Assignment 6 TIM 207, Random Process Models in Engineering

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1. Durrett 9.44 pp 95. However, rather than computing a Queen's random walk, compute a Rook's random walk. A rook can move any number of squares vertically or horizontally, but cannot move diagonally. Assume each legal move is equally likely. Hint: Read about example 5.6 on pg. 64 carefully.
2. Durrett 5.2 pp 121
3. Durrett 5.14 pp 121 HINT: This problem shoes how one can use cleverly chosen Martingales to find closed form results about random walks that might otherwise be very hard to compute. Define Y_n to be equal to the expression given in part (a). Show with algebra that $E[Y_{n+1} - Y_n | Y_n] = 0$ and hence Y_n is a Martingale. Note that $Y_0 = S_0^2$ and that $E_{S_0=x} Y_0 \triangleq E_x Y_0 = E_x Y_{V_0 \wedge n} = E_x S_0^2 = x^2$ where V_0 is the stopping time when the Gambler goes bankrupt, and n is any nonnegative integer. This is due to Theorem 3.5 which we proved in class. This will give you an equation involving $E_x(V_0 \wedge n)$, $E_x(S_{V_0 \wedge n} - (p-q)V_0 \wedge n)^2$, and x^2. Since $E_x V_0$ is finite (you are given an expression for it) and $P(V_0 < \infty) = 1$ it must be that $\lim_{n\to\infty} E_x(V_0 \wedge n) = E_x V_0$. Similar reasoning allows you to say that $\lim_{n\to\infty} E_x(S_{V_0 \wedge n} - (p-q)V_0 \wedge n)^2 = E_x(S_{V_0} - (p-q)V_0)^2$. Also note that $S_{V_0} = 0$ by the definition of V_0.
4. Durrett 7.7 pp 152
5. Durrett 7.11 pp 153
6. Durrett 7.44 pp 157
7. Suppose a server can accommodate at most 2 people logged into it. Clients try to log in according to a Poisson process with a rate of 1 per hour. If 2 people are already logged in, clients are rejected and they go away. Clients who succeed in logging in stay logged in an exponential amount of time with an average stay of 15 minutes. In steady state, what is the distribution of the number of clients logged in? What fraction of arriving clients are rejected?
8. Durrett 8.12 pp 202