S2014, BME 101L: Applied Circuits Lab 5a
Characterizing loudspeaker

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1 Design Goal

This is a measurement and modeling lab. The goal is to provide an equivalent circuit for the loudspeaker in your parts kit, so that we can later design audio amplifiers for it.

2 Background

2.1 Inductors

Since we are not using a textbook this quarter, I have put a lot of "textbook" material about inductors here, but I've also tried to indicate which of the material is just to be read once and which you need to really internalize.

Inductance results from combining two properties of conductors and magnetic fields:

- When a current passes through a conductor, it generates a magnetic field perpendicular to the conductor. When the current changes, the magnetic field changes proportionately.

- When a conductor is in a time-varying magnetic field perpendicular to the conductor, a voltage is produced that is proportional to the rate of change of the field.

That means that for any conductor the voltage from one end of the conductor to the other is proportional to the change in current:

$$v(t) = L\frac{di(t)}{dt} \ .$$

The constant of proportionality, usually written as L, is called the *inductance* (or, more pedantically, *self-inductance*) of the conductor, and the unit of inductance is the *henry*, named after the 19th century physicist Joseph Henry, written as a capital H. The henry can be expressed in terms of various other electrical units:

$$H = \frac{V}{A/s} = \frac{Vs}{A} = \Omega s = \frac{s^2}{F} = \frac{Tm^2}{A} \ ,$$

where V is volts, A is amperes, s is seconds, Ω is ohms, F is farads, T is teslas, and m is meters.

Inductance of a conductor depends mainly on the geometry of the conductor. For example, if we take a pair of parallel straight wires connected to form a loop, the inductance is

$$L = \frac{\mu_0}{\pi} w \left(\ln\left(\frac{d}{r}\right) + \frac{Y}{2} \right) \ ,$$

where L is the inductance in henries, w is the length of each wire in meters, $\mu_0 = 4\pi 10^{-7}\frac{Tm}{A}$ is the magnetic constant, d is the distance between the wires, r is the radius of the wire, and Y is a constant for modeling "skin effect" that varies with frequency. At low frequency in thin wires (most of what we do in this class), the skin effect is negligible and $Y = 1/2$. At high frequencies (radio electronics) or in thick wires (power-grid electronics), the current mainly travels on the surface of the wire, and Y drops to 0.

(Actually, at high frequencies things get even more complicated, but we'll only deal with low frequencies in this class.)

I don't expect anyone to remember this formula for inductance, and we won't use it directly—the main thing to notice is that inductance increases linearly with wire length. When you need to keep inductance to a minimum (for example, to prevent ringing in the ground wire of an oscilloscope probe), keep wires short.

Inductance can be greatly increased if we coil up the wire to make the magnetic field more intense. For example, a *solenoid* is a long, thin helix of wire (that is, the length of the helix l is much larger than the diameter of the helix d). The inductance of a solenoid is approximately

$$L = \mu_0 N^2 A / l \ ,$$

where $\mu_0 = 4\pi 10^{-7} \frac{Tm}{A}$ is the magnetic constant, N is the number of turns of the helix, A is the cross-sectional area of the cylinder in meters, and l is the length of the helix in meters. Note that the wire length w is approximately $N\pi d$ and $A = \pi d^2/4$, so we could write the inductance as

$$L = \frac{\mu_0 N^2 \pi d^2}{4l} = \frac{\mu_0}{4\pi} \frac{w^2}{l} \ .$$

Coiling up a wire of length w has increased the inductance by a factor proportional to w/l—the shorter the helix relative to the wire length, the more the inductance. We won't be using this formula in this class—but you should be aware that coiling up a wire greatly increases the inductance.

In practice, long, thin solenoids are an impractical configuration to work with, and formulas have been devised for computing the inductance of many different conformations of inductors, including flat ones that can be built directly on circuit boards. (See, for example, "Simple Accurate Expressions for Planar Spiral Inductances" by S. Mohan, M. Hershenson, S. Boyd, and T. Lee *IEEE Journal of Solid-State Circuits*, 34(10):1419–1424, October 1999. `http://www.stanford.edu/~boyd/papers/inductance_expressions.html`.) Again, we won't be computing inductances for layouts of wires in this class, but you should be aware that it is possible to design inductors to have specific inductances.

The air-coil inductors mentioned above often don't have enough inductance, either for their size or for the resistance of the wire, so another technique is used to increase inductance: adding a ferromagnetic "core" to the coil. The ferromagnetic material concentrates the magnetic field and greatly increases the inductance. (See `http://en.wikipedia.org/wiki/Magnetic_core`.) We'll be using a ferrite core inductor later this quarter as part of the power amplifier.

Please read big chunks of `http://en.wikipedia.org/wiki/Inductance`. We won't be dealing with mutual inductance and transformers in this class, nor will I be quizzing you on the physics beyond the vague intuitive ideas presented above, but you should have some understanding of self inductance.

Figure 1: We often model actual inductors as an inductance (L_1) in series with the resistance of the wire used to make the inductor (R_1).

2.2 Impedance of inductors

Most of the math in Section 2.1 was stuff to read once and forget—it was there just to give you two concepts: increasing wire length increases inductance and coiling up wires increases inductance. The math in this section is more important—you will use the impedance of inductors for design problems. The derivation of the impedance is something that you should understand, and the formula itself is important enough to memorize.

So, let's derive impedance for an inductor. Remember that impedance is the ratio of voltage to current, and that we model both voltage and current with complex-valued sinusoids. For simplicity, let's say that the current through an inductor is $i(t) = e^{j\omega t}$. Then the voltage across the inductor is

$$\begin{aligned} v(t) &= L\frac{di(t)}{dt} \\ &= L\frac{de^{j\omega t}}{dt} \\ &= j\omega L\, e^{j\omega t} \\ &= j\omega L\, i(t) \;, \end{aligned}$$

and the impedance of an inductor is

$$Z_L = j\omega L \;.$$

Note that for DC ($\omega \to 0$), the impedance is 0, and at very high frequencies the impedance is large.

Because long, thin wires are used to make inductors, and wires are not perfect conductors, we often end up having a significant resistance in the wire, so it is common to model inductors as an inductance and a resistance in series, as shown in Figure 1.

The magnitude of the impedance is dominated by the resistance at low frequencies and by the impedance at high frequencies. We have a "corner frequency" where the two contribute equally: ($R = |j\omega L|$ or $f = \frac{R}{2\pi L}$. See Figure 2 for how the magnitude of impedance changes with frequency for a typical ferrite-core inductor (an air-core inductor would usually have a larger resistance for the same inductance). The plot was created with the gnuplot script in Figure 3.
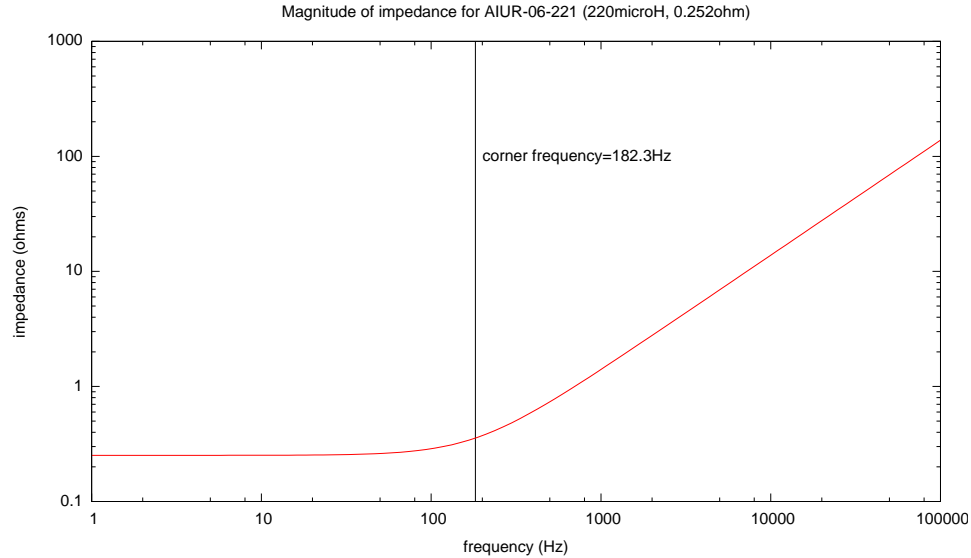
Figure 2: Using the model of Figure 1 for the inductor in your parts kit with $L_1 = 220\mu H$ and $R_1 = 0.252\Omega$, we can compute the impedance for any frequency. Note that below the corner frequency $R/(2\pi L)$, the impedance is mainly from the R_1 resistance, and above the corner frequency, the impedance is mainly from the L_1 inductance.

```
j=sqrt(-1)

# impedance of two impedances in parallel
parallel(z1,z2) = z1*z2/(z1+z2)
# note: there is no need for a "series" function, since that is just z1+z2

# impedance of a capacitor of C farads at f Hz
Z_C(f,C) = 1./(j*2*pi*f*C)
# impedance of an inductor of L henries at f Hz
Z_L(f,L) = j*2*pi*f*L

# values for inductor AIUR-06-221
L=220E-6
R=0.252

set title sprintf("Magnitude of impedance for AIUR-06-221 (%.0fmicroH, %.3fohm)",L*1e6,R)
set ylabel "impedance (ohms)"
set xlabel "frequency (Hz)"
set logscale xy
set xrange [1:1e5]

corner_freq = R/(2*pi*L) # compute and show corner frequency
unset arrow
unset label
set arrow nohead from corner_freq,0.1 to corner_freq,1000
set label sprintf("corner frequency=%.1fHz",corner_freq) at corner_freq*1.1, 100

plot abs(R+Z_L(x,L)) notitle
```

Figure 3: This is the gnuplot script used to generate Figure 2. Note that I defined some general-purpose constants and functions at the beginning of the script (j, parallel(z1,z2), Z_C(f,C), and Z_L(f,L)) that will be useful in almost any gnuplot script that plots or fits complex impedance. Note also that I am plotting the *magnitude* of impedance: abs(R+Z_L(x,L)).
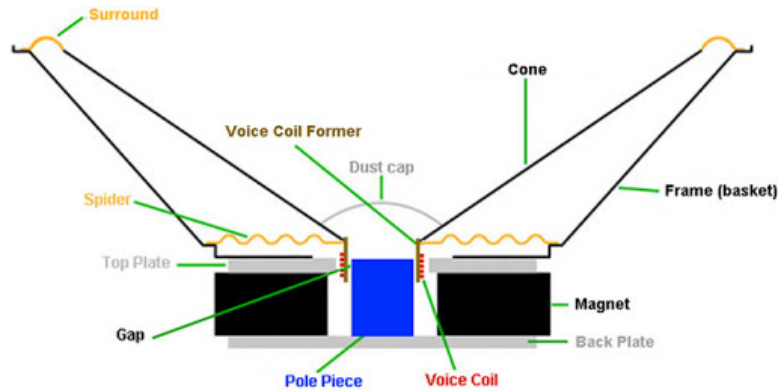
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Figure 4: Cross-sectional view of a typical loudspeaker, showing the cone, the voice coil, and the permanent magnet. Image copied from a 2011 May 8 post by "Djim" at `http://www.diyaudio.com/forums/subwoofers/` `177905-thread-those-interested-ppsl-enclosures-6.html` I do not know who drew this image, nor its copyright status.
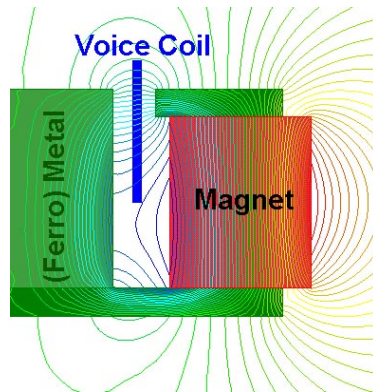

Figure 5: Cross-sectional view of the magnetic field in the magnetic gap where the voice coil sits. Note that the field is radial (always perpendicular to the wires of the voice coil). Image copied from a 2011 May 8 post by "Djim" at `http://www.diyaudio.com/forums/subwoofers/` `177905-thread-those-interested-ppsl-enclosures-6.html` I do not know who drew this image, nor its copyright status.

2.3 How loudspeakers work

A loudspeaker consists of *voice coil*, a low-mass coil of wire, attached to a stiff, low-mass (often paper) cone. Movement of the coil moves the cone, which in turn moves the air, producing sound. The voice coil sits in a magnetic field from a permanent magnet that is radially oriented. See the illustrations in Figures 4 and 5.

When current is passed through the coil, it produces a force that is perpendicular to both the wire and the magnetic field—that is a vertical force in the orientation shown in Figure 4. The force produced is approximately proportional to the current (not exactly, because the magnetic field is not really uniform).

This force moves the voice coil and the cone up or down. The supports for the cone (the "spider" and "surround") act as a spring to provide a restoring force, so that the displacement of the cone is roughly proportional to the current.

See `http://en.wikipedia.org/wiki/Voice_coil` for more information about how loudspeakers work.

Information about the loudspeakers in your parts kit is available from the distributor at `http://www.parts-express.com/jamo-30466-30462-2-1-2-x-4-shielded-speaker--299-919`

Note: that the data sheet gives two different values in ohms: the impedance (actually, the magnitude of the impedance) and the DC resistance. In this lab you will measure both. The impedance is not really a single number though, but a function of frequency, and you will model the impedance of the loudspeaker as a combination of different linear components (inductors, resistors, and capacitors).

The simplest model of a loudspeaker is that it is a constant impedance (the 8Ω value on the data sheet). A slightly more sophisticated model has a voice-coil inductance in series with a DC resistance (as in Figure 1). We'll gather data to fit both of those models, but also a more complicated one.

One of the other important properties of a loudspeaker has to do with its mechanical design: the mass of the voice coil and cone and the stiffness of the spring supporting them. As you should remember from your physics class, a mass and spring form a *harmonic oscillator* that vibrates at a resonant frequency. At that resonant frequency, even small currents produce large motions of the voice coil, in turn producing large voltages—that is, the impedance at that frequency is large. We will model that resonance as well. (Note: loudspeakers are usually used only at frequencies above their mechanical resonant frequency, both to avoid distortion of the sound and to avoid damage to the loudspeaker.)

Even if we avoid the resonant frequency of a loudspeaker, providing too large a current can have two damaging effects: heating up the voice coil too much ("burning out" the speaker) or pushing the cone past the limits that the supports can tolerate, tearing the cone or the supports. The limits are usually expressed in terms of the amount of power the loudspeaker can accept. To avoid damaging the loudspeaker, you do not want to exceed the RMS power limit for long and never more than the max power limit. That won't be a problem for this lab, but it will be an important constraint for the power-amp lab in a few weeks.

3   Pre-lab assignment

Read the data sheet and determine the power limit(s), the DC resistance, and the inductance that the speaker is supposed to have. Use gnuplot to plot the magnitude of the impedance of the loudspeaker from 1Hz to 1MHz, modeling it as just a series inductor and resistor with the values on the data sheet (this should be a log-log plot). What is the lowest impedance over this range? the highest?

Look up the data sheets for the Agilent 34401A multimeters we have in the lab to determine what frequencies and voltage or current ranges you can reasonably use.

Come up with a circuit for measuring the magnitude of impedance of a loudspeaker at an arbitrary frequency using two AC voltmeters, a sine-wave generator, and whatever components from your kit you need. (Why do I say to use two voltmeters, rather than a voltmeter and an ammeter?) Note that each measurement will have at least three pieces of data: a frequency and two voltages.

You want to be able to measure the impedance both at low frequencies and at high frequencies. Although you can change your test setup for different frequency ranges, it will be easier to do the lab if you can use the same setup for the whole frequency range.

Write a gnuplot script that can plot the impedance of your loudspeaker as a function of frequency, from the data you will collect in the lab. You will want to view this plot as you collect data, so have the script written before lab. You'll be entering the collected data by hand, so you'll want to figure out the format you'll be using. If your setup is going to change for different frequency ranges, you'll want an extra column for each parameter that you plan to change. If you will be using a single setup, then you can use gnuplot variables to name the parameters and set them once in the script (like the way that L and R are set in Figure 3).

4 Parts, tools, and equipment needed

Parts for this lab from kit:

- loudspeaker
- resistors or other passive components
- breadboard
- double-ended male headers (to connect loudspeaker to breadboard)

Equipment in lab:

- function generator
- two voltmeters

5 Procedures

5.1 Characterizing loudspeaker

Measure the DC resistance of your loudspeaker.

Measure any other components you will use in your measurement circuit.

Measure the magnitude of impedance of your loudspeaker (using the circuit you devised in pre-lab), at frequencies that are roughly equally spaced on a log scale (say 10, 15, 20, 30, 50, 75, 100, ..., or 10, 12, 15, 18, 22, 27, 33, 39, 47, 56, 68, 82, 100, ...) over the entire frequency range.

Measure the magnitude of impedance of your loudspeaker at frequencies close to the resonant frequency of your loudspeaker. You want several measurements at very close frequencies, to get the shape of the peak—try to get the resonant peak itself to within 1Hz. The resonance may not be where the spec sheet says it is, as it is dependent on how the speaker is mounted and even minor variations in the stiffness of the paper cone. The loudspeaker may also vary in its properties as it ages (the paper springs may become less stiff as they are flexed a lot, for example).

If you finish this lab quickly, then try measuring the magnitude of impedance as a function of frequency for the AIUR-06-221 inductor in your parts kit. You may need to change your test setup slightly to measure this part, as its inductance and resistance are rather different from the loudspeaker. Does the data fit the model in Figure 2?

If you have even more time on your hands, try measuring the impedance of a 0.1μF capacitor in parallel with a 1kΩ resistor.

6 Demo and writeup

We will look at models for loudspeakers that include the resonant peak in class on Wednesday.

Your report should include the schematic diagram of your measuring circuit, and schematics of any models of the loudspeaker that you fit to your data. Be sure to include the values of the components in your schematics!

Some things to discuss:

- Which models fit the data best?
- When would you use the simpler models?

- What RMS voltage can you apply to your loudspeaker without exceeding the power limit? (It may be a function of frequency.)

- What is the largest peak-to-peak square wave that you can apply to your loudspeaker without exceeding the RMS power limit?

- How would you modify the measurement technique to get the full complex impedance, rather than just the magnitude of it?

7 Design Hints

Since this is a measurement and modeling lab, rather than a design lab, the main things to think about are how to make and record the large number of measurements efficiently.

You also want to be sure that all the voltages you measure are in a reasonable range for the multimeter. If one of the voltages gets very small, you may have difficulty getting accurate measurements. You should be able to design your test setup ahead of time to ensure that your voltages stay within a reasonable range.