Assignment 3 TIM 207, Random Process Models in Engineering

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Due: May 1, 2011

1. (a) Suppose that Y is J.G. with zero mean and covariance

$$K_Y = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

Find a matrix A such that $W := A^{-1}Y$ satisfies $W \sim N(0, I)$.

(b) what is the probability that $1y_1 + 2y_2 > 4$?

2. Consider the matrix

$$K_Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{bmatrix}.$$

Find a matrix A and a 2 dimensional JG random vector Z such that the covariance matrix of $Y = AZ$ is K_Y and $det(K_Z) \neq 0$.

3. Let

$$H(\omega) = \begin{cases} 1 & \text{for } |\omega| \in [\omega_0 - \epsilon/2, \omega_0 + \epsilon/2] \\ 0 & \text{otherwise} \end{cases}$$

Show that the inverse Fourier transform of $H(\omega)$ is

$$h(t) = 2\cos(\omega_0 t)\frac{\sin(\epsilon t/2)}{\pi t}.$$

4. (a) Suppose that $W(t)$ is White Gaussian Noise with p.s.d. $S_W(\omega) = N_0/2$. Suppose that $d(t)$ is a deterministic function satisfying $\int_{-\infty}^{+\infty} |d(t)|^2 dt = D$. Let

$$Z = \int_{-\infty}^{+\infty} d(t)W(t)dt.$$

What is the distribution of Z?

(b) Suppose that your friend sends a 1 represented by the signal $X(t) \equiv d(t)$ with probability 0.5 or sends a 0 represented by the signal $X(t) \equiv 0$ with probability 0.5. In either case, you receive the signal $Y(t) = X(t) + W(t)$. Suppose you decide to build a detector. Your detector starts by computing the random variable $Z := \int_{-\infty}^{+\infty} d(t)Y(t)dt$. What is the distribution of Z conditioned on the event that your friend sends a 1? What is the distribution of Z conditioned on the event that your friend sends a 0?

(c) Write the MAP detector, based on the observation Z. Use the results from your last homework assignment here.

(d) What is the probability of making an error if $N_0/2 = 1$ and $D = 3$?

(e) Suppose now that you build your detector differently. Instead you compute $Z' := \int_{-\infty}^{+\infty} d'(t)Y(t)dt$, where $d'(t)$ is a deterministic signal that is not equal to $d(t)$ and satisfies

$$\int_{-\infty}^{+\infty} |d'(t)|^2 dt = D$$

and

$$\int_{-\infty}^{+\infty} d'(t)d(t)dt = D/4.$$

What is the distribution of Z' conditioned on the event that your friend sends a 1? What is the distribution of Z' conditioned on the event that your friend sends a 0?

(f) Write the MAP detector, based on the observation Z'.

(g) What is the probability of making an error if $N_0/2 = 1$ and $D = 1$?

(h) Which of the two detectors is better?