Verified Causal Broadcast with Liquid Haskell

Patrick Redmond Gan Shen Niki Vazou Lindsey Kuper

IFL 2022
Copenhagen, Denmark
31 August 2022

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institute imdea software

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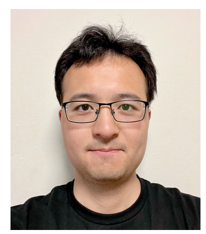

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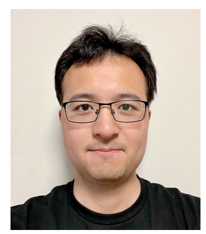

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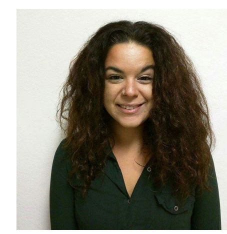

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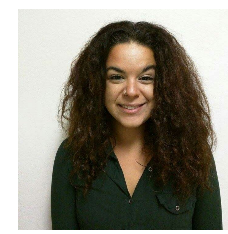

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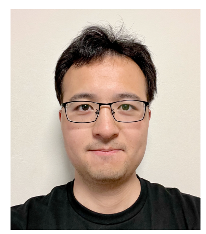

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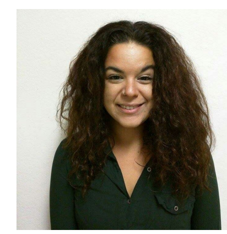

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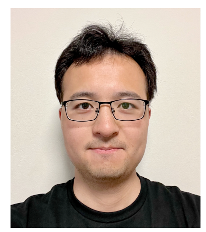

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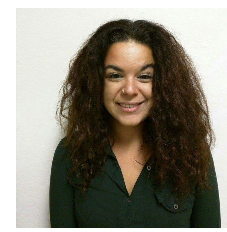

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 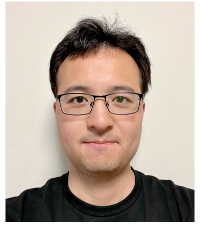  

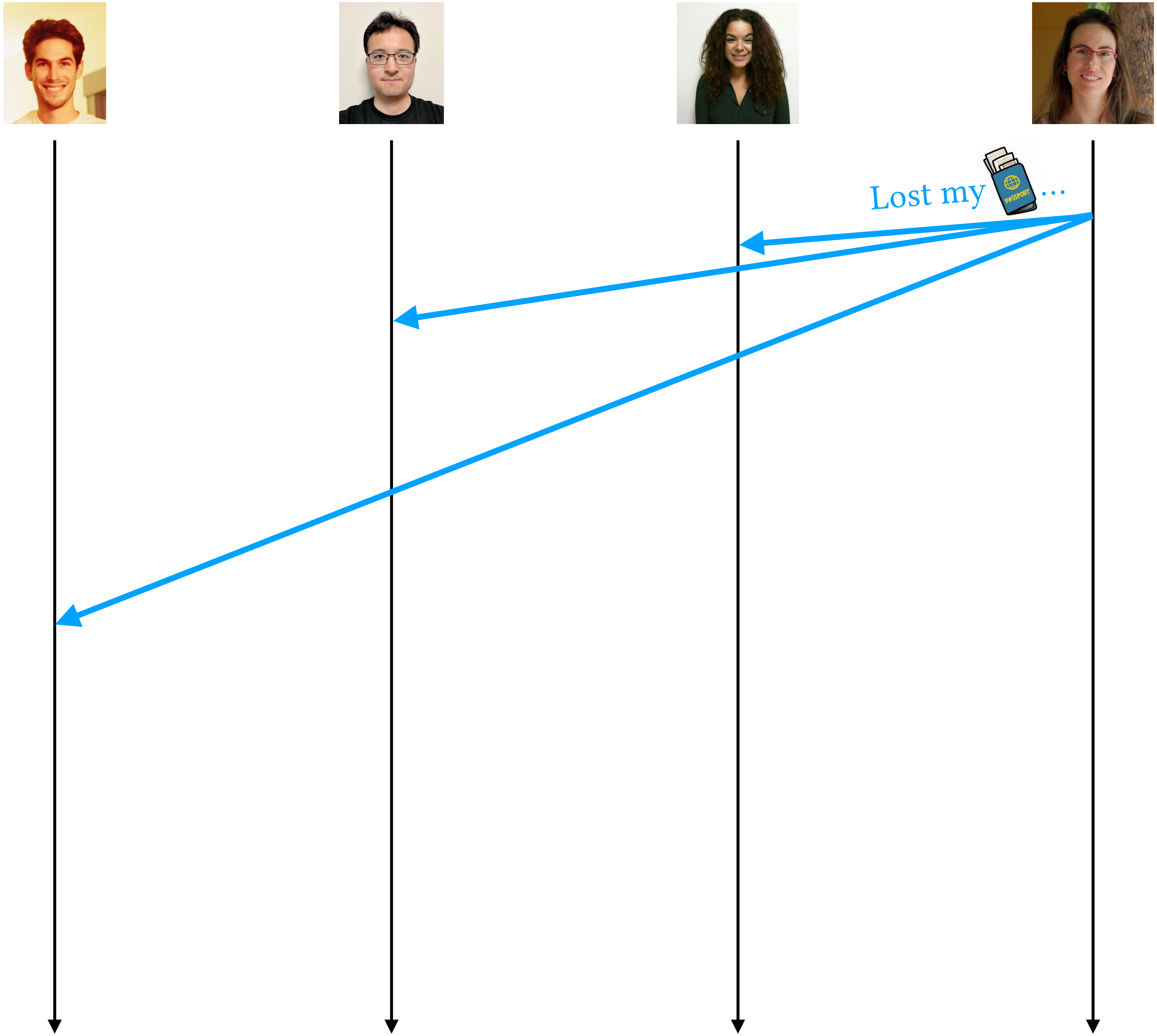
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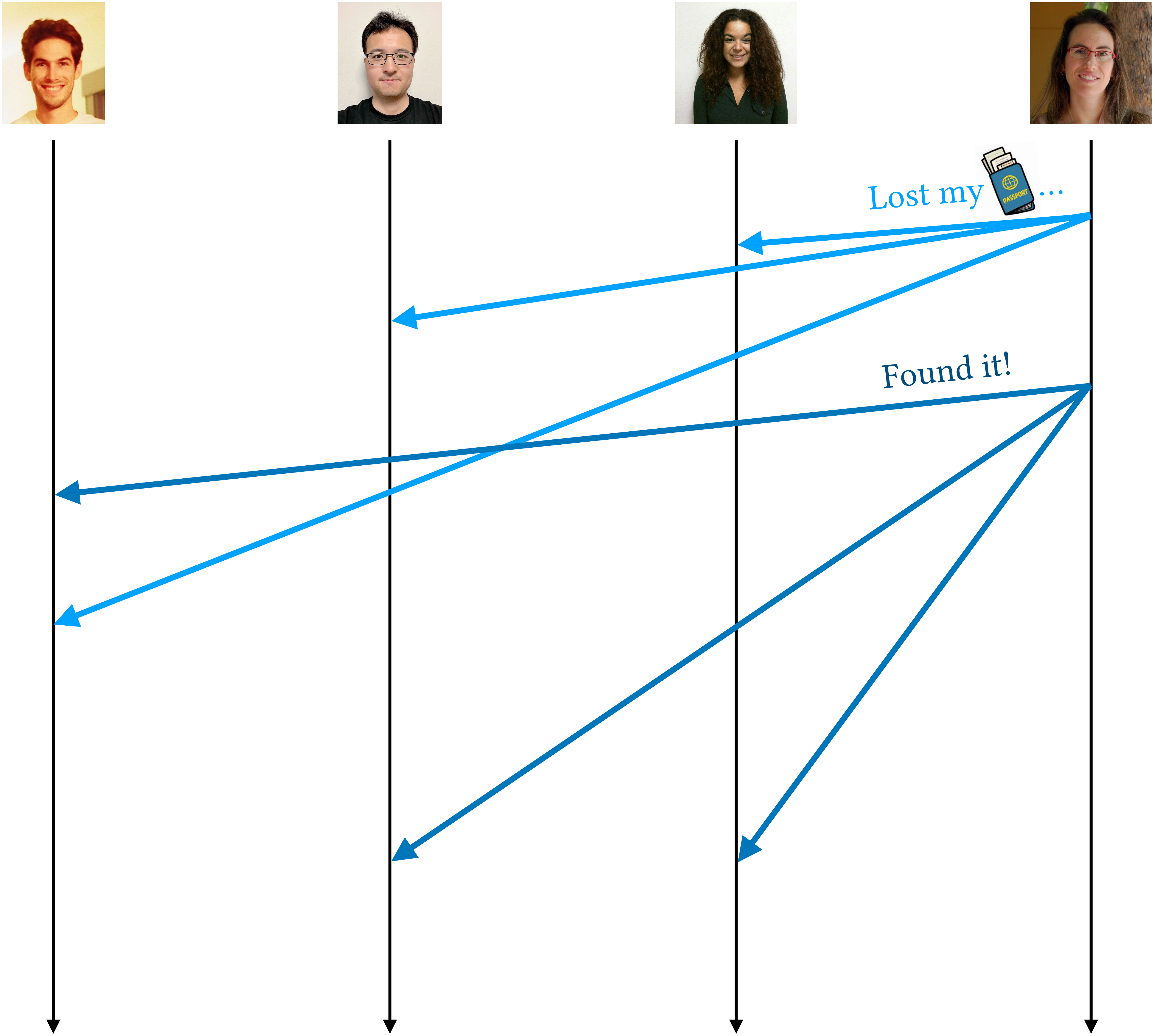
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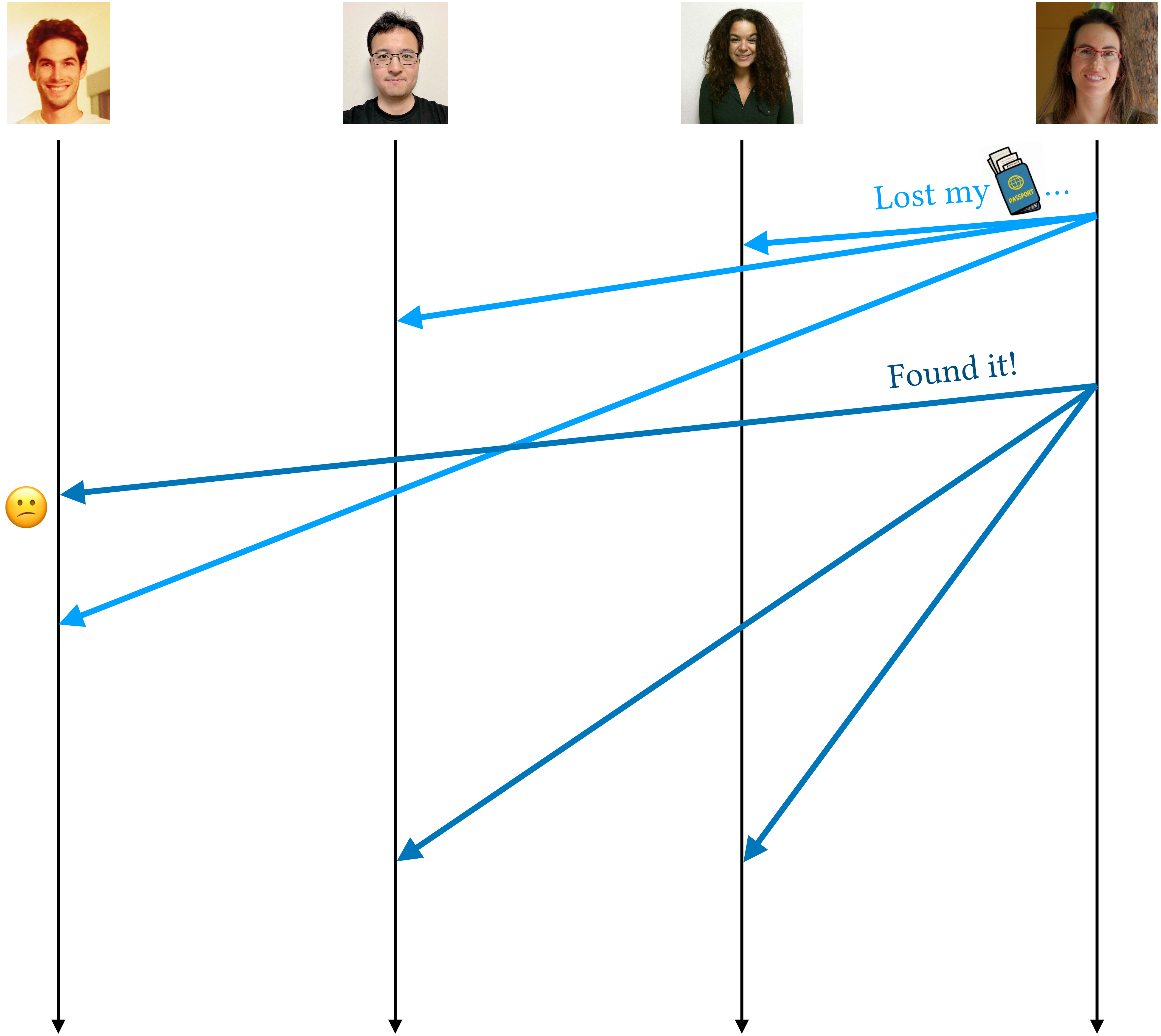

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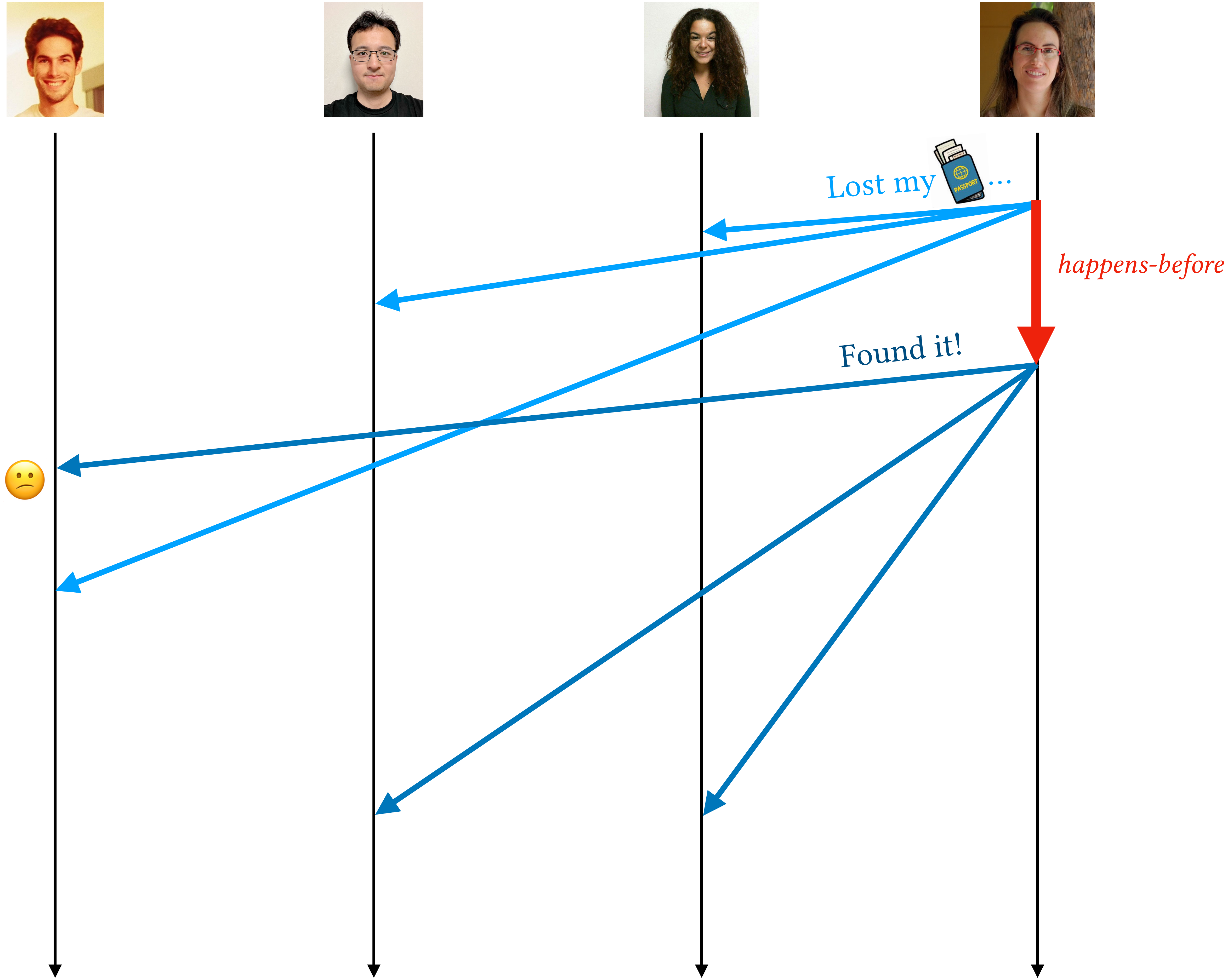
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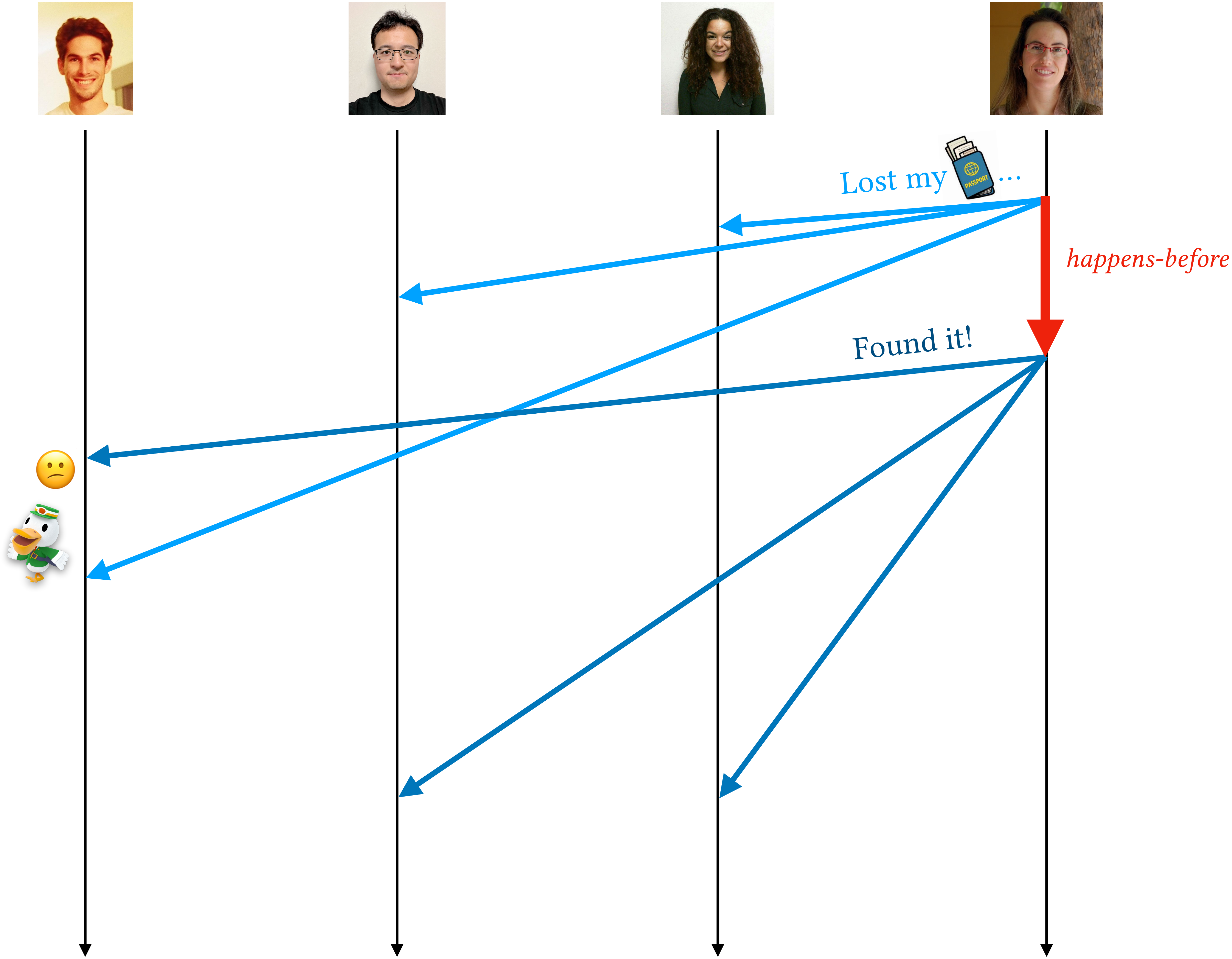
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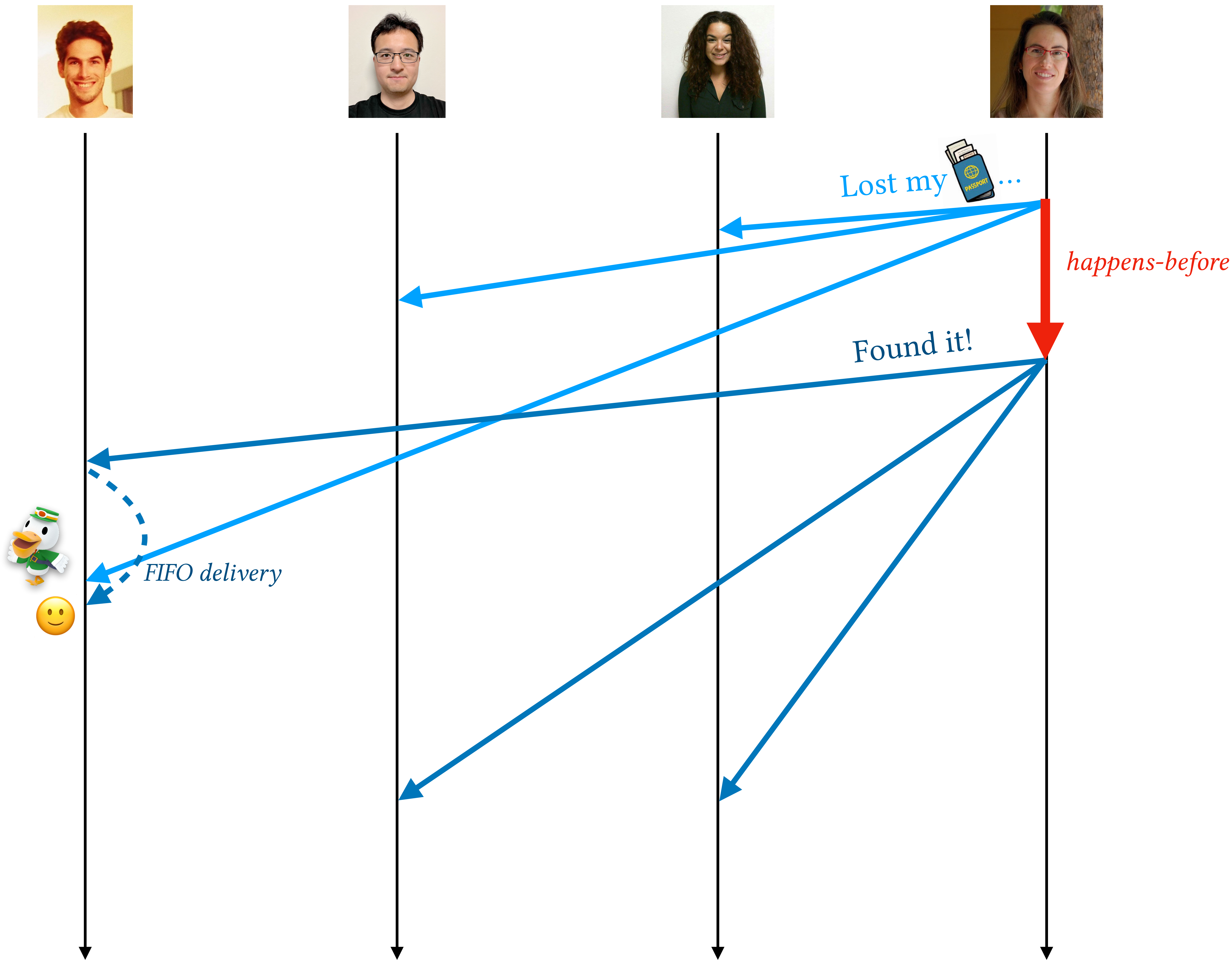
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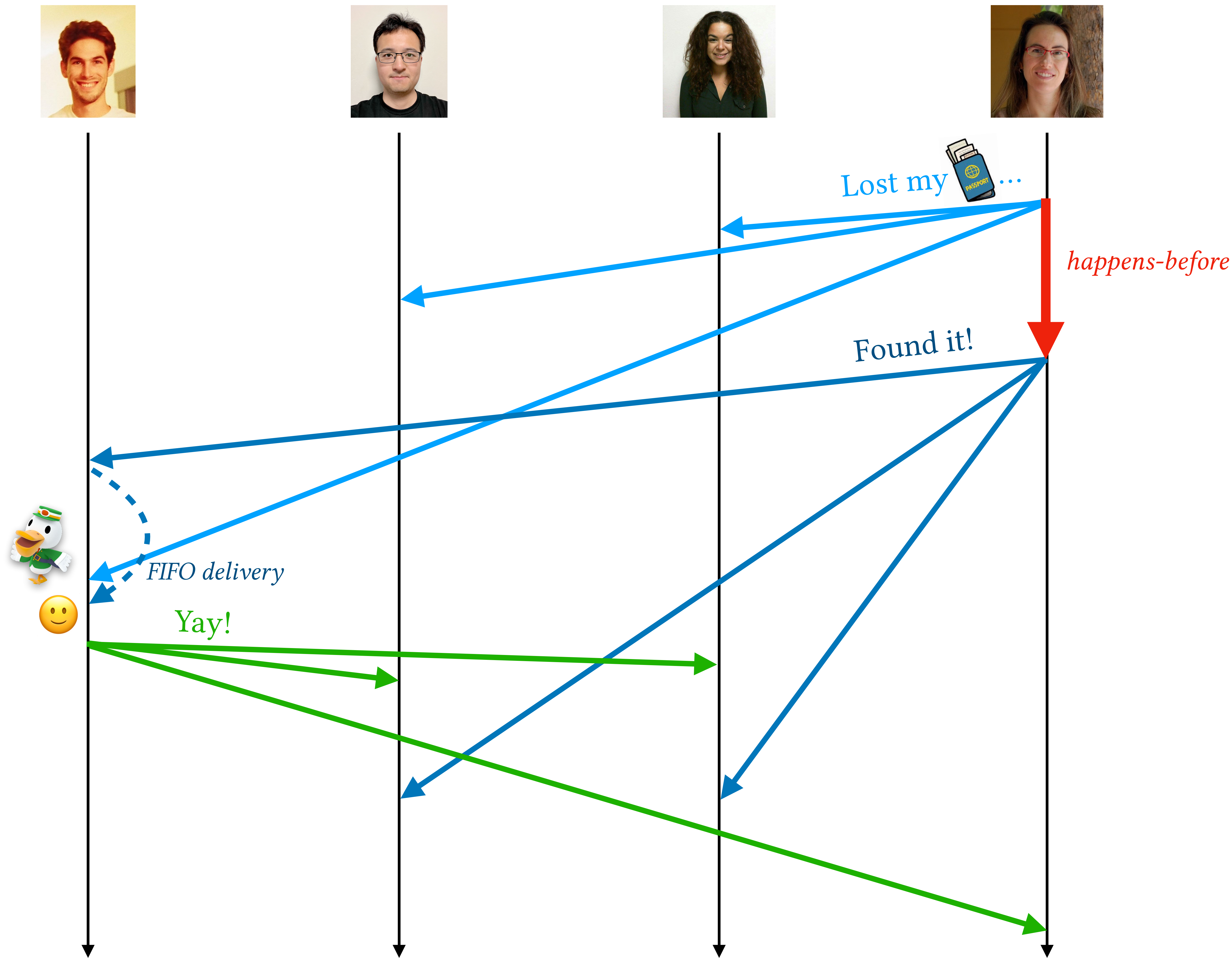
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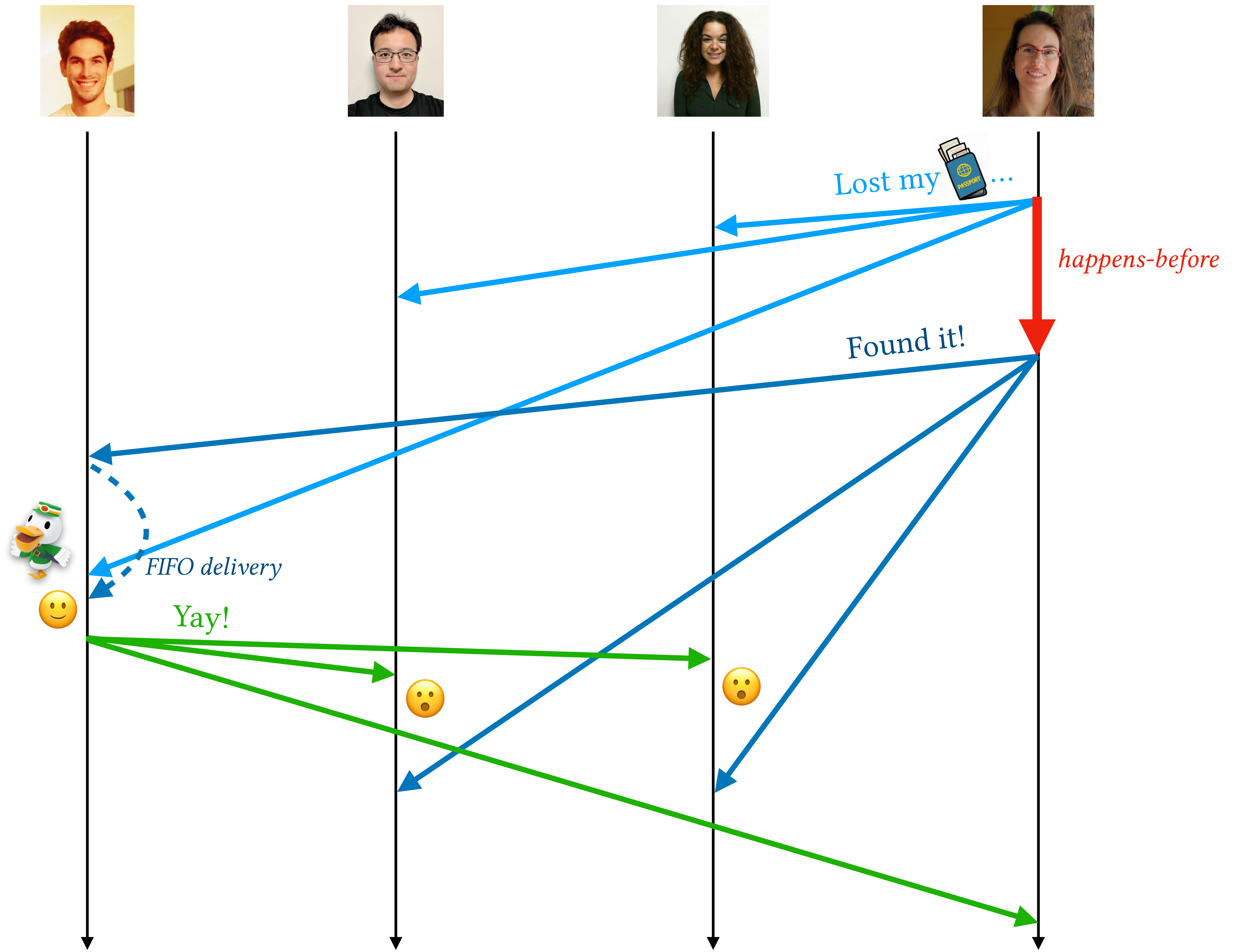

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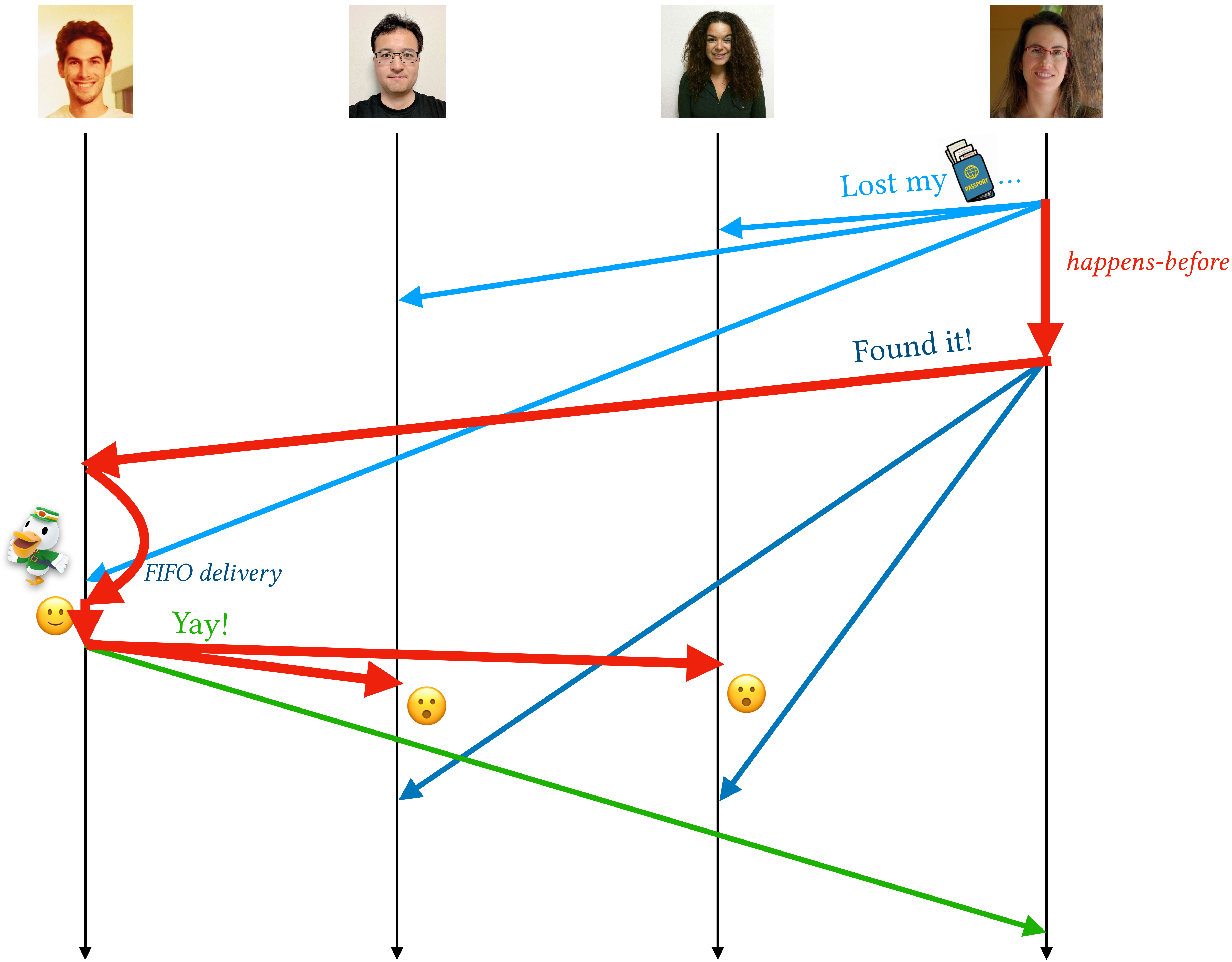

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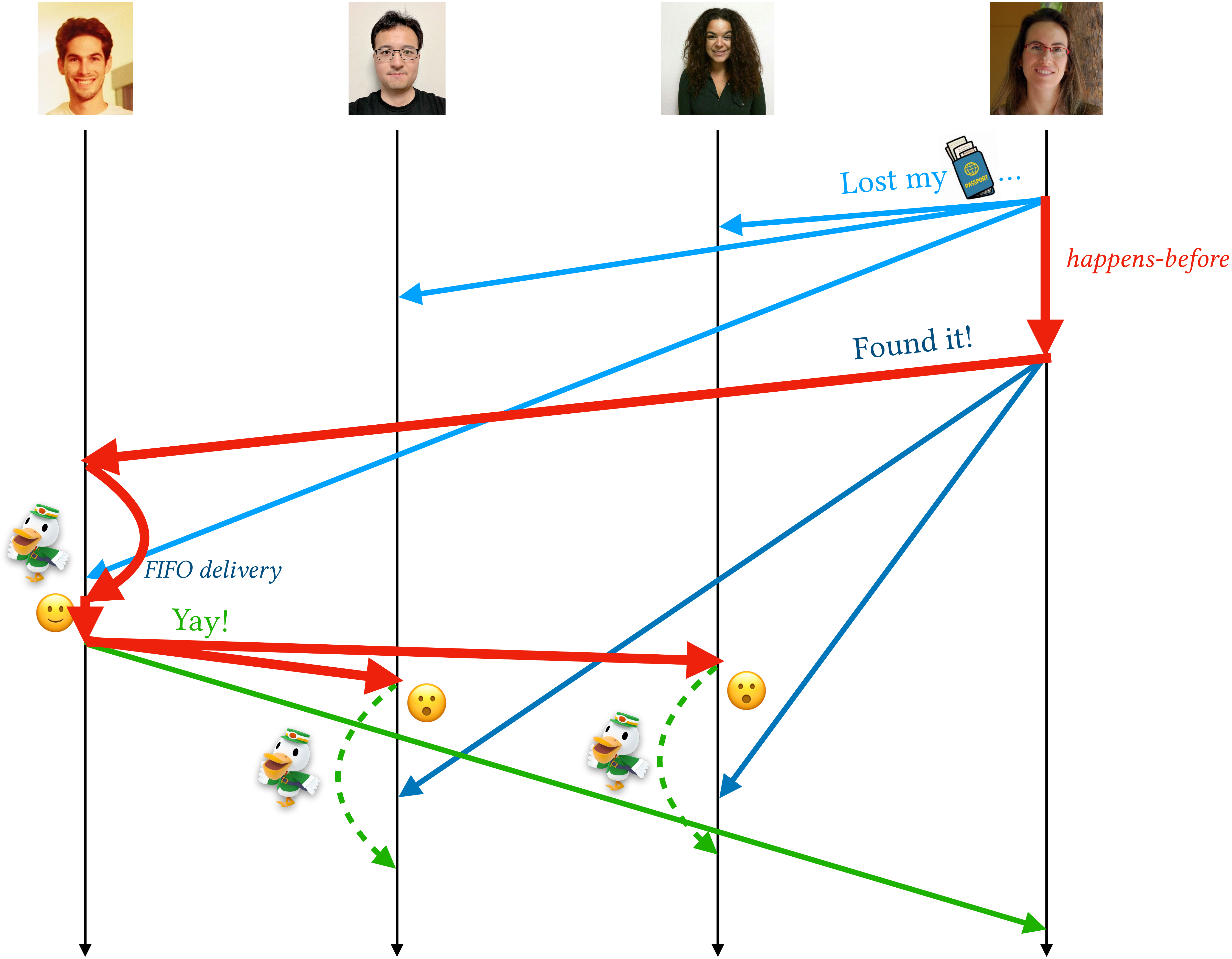

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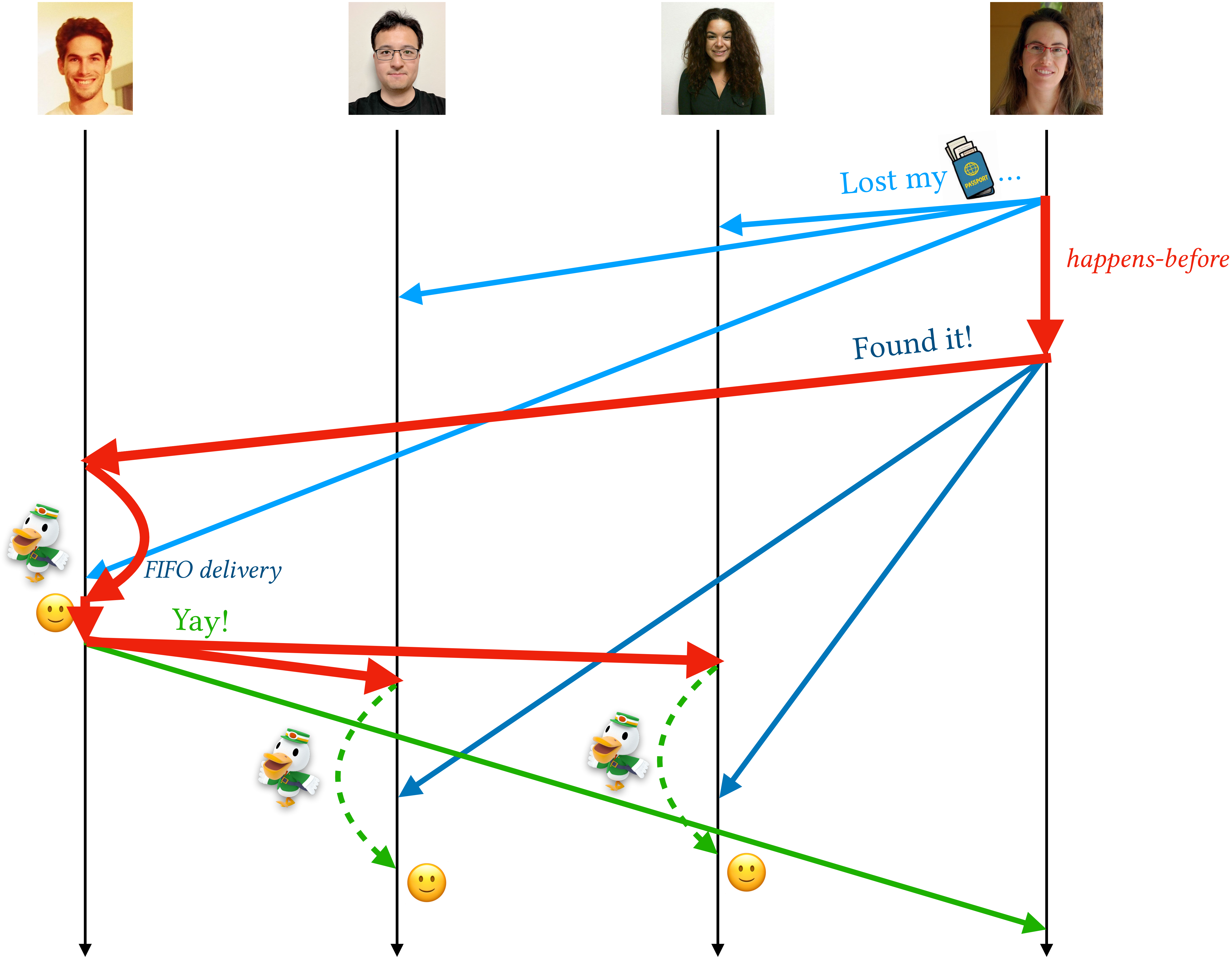
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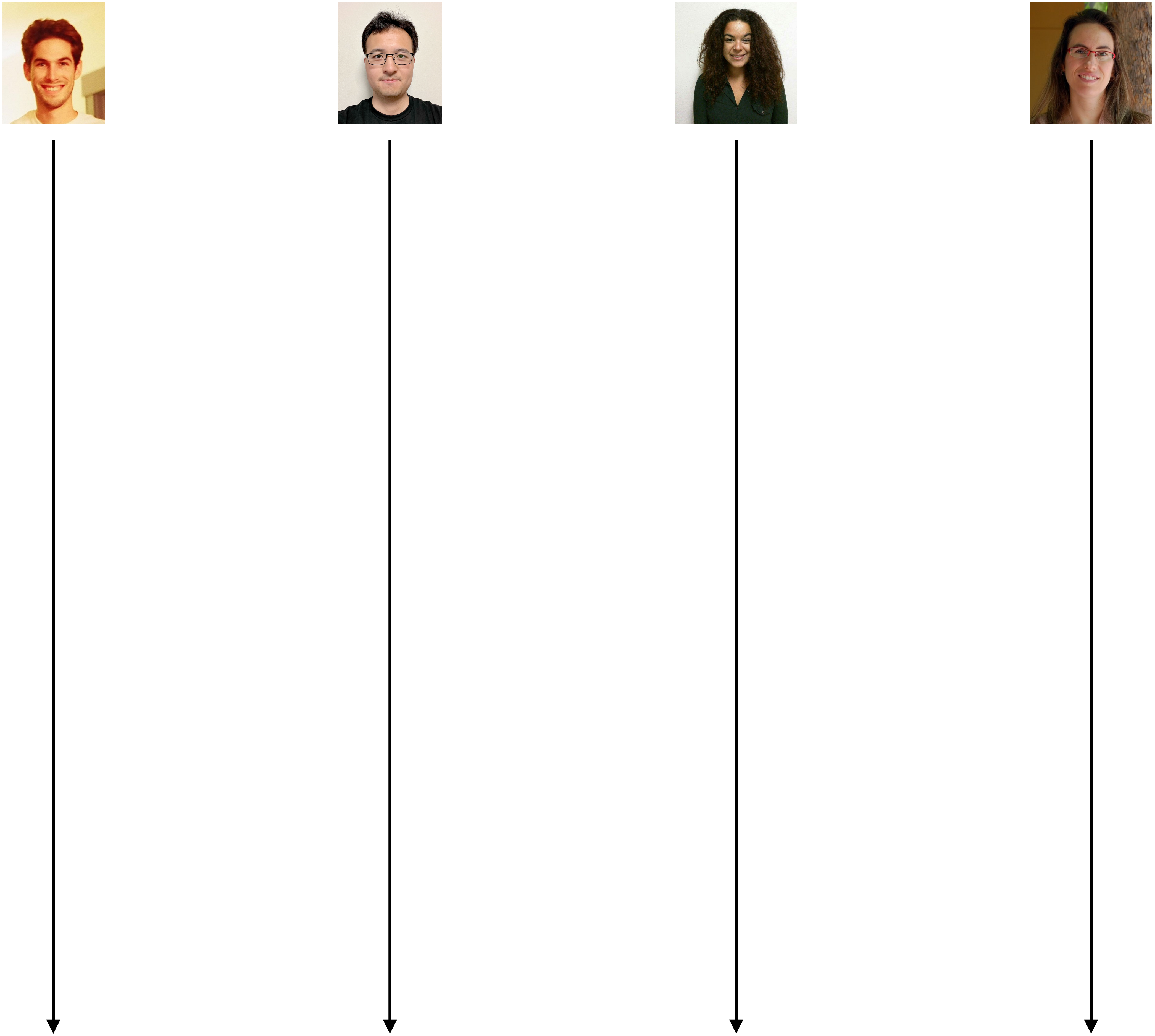
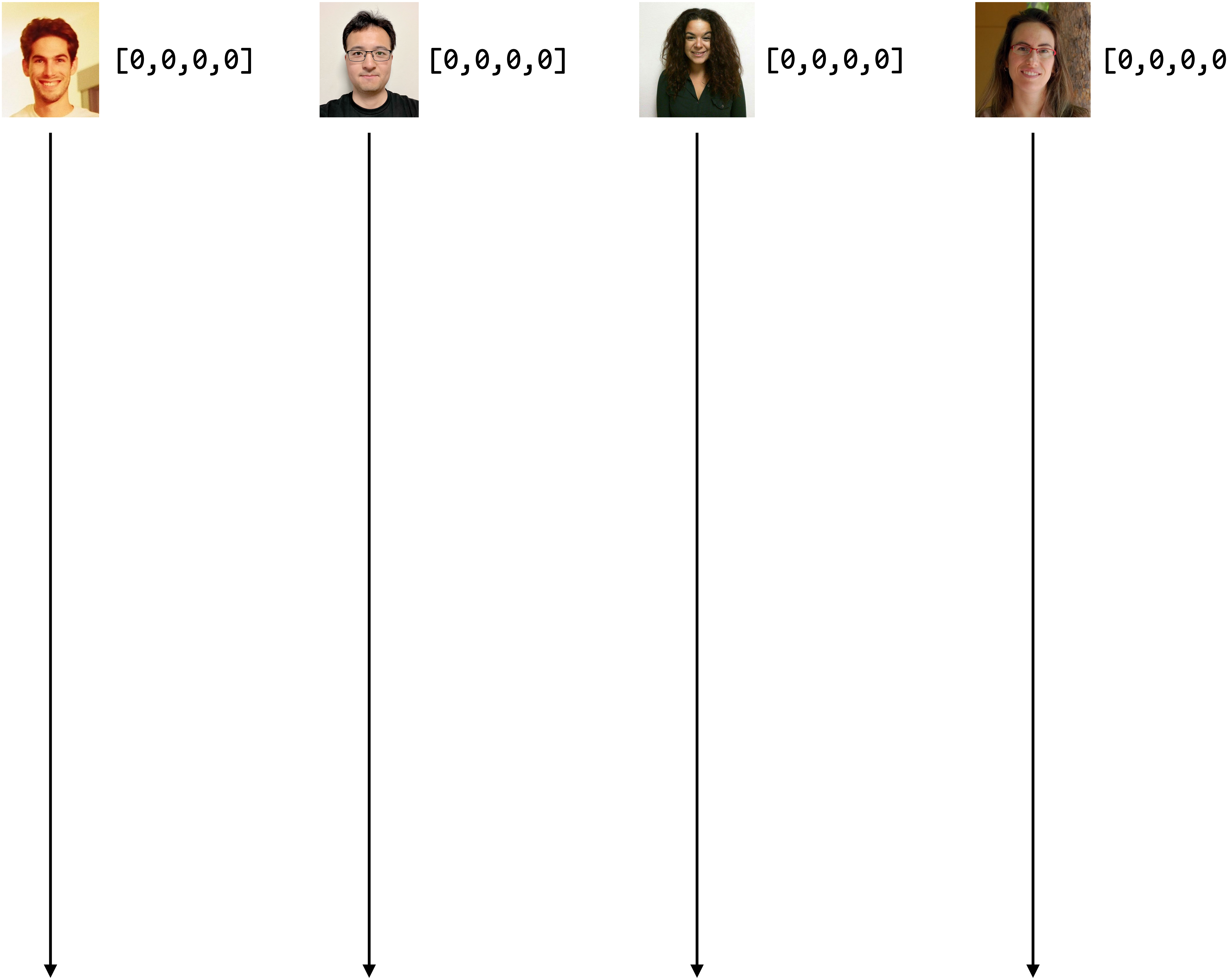


Causal broadcast with vector clocks [Birman et al., 1991]



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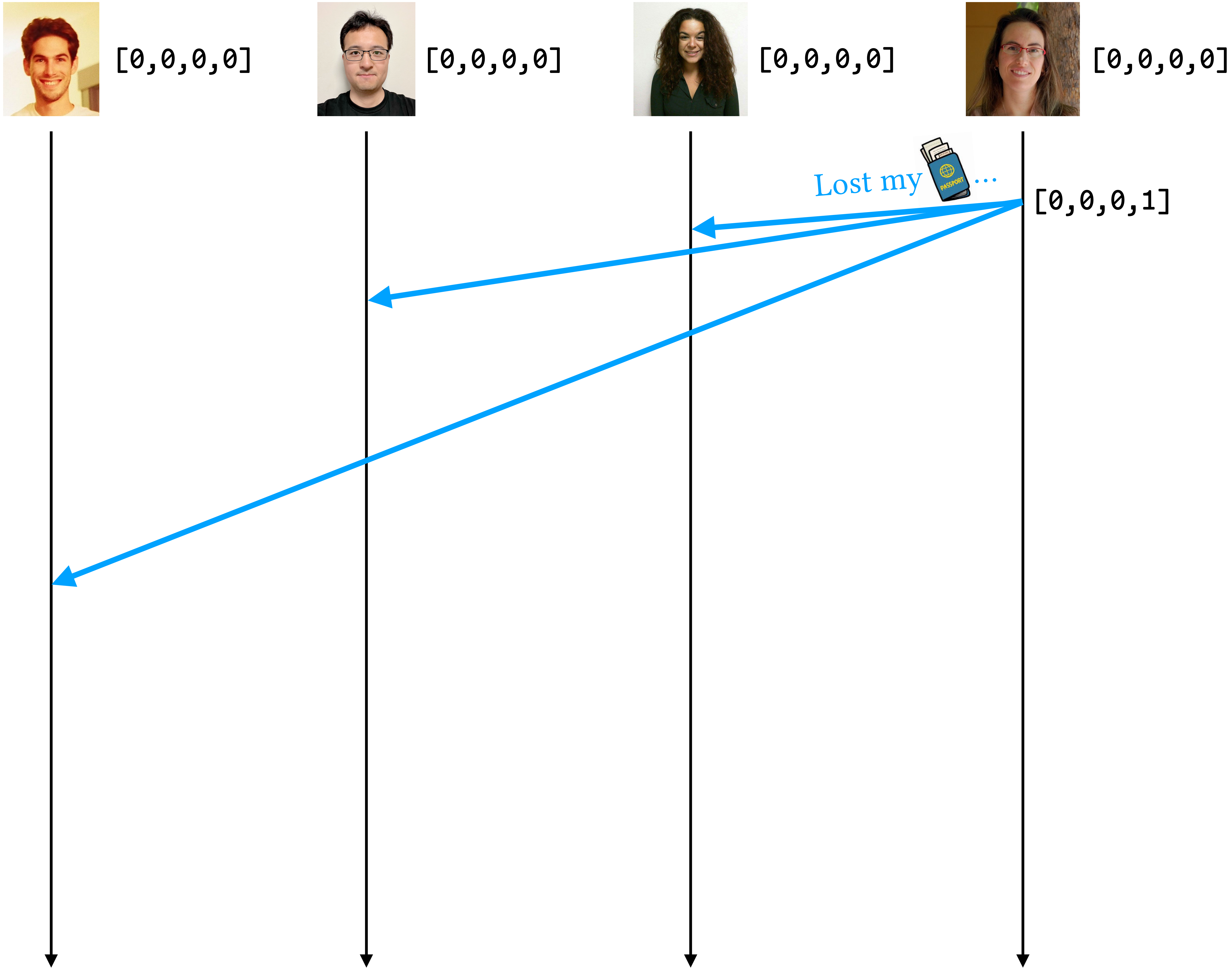

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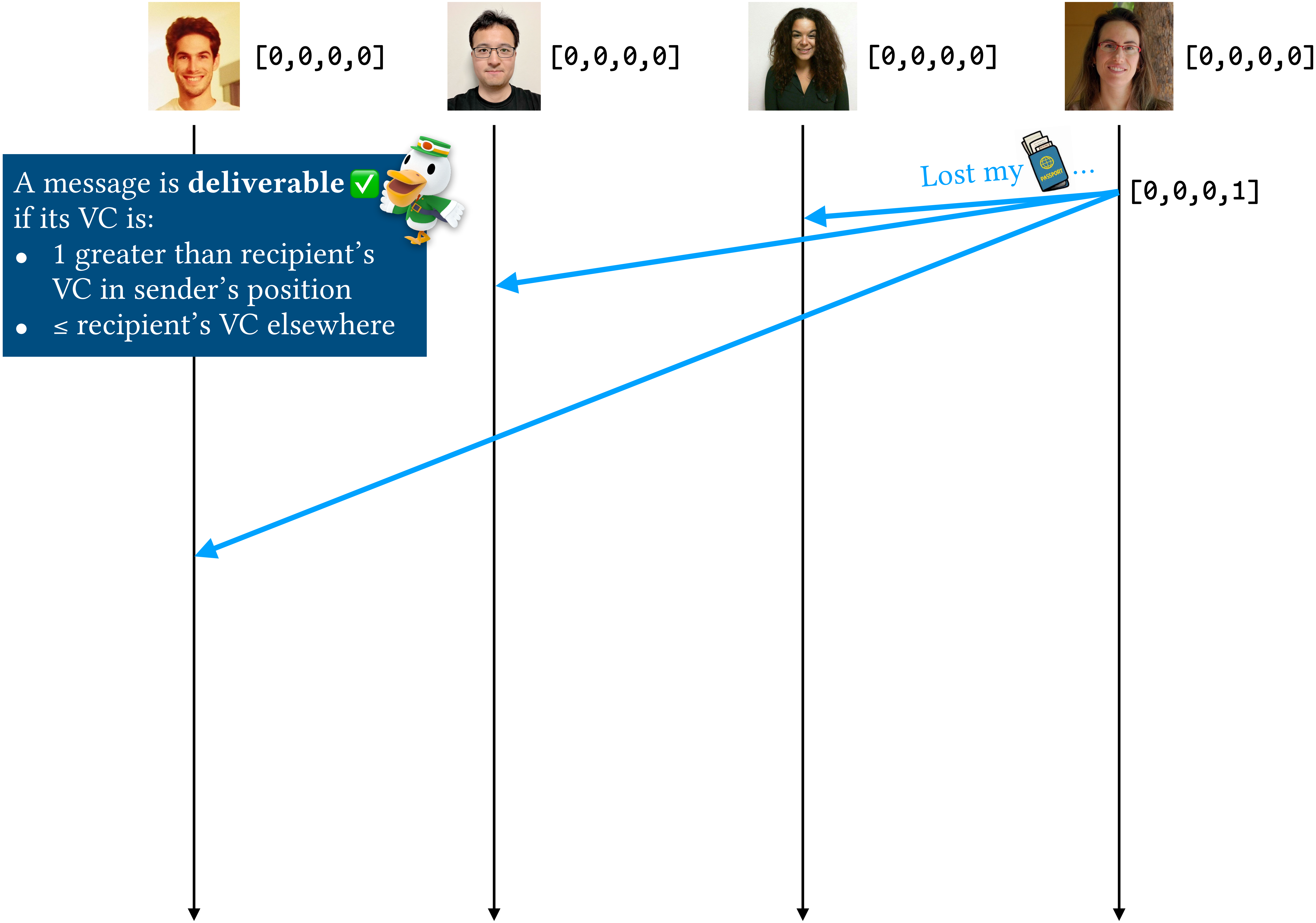

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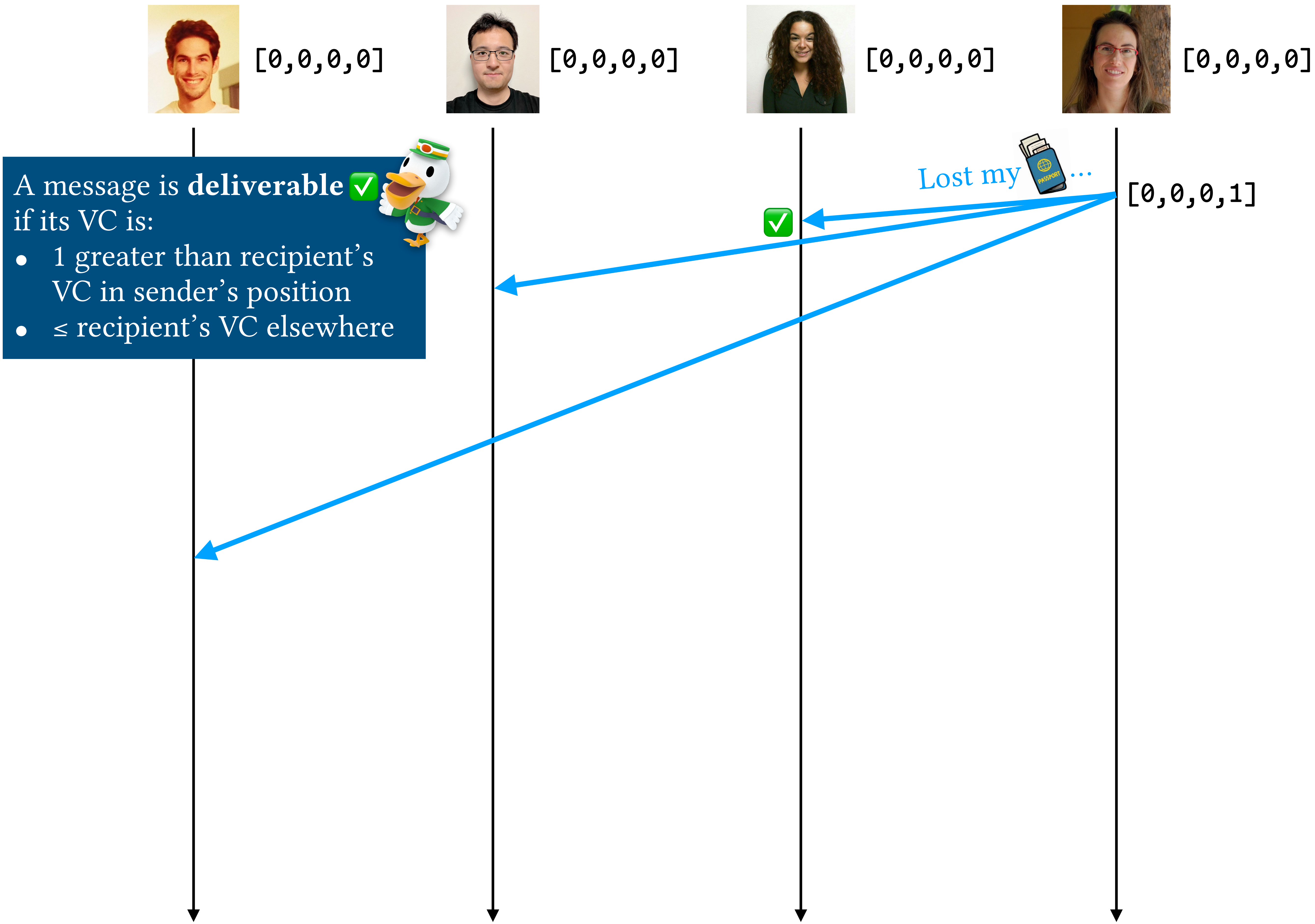

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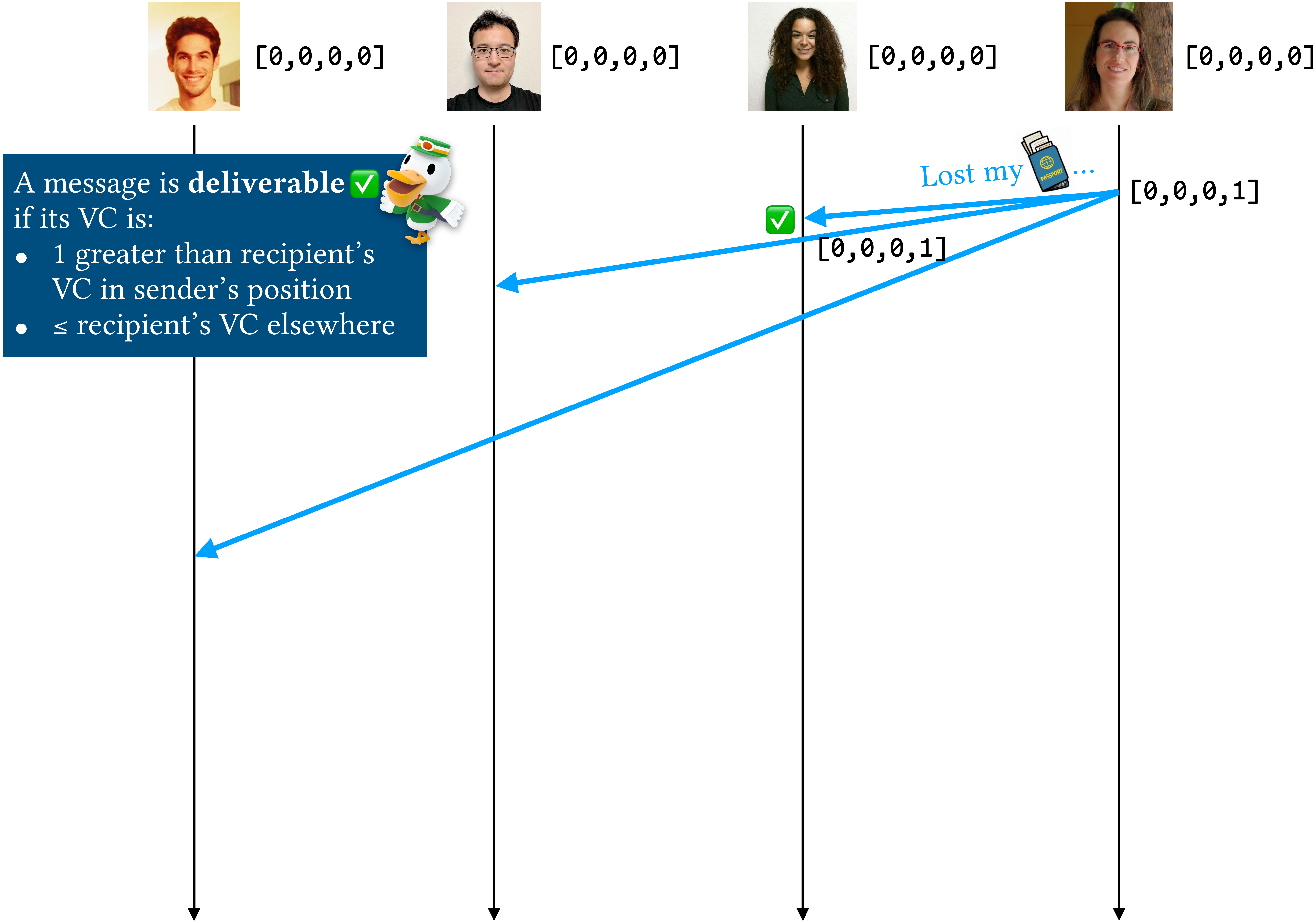

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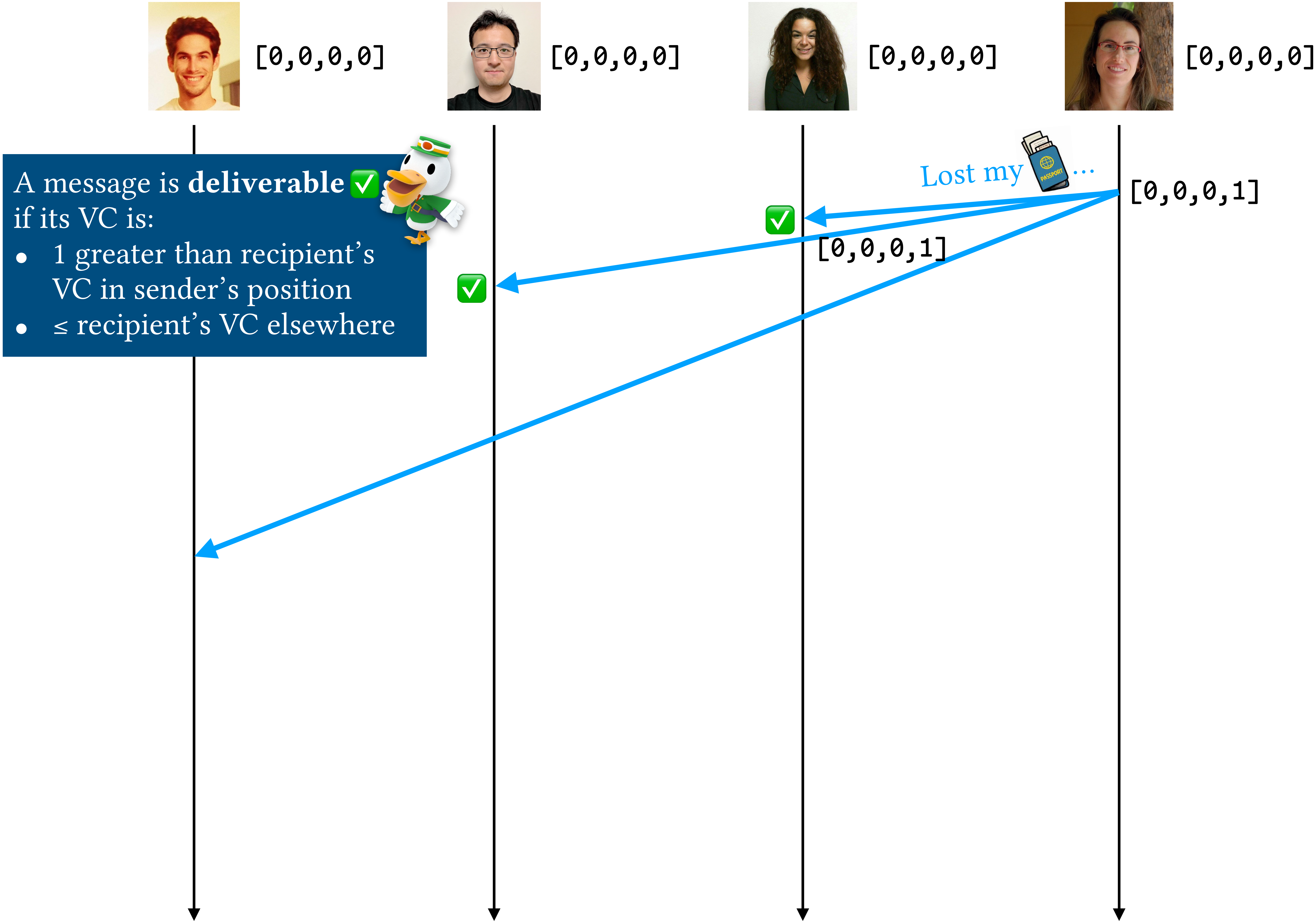

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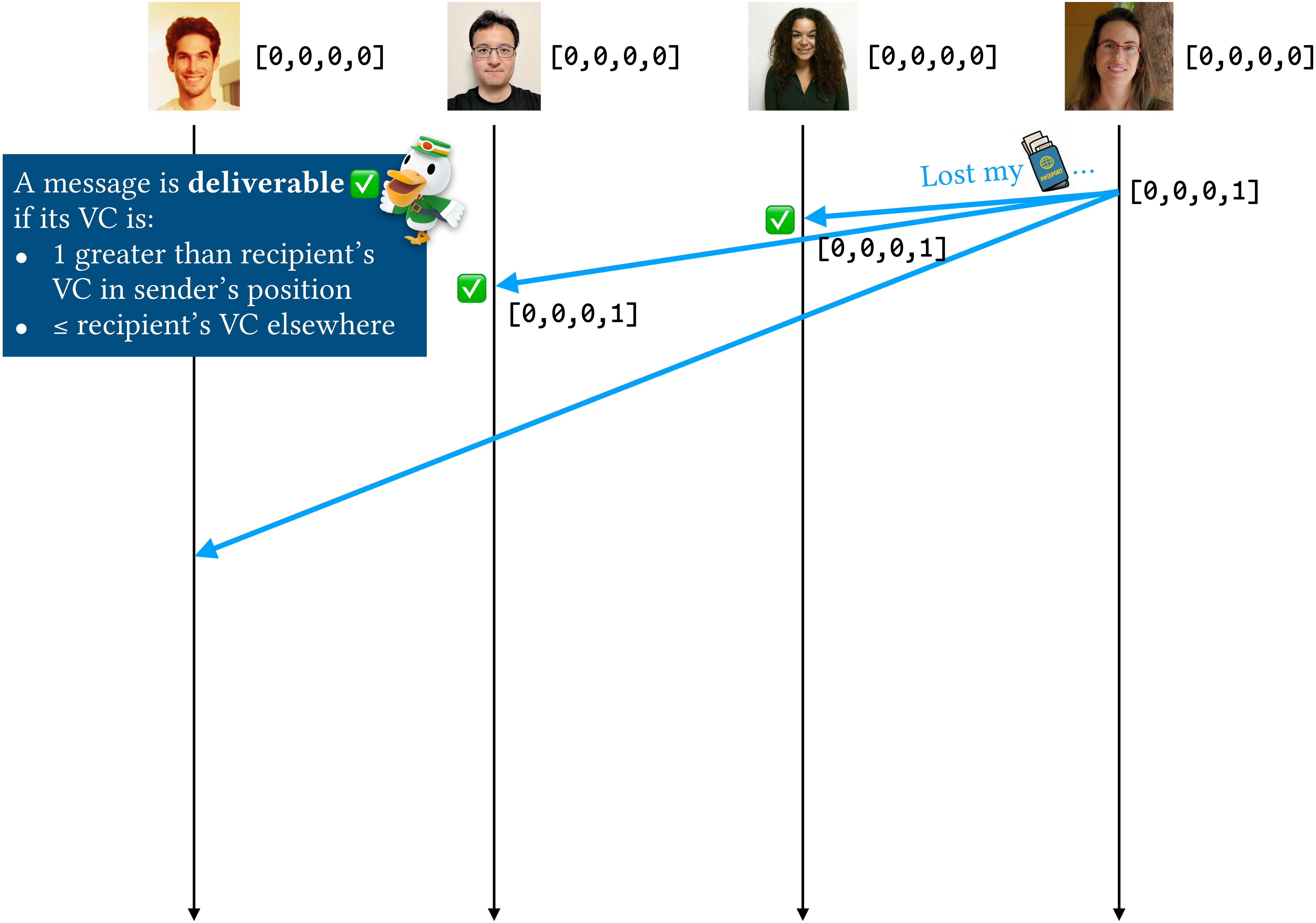

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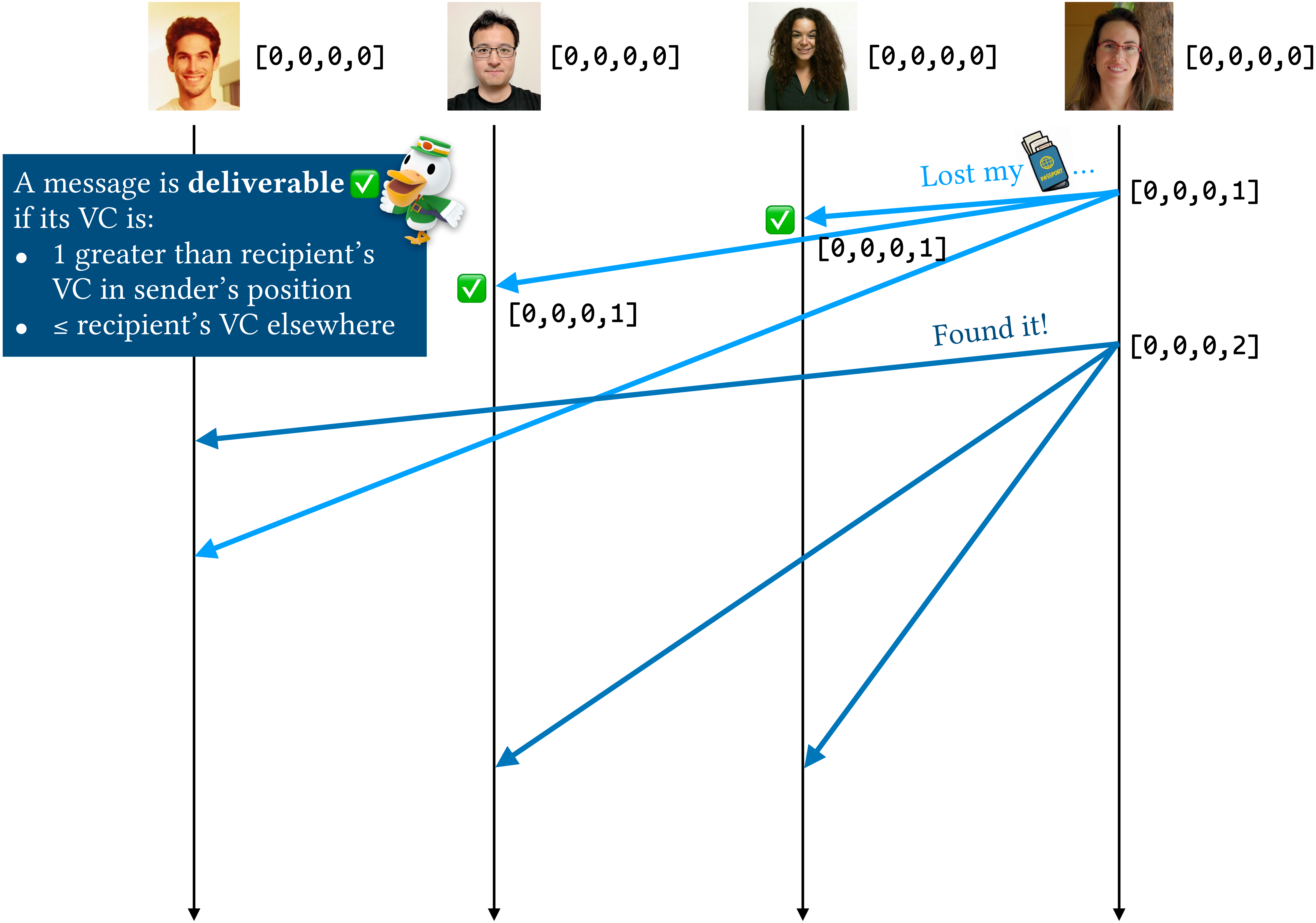

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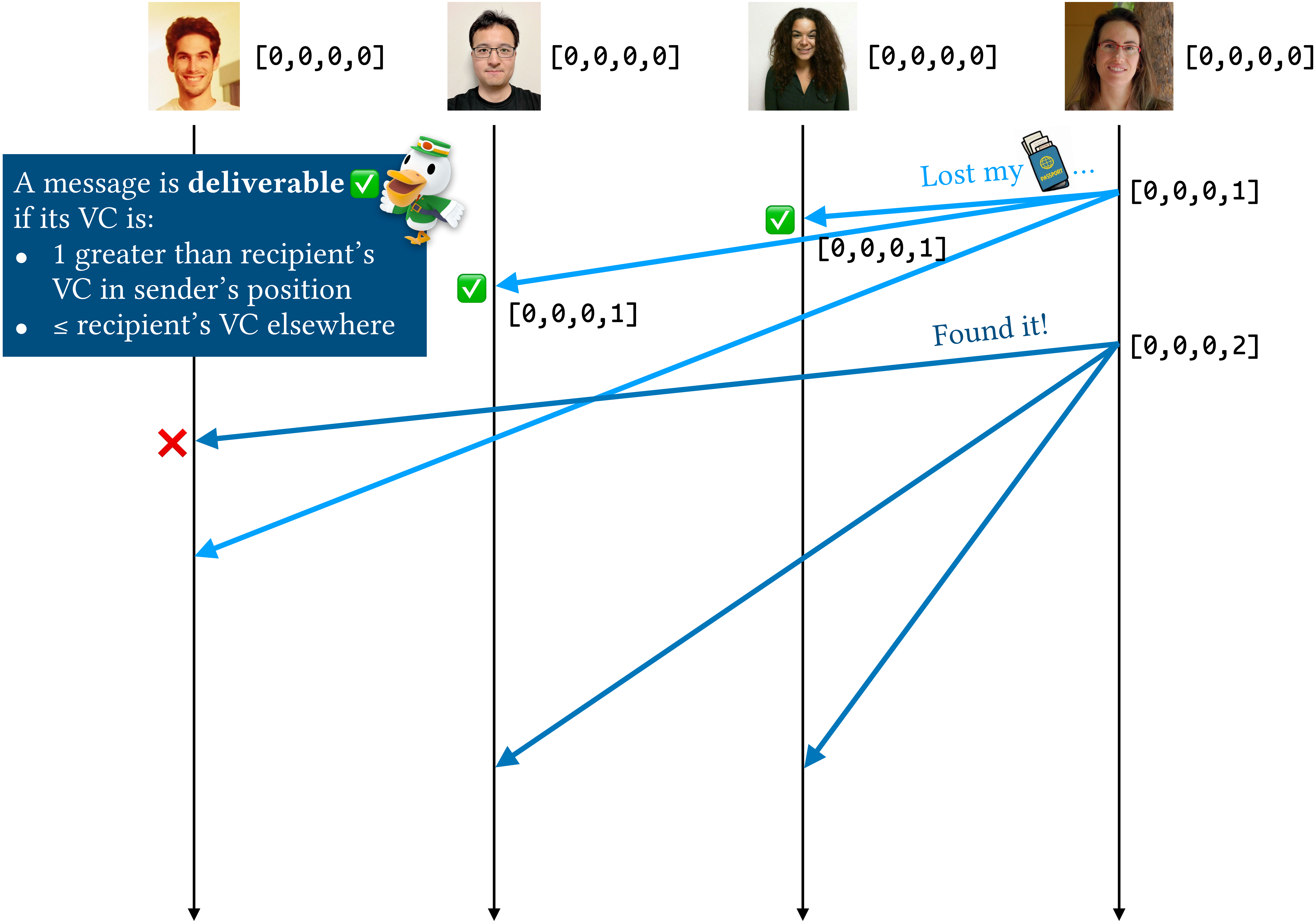

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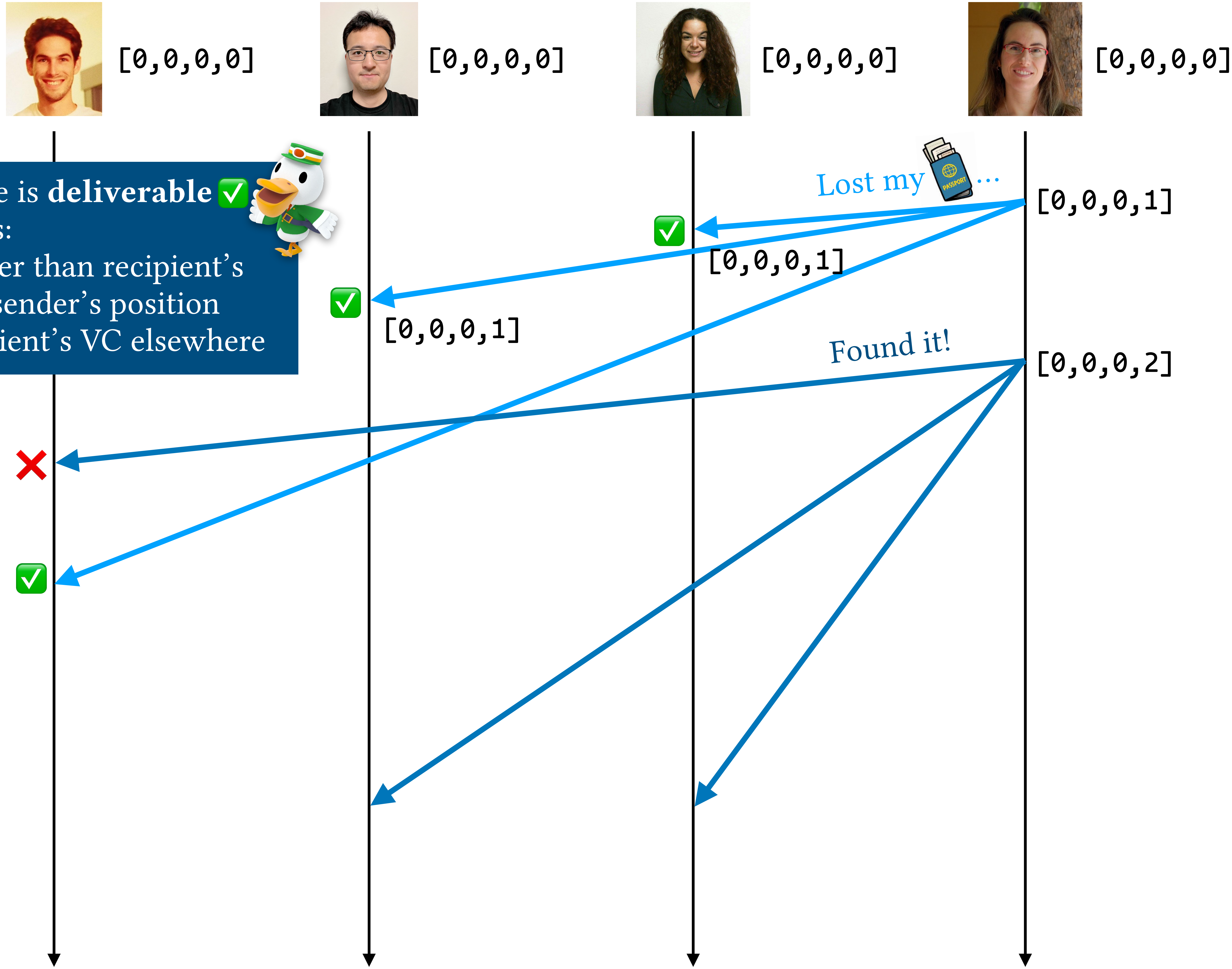

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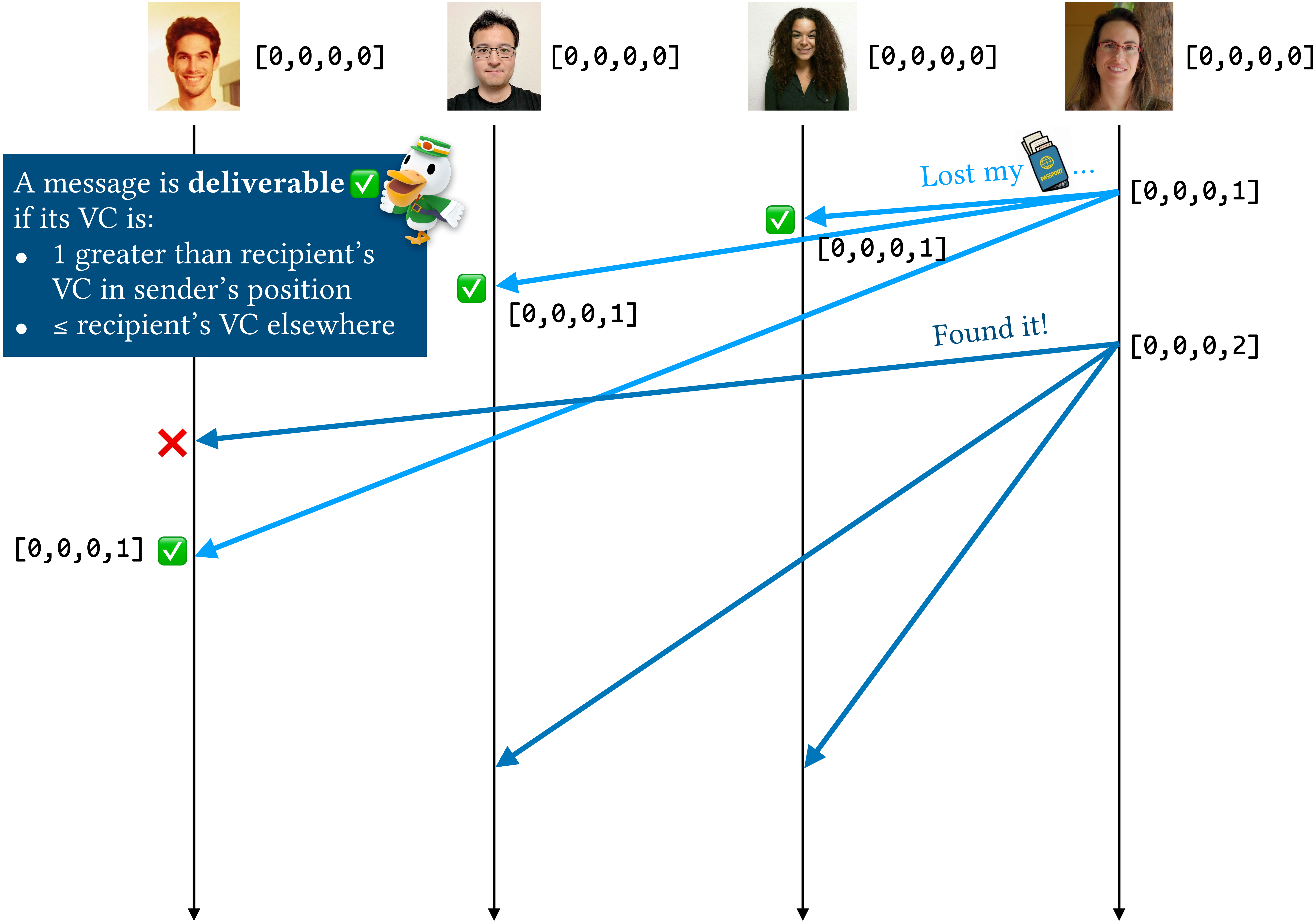

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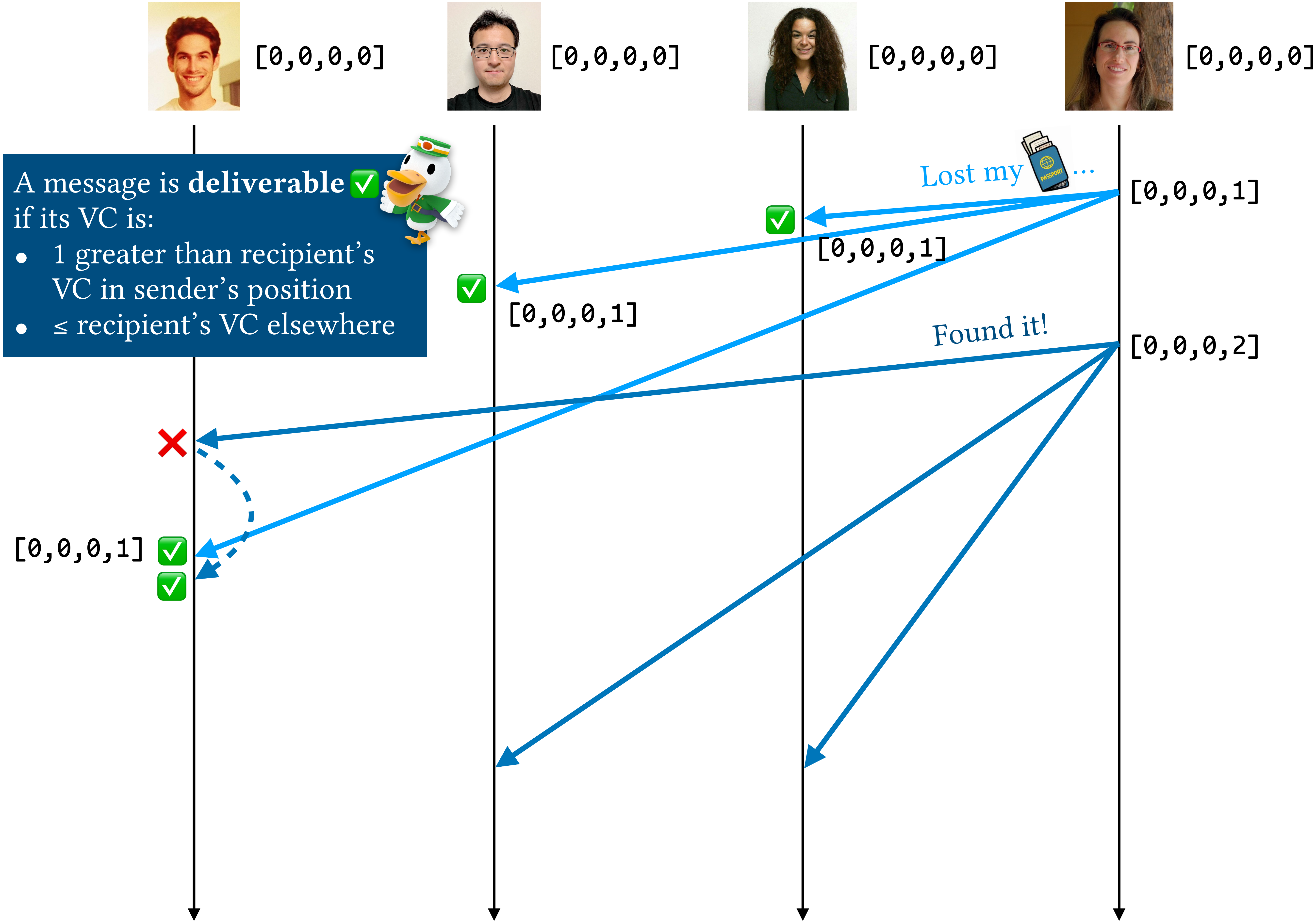

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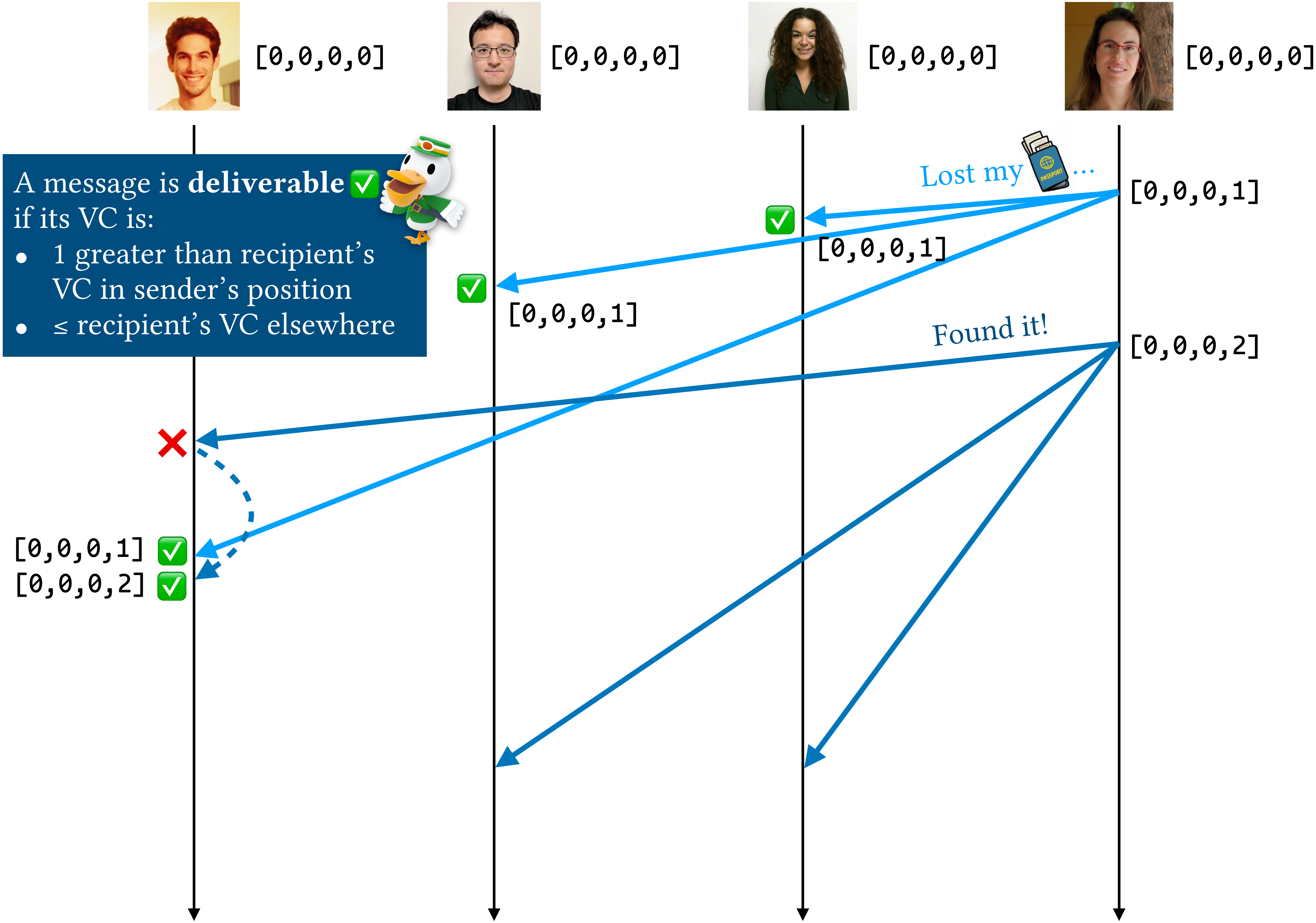

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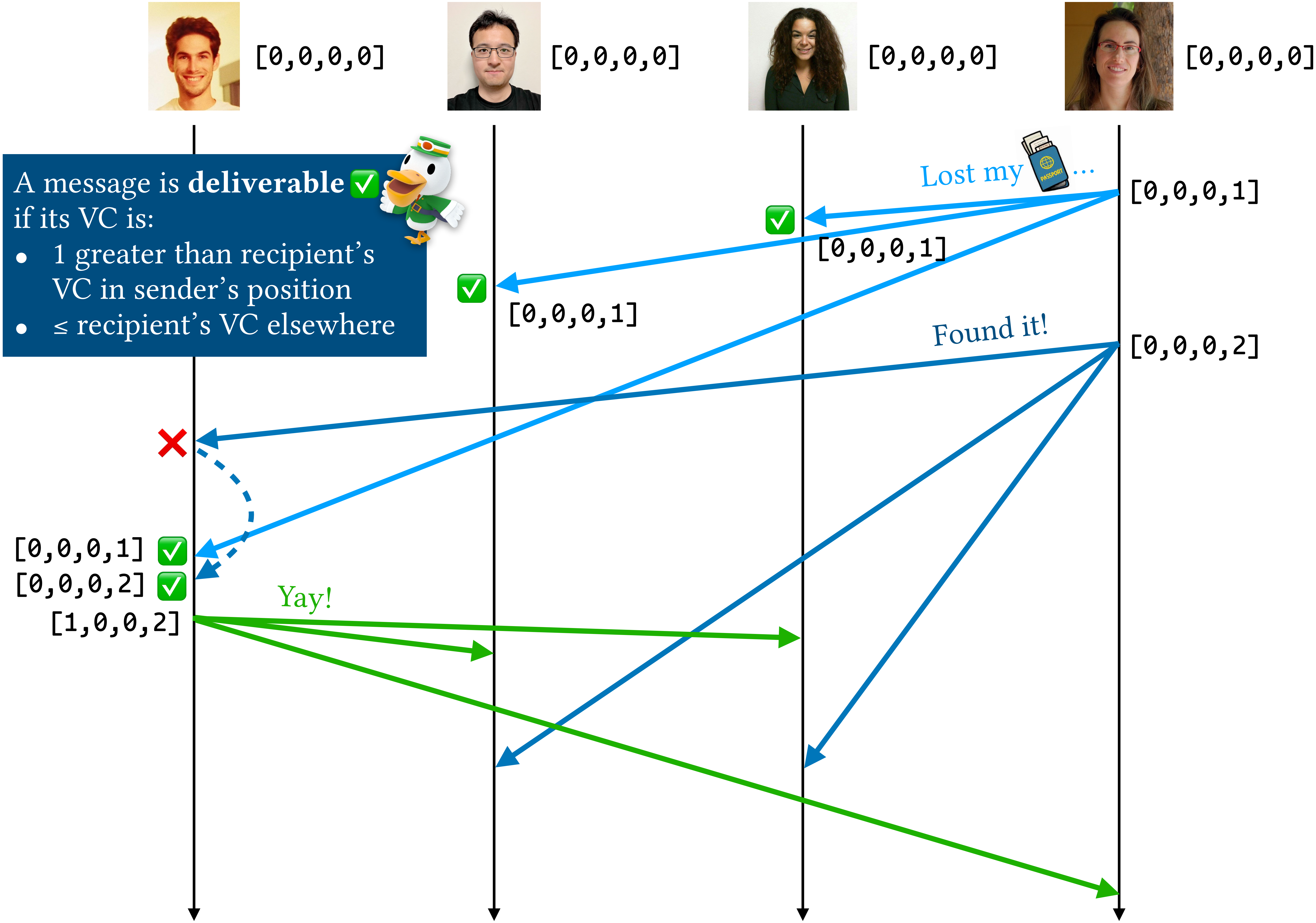

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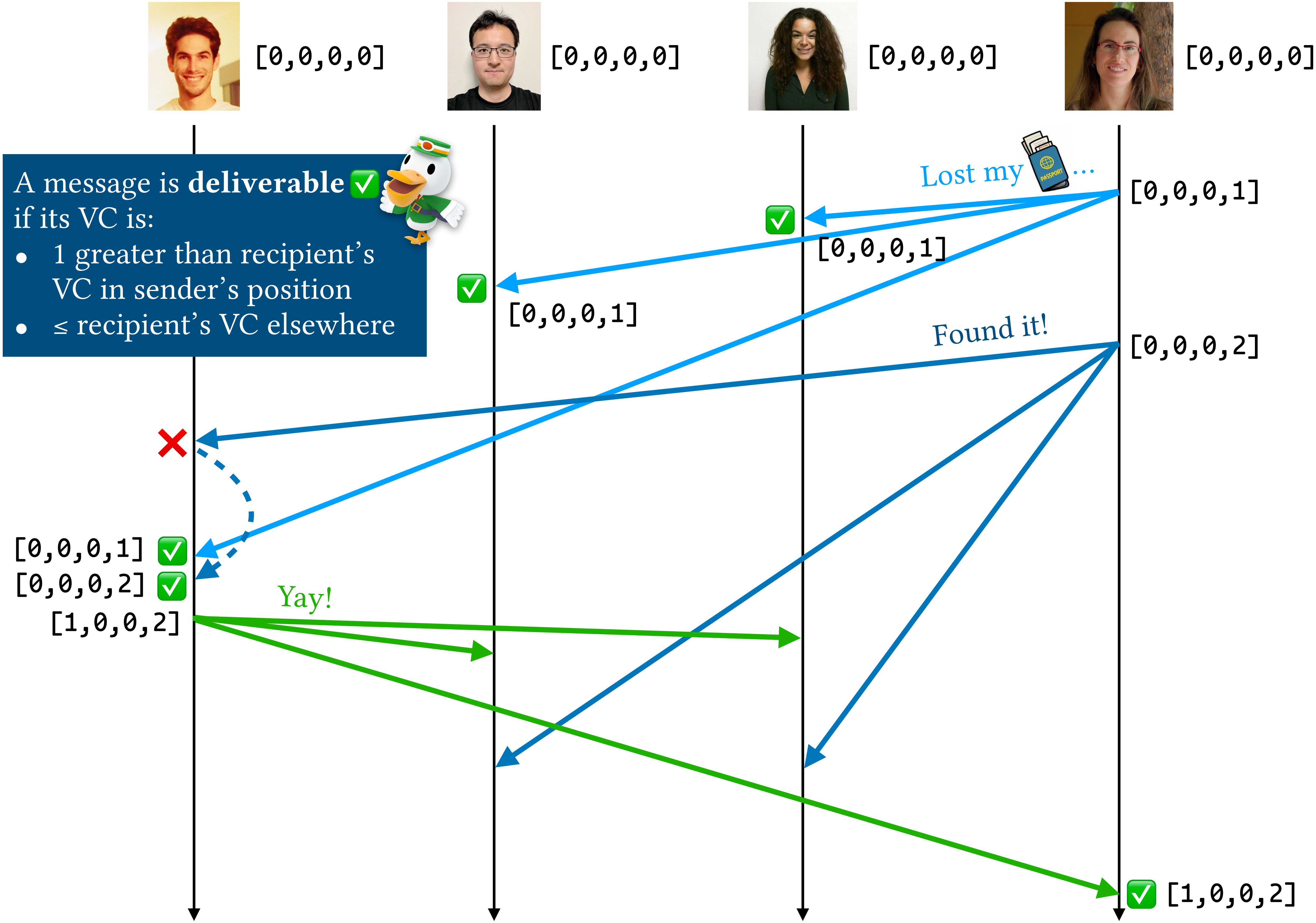

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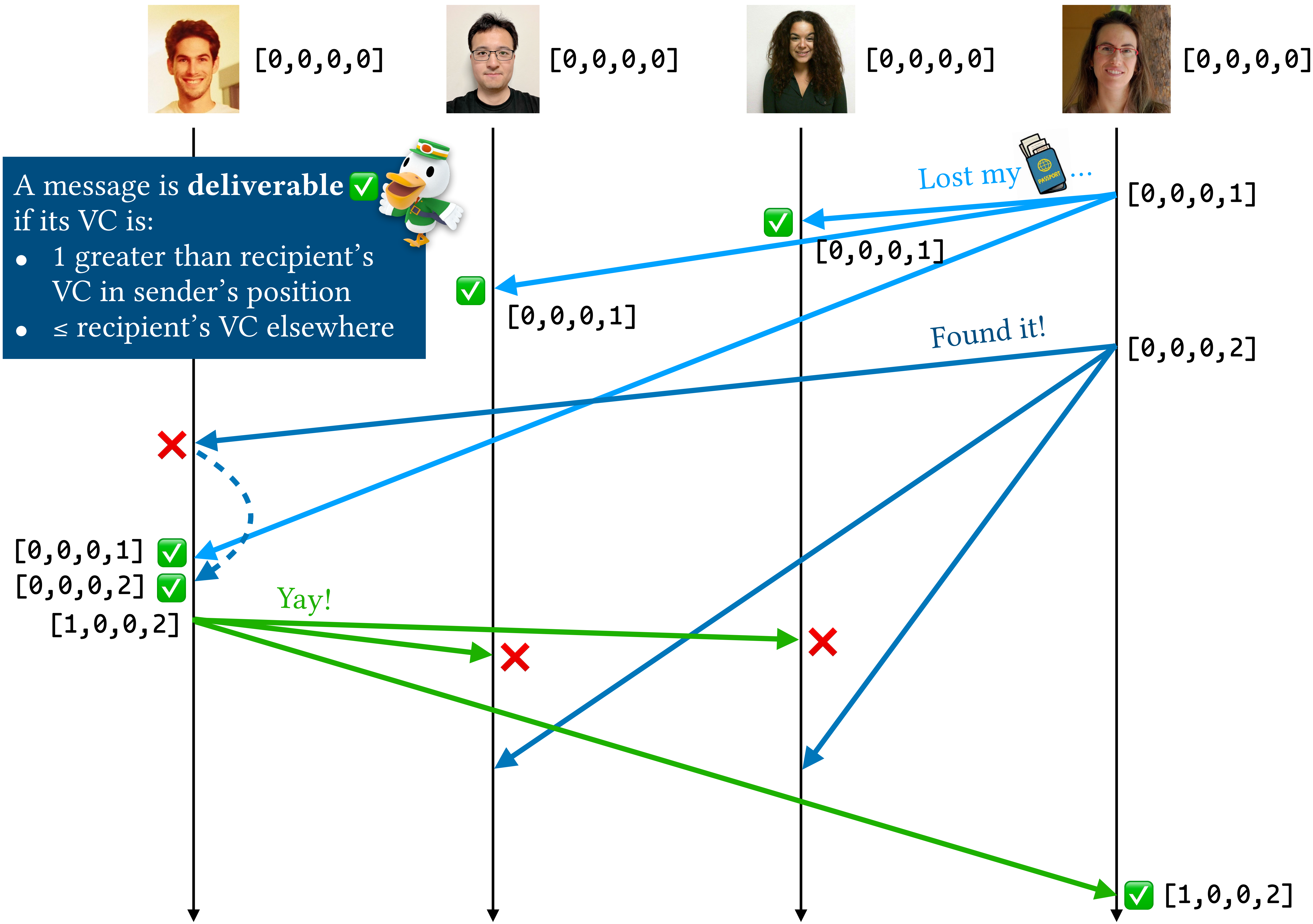

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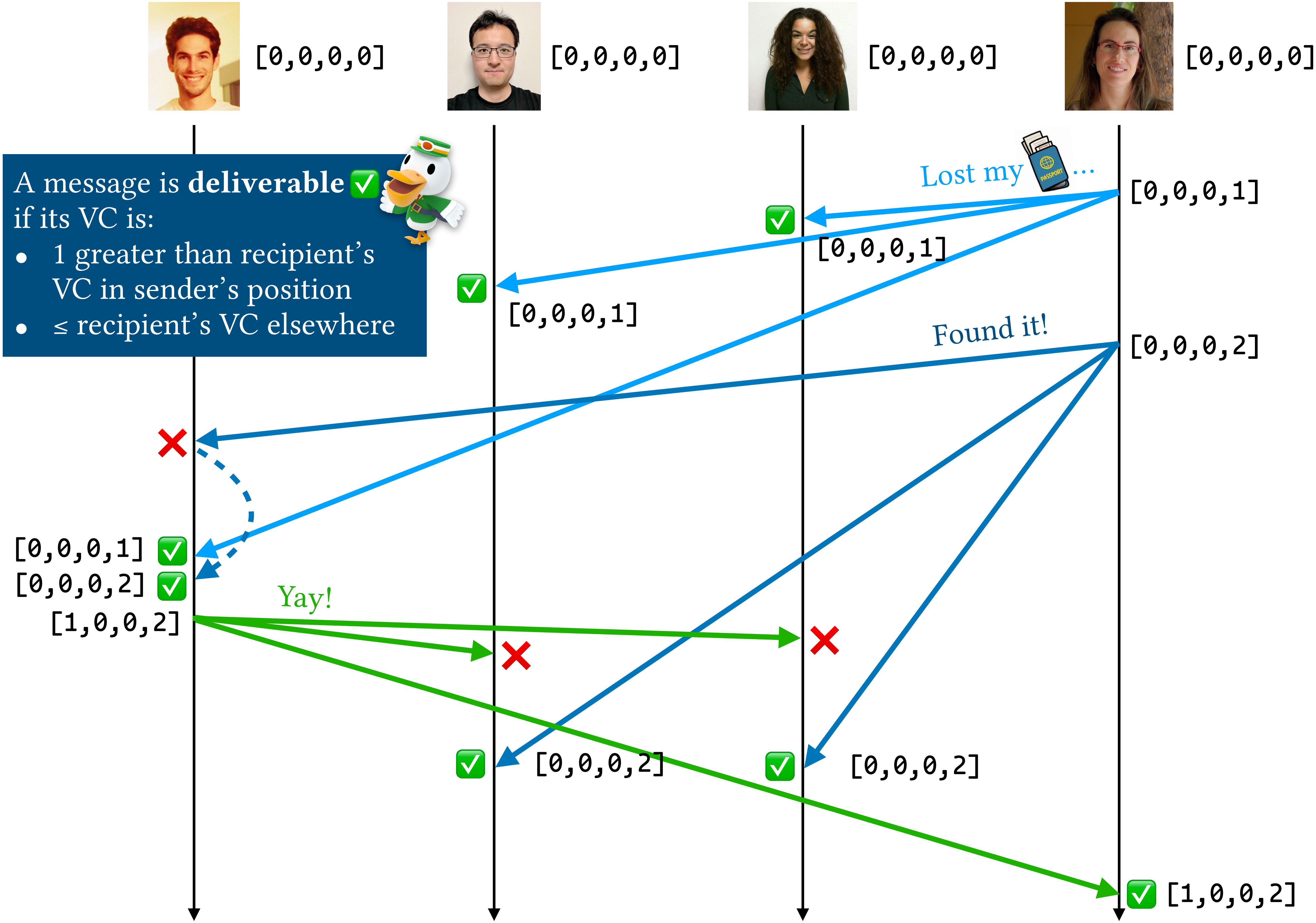

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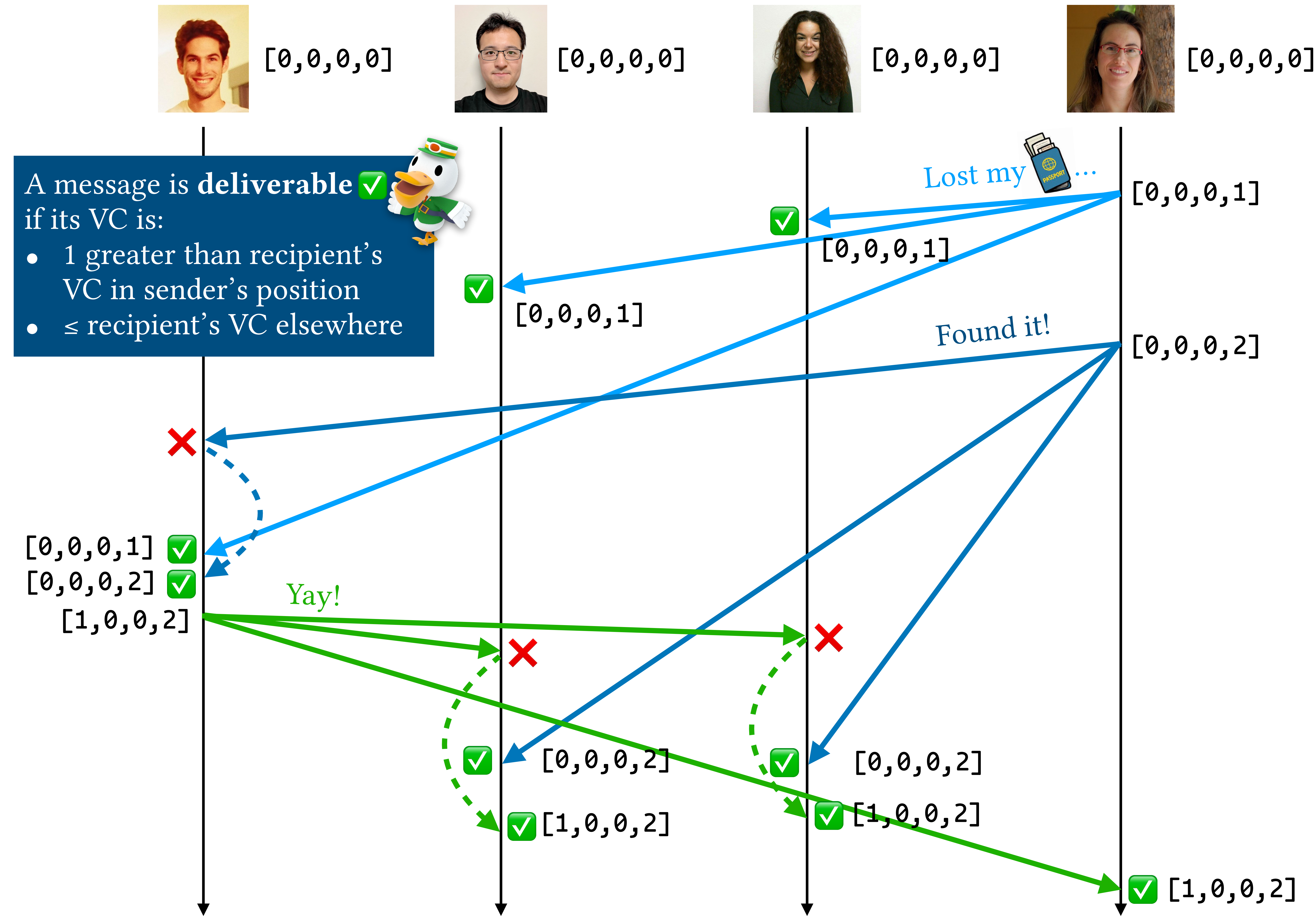

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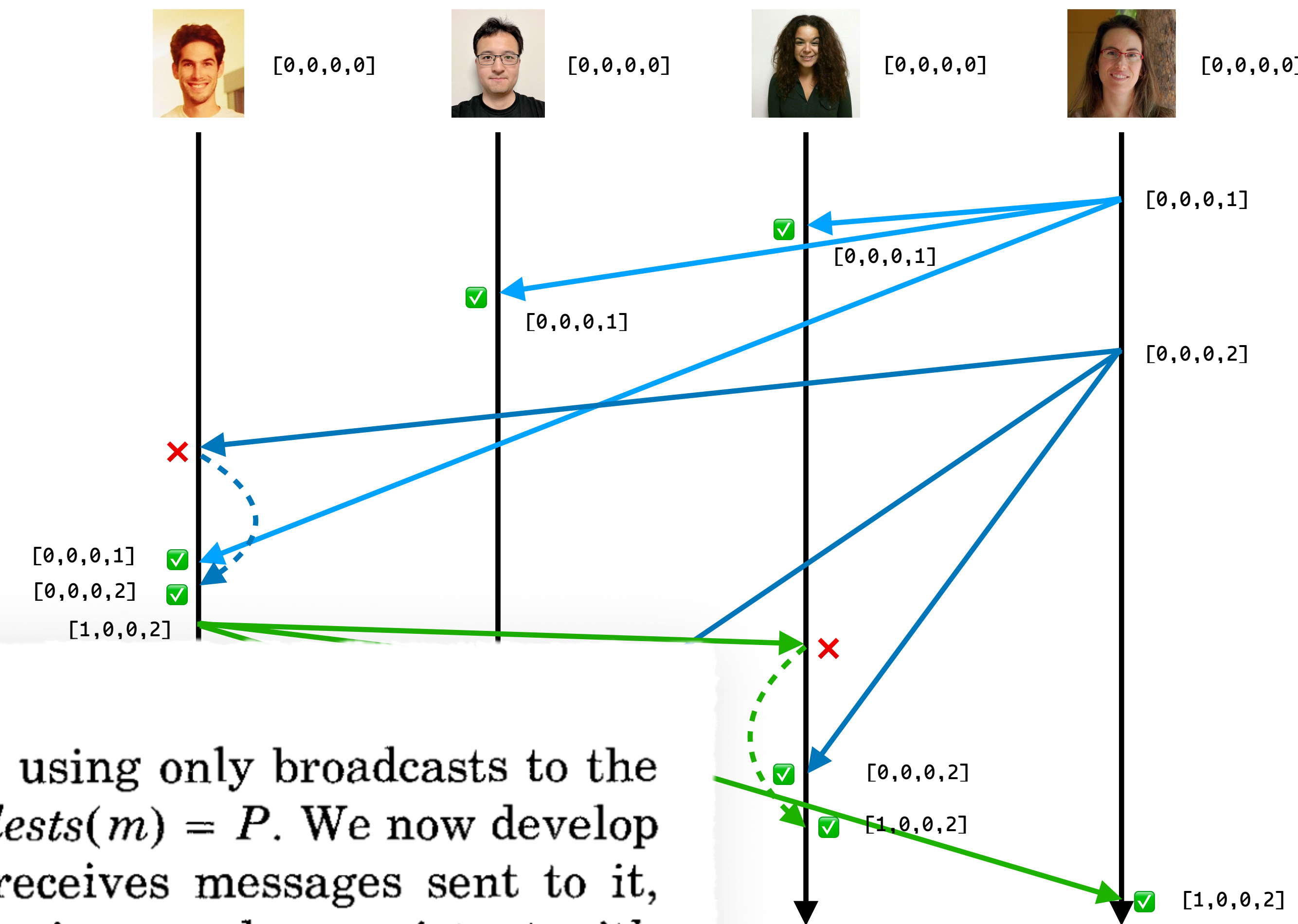
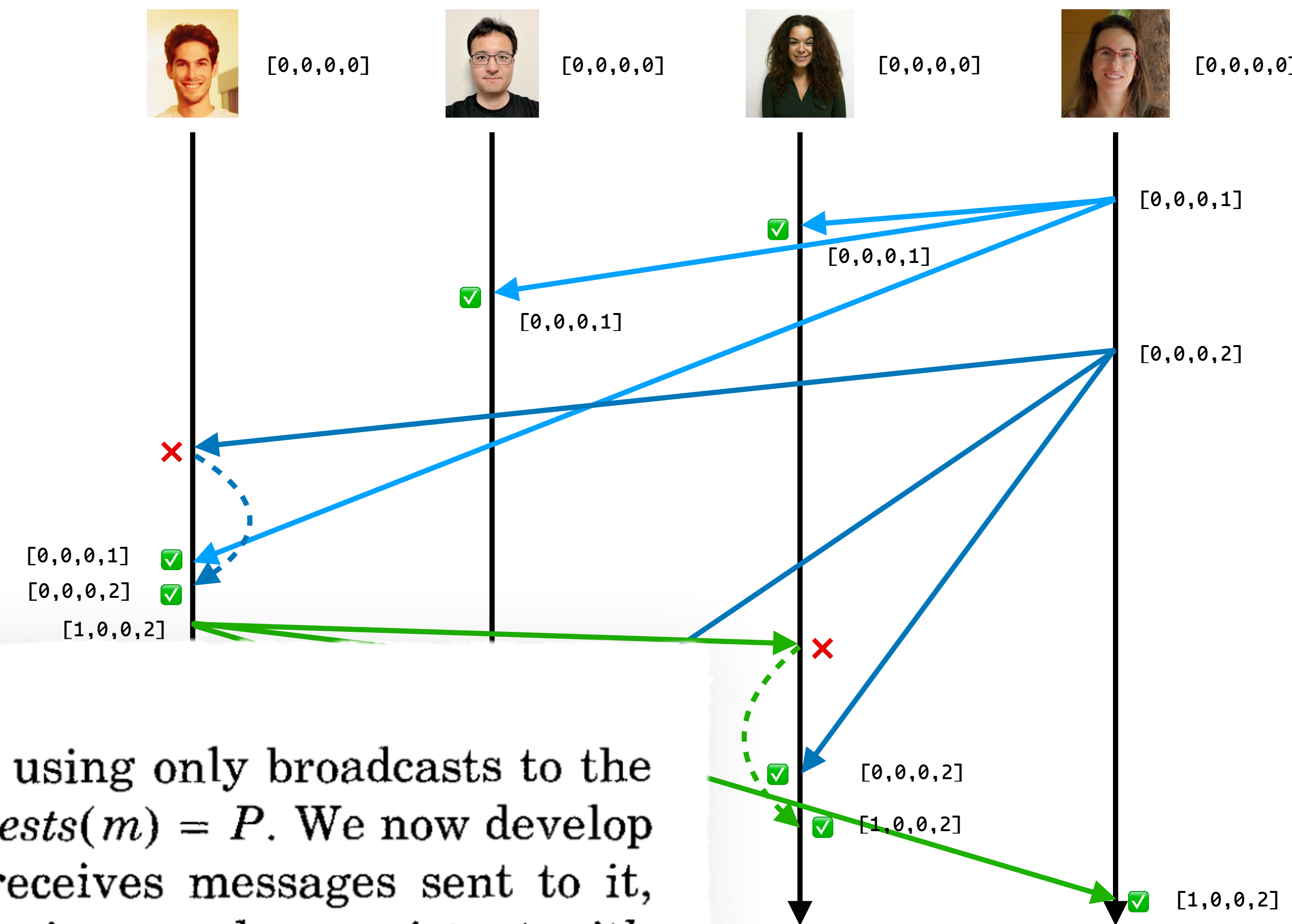


5.1 CBCAST Protocol

Suppose that a set of processes P communicate using only broadcasts to the full set of processes in the system; that is, $\forall m$: *dests*$(m) = P$. We now develop a *delivery protocol* by which each process p receives messages sent to it, delays them if necessary, and then delivers them in an order consistent with causality:

$$m \rightarrow m' \Rightarrow \forall p\colon \mathit{deliver}_p(m) \xrightarrow{p} \mathit{deliver}_p(m').$$

Birman et al., "Lightweight Causal and Atomic Group Multicast"
ACM *TOCS*, 1991



5.1 CBCAST Protocol

Suppose that a set of processes P communicate using only broadcasts to the full set of processes in the system; that is, $\forall m$: $dests(m) = P$. We now develop a *delivery protocol* by which each process p receives messages sent to it, delays them if necessary, and then delivers them in an order consistent with causality:

$$m \rightarrow m' \Rightarrow \forall p\colon deliver_p(m) \xrightarrow{p} deliver_p(m').$$

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```
type CausalDelivery p =
    { m : Message | elem (Deliver m ) (pHist p) }
  -> { m': Message | elem (Deliver m') (pHist p)
                     && causallyBefore m m' }
  -> { _: Proof | ordered (pHist p) (Deliver m) (Deliver m') }
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data Op = OpReceive (Message) | OpDeliver | OpBroadcast

step :: Op -> Process -> Process
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causalDeliveryPreservation :: ops : [Op]
                           -> p : Process
                           -> CausalDelivery p
                           -> CausalDelivery (foldr step p ops)
causalDeliveryPreservation = … – a few hundred lines
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[HATRA 2021]

Toward Hole-Driven Development in Liquid Haskell

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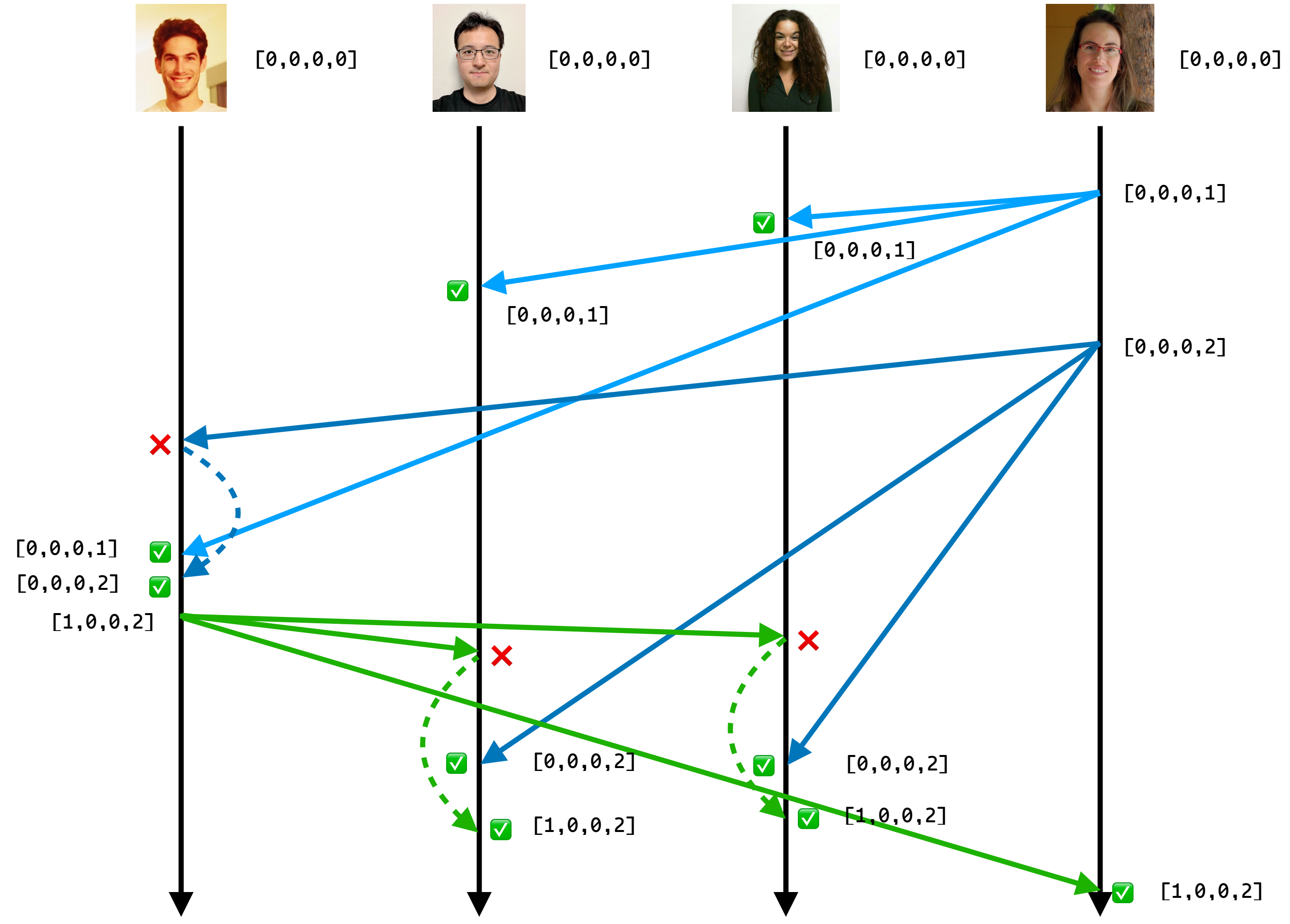
LINDSEY KUPER, University of California, Santa Cruz, USA

Liquid Haskell is an extension to the Haskell programming language that adds support for *refinement types*: data types augmented with SMT-decidable logical predicates that refine the set of values that can inhabit a type. Furthermore, Liquid Haskell's support for *refinement reflection* enables the use of Haskell for general-purpose mechanized theorem proving. A growing list of large-scale mechanized proof developments in Liquid Haskell take advantage of this capability. Adding theorem-proving capabilities to a "legacy" language like Haskell lets programmers directly verify properties of real-world Haskell programs (taking advantage of the existing highly tuned compiler, run-time system, and libraries), just by writing Haskell. However, more established proof assistants like Agda and Coq offer far better support for interactive proof development and insight into the proof state (for instance, what subgoals still need to be proved to finish a partially-complete proof). In contrast, Liquid Haskell provides only coarse-grained feedback to the user — either it reports a type error, or not — unfortunately hindering its usability as a theorem prover.

In this paper, we propose improving the usability of Liquid Haskell by extending it with support for Agda-style *typed holes* and interactive editing commands that take advantage of them. In Agda, typed holes allow programmers to indicate unfinished parts of a proof, and incrementally complete the proof in a dialogue with the compiler. While GHC Haskell already has its own Agda-inspired support for typed holes, we posit

Tak!

Languages, Systems, and Data Lab: lsd.ucsc.edu
Lindsey's research blog: decomposition.al



github.com/lsd-ucsc/cbcast-lh