HW 6 Answer Key

Section 3.2

55. Let $P(n)$ be the statement that if x_1, x_2, $\cdots x_n$ are n distinct real numbers, n-1 multiplications are required to form their product.The basis case $P(1)$ is true since $1-1=0$ multiplications are required to find the product of x_1, as there is only one factor. Suppose that $P(k)$ is true for $1 \leq k \leq n$. The last multiplication used to to find the product of $n+1$ terms, is a multiplication of the product of the first k of these numbers for some k and the product of the last $n+1-k$ of them. By the inductive hypothesis, $k-1$ multiplications are used to find the product of the k of the numbers, and $n-k$ multiplications are used to find the product of $n+1-k$ of them. Since one more multiplication is used to find the productof all of them the total no. of multiplications used equals $(k-1)+(n-k)+1=n$. Hence $P(n+1)$ is true.

Section 3.3

4a.$f(2) = f(1) - f(0) = 1 - 1 = 0$
$f(3) = f(2) - f(1) = 0 - 1 = -1$
$f(4) = f(3) - f(2) = -1 - 0 = -1$
$f(5) = f(4) - f(3) = -1 + 1 = 0$
b. Clearly $f(n) = 1$ for all n, since $1 \cdot 1 = 1$.
c. $f(2) = f(1)^2 + f(0)^3 = 1^2 + 1^3 = 2$
$f(3) = f(2)^2 + f(1)^3 = 2^2 + 1^3 = 5$
$f(4) = f(3)^2 + f(2)^3 = 5^2 + 2^3 = 33$
$f(5) = f(4)^2 + f(3)^3 = 33^2 + 5^3 = 1214$
d. Clearly $f(n) = 1$ for all n, since $1/1 = 1$.

6 Many answers are possible
6a. Each term is 4 more than that term before it. We can therefore define the sequence by $a_1 = 2$ and $a_{n+1} = a_n + 4$
6b. We note that the terms alternate: 0,2,0,2, and so on.
Thus we could define the sequence by $a_1 = 0$, $a_2 = 2$, and $a_n = a_{n-2}$ for all $n \geq 3$
6c. The sequence starts out 2, 6, 12, 20, 39, and so on. The differences between successive terms are 4, 6, 8, 10, and so on. Thus the n^{th} term is 2n greater that the term preceding it; In symbols: $a_n = a_{n-1} + 2n$. Together with the initial condition of $a_1 = 2$.
6d. The sequence starts out 1, 4, 9, 16, 25, and so on. The differences between successive terms are 3, 5, 7, 9, and so on - the odd numbers.
The n^{th} term is 2n-1 greater than the term precedint it; In symbols: $a_n = a_{n-1} + 2n - 1$. Together with the initial condition $a_1 = 1$, this defines the sequence recursively.

10 The base case for $n = 1$ is clear since $f_1^2 = f_1 f_2 = 1$.
Assume the inductive hypothesis. Then $f_1^2 + f_2^2 + \cdots f_n^2 + f_{n+2}^2$
$= f_n f_{n+1} + f_{n+1}^2$
$= f_{n+1}(f_n + f_{n+1})$
$= f_{n+1} f_{n+2}$ as desired.

12 The base case for $n = 1$ is clear since $f_2 f_0 - f_1^2 = 1 \cdot 0 - 1^2 = -1 = (-1)^1$.
Assume the inductive hypothesis. Then $f_{n+2} f_n - f_{n+1}^2 = (f_{n+1} + f_n) f_n - f_{n+1}^2$
$= f_{n+1} f_n + f_n^2 - f_{n+1}^2$

$= -f_{n+1}(f_{n+1} - f_n) + f_n^2$
$= -(f_{n+1}f_{n-1} - f_n^2)$
$= -(-1)^n = (-1)^{n+1}$.

22 a. Odd integers are obtained from other odd integers by adding 2.
We can define this set S as follows $1 \in S$; and if $n \in S$, then $n + 2 \in S$
b. Powers of 3 are obtained from other powers of 3 by multiplying by 3.
We can define this set S as follows: $3 \in S$ (this is 3^1); and if $n \in S$, then $3n \in S$.
c. There are several ways to do this. We will assume that the variable for these polynomials is the letter x. All integers are in S (this base case gives us all the constant polynomials); if $p(x) \in S$ and n is any integer, then $xp(x) + n$ is in S.
Another method constructs the polynomials term by term.
Its base case is to let 0 be in S; and its inductive step is to say that $p(x) \in S$, c is an integer, and n is a nonnegative integer, the $p(x) + c(x)^n$ is in S.

29. $w^0 = \lambda$ and $w^{n+1} = ww^n$ where w^i represents the concatenation of i copies of w.

32.There can be several different correct answers for this one.
The base case would be that 0 belongs to the set.
The key fact in the recursive case is that any string containing more 0's than 1's could be a concatenation of these strings, or a concatenation with a 1 before, in between or after them. If x and y are in the set then so are xy, x1y, xy1, 1xy.

33. Let P(i) be "$l(w^i) = i.l(w)$."
$P(0)$ is true since $l(w^0) = 0 = 0.l(w)$.
Assume P(i) is true. Then $l(w^{i+1}) = l(ww^i) = l(w) + l(w^i) = l(w) + i.l(w) = (i+1).l(w)$ which gives the required proof.

36a. $A(1, 0) = 0$ by the second line of the definition.
b. $A(0, 1) = 2$ by the first line of the definition.
c. $A(1, 1) = 2$ by the third line of the definition.
d. $A(2, 2) = A(1, A(2, 1)) = A(1, 2) = A(0, A(1, 1)) = A(0, 2) = 4$