WiFi Access Point Pricing as a Dynamic Game

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1. Introduction

In this work we study the relationship between a WLAN owner acting as a wireless access provider and a paying client. We model the interaction as a dynamic game in which the players have asymmetric information – the client knows more about her utility function than the access provider knows. In earlier work [1], we found that if a client has a "web browser" utility function (a temporal utility function that grows linearly), it is a Nash Equilibrium for the provider to charge the client a constant price per unit time. While if the client has a "file transferor" utility function (a temporal utility function that is a step function), and also has an a priori bounded file length, the client would be unwilling to pay until the final time slot of the file transfer. In this new work, we show that the equilibrium strategy profile for web browser clients is unique under certain assumptions. We also find that the earlier result for file transferor clients can extend to a more general class of utility functions. Finally we extend our web browser result to a three player game with multiple wireless hops, and show in this scenario that constant price is a Nash equilibrium.

2. Basic Model

We formulate the interaction between a base station and paying client using a simple two-player game model, that we first introduced in [1]. The game progresses in discrete time slots or "periods." At the beginning of the first time slot, the base station proposes an access price, p_1 for access during the first time slot. The client can either accept the price and connect, or reject the price and not connect. If the price is rejected, the game ends and both client and base station receive zero payoff.

In general, the base station offers connectivity at the beginning of time slot i at price p_i. The game ends the first time the client rejects the base station's proposal.

On completion of the game, the client's utility function $f(t, K)$ is a function of the number of time slots the client was connected, and a parameter K which we call the client's intended session length. The client's net payoff is $f(t, K) - \sum_{i=1}^{t} p_i$. The base station's net payoff is simply $\sum_{i=1}^{t} p_i$. The underlying assumption is that the base station's marginal cost to provide the service to the client is negligible. We study the Nash equilibria of this game under different assumptions of the structure of the utility function $f(t, K)$.

3. Web Browser Model

In a version of the basic model we call the web browsing model, the client's utility is proportional to the length of time t connected, up until the maximum intended session length K.

$$f(t, K) = U \cdot \min(t, K) \tag{1}$$

The parameters U and K of expression 1 may have a random distribution. The client knows the sample value, the base station just knows the distribution.

Theorem 1 *Consider a web browser client with utility defined by expression (1). Suppose that U and K are independent and finite mean. Then the following strategy profile is a perfect Bayesian equilibrium (PBE):*

- *The client connects or remains connected in each slot iff $i \leq K$ and $p_i \leq U$. (We refer to this as the "myopic strategy.")*
- *The base station charges price p^* in all time slots, where p^* is a maximizer of $xP(U \geq x)$.*

The proof is presented in [1]. It is somewhat surprising that the base station would keep its price constant. Whenever a myopic client accepts price p_i, the base station can refine its conditional distribution of U by lower bounding it by p_i. One might have expected that a base station might want to try charging a higher price than p_i after learning that the client's utility is at least p_i. The theorem shows that this intuition is not correct.

Theorem 1 shows that a strategy profile with the client playing a myopic strategy is a PBE, however it does not show that this strategy profile is an unique PBE.

In this new work, we show that in the special case that the intended session length, K, has a finite distribution on $\{1, ..., N\}$, then the myopic strategy is the unique client strategy in PBE.

Lemma 1 *Suppose K is distributed on $\{1, ..., N\}$, and U is finite mean, then the following characterizes all PBE:*

- *The client follows a myopic strategy, connecting iff $i \leq K$ and $p_i \leq U$.*
- *The base station charges a non-decreasing sequence of prices $\{p_i\}$ such that $p_i \in \mathbf{X}$ where $\mathbf{X}$ is the set of maximizing values of $xP(U \geq x)$.*

The proof is left for the full paper.

4. File Transfer Model

In an instance of the basic model that we call the file transfer model, the client's utility function is a step function. The client must remain connected for the entire intended session length, and complete his file transfer, to get any utility.

$$f(t, K) = \begin{cases} 0 & \text{if } t < K \\ U \cdot K + \epsilon & \text{if } t = K \end{cases} \tag{2}$$

The ϵ of expression (2) is assumed to be positive and smaller than the smallest unit of payment. The following theorem was shown in [1]:

Theorem 2 *Suppose the client has a file transfer utility function as in expression (2), but with $U \equiv 1$. The session length K, is distributed on $\{1, ..., N\}$, with a sample value* known *to the client, and* unknown *to the base station. We also assume that the function $iP(K = i)$ has a unique maximizer, $\hat{i}$. Then the following strategy profile is the unique, perfect Bayesian equilibrium:*

- *The client accepts the base station's offer if $p_i = 0$ when $i < K$, and $p_i \leq K$ when $i = K$. We refer to this as the "pessimistic" strategy.*
- *The base station charges:*

$$p_i = \begin{cases} 0 & \textit{if } i < \hat{i} \\ i & \textit{otherwise} \end{cases}$$

The result is proved in [1]. In this new work, we prove the following more general result:

Theorem 3 *Suppose the client has a file transfer utility function as in expression (2), with U a continuous random variable on $[l, h]$, and the session length K, distributed on $\{1, ..., N\}$. Both U and K have sample values* known *to the client, and* unknown *to the base station. We also assume that the function $uiP(U > u, K = i)$ has a unique maximizing pair, (u^*, i^*). Then the following strategy profile is the unique, perfect Bayesian equilibrium:*

- *The client plays a "pessimistic" strategy: The client accepts the base station's offer if $p_i = 0$ when $i < K$, and $p_i \leq U \cdot K$ when $i = K$.*
- *The base station charges:*

$$p_i = \begin{cases} 0 & \textit{if } i < i^* \\ u^* i^* & \textit{otherwise} \end{cases}$$

When the function $uiP(U > u, K = i)$ does not have a unique maximizing pair, Theorem 3 can be modified to say that the described strategy profile is a (not unique) PBE for any choice of (u^*, i^*) that maximizes $uiP(U > u, K = i)$.

4..1 Multiple Hops

In this new work, we consider a scenario where root base station sells service to a reseller, while the reseller resells the service to an end client. The reseller charges the end client a price of p_i in slot i, while the root base station charges the reseller a price of c_i in slot i. The reseller's payoff is simply $\sum_{i=1}^{t} (p_i - c_i)$

Theorem 4 *The following strategy profile is a PBE:*

- *The client follows a myopic strategy, connecting iff $i \leq K$ and $p_i \leq U$.*
- *The reseller charges a price that is a static function of the root base station price.*

$$p_i^*(c_i) = \arg\max_p (p - c_i) P(U > p).$$

- *The root base station charges a constant price.*

$$c^* = \arg\max_c [c \cdot P(U > p^*(c))]$$

The proof is left for the full paper.

References

[1] J. Musacchio, and J. Walrand, "Game Theoretic Modeling of WiFi Pricing," *41st Allerton Conference on Communication and Control*, Monticello, IL, October 2003.