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Network Platform Competition in a Two-Sided Market: Implications to the Net Neutrality Issue

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Abstract

We consider a model for network platform competition between the current "Best Effort" (BE) network and a hypothetical "Next Generation Network" (NGN). We suppose there are indirect network externalities between content providers (CPs) and users of each platform. Moreover, we suppose that NGN users can access BE content, thus there is a cross-platform, indirect network externality. This feature is not symmetric – BE users cannot enjoy NGN content. Both users and content providers are distributed in their relative affinity for each platform type. We find that depending on the strength of the cross-platform network externality, there might either be one stable equilibrium in which CPs and users distribute across both platforms, or there might be 2 stable equilibria corresponding to the market tipping in favor of either NGN or BE. In both cases, we study the pricing power of a monopoly provider of both platforms, and then we study the possible effects of net-neutrality like regulation that constrains the price premium for NGN. We find that for most parameter choice of the model, the monopolist chooses price so that the user population "tips" to all using NGN.

1 Introduction

A key issue in moving from the current Best Effort (BE) network architecture to a new architecture that is capable of providing higher Quality of Service (QoS) for (possibly) higher prices is whether the market participants would adopt this new architecture. One example of such a new architecture would be the "Next Generation Network (NGN)" proposed by the ITU (2007). For the advantages of such a network to be fully realized, content providers (CPs) would have to develop content specially designed to use the NGN's capabilities. Likewise, users would have to be willing to subscribe to a NGN network provider, and possibly pay a price premium over the old BE service. Thus, the NGN can be viewed as a competing platform to the current BE network in a two-sided market, and therefore ideas from two-sided markets can be applied to understand the dynamics of this competition.

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In such a situation, there might be several stable states – for instance if all users were subscribing to BE, no CP would have an incentive to develop NGN content. Of course without NGN content, there would be no reason for a BE user to switch to NGN, and so this situation would persist. Yet it might also be stable for both users and CPs to use NGN, since once this situation were reached, neither CPs or users would want to move back to the inferior BE platform. From a policy perspective, it perhaps would be desirable to structure the market in such a way as to encourage the market to tip in favor of NGN. For instance it might be that if a single, unregulated monopoly network provider operated both platforms, such a provider might find it advantageous to price the NGN platform "too high" from a user welfare perspective. In particular, the price might be so high that a significant fraction, or even most, of the users and CPs avoid the NGN. If this were the case, it would support an argument in favor of regulating the price differential between NGN and BE, so that the benefits of the NGN platform would end up being enjoyed by more users and CPs.

One of key structural characteristics that make BE vs. NGN platform competition different from other platform competitions such as Nintendo Wii vs. Sony PlayStation, is that subscribers to NGN ought to be able to use BE content, but NGN contents (for example, IP-HDTV) cannot be enjoyed by BE subscribers. This asymmetry in the indirect network externality is one of the key features of our model. Though there have been some models suggested to incorporate the two-sided market perspective into modeling the Internet industry, to our knowledge none of these other models study this asymmetry explicitly.

We develop a two-platform (BE & NGN) model that explicitly incorporates the effects of indirect network externalities between CPs and users. Our model assumes a single monopolistic or dominant NP, who determines the prices to charge for each platform. We also use a variant of Hotelling's spatial differentiation to model the relative affinity of users and CPs to different platforms. For instance, a less "tech-savvy" person might prefer the older platform, all other factors being equal. With these settings, users and CPs play a platform subscription game with well-defined payoffs based on the indirect network externalities. Users of the NGN service enjoy both content designed for the premium service and content for the lower, "older" generation service. Parameters β and δ modulate the magnitude of this positive, indirect externality that users enjoy from the presence of NGN and "older" generation content respectively. For instance when $\delta = 0$, users see no benefit from older content; when $\delta = \beta$ users enjoy older and NGN content equally. (The precise form of our model follows in section 3.)

Given a choice of prices for each platform, analysis of the model determines an equilibrium fraction of users and content providers using each service. Net-neutrality policy enters the model in terms of possible restrictions on the pricing profile. For instance, it might be mandated that NGN content not be priced higher than BE content.

2 Related Work

The issues we address in this work are part of the larger debate on network neutrality, which includes diverse issues such as whether service differentiation should be allowed, or whether charges for content constitute an impingement of freedom of speech (see for example, Odlyzko, 2009; Hahn and Wallsten, 2006; van Schewick, 2007; and Wu and Yoo, 2007).

Our model is based on the ideas of two-sided markets, and there is a large literature on the subject. For a survey of two-sided markets, see for example, Rochet and Tirole (2006) and

Armstrong (2006). The two-sided market literature studies markets in which a platform provider needs to attract two types of participants, and the presence of more of one type makes the platform more valuable to the other type. The benefit one type of participant derives from the presence of more of the other type of participant is usually referred to as an "indirect network externality," although others use slightly different terminology. For instance Rochet and Tirole (2003) referred to the classical network externality and the indirect externality as the membership externality and usage externality, respectively.

The model we study is driven by the indirect (usage) externality between CPs and end users, with ISPs serving as a platform. Other examples of work examining effects driven by indirect (usage) externalities include Caillaud and Jullien (2003), Chakravorti and Roson (2004), and Parker and van Alstyne (2005). Pricing structure on both sides of two-sided market can be important; for instance it might be necessary to lower the price for one type of participant to attract more of the other. Armstrong (2006), Eisenmannn, Parker and van Alstyne (2006), Evans (2003), Parker and van Alstyne (2005), Rochet and Tirole (2003, 2006) all address these kinds of issues.

Other past work has also used the two-sided market and indirect network externality ideas to study questions of network neutrality in particular. For instance Economides and Tag (2007) discuss the benefits of net-neutrality regulation in the context of a two-sided market model. They suppose that net-neutrality regulation is equivalent to an imposition of a zero-fee in their model. When the access market is monopolized, they find that generally net neutrality regulation increases total industry surplus compared to the fully private optimum at which the monopoly platform imposes positive fees on CPs. They also model a platform competition in a platform duopoly situation. Similarly, imposing net neutrality in a duopoly increases total surplus compared to duopoly competition between platforms that charge positive fees on CPs. They also discuss the incentives of duopolists to collude in setting the fees on the other side of the Internet while competing for Internet access customers since under net neutrality conditions, the regulator will choose a negative fee to CPs (while a monopolist or duopolists choose positive fees).

Other researchers have used the ideas of two-sided markets to study network neutrality. According to Hagiu (2006), the effectiveness of net neutrality policy is controversial. He also uses a two-sided market model to study the Internet services market. His model shows a possibility that the profit maximization behavior of monopolistic platform provider fully internalizes the indirect network externality, thereby increasing the overall social welfare by reducing transaction costs and expanding both markets. Hermalin and Katz (2006) model network neutrality as a restriction on the product space, and consider whether ISPs should be allowed to offer more than one grade of service. Hogendorn (2007) studies two-sided markets where intermediaries sit between "conduits" and content providers. In his context, net-neutrality means content has open access to conduits where an "open access" regime affords open access to the intermediaries. Weiser (2007) discusses policy issues related to two-sided markets. Musacchio, Schwartz, and Walrand (2009) use a two-sided market model to study the investment decisisions of content providers and ISPs, and then use this model to study the welfare effects from either allowing or not allowing ISPs to charge content providers for the right to deliver content to end-users. Njoroge, Ozdaglar, Stier, and Weintraub (2009) study competion between independently owened platforms that select prices and quality levels to offer a user and content provider market.

2.1 Key similarities and differences between our work and past work

Our work captures the effect that there will be competing generations of network platforms – what we call "BE" and "NGN." Even if the market were to fully transition to one type of platform, the time it would take to make this transition cannot be ignored. Therefore, it is important to study the behavior of an operator simultaneously operating platforms of both kinds.

Another distinguishing feature is the incorporation of the cross-platform (indirect) network externality in our model. In our context, this feature appears because users of the NGN platform can see content from BE content providers but not the other way around. A nice feature of our model is that the strength of this effect can be adjusted with a single parameter. By setting $\delta=0$, the platforms become independent.

Our model assumes a single monopolistic or dominant NP that determines the prices of multiple platforms, just as in other work (Economides & Tag, 2007; Hagiu, 2006; Laffont, Marcus, Rey & Tirole, 2003). We also employ a spatial differentiation model similar to the classic Hotelling model (as in Armstrong, 2006; Economides & Tag, 2007; Hagiu, 2006) to model the diversity of users and content providers in their relative preferences for platforms.

The model we pose results in the existence of both stable and unstable equilibria to the platform selection game. We suppose that a network provider cannot maintain a market in an unstable equilibrium because the market will have a tendency to "tip" to a stable equilibrium if perturbed. To study this important effect, we carry out a stability analysis to characterize the stability of the equilibria. Our stability analysis has some similarities to that of the recent work Sen, Jin, Guerin, and Hosanagar (2009). However the work of Sen, Jin, et al. (2009) studies a model driven by direct network externalities between users while our model is driven by indirect network externalities between users and content providers. Also Sen, Jin, et al. (2009) studies the role of "converters" in allowing a new platform to leverage the direct network externalities of an existing one. Our cross platform network externality is similar in character, but the fact that our model looks at indirect network externalities, and not direct, makes our model quite different.

3 Model

In our model there are two platforms, A and B. We suppose Platform B (NGN) has more technical capabilities than platform A (BE). A monopoly network provider operates both platforms. Figure 1 illustrates some of the features of the model, which we now describe.
Each Content Provider (CP) must to specialize in providing services either over platform A (BE) or B (NGN). Services designed for platform A (BE) can be also accessed via platform B (NGN) since platform B (NGN). However, services designed for the NGN platform cannot be accessed via the less sophisticated BE platform.

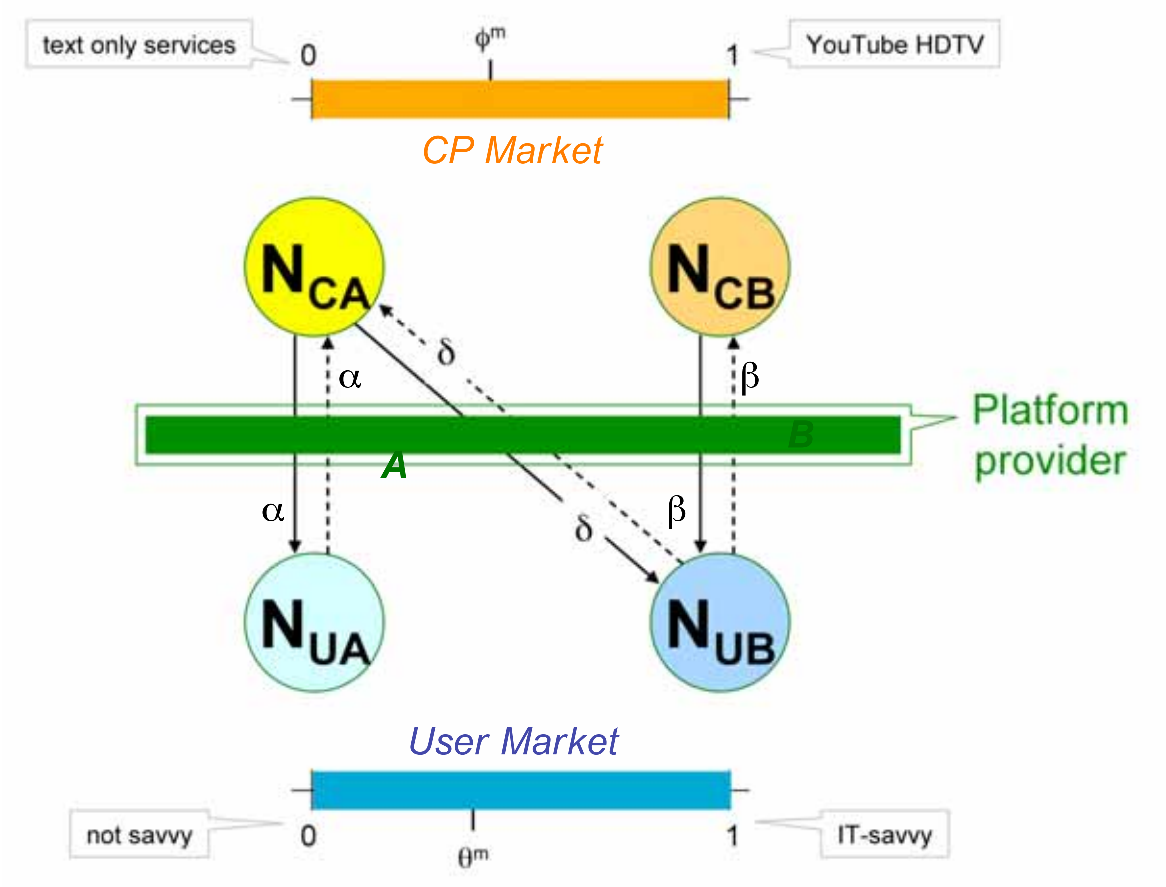

Figure 1: *An illustration of the model for platform competion, and the direction of the cross-platform network externality.*

In a model like Hotelling's spatial differentiation (Armstrong, 2004, Hagiu, 2006), we suppose that there are a continuum of both users and content providers, and therefore each individual CP or user represents a negligible fraction of the total population of users or content providers. Each CP is endowed with a parameter φ that comes from a uniform distribution on [0,1]. The larger φ, the more that CP is inclined to use the more sophisticated platform B (NGN), in a way we make precise later. Similarly, each user is endowed with a parameter θ, which comes from a uniform distribution on [0,1] and this parameter determines that user's inclination, before considering the effect of network externalities, to subscribe to platform B (NGN).

We suppose that in any state of the system, there will always be some threshold $\phi^m \in [0,1]$ (the *m* is a mnemonic for "marginal") such that all CPs with ϕ larger than ϕ^m use platform B (NGN) and all CPs with $\phi \le \phi^m$ use platform A (BE). Under this assumption, the number of CPs using A and B is ϕ^m and $1-\phi^m$ respectively. The payoff (net-utility) to a user of type θ is either

$$U_A(\theta,\ \phi^m) = 1-\theta+\alpha\phi^m - P^A \qquad \text{if he connects to A (BE),}$$

$$U_B(\theta,\ \phi^m) = \theta+\beta\left(1-\phi^m\right)+\delta\phi^m - P^B \qquad \text{if he connects to B (NGN).}$$

In these expressions:

- α is a parameter that characterizes the magnitude of the indirect network externality platform A users enjoy from platform A content and vice-versa.
- β is a parameter that characterizes of the indirect network externality platform B (NGN) users enjoy from platform B (NGN) content and vice-versa. We suppose $\beta > \alpha$.
- δ is a parameter that characterizes of the indirect network externality that service B (NGN) users enjoy from the presence of platform A (BE) CPs and vice-versa. We suppose that $\delta \le \max(\alpha, \beta)$ -- that the cross-platform externality coefficient is weaker then the strongest within platform externality coefficient.
- P^A and P^B are the prices of platform A and platform B respectively.

Note that the above expression for $U_A(\theta,\ \phi^m)$ reflects our assumption that the more content providers that use platform A, the more attractive platform A will be for users who connect to A. Our expression for $U_B(\theta,\ \phi^m)$ likewise reflects our assumption that users of B (NGN) derive an indirect externality both from the number of CPs on B, $(1-\theta^m)$, as well as the CPs using A, ϕ^m.

As we did for content providers, we suppose that in any state of the system all users with θ larger than a threshold θ^m use platform B (NGN) and all users with θ smaller than the same threshold use platform A (BE). The payoff to a CP of type ϕ is

$$\pi_A(\phi,\ \theta^m) = 1 - \phi + \alpha\theta^m + \delta(1-\theta^m) - P^A \qquad \text{if the CP connects to A (BE),}$$
$$\pi_B(\phi,\ \theta^m) = \phi + \beta(1-\theta^m) - P^B \qquad \text{if the CP connects to B (NGN).}$$

These expressions reflect our assumption that CPs using platform A (BE) derive indirect network externalities from users of both platforms, while CPs using platform B (NGN) benefit only from the users of the platform B (NGN).

We suppose that users and CPs select the platform that gives them the highest payoff. If a player (either a user or a CP) finds that both options give negative payoff, they select the platform that has the least negative payoff. This assumption simplifies the analysis, and perhaps is justified by the observation that not having Internet connectivity is not an option in today's society. Since all players have to subscribe to a platform, they must pay the provider at least the minimum of the two prices no matter what they do. Therefore, the behavior of the players only depends on the price difference, or premium

$$\Delta = P^B - P^A.$$

Therefore from now on only consider the price premium Δ and not the prices of each platform.

4 Analysis

We first consider the behavior of the system with fixed premium Δ. We call this the *platform selection game*, as the main dynamic is the selection of platforms by users and content providers. After we analyze the platform selection game, we consider how a monopoly network provider would choose prices to steer the outcome of the platform selection game to an equilibrium that yields the highest revenue.

4.1 Platform Selection Game Equilibrium

In our model, an "interior" equilibrium is characterized by a pair of thresholds (θ^m, ϕ^m) such that a user of type θ^m will be indifferent between using either platform and likewise a CP of type ϕ^m will also be indifferent between using either platform. "Indifferent" means that both platforms give the player the same payoff. Since users with $\theta < \theta^m$ will use platform A (BE), the pair of thresholds (θ^m, ϕ^m) also specifies the fractions of users and content providers respectively that are using platform A (BE).

"Boundary" equilibria in which one or both types of players choose one platform exclusively are possible. For instance if in the equilibrium (θ^m, ϕ^m), all users use platform A (NE), it must be that the user least inclined to use platform A (a user of type $\theta = 1$) finds $U_A(1,\ \phi^m) \geq U_B(1,\ \phi^m)$. All the possible cases for boundary and interior equilibria are covered in the following definition.

Definition 1: A pair (θ^m, ϕ^m) is an equilibrium to the platform selection game if and only if the following conditions are satisfied:

$$
\begin{aligned}
&U_A(\theta^m,\ \phi^m) = U_B(\theta^m,\ \phi^m) \ \text{ if } \ \theta^m \in (0,1),\\
&U_A(\theta^m,\ \phi^m) \geq U_B(\theta^m,\ \phi^m) \ \text{ if } \ \theta^m = 1,\\
&U_A(\theta^m,\ \phi^m) \leq U_B(\theta^m,\ \phi^m) \ \text{ if } \ \theta^m = 0,\\
&\pi_A(\phi^m,\ \theta^m) = \pi_B(\phi^m,\ \theta^m) \ \text{ if } \ \phi^m \in (0,1),\\
&\pi_A(\phi^m,\ \theta^m) \geq \pi_B(\phi^m,\ \theta^m) \ \text{ if } \ \phi^m = 1,\\
&\pi_A(\phi^m,\ \theta^m) \leq \pi_B(\phi^m,\ \theta^m) \ \text{ if } \ \phi^m = 0.
\end{aligned}
$$

4.2 Off-Equilibrium Dynamics of Platform Selection Game – Specification

Here we consider how the state of the platform selection game evolves if it is started in an arbitrary state (not necessarily an equilibrium). This analysis is important because it will allow us to study whether equilibria are stable or unstable. To study off-equilibrium dynamics, we first must define how the system behaves over time when started in an off-equilibrium state.

Consider a system beginning in the state (θ^m, ϕ^m) - meaning that all users with $\theta < \theta^m$ are using platform A (BE), all CPs with $\phi < \phi^m$ are also using platform A, and the remaining users and CPs are using platform B (NGN). If the marginal user with type θ^m finds that $U_A(\theta^m, \phi^m) > U_B(\theta^m, \phi^m)$, he and some group of users with types $[\theta^m, \theta^m + \varepsilon]$ for some small enough ε, will find it beneficial to switch to platform A. Thus we should expect the system to evolve toward more users using platform A, or in other words θ^m should increase. Thus we can suppose the rate of change of θ^m, which we call $\dot{\theta}^m$, is proportional to the payoff difference between platform A and B. We can suppose a similar process for $\dot{\phi}^m$. We can therefore express the system's dynamics as

$$\dot{x} = Ax + B \qquad \text{if} \quad x \in (0,1)\times(0,1) \tag{1}$$

where $$A = \begin{bmatrix} -2 & \alpha+\beta-\delta \\ \alpha+\beta-\delta & -2 \end{bmatrix},\quad B = \begin{bmatrix} 1-\beta+\Delta \\ 1-\beta+\delta+\Delta \end{bmatrix},\quad x = \begin{bmatrix} \theta^m \\ \phi^m \end{bmatrix},$$

and Δ is the price premium P^B-P^A charged for platform B (NGN). Expression (1) is qualified by the statement "if $x \in (0,1)\times(0,1)$" because we have not yet considered the behavior on the boundaries of the allowable state space.

For a state *x* on one of the boundaries (the market is currently tipped to one or both kinds of players using one platform exclusively) the dynamics are the same as (1) except that the state is stopped from leaving the boundaries.

$$z = Ax + B, \qquad \text{for } i\text{=1,2,} \quad \dot{x}_i = \begin{cases} \max(z_i,0) & x_i = 0, \\ z_i & x_i \in (0,1), \\ \min(z_i,0) & x_i = 1. \end{cases} \tag{3}$$

Equation (3) has the interpretation that if, for example, all users are using platform A and platform A's payoff is higher than B's for all users, then no more users can switch to A because everyone is already using it.

4.3 Off-Equilibrium Dynamics of Platform Selection Game – Analysis

The dynamics we defined in Section 4.2 are those of a linear dynamical system with state constraints. We are interested in the stability of equilibria of the platform selection game. We make this more precise with the following definitions.

Definition 2: (stable equilibrium) A *locally stable equilibrium x* is an equilibrium such that there exists some region of states surrounding *x* for which if the system were started in that state, the system would approach equilibrium *x* (and get arbitrarily close to it over time). A *globally stable equilibrium x* is such that from any starting state, the system approaches *x*.

Definition 3: (unstable equilibrium) An *unstable equilibrium x* is an equilibrium such that for any arbitrarily small region of states surrounding *x*, there exists a state *y* for which if the system were started in that state, the system would be drawn further away from equilibrium *x*.

(These definitions are standard notions from systems theory. Any standard reference such as Callier and Desoer (1991) would supply these definitions in a more mathematically precise way. We chose to state the definitions as above to make the analysis more readable to a wide audience.)

An interior equilibrium point requires that the system is in a state *x* for which the marginal player of both kinds (users and CPs) finds the payoff in both platforms to be the same – equivalently that $Ax+B=0$. Therefore an interior equilibrium, if it exists, occurs at the vector

$$y = A^{-1}B. \tag{2}$$

Note that depending on parameters, the vector *y* might be outside the allowed boundaries of [0,1] x [0,1]. Also, if *A* is not invertible (has a 0 eigenvalue), *y* is not defined.

The stability of a linear dynamical system is determined by whether the eigenvalues of the matrix *A* are negative. Simple algebra reveals that the two eigenvalues are

$$\lambda_{1,2} = -2 \pm (\alpha + \beta - \delta).$$

Thus, if $\alpha + \beta - \delta < 2$, both eigenvalues are negative. However, if $\alpha + \beta - \delta > 2$, there will be one positive eigenvalue. If the eigenvalues of *A* are negative, the state of the system will tend to approach *y* (Callier and Desoer 1991). Therefore if $y \in [0,1] \times [0,1]$, there will be one stable equilibrium of the platform selection game at the point *y*. When the point *y* is outside the region $[0,1] \times [0,1]$ the argument is slightly more complex. Here we can show that the distance from the point *y*, as measured by a metric called a Lyapunov function in control theory, always decreases.

Theorem 1: Suppose that the eigenvalues of *A* are both negative (equivalently $\alpha + \beta - \delta < 2$), then:

- If, $y \in [0,1] \times [0,1]$ there exists a unique stable interior equilibrium $y = -A^{-1}B$.
- Otherwise there exists a stable boundary equilibrium in which one or both kinds of players use one of the platforms exclusively.

Proof: (Sketch) Consider the Quadratic Lyapunov function $L(x) = (x - y)^T A^{-1^T} A^{-1}(x - y)$. Note that $A^{-1^T} A^{-1}$ is positive definite. One can show at every state $x \neq y$, the Lyapunov function is nonincreasing. For instance for $x \in (0,1) \times (0,1)$, $\dot{L}(x) = (x - y)^T A^{-1}(x - y) < 0$ for all $x \neq y$. Similar analysis shows that the Lyapunov function is non-increasing on the boundaries of the feasible region [0,1]x[0,1]. Because *L(x)* never increases, and the Lyapunov function can not be negative by construction, it follows that the system must approach an equilibrium point. ■

Now we consider the case for which one of the eigenvalues is positive (equivalently $\alpha + \beta - \delta > 2$). If *y* is in the interior (i.e. $y \in (0,1) \times (0,1)$) then the point *y* will be an unstable equilibrium, which is easily shown by standard linear systems theory techniques. This means that if one starts in state *y*, and one perturbs the system by moving a small fraction of users (or CPs) to the other platform, then the platform the players are moved to becomes more attractive to players of the other type, some of those players move, and the system moves even further from *y*. Since equilibrium *y* is so "fragile," and tends to tip in favor of one platform or the other when perturbed, it is also important to study the boundary equilibria, which turn out to be more stable.

In the following theorem, we outline the possibilities regarding equilibria. Note that the system may have multiple equilibria. For instance, there might simultaneously be a locally stable equilibrium for which everyone uses NGN and another one for which everyone uses the BE platform.

Theorem 2: The following statements are true:

1. For price premium Δ satisfying $\Delta < \beta - 1 - \delta$, the state (0,0) - all CPs and users using platform B (NGN) - is a stable equilibrium.
2. For price premium Δ satisfying $\Delta > 1 - \alpha + \delta$, the state (1,1) – all CPs and users using platform A (BE) is a stable equilibrium.
3. For price premium Δ satisfying
$$\Delta \in \left[\beta - 1 - \delta,\ \beta - 1 - \delta + \frac{2\delta}{2 + \alpha + \beta - \delta}\right)$$
the state (0, ϕ^m) where
$$\phi^m = \min\left(\frac{1}{2}\left[\Delta - (\beta - 1 - \delta)\right], 1\right) \tag{4}$$
is a stable equilibrium.
4. There is no equilibrium of the form (θ^m,0) with $\theta^m \in (0,1)$.
5. If $y \in (0,1) \times (0,1)$, then y is an equilibrium, and it is unstable if one of the eigenvalues of A is positive (equivalently $\alpha + \beta - \delta > 2$).

Proof: (Sketch) The suggested equilibria in statements 1 through 4 can be shown to be equilibria by verifying the conditions in Definition 1. For statements 1 and 2, the stability properties of the boundary equilibria x can be verified (roughly) by checking whether the components of $\dot{y}$ have the correct signs to "push" the component of x that is supposed to be on the boundary towards the boundary, while the other component is being "pushed" toward the value that is supposed to be a stable equilibria. The equilibrium of statement 3 puts conditions on price Δ such that: i) users will not want to switch from platform B, ii) the marginal CP of type ϕ^m (given by (4)) finds the same payoff in both platforms, and iii) there exists a region surrounding ϕ^m from which the state is attracted to the claimed equilibrium. Statement 4 is shown by finding ϕ^m as a function of Δ using that $\dot{\phi}^m$=0, which gives a lower bound of Δ to support $\phi^m > 0$. There is an upper bound of Δ that supports θ^m. One finds the lower and upper bounds contradict. The instability of the equilibrium in statement 5 results from one eigenvalue being positive. ■

4.4 Optimal Pricing of Monopoly Network Provider

Now we address the price choice of the monopoly network provider. As we have demonstrated, the network provider's choice of price premium Δ determines the possible equilibria of the platform selection game. We suppose that a network provider cannot drive the system to an unstable equilibrium because such equilibria are so fragile. From most starting states, the system will not approach an unstable equilibrium, and if a system is in unstable equilibrium, the slightest perturbation could pull it away from equilibrium.

With these observations in mind we suppose that a monopoly provider will consider the highest revenue (locally) stable equilibrium associated with each possible price choice of Δ. For short we will call this the "best" equilibrium for price Δ. Then the monopoly provider will choose Δ to maximize the revenue of the "best" equilibrium.

This begs the question why ought the monopolist try to maximize the revenue of his "best" equilibrium and not say, the worst. We believe the "best" is more important by the following argument. We suppose that the monopoly provider is interested in the "best" equilibrium and not the "worst" equilibrium, because the monopolist can use temporary price breaks to drive the system near his target equilibrium. For instance, a monopoly provider can always use promotional pricing, such as charge a 0 or even negative price to attract players to NGN. Clearly for a negative price of large enough magnitude, the all NGN state can be made the only stable equilibrium. Once the system is tipped the way the provider likes it (for instance all players using NGN), then he can raise the price to the level that maximizes revenue.

4.4.1 Finding the Revenue Maximizing Equilibrium

For each of the 5 types of equilibria given by Theorem 2, one can write closed-form expressions for revenue as a function Δ of as well as closed form necessary and sufficient conditions on Δ to allow each type of stable equilibrium to exist. Then for each type of equilibria, one can solve a constrained optimization problem to find the value of Δ that maximizes revenue.

In some cases the conditions on Δ to have a stable equilibrium are such that it must be strictly less than some threshold. Consequently there may be no "best" stable equilibrium in a mathematical sense, since the equilibria associated with Δ may get better as it approaches the threshold, but when Δ equals the threshold, stability is lost. In these cases, we find the limit of this sequence of equilibria. The following definition makes this notion precise.

Definition 4: (nearly stable equilibrium) An equilibrium price pair (x, Δ) is *nearly stable* if x is an equilibrium of the platform selection game with price Δ, and there exists a sequence $\{(x^i, \Delta^i)\}$ such that: 1) for each i, x^i is a stable equilibrium for the platform selection game with price Δ^i, and 2) $x^i \to x$ and $\Delta^i \to \Delta$.

Note that any stable equilibrium, price pair is also a "nearly" stable equilibrium by the above definition. To state the optimization problem of the network provider, define the following set:

$$S = \left\{ (x,\ \Delta) : x \text{ is a nearly stable equilibrium when the price is } \Delta \right\}.$$

The set S is constructed so that it contains all possible nearly stable equilibrium, price pairs. Using this set S, we can now precisely state the network provider's revenue optimization problem as

$$\max_{(x,\ \Delta)\in S} \Pi(x,\ \Delta) \qquad (5)$$

where

$$\Pi(x,\ \Delta) = \Delta \begin{bmatrix} 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - x \right)$$

is the revenue from operating in state x with price Δ.

4.4.2 Conditions for Revenue Maximizing Equilibrium to be an Interior Point

It turns out that for much of the space of problem parameters (α, β, δ), the revenue maximizing equilibrium (what we have been calling "best" equilibrium for short), occurs on the boundary of the state space – particularly the position described in Theorem 2, type 3. In this subsection we determine the region of the problem parameter space for which the revenue maximizing equilibrium is an interior point.

Theorem 3: The following set of conditions are necessary and sufficient for (x*, Δ*), the maximizing value of problem (5) to be an interior point (i.e. $x \in (0,1)\times(0,1)$):

$$\alpha+\beta-\delta \le 2,$$

$$\Delta = \frac{2+\delta-2\alpha}{4} \ ,$$

$$\Delta < \frac{2 - \alpha^2 - \alpha\beta + 2\alpha\delta + \beta\delta - \delta^2 - \alpha + \beta - \delta}{2+\alpha+\beta-\delta},$$

$$\Delta > \frac{-2+2\beta+(-\alpha-\beta+\delta)(1-\beta+\delta)}{2+\alpha+\beta-\delta},$$

$$\alpha \le \beta,\ \ \delta \le \beta.$$

Proof: (Sketch) The first condition makes the eigenvalues nonpositive – otherwise any interior point would be unstable. The second condition results from observing that an interior equilibrium, if it occurs, is at $-A^{-1}B$, which inturn, allows the revenue to be expressed in closed form as a function of Δ. The second condition results from the first order condition on this revenue function. The third and fourth conditions result from algebraic manipulation of the requirement that $-A^{-1}B \in [0,1]\times[0,1]$. The last conditions are actually assumptions we made earlier about the valid parameter ranges for our model. ■

From the conditions in Theorem 3, we can evaluate the parameter ranges for which the revenue maximizing equilibrium is an interior point, and when it is not. Figure 3 illustrates that for most values of (α,β,δ), the revenue maximizing equilibrium is not an interior point. The figure also shows that the parameter region in which the revenue maximizing equilibrium is an interior point corresponds to a region where all the network externality coefficients are relatively small. When all these values are small, the player's (user's or CP's) relative preferences driven by their type variable are strong compared to the strength of the network effect.

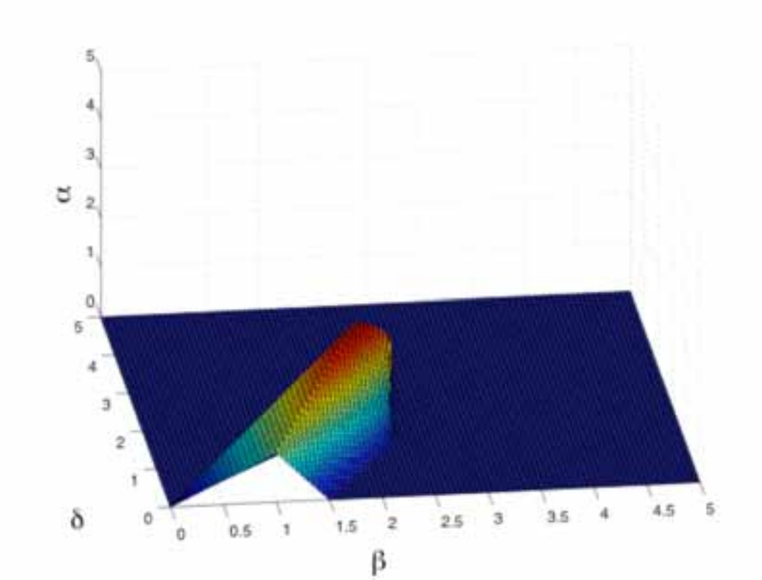

Figure 3 *A polytope in the space of parameters (α,β,δ) found from the conditions of Theorem 3. The volume under the curve shows where the revenue maximizing stable equilibrium point is an interior point. The polytope shows that for most of the parameter space, the revenue maximizing stable equilibrium point is not an interior point, but instead is a boundary point.*

5 Results

We now consider results using the representative case with $\alpha = 1$, $\beta = 3$. First we consider the dynamics of the system with prices fixed at $\Delta = 1$ and cross-platform network externality $\delta = 0$. Figure 4 illustrates the dynamics of the system with a diagram called a phase portrait. The plot illustrates the entire state space, the possible values of (θ^m, ϕ^m). In a grid there are arrows that indicate in which direction, and at what magnitude, the state space of the system would change if started there. This style of diagram is especially useful for visualizing the system's dynamics. The diagram shows that there are 3 equilibria, 2 stable ones at (0,0) and (1,1) (all NGN and all BE respectively), and an unstable one at (.5, .5).

As we discussed in section 4.4, we suppose that the monopoly network provider would choose his price to get to the revenue maximizing equilibrium – the solution to problem (5). We again consider the case with $(\alpha=1, \beta=3, \delta=0)$. Figure 5, illustrates the results. The revenue maximizing equilibrium for the network provider occurs at (0,0), with a price set at 2. As we discussed in section 4.4.1, there are cases for which the price must be strictly less than a threshold to have local stability. The right panel of Figure 5 illustrates that by reducing the price to 1.99, local stability is achieved.

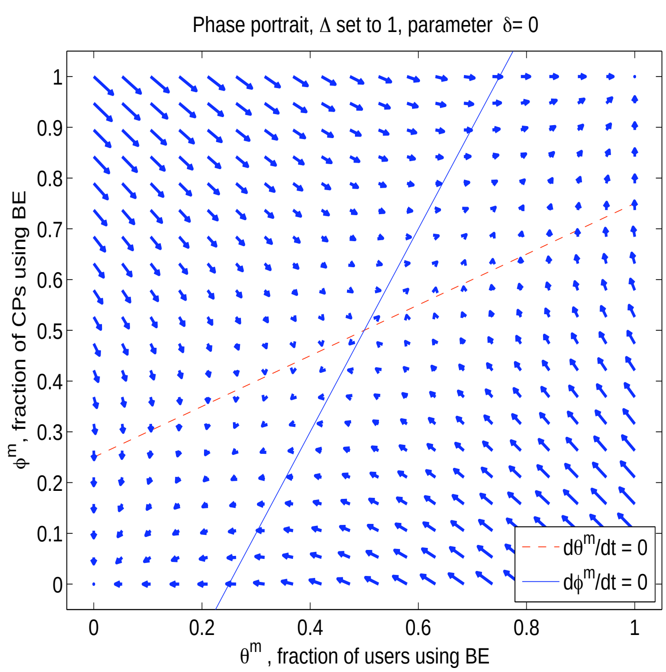

Figure 4: *The "phase portrait" of the system for parameters $\alpha=1$, $\beta=3$, $\delta=0$, and price premium $\Delta=1$. The arrows indicate the direction the state would evolve if the system were started from that state. The blue (solid) line indicates the states for which the rate of change of the platform distribution of CPs. The red (dashed) line indicates the same for users. The intersection of the lines is equilibrium, but is unstable. The corners (0,0) and (1,1), all players NGN or BE respectively are both stable equilibria.*

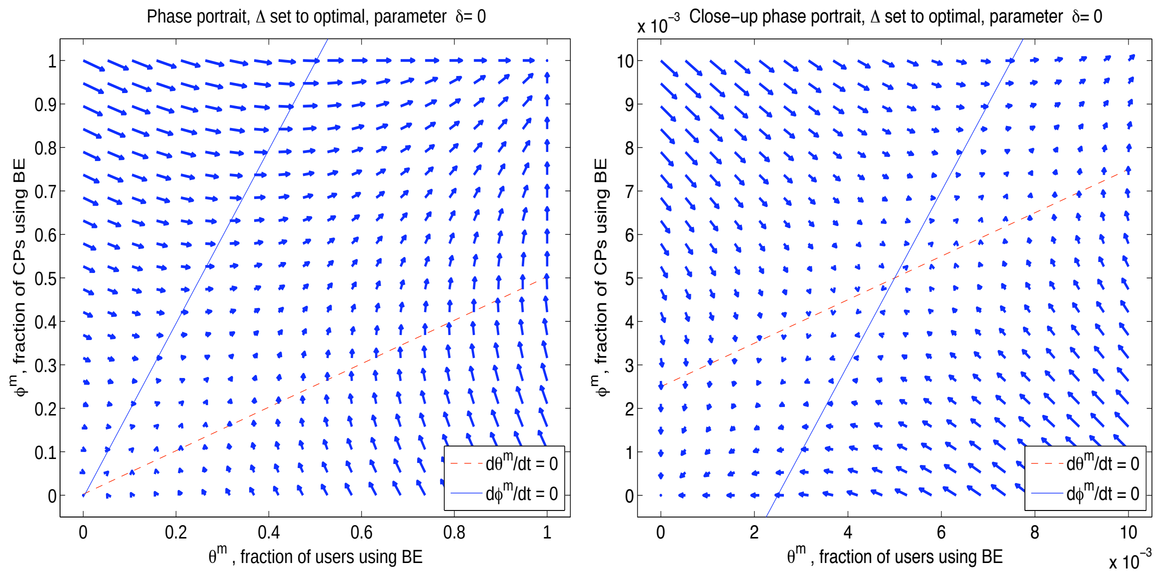

Figure 5: *The phase portrait after the network provider has chosen the revenue maximizing equilibrium as in expression (5). The model parameters are α=1, β=3, δ=0. By raising price, the intersection of the blue and red lines has shifted closer to (0,0). To maximize revenue while preserving (local) stability, it turns out that the network provider wants to charge a price slightly less (by an arbitrarily small amount) than the level at which the blue and red lines would intersect on the boundary. The close-up phase portrait on the right taken with Δ set to be 0.01 less than 2, the amount that would cause the intersection to happen on the boundary. The figure illustrates that (0,0) is a stable equilibrium for this price choice.*

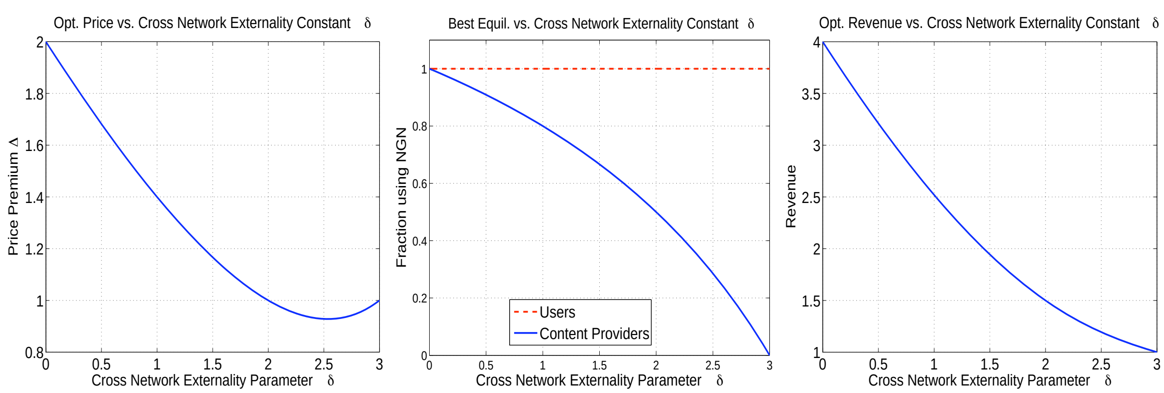

Figure 6: *Optimal price, the platform choice of users and CPs, and revenue in the revenue maximizing equilibrium for δ varied and α=1, β=3. Recall δ characterizes the magnitude of the cross-platform externality.*

We now consider the prices and revenue of the monopoly provider's revenue maximizing equilibrium as we vary δ, the parameter that characterizes the magnitude of the cross-platform network externality. Figure 6, illustrates our results. The first panel of Figure 6 shows that the price premium the network provider charges to achieve the "best" equilibrium drops as the cross-platform network externality grows. This is intuitive because with a larger δ, the more adequate the BE platform is for CPs to reach users, which in-turn, erodes the pricing power of the platform provider. The middle panel of Figure 6 shows that the "best" equilibrium occurs where all users have tipped to the NGN platform, while some CPs stick to the BE platform.

Moreover the number of CPs that use the BE platform increases as δ is increased. The right panel shows the erosion of revenue as δ increases.

Figure 7 shows phase portraits for several values of δ corresponding to the δ's studied in Figure 6. The progression of phase portrait diagrams illustrates the "best" equilibrium moving up the "y-axis" boundary as δ increases.

Although we have presented results for the α=*1*, β=*3* case only, our analysis in section 4.4.2 showed that for most of the parameter space, the revenue maximizing equilibrium is on the boundary. For such cases, we found that the results are qualitatively similar to the α=*1*, β=3 case.

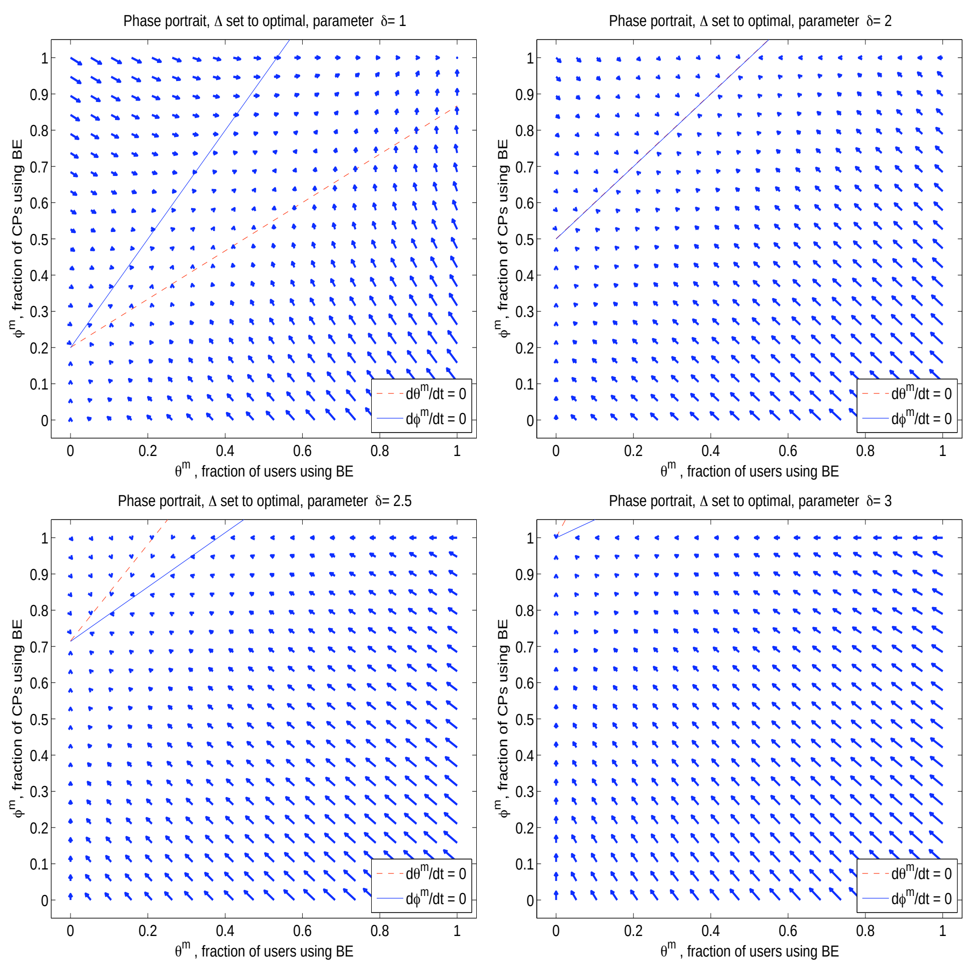

Figure 7: *The phase portraits of the system dynamics, for different values of δ (α=1, β=3). The top left plot illustrates the best equilibrium for δ=1 occurs for (θ^m, ϕ^m)=(0,0.2) with D=1.4 (stability requires that the price be an arbitrarily small amount below 1.3. The top right plot is for δ=2, here one of the eigenvalues of the matrix A is 0. Consequently there is an entire line of equilibria. The bottom left plot is for δ=2.5. For δ>2, all eigenvalues of A are negative. Consequently the intersection of the red (dashed) and blue lines is stable globally. For δ=3, the bottom right plot, the revenue maximizing equilibrium is (0,1). This is partly because with δ= β, content providers can interact with NGN users just as well by subscribing to BE as they can by subscribing to NGN.*

6 Result Interpretation and Policy Implications

The results demonstrate that for most values of α, β, and δ, the network provider maximizes his revenue by driving the system to a state in which all users use NGN and CPs use a mix of NGN and BE. Therefore, if the goal of net-neutrality restrictions were to prevent a network provider from charging prices so high as to prevent a significant share of the user market from using NGN, it might not be necessary to impose price regulation to achieve this. It appears to be in the interest of the network provider to charge prices in such a way to get the user market to "tip" in favor of NGN, and then to charge as high a price as possible and still have the market remain "tipped."

For the CP market, the situation is slightly different. The CPs can use the BE platform to access NGN users, and the quality of that access increases with δ. The network provider could get all the CPs to use NGN if he made the price sufficiently low, but the network provider can achieve a higher revenue by not doing this, and allowing some CPs to "leak" over to the BE platform. As δ increases, the number of CPs that use BE increase. Of course when $\delta=\beta$, the CPs can access the NGN users via the cheaper BE platform as they can with the NGN platform. Therefore in that case, all CPs use the BE platform.

In future work we plan to expand on the simple model we presented here. For instance, we will explore extensions such as allowing the network to select the user and content prices separately. It is an important research question to see if the tendency of the network provider to force the user market to tip to NGN remains when other features are added to our basic model.

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