HW 8 Answer Key

SECTION 4.3

46. The largest binomial coefficient occurs for $r = \lfloor n/2 \rfloor$. To prove this, first note that by symmetry(C(n,r) = C(n,n-r)) it is enough to show that for $r < \lfloor n/2 \rfloor, C(n, r+1) > C(n, r)$. Now we have $C(n, r+1) = \frac{n!}{(r+1)! \cdot (n-r-1)!} = \frac{n! \cdot (n-r)}{(r+1) \cdot r! \cdot (n-r)!} = \frac{n-r}{r+1} \cdot C(n, r)$.
Thus it is sufficient to show that $n - r > r + 1$. This is equivalent to $r < n/2 - 1/2$. But this follows from our original assumption that $r \leq \lfloor n/2 \rfloor - 1$.

57. The number of paths of length n of the type described equals the number of strings of length n which is 2^n. A path of length n of the type described must end at a point that has n as the sum of it's coordinates, say $(n-k, k)$ for some k between 0 and n inclusive.The number of such paths ending at (n-k,k) equals $C(n-k+k, k) = C(n, k)$. Hence $\sum_{k=0}^{n} C(n, k) = 2^n$

SECTION 4.4

6. There are 16 cards that qualify as being an ace or a heart, so the answer is $16/52 = 4/13 \approx 0.31$.
We could also compute this from Theorem 2 as $4/52 + 13/52 - 1/52$.

8. $\frac{C(51,4)}{C(52,5)} = \frac{5}{52}$
There are C(52,5) possible poker hands. There are C(51,4) possible hands with the ace of hearts. The ace takes up one of those cards, then pick 4 more from the remaining deck.

12. $C(4,1)C(48,4)/C(52,5)$
There are 4 ways to specify the ace. One the ace is chosen for the hand, there are C(48,4) ways to choose nonaces for the remaining four cards. Therefore there are $4C(48,4)/C(52,5) \approx 0.30$

14. $1317888/2598960 = 2112/4165 \approx 0.51$ There are $C(52,5) = 2,598,960$ different hands. We need to count the number of hands that have 5 different kinds/ranks. There are C(13,5) ways to choose the kinds. For each card, there are then 4 ways to choose the suit for each of those cards. Therefore there are $C(13,5) \cdot 4^5 = 1,317,888$ ways to choose the hand.

16. $5148/2598960 = 33/16660 \approx 0.0020$
There are $C(52,5) = 2,598,960$ different hands. $C(4,1) \cdot C(13,5) = 5148$ are flushes, since we can specify a flush by choosing a suit and then choosing 5 cards from that suit.

18. There are clearly only $10.4 = 40$ straight flushes, since all we get to specify for a straight flush is the starting kind in the straight(anything from ace upto 10) and the suit. Therefore the answer is $40/C(52,5) = 40/2598960 = 1/64974$.

20. $4/C(52,5) = 4/2598960 = 1/649740$.
There are 4 royal flushes, one in each suit.

24. In each case, if the numbers are chosen from integers from 1 to n, then there are $C(n,6)$ possible entries, only one of which is the winning one, so the answer is $1/C(n,6)$
a. $1/C(30,6) = 1/593775 \approx 1.7\ 10^{-6}$
b. $1/C(36,6) = 1/1947792 \approx 5.1\ 10^{-7}$
c. $1/C(42,6) = 1/5245786 \approx 1.9\ 10^{-7}$
d. $1/C(48,6) = 1/12271512 \approx 8.1\ 10^{-8}$

32. It is more likely with two dice. $21/216 = 0.097$ vs. $5/36 = 0.139$

There are $6^2 = 36$ equally possible outcomes of the roll of two dice.
There are 5 ways to get a total of 8 when two dice are rolled:
(6,2), (5,3), (4,4), (3,5), (2,6).
There are $6^3 = 216$ equally possible outcomes of the roll of three dice.
There are 21 ways to get a total of 8 when three dice are rolled:
The first die could turn out to be a 6, giving rise to the 1 triple (6,1,1). The first die could be a 5, giving rise to the 2 triples (5,2,1) and (5,1,2). Continuing in this way, we see that there are 3 triples giving a total of 8 when the first die shows a 4, 4 triples when it shows 3, 5 triples when it shows a 2, and 6 triples when it shows a 1 (namely (1,6,1), (1,5,2), (1,4,3), (1,3,4), (1,2,5), and (1,1, 6)) Therefore the are $1 + 2 + 3 + 4 + 5 + 6 = 21$ possible outcomes giving a total of 8.