HW 10 Answer Key

Section 5.2

4a. $r^2 - r - 6 = 0 r = -2, 3$
$a_n = \alpha_1(-2)^n + \alpha_2 3^n$
$3 = \alpha_1 + \alpha_2$
$6 = -2\alpha_1 + 3\alpha_2$
$\alpha_1 = 3/5, \alpha_2 = 12/5$
$a_n = 3/5 \cdot -2^n + 12/5 \cdot 3^n$
c. $r^2 - 6r + 8 = 0 r = 2, 4$
$a_n = \alpha_1 2^n + \alpha_2 4^n$
$4 = \alpha_1 + \alpha_2$
$10 = 2\alpha_1 + 4\alpha_2$
$\alpha_1 = 3, \alpha_2 = 1$
$a_n = 3 \cdot 2^n + 4^n$
e. $r^2 - 1 = 0 r = -1, 1$
$a_n = \alpha_1(-1)^n + \alpha_2 1^n$
$5 = \alpha_1 + \alpha_2$
$-1 = -\alpha_1 + \alpha_2$
$\alpha_1 = 3, \alpha_2 = 2$
$a_n = 3 \cdot (-1)^n + 2$
g. $r^2 + 4r - 5 = 0 r = 1, -5$
$a_n = \alpha_1(-5)^n + \alpha_2 1^n$
$2 = \alpha_1 + \alpha_2$
$8 = (-5)\alpha_1 + \alpha_2$
$\alpha_1 = -1, \alpha_2 = 3$
$a_n = -(-5)^n + 3$

12. The characteristic equation is $r^3 - 2r^2 - r + 2 = 0$
This factors as $(r-1)(r+1)(r-2) = 0$.
The roots are 1, -1, and 2. Therefore the general solution is $a_n = \alpha_1 + \alpha_2(-1)^n + \alpha_3 2^n$.
Plugging in initial conditions gives $3 = \alpha_1 + \alpha_2 + \alpha_3, 6 = \alpha_1 - \alpha_2 + 2\alpha_3$, and $0 = \alpha_1 + \alpha_2 + 4\alpha_3$. The solution to this system of equations is $\alpha_1 = 6, \alpha_2 = -2, \alpha_3 = -1$. Therefore the answer is $a_n = 6 - 2(-1)^n - 2^n$

14 $a_n = 1 + (-1)^n + 2^n$
The characteristic equation is $r^4 - 5r^2 + 4 = 0$. This factors as $(r^2-1)(r^2-4) = (r-1)(r+1)(r-2)(r+2) = 0$ so the roots are 1, -1, 2, -2. Therefore the general solution is $a_n = \alpha_1 + \alpha_2(-1)^n + \alpha_3 2^n + \alpha_4(-2)^n$ Plugging in initial conditions gives $3 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$, $2 = \alpha_1 - \alpha_2 + 2\alpha_3 - 2\alpha_4$, $6 = \alpha_1 + \alpha_2 + 4\alpha_3 + 4\alpha_4$, and $8 = \alpha_1 - \alpha_2 + 8\alpha_3 - 8\alpha_4$. The solution to this system of equtions is $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and $\alpha_4 = 0$

Section 6.1

2a. (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (5,5), (6,6)
b. need figures
c. need figures

4a. Being taller than is not relexive (I am not taller than myself), nor symmetric (I am taller than my friend,

but she is not taller than I). It is antisymmetric (vacuously, since we never have A taller than B, and B taller than A, even if A=B). It is clearly transitive.
b. This is clearly reflexive, symmetric, and transitive. It is not antisymmetric, since twins, for example, are unequal people born on the same day.
c. This has exactly the same answers as (b), since having the same first name is just like having the same birthday.
d. This is clearly reflexive and symmetric. It is not antisymmetric, since my cousin and I have a common grandparent, and I and my cousin have a common grandparent, but I am not equal to my cousin. This relation is not transitive. My cousin and I have a common grandparent; my cousin and her cousin on the other side of her family have a common grandparent.My cousin's cousin and I do not have a common grandparent.

8. Only the relation in part (a) is irreflexive (the others are all reflexive)

11. a is asymmetric

12. An asymmetric relation must be antisymmetric, since the hypothesis of the condition of antisymmetry is false if the relation is asymmetric. The relation (a,a) on a is antisymmetric but not asymmetric, however, so the answer to the second question is no.

13. 2^{mn}

20.Since $(1,2) \in R$ and $(2,1) \in S$, we have $(1,1) \in S \circ R$. We use similar reasoning to form the rest of the pairs in the composition, giving us the answer $\{(1,1),(1,2),(2,1),(2,2)\}$.

22 There are just the 16 different subsets.
1. ϕ
2. $\{(0,0)\}$
3. $\{(0,1)\}$
4. $\{(1,0)\}$
5. $\{(1,1)\}$
6. $\{(0,0),(0,1)\}$
7. $\{(0,0),(1,0)\}$
8. $\{(0,0),(1,1)\}$
9. $\{(0,1),(1,0)\}$
10. $\{(0,1),(1,1)\}$
11. $\{(1,0),(1,1)\}$
12. $\{(0,0),(0,1),(1,0)\}$
13. $\{(0,0),(0,1),(1,1)\}$
14. $\{(0,0),(1,0),(1,1)\}$
15. $\{(0,1),(1,0),(1,1)\}$
16. $\{(0,0),(0,1),(1,0),(1,1)\}$

24 a. 8, 13, 14, 16
b. 1, 3, 4, 9
c. 1,2,5,8,9,12,15,16
d. 1,2,3,4,5,6,7,8,10,11,13,14
e. 1,3,4
f. 1,2,3,4,5,6,7,8,10,11,13,14,16