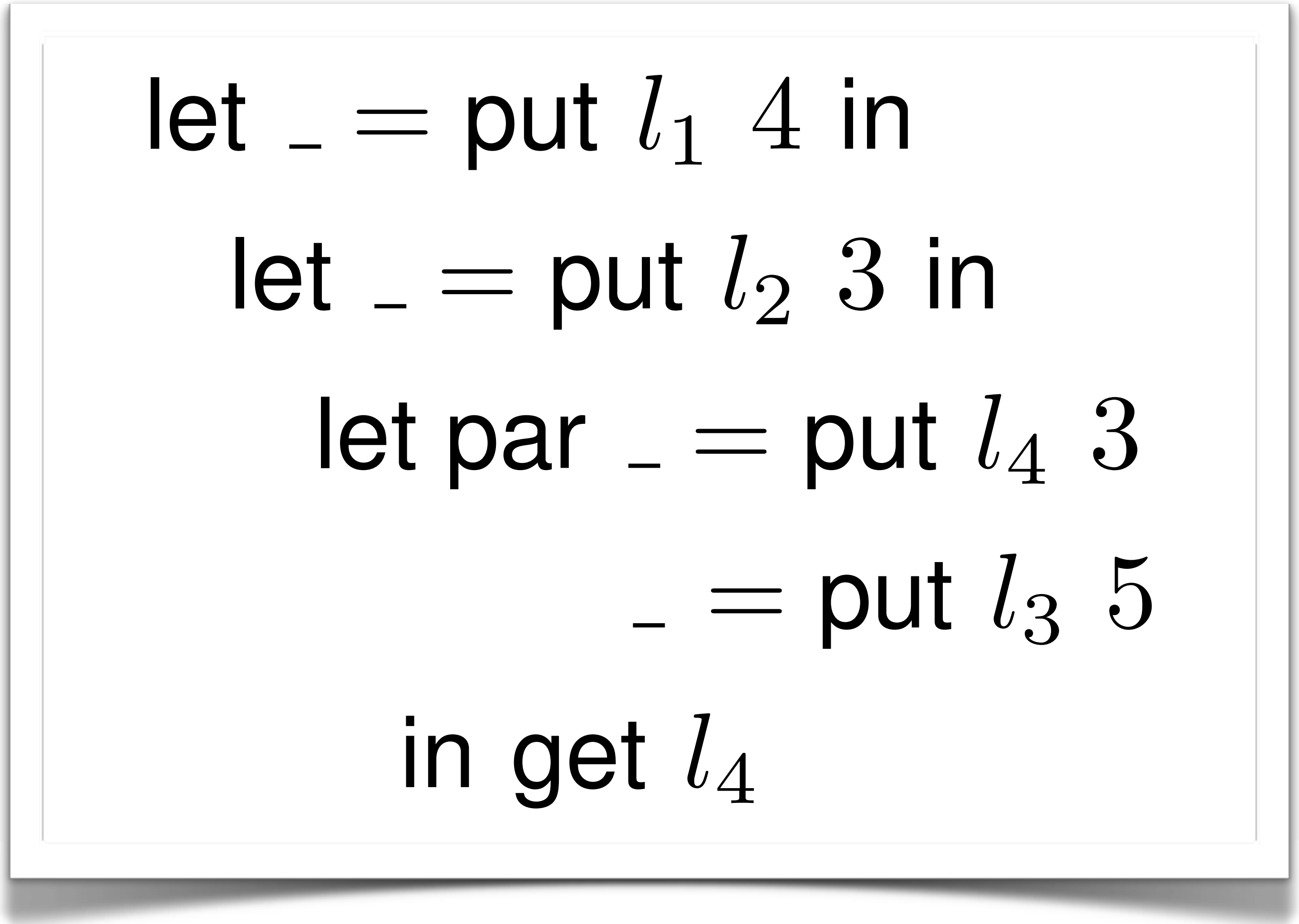

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| $l_4$ | $3$ |

For stores, the ≤ relation is ⊆:

$$\{l_1 \rightarrow 4,\ l_2 \rightarrow 3\} \subseteq \{l_1 \rightarrow 4,\ l_2 \rightarrow 3,\ l_3 \rightarrow 5\} \Longrightarrow$$

$$\{l_1 \rightarrow 4,\ l_2 \rightarrow 3,\ l_4 \rightarrow 3\} \subseteq \{l_1 \rightarrow 4,\ l_2 \rightarrow 3,\ l_3 \rightarrow 5,\ l_4 \rightarrow 3\}$$

# Generalizing our notion of monotonicity

For stores, the $\leq$ relation is $\subseteq$:

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- Given stores $S$ and $S'$, we say that $S \leq S'$ iff:
  - dom($S$) $\subseteq$ dom($S'$), and
  - for all locations $l$ in dom($S$), $S(l) = S'(l)$

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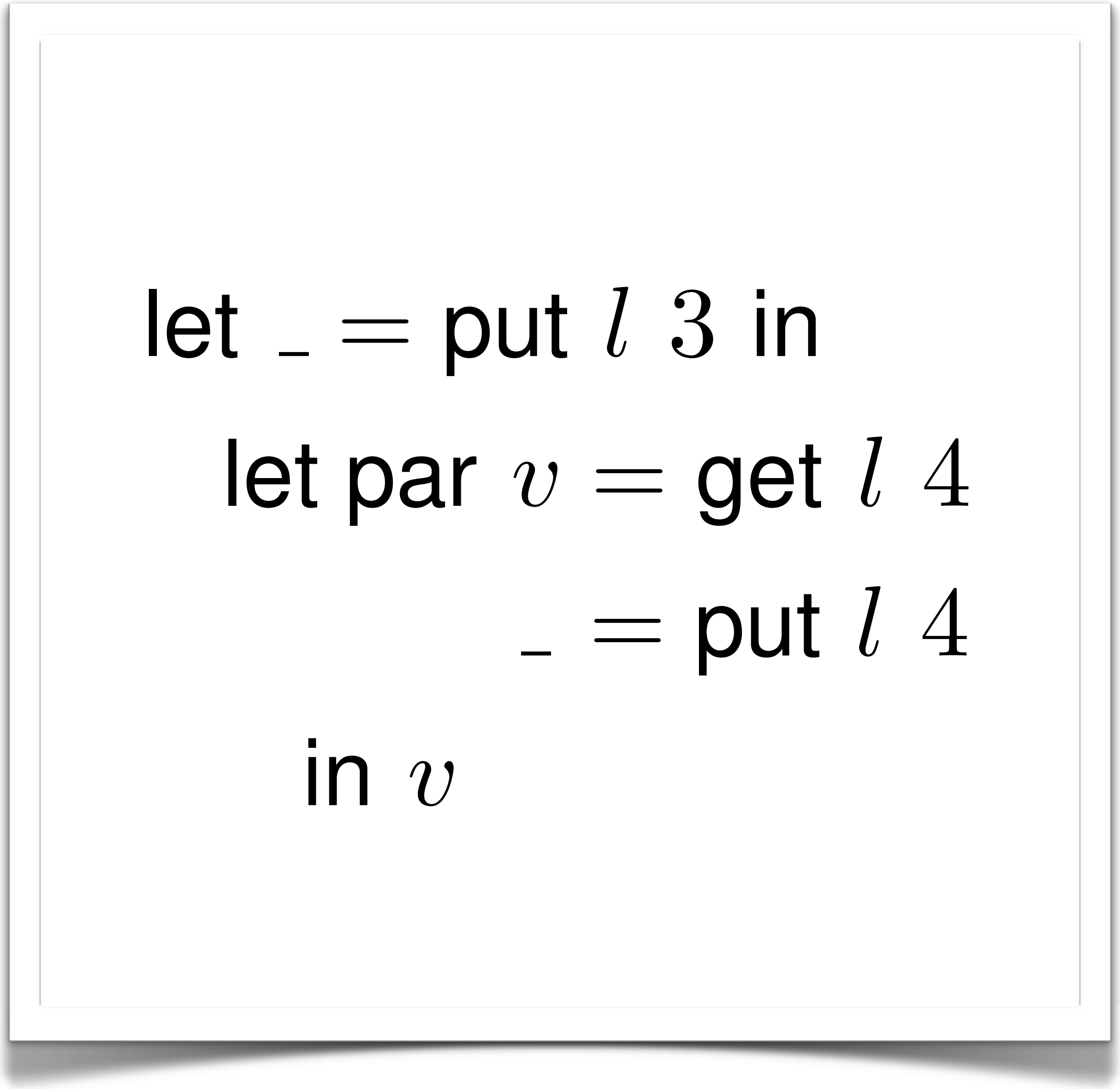
$\leq$

user-specified

# Idea: restrict reads to “threshold” reads



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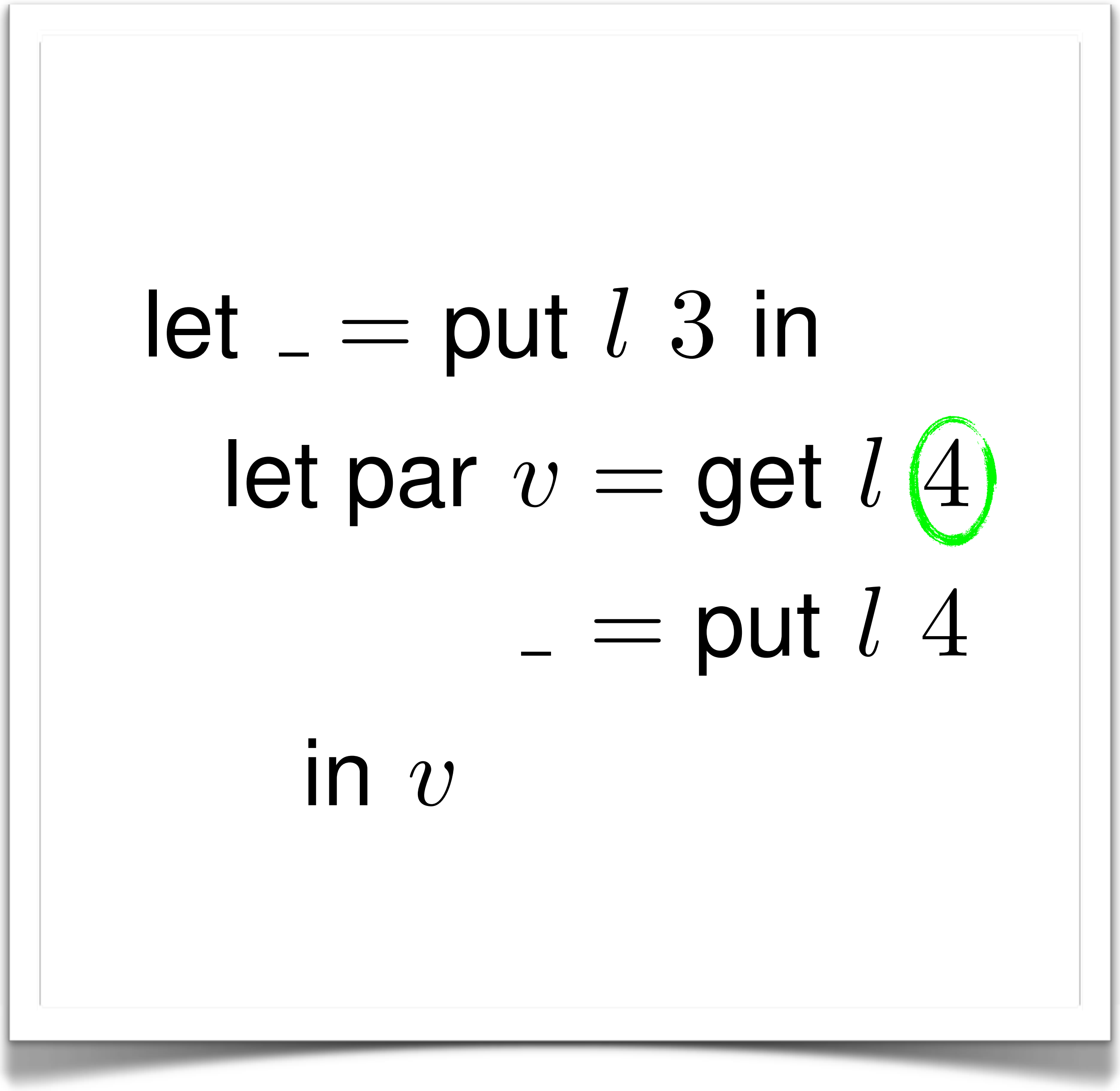

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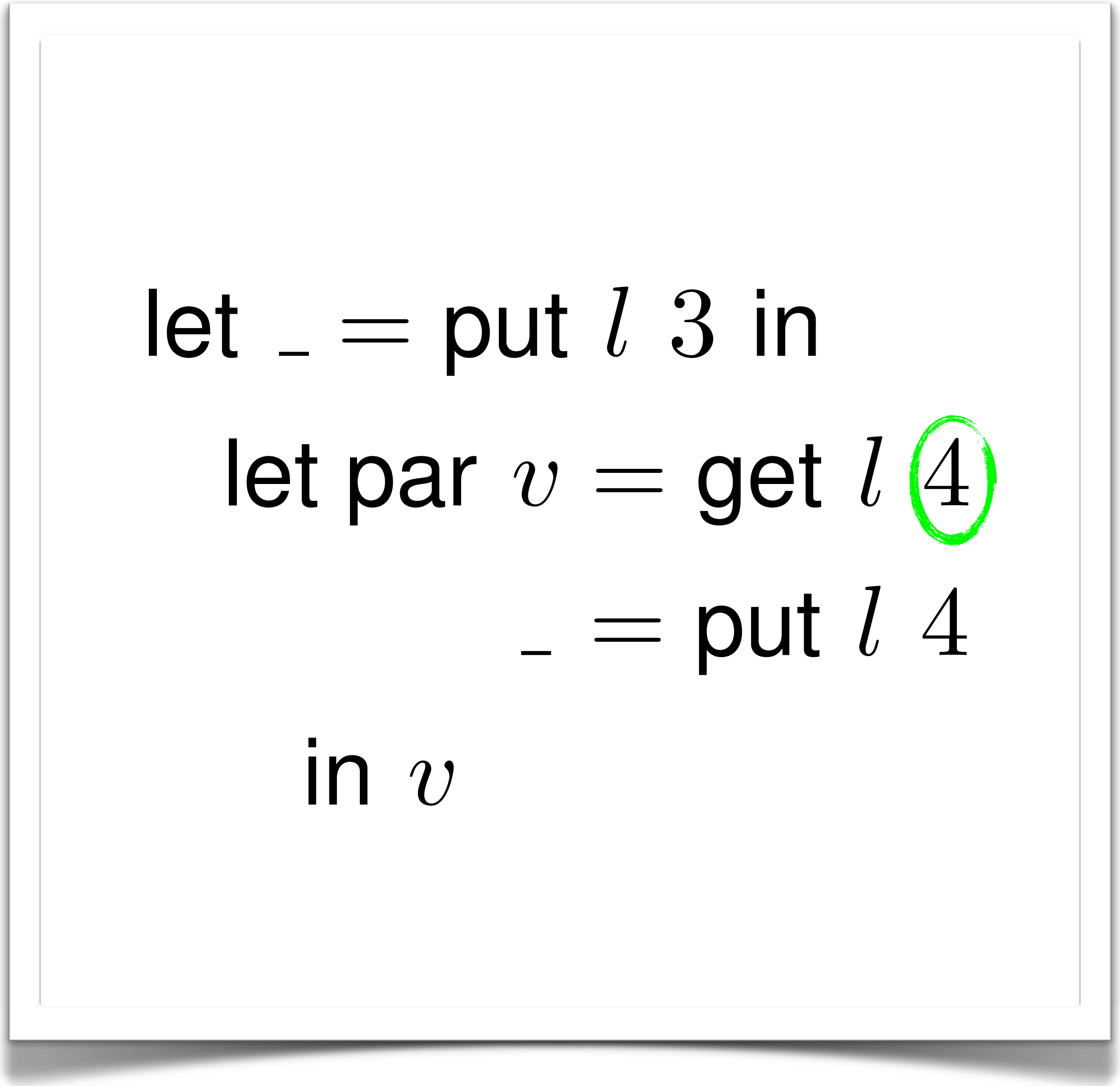

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$\_ =$ put $l\ 4$

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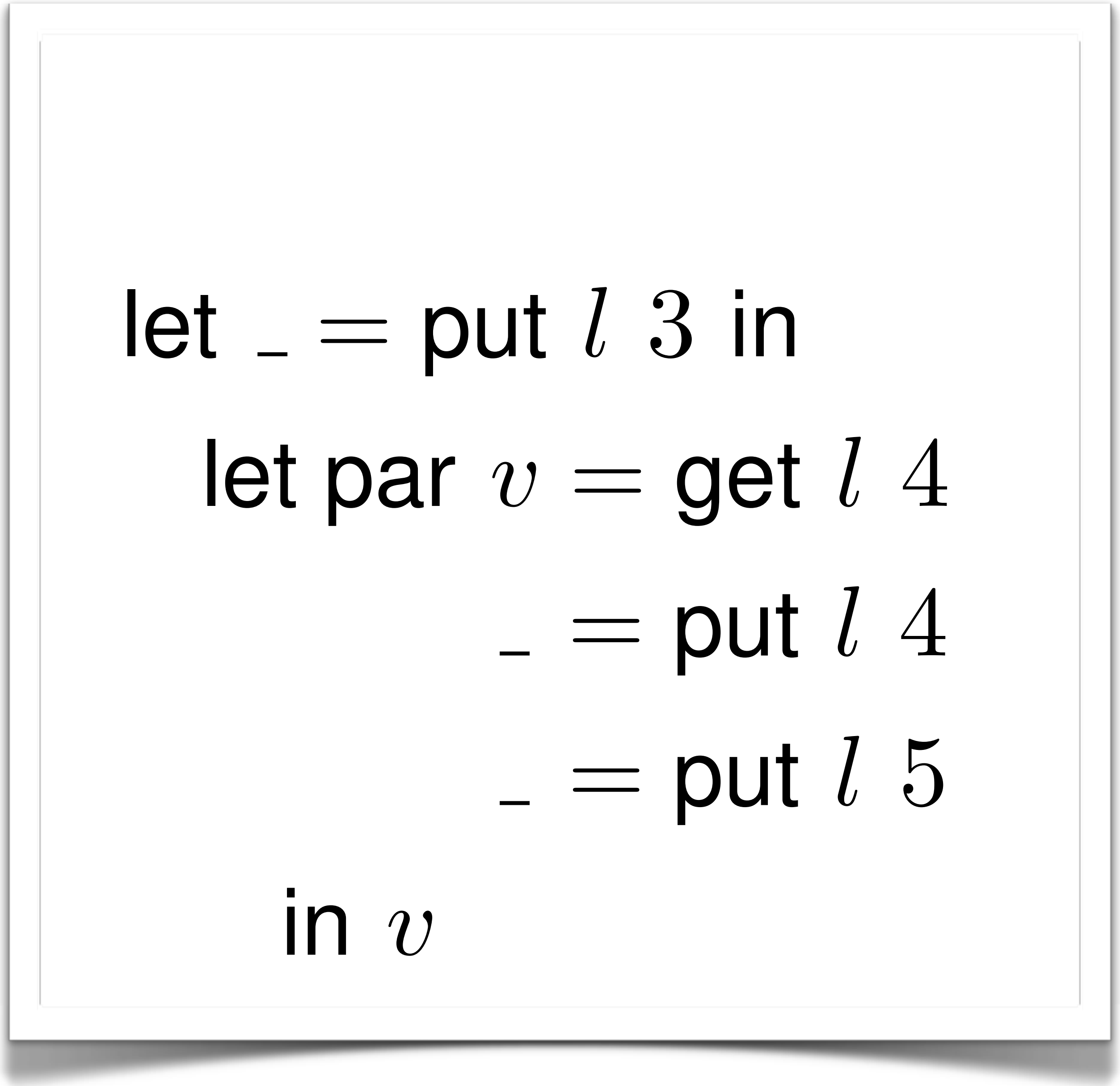

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in $v$

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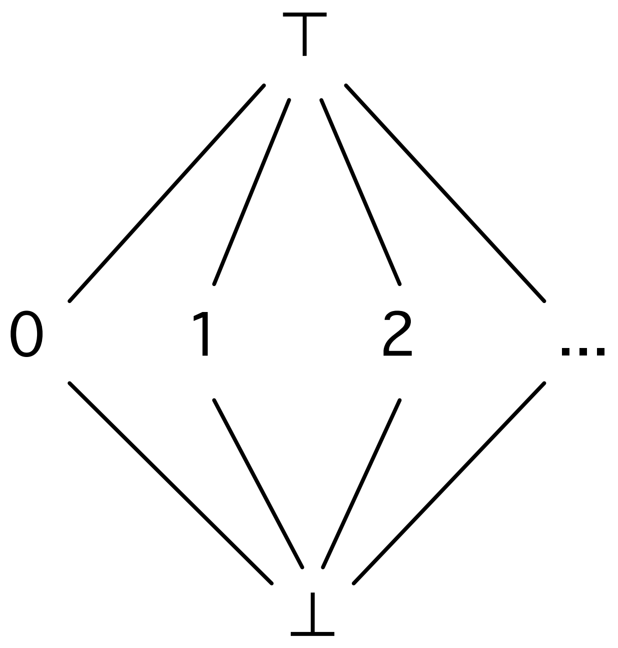
$$
\begin{array}{l}
\text{let}\ \_ = \text{put}\ l\ 3\ \text{in} \\
\quad \text{let par}\ v = \text{get}\ l\ \boxed{4} \\
\quad \phantom{\text{let par}\ v} \_ = \text{put}\ l\ 4 \\
\quad \text{in}\ v
\end{array}
$$

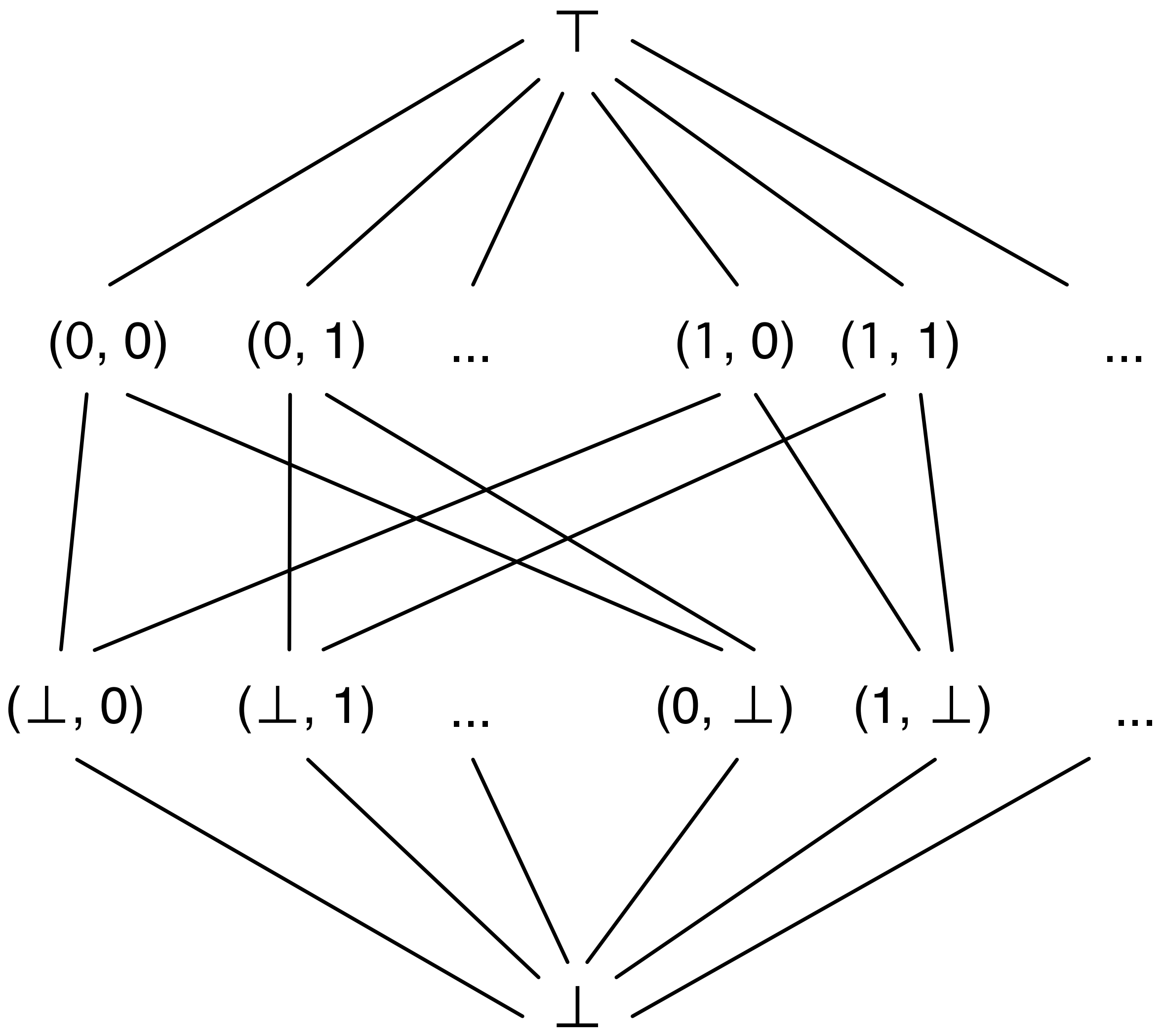
$$
\begin{array}{l}
\text{let}\ \_ = \text{put}\ l\ 3\ \text{in} \\
\quad \text{let par}\ v = \overset{\text{return 4}}{\boxed{\text{get}\ l\ 4}} \\
\quad \phantom{\text{let par}\ v} \_ = \text{put}\ l\ 4 \\
\quad \phantom{\text{let par}\ v} \_ = \text{put}\ l\ 5 \\
\quad \text{in}\ v
\end{array}
$$

Monotonically increasing writes
+ threshold reads
= deterministic parallelism

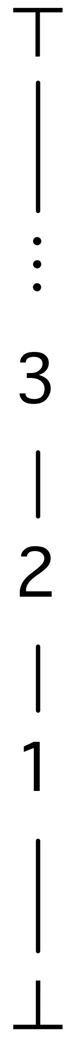
# Parameterizing our language: “LVars”



IVar

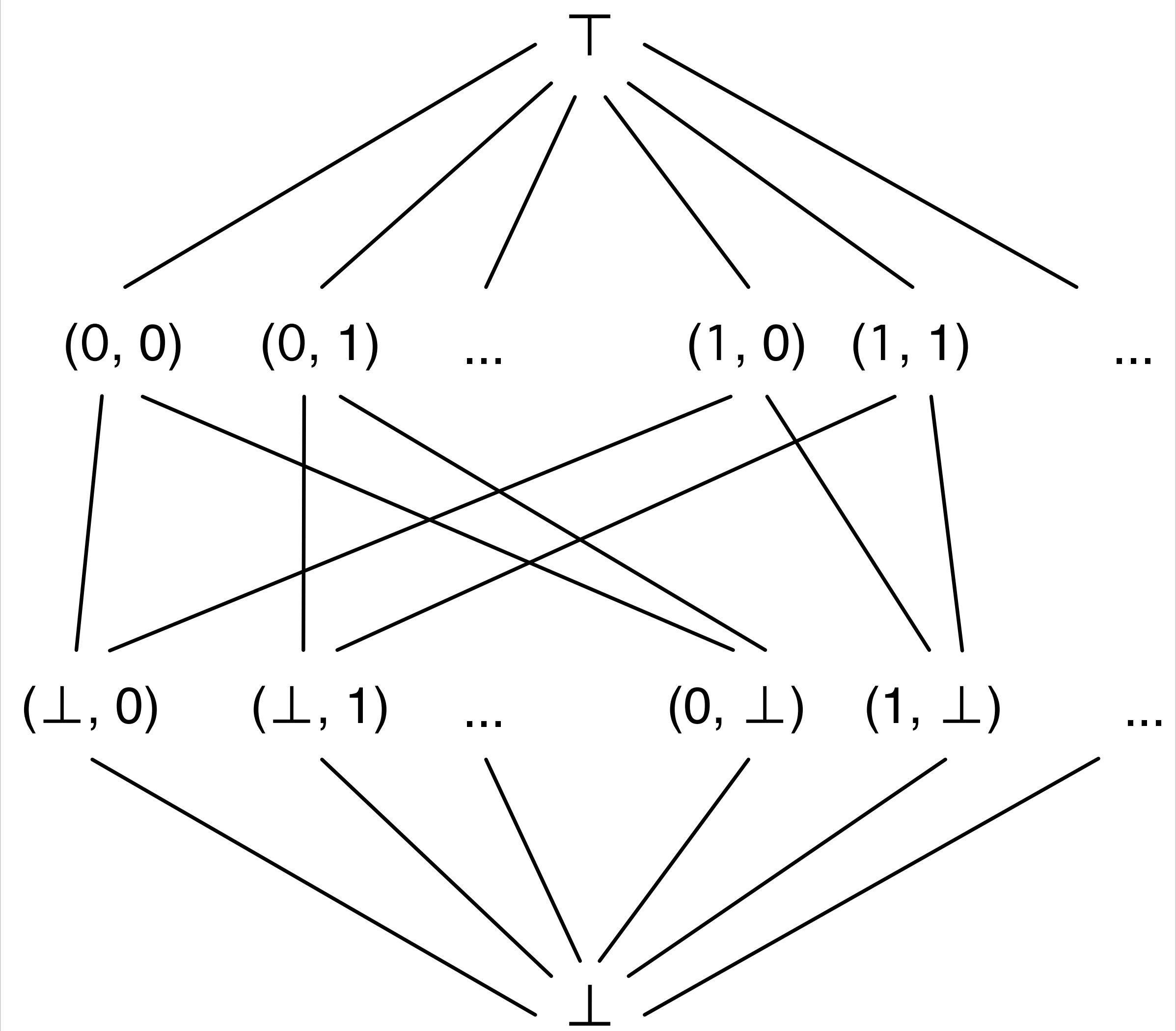


Pair of IVars



Counter

# Parameterizing our language: “LVars”



Pair of IVars

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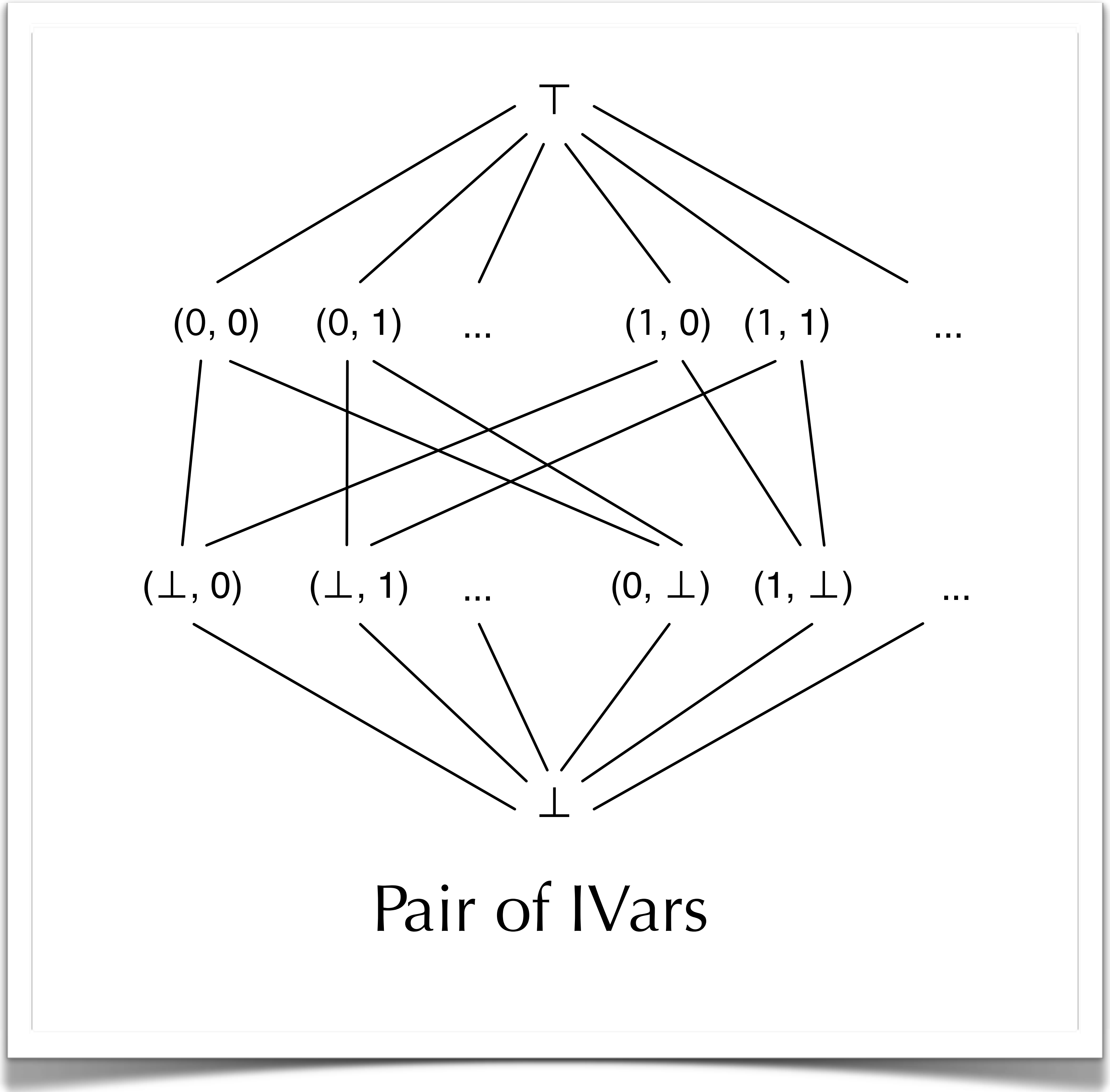

Pair of IVars

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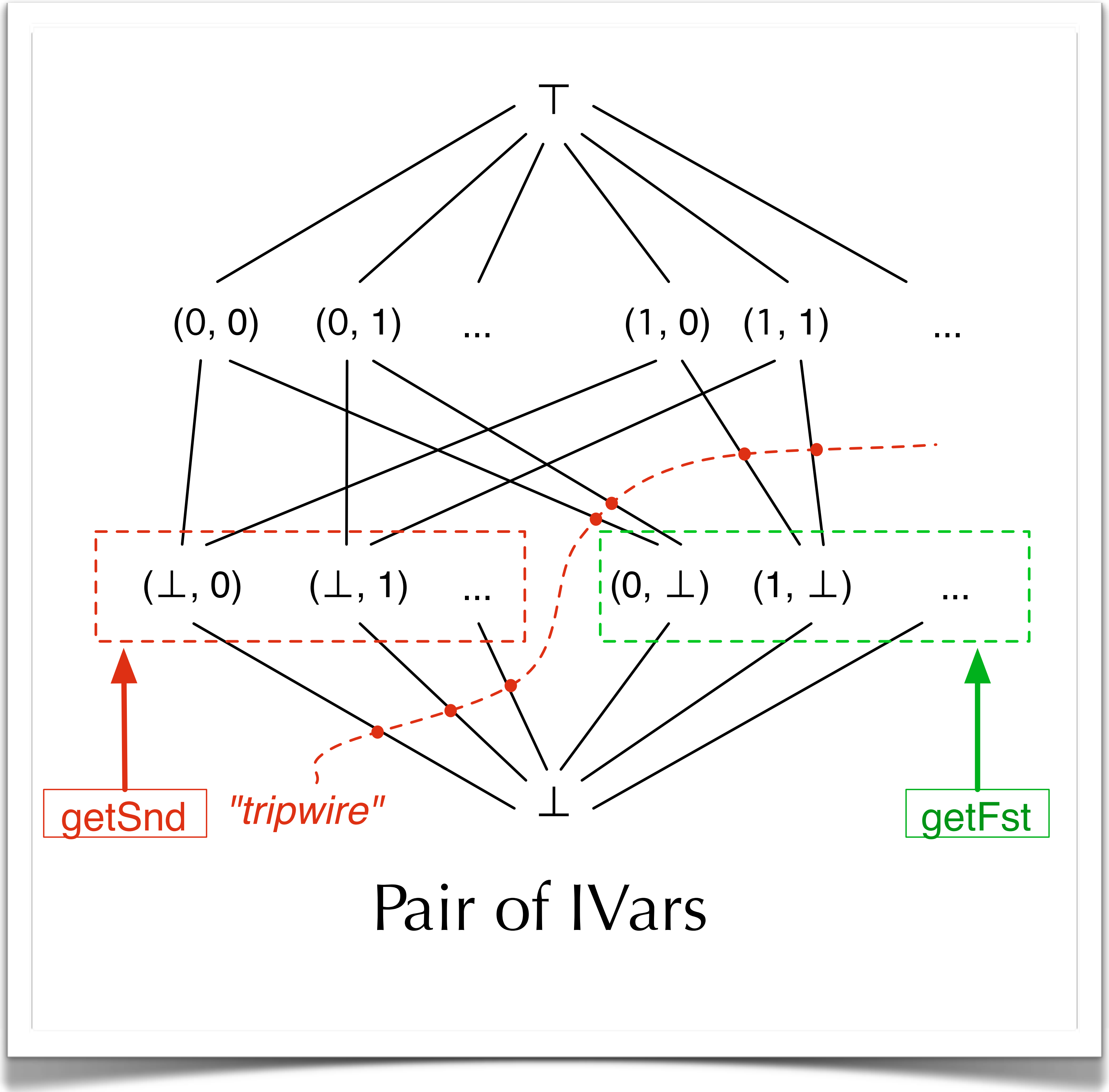


# Parameterizing our language: “LVars”

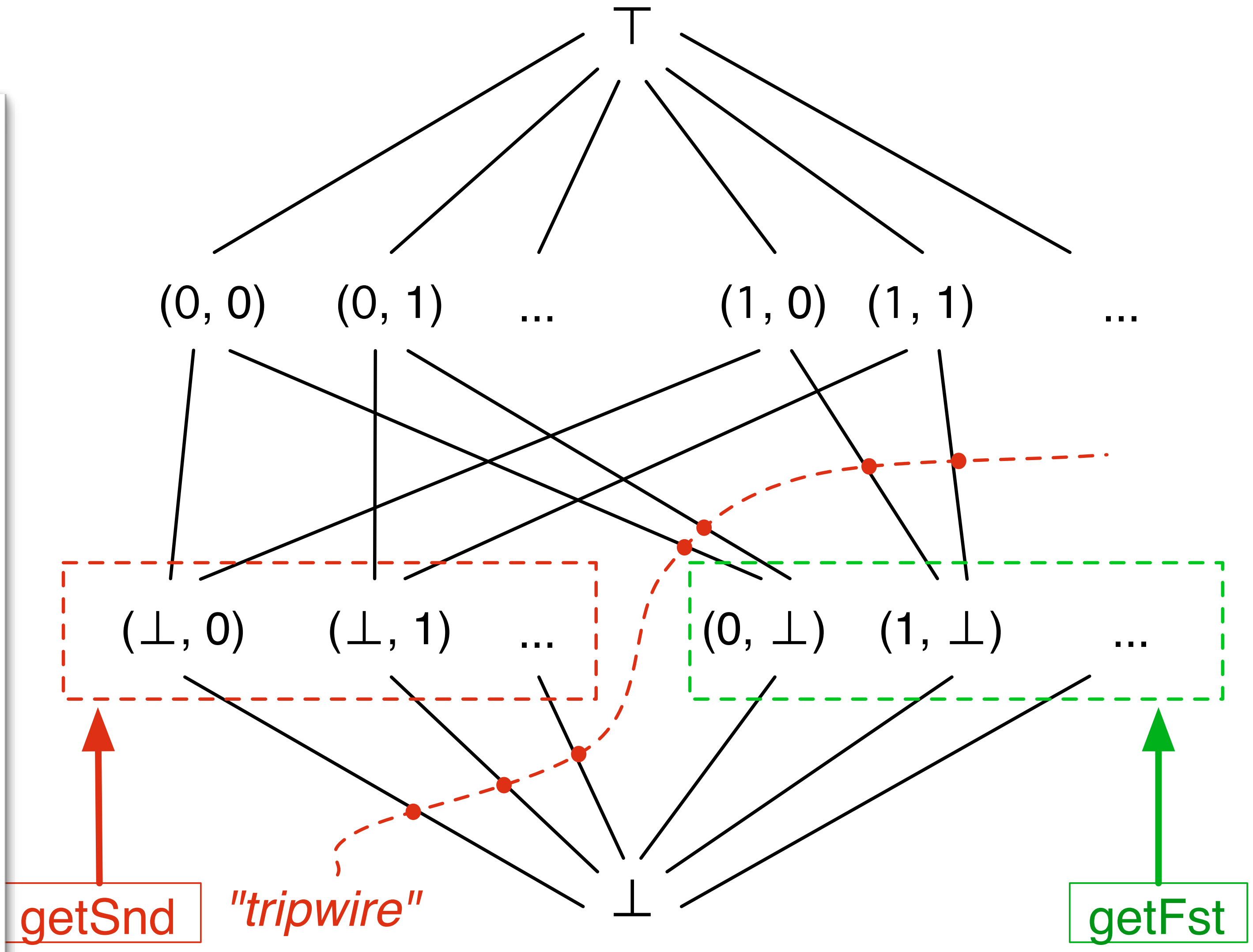
let $p$ = new in

  let $\_$ = put $p$ $\{(\bot, 4)\}$ in

    let par $v_1$ = getFst $p$

          $\_$ = put $p$ $\{(3, 4)\}$

     in $\ldots v_1 \ldots$



Pair of IVars

# Two take-aways

Monotonicity causes deterministic parallelism

Monotonically increasing writes
+ threshold reads
= deterministic parallelism

# More in our paper draft

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- Subsuming existing models
  - KPNs, CnC, monad-par
- Support for controlled nondeterminism
  - “probation” state

# Tak!

**Email:**
lkuper@cs.indiana.edu
**Twitter:** @lindsey
**Web:** cs.indiana.edu/~lkuper
**Research group:**
lambda.cs.indiana.edu

Photo by kakadu on Flickr.  Thanks!