Application of Randomization Techniques to Space-time Convolutional Codes

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Abstract—In this paper, we will demonstrate that a system employing a space time convolutional code (STCC) can be implemented with only a single transmit antenna when there are multiple receive antennas. The idea is to transmit more than one symbol from a single transmit antenna during a symbol period by superimposing the encoded symbols on top of each other. This objective is achieved by inducing randomness into the system, that creates additional channel paths, called virtual paths. The design of such an approach is studied for slow Rayleigh fading channels. One immediate application of this approach is to model an $n \times m$ multiple-input multiple-output (MIMO) system as an equivalent group of n distinct $1 \times m$ systems. Consequently, we demonstrate that STCCs with high spectral efficiencies can be designed utilizing QPSK STCCs as component codes for each $1 \times m$ system. Simulation results evaluate the performance of this technique for the case of two transmit antennas and several different number of receive antennas, a spectral efficiency of 4 bits/s/Hz, and slow Rayleigh fading channels.

I. Introduction

Multiple antennas are very important to increase capacity and reliability of wireless channels. It is a common belief that future wireless systems will have multiple antennas at both transmitter and receiver end to be able to transmit high data rate video, data, and voice. A system with multiple-input multiple-output (MIMO) capability has much higher spectral efficiency than singleinput single-output (SISO) and single-input multipleoutput (SIMO) systems [1]. Recent research results have shown that not only is the channel capacity of MIMO systems very high [1], [2], but large fractions of this can actually be acheived in implementations [3], [4]. One practical system called Vertical Bell Labs Layered Space-Time (V-BLAST), is capable of carrying tens of bits per second per hertz (b/s/Hz). For example, it has been shown in [5] that with multi-element array (MEA) technology, one can achieve 42 b/s/Hz with 8 transmit and 8 receive antennas and 1% outage capacity at 21-dB average signal-to-noise ratio (SNR).

The main idea behind the high capacities of MIMO channels stems from their mathematical equivalence to a set of parallel independent channels. We can exploit this same idea to send redundant data to achieve diversity to combat channel fading and increase transmission reliability. These two aspects of MIMO channels, namely diversity and multiplexing and their tradeoffs, were investigated by many researchers [6], [7]. Thus [13], [5], the concept of sending different data from separate transmit antennas (spatial multiplexing) or coded data from more than one transmit antenna (spatial diversity) has been explored. In this context, for n transmit antennas, one can at most transmit n different symbols at a time. One can also view an $n \times m$ MIMO system as an equivalent group of n distinct $1 \times m$ systems and a natural question would be how can we utilize these n different $1 \times m$ systems to transmit data reliably using existing STCC designs? In this paper, we propose a technique to transmit data reliably, by using a STCC, with only a single transmit antenna. Hence, with n transmit antennas, we are able to transmit using n STCCs from these transmit antennas. In a typical cellular communication system, the number of antennas at the base station is larger than the number of antennas at the mobile station, i.e., n < m so considering cases with $n \neq m$ can be important.

Inducing randomness into a physical channel has been proposed by many authors [8], [9], [10], [11]. The main objective of these techniques is to induce more fluctuations

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into the channel. In [8], the authors induce randomness into the downlink of a wireless communication channel to create more fluctuations into an environment where slow fading or little scattering may occur. By taking advantage of the multiuser diversity effect [12], it can be shown that (opportunistic) beamforming gains can be achieved when there are a large number of users in the cell. In [9], the idea of inducing intentional frequency offset was introduced to create a fast fading environment with the goal of creating point-to-point time diversity in slow fading environments. The randomization concept has been also proposed for space-time code applications [10], [11]. The main idea behind these works is to increase fluctuations in the channel similar to the previous work. For example in [11], for stationary users with line-of-sight scenarios, a phase randomization strategy improves performance of the system significantly. In this paper, we propose to induce randomness into the physical channel in what appears to be a new way. The goal is to explore the rich diversity capabilities of MIMO systems. We will derive the conditions under which maximum coding and diversity gain can be attained for fading channels. We find the optimum randomization scheme for our approach. Our optimality criterion is based on the upper bound on the pairwise block error probability.

The landmark paper [13] defined the first systematic way to design channel codes for reliable transmission of data over wireless fading channels utilizing multiple transmit antennas. By formulating an upper bound on the pairwise block error probability, they devised several space-time codes for various number of states and symbol constellation sizes. We will use those results to design systems with similar capabilities when there is only a single transmit antenna.

The outline of the paper is as follows. In section II, we will review the system model and STCC design [13] for slow fading wireless channels. The proposed algorithm for utilizing a STCC for the single transmit antenna case is formulated in section III. The optimum induced randomization for slow Rayleigh fading channels is described in section IV. It is shown in this section, that this optimum randomization cannot be obtained for all error patterns. In section V, we design two randomization approaches to approximate the optimum approach as closely as possible. Simulation results on the performance of the proposed algorithm are given in section VI. Section VII contains the conclusion.

II. System Model

We consider a wireless communication system utilizing n transmit and m receive antennas. The channel path gain from transmit antenna i to receive antenna j is denoted by $h_{i,j}$ and is a complex Gaussian random variable with zero mean and variance 0.5 per complex dimension (realimaginary parts). We assume that different channel path gains are statistically independent. We also assume that the channel coefficients are constant during one block of data and change independently from one block to another. The received data r_t^j at antenna j and time t (slow fading channel) can be written as

$$r_t^j = \sum_{i=1}^n h_{i,j} c_t^i \sqrt{E_s} + n_t^j, \quad 1 \le j \le m$$
(1)

where c_t^i is the complex transmit symbol with unit average power sent from antenna i at time t, n_t^j is the additive Gaussian noise sample with zero mean and variance $\frac{N_o}{2}$ per dimension, and E_s is the contraction factor of the signal constellation. A block error occurs when the decoded data sequence

$$\underline{\mathbf{E}} = e_1^1 \dots e_1^n \dots e_N^1 \dots e_N^n$$

is different from the transmit sequence

$$\underline{\mathbf{C}} = c_1^1 \dots c_1^n \dots c_N^1 \dots c_N^n,$$

where N is the number of symbols in one block. It is shown in [13] that for a maximum likelihood receiver, an upper bound on the conditional pairwise block error probability (slow fading channel) is

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}} \mid h_{i,j}, 1 \le i \le n, 1 \le j \le m, 1 \le t \le N)$$
$$\le \prod_{j=1}^{m} \exp\left(-\Omega_{j}B_{s}(\underline{\mathbf{C}}, \underline{\mathbf{E}})\Omega_{j}^{*}\frac{E_{s}}{4N_{o}}\right), \quad (2)$$

where $\Omega_j = (h_{1,j}, h_{2,j}, \dots, h_{n,j}), B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$ (slow fading) is an $n \times n$ matrix whose elements are defined as $B_{s,mn}(\underline{\mathbf{C}}, \underline{\mathbf{E}}) = \sum_{t=1}^{N} (c_t^m - e_t^m)(c_t^n - e_t^n)^*$ and * denotes the complex conjugate transpose operation. It can be shown that [13] $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}}) = V^*DV$ is a Hermitian matrix, Vis a unitary matrix whose rows $v_i, 1 \leq j \leq n$ are the eigenvectors of $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$ and $D = diag(\lambda_1, \dots, \lambda_n)$ where the λ_j 's are the eigenvalues of $B_{s,mn}(\underline{\mathbf{C}}, \underline{\mathbf{E}})$. If a vector β_j is defined as $\beta_j = [\beta_{1,j} \dots \beta_{n,j}] = \Omega_j V^*$, then

$$\Omega_j B_{s,mn}(\underline{\mathbf{C}}, \underline{\mathbf{E}}) \Omega_j^* = \sum_{i=1}^n \lambda_j |\beta_{i,j}|^2.$$
(3)

By substituting (3) into (2), one can average over $\mid \beta_{i,j} \mid \forall i, j$ to arrive at

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}}) \le \left(\frac{1}{\prod_{t=1}^{n} (1 + \frac{E_s}{4N_o}\lambda_i)}\right)^m.$$
(4)

III. Problem Formulation

STCCs were originally designed to achieve diversity and coding gain in wireless fading channels utilizing multiple transmit antennas. The search space for these codes increases exponentially with the constellation size. In this paper, we will first show that one can apply STCCs in systems with only a single transmit antenna by using randomization techniques as long as we have multiple receive antennas. Then we use this approach to design STCCs with high spectral efficiencies using smaller constellation size STCCs such as QPSK STCC. Therefore, we assume for now that the number of transmit antennas is equal to 1, i.e., n = 1. Using this assumption, (1) can be written as

$$r_t^j = h_{1,j} C_t \sqrt{E_s} + n_t^j, \quad 1 \le j \le m.$$
 (5)

Here again, the physical channel path gains, the $h_{1,j}$'s, are independent complex normal random variables with zero mean and variance 0.5 per dimension. How can we modify the transmit signal (C_t) such that the system can use a STCC? We propose to use as the transmitted signal

$$C_t = A_1 c_t^1 + A_2 c_t^2 + \ldots + A_n c_t^n,$$
 (6)

where the induced random variables, the A_i 's, are statistically independent random variables and the c_t^i 's are symbols chosen from the output of a STCC encoder. The A_i 's are also independent of the physical channel path gains, the $h_{1,j}$'s. Combining (5) and (6) leads to

$$r_t^j = \sum_{i=1}^n h'_{i,j} c_t^i \sqrt{E_s} + n_t^j, \quad 1 \le j \le m,$$
 (7)

where $h'_{i,j} = h_{1,j} A_i$ is called a virtual path gain. We call this the virtual path gain because only m physical paths exist in this system and by introducing random data at the transmitter, we have created $n \times m$ virtual paths. Of course, some of these virtual paths are statistically dependent, but this approach will allow us to employ a STCC in a setting with a single transmit antenna and numerical results to be presented will demonstrate the gains that can be achieved. One way to interpret these gains is to recall that STTCs can provide gains in channels with correlated path gains, provided the correlation is not too close to unity.

One immediate application of the approach outlined above is to model an $n \times m$ MIMO system as an equivalent group of n distinct $1 \times m$ systems. Figure 1 compares a 2×2 MIMO system with its equivalent SIMO system and with a group of 1×2 systems that illustrate our approach. Note that we do not claim that such an approach can change a SIMO channel into a MIMO channel with independent path gains, nor do we claim that the rank of the new channel matrix changes. The induced random variables $(A_i$'s) can either change from symbol to symbol or they can be constant during one data frame. Our intention is to derive conditions under which one can obtain the minimum upper bound on the block error probability.

Applying (2) and (3) to the virtual paths $(h'_{i,j})$'s), we see that the conditional pairwise upper bound block error probability is

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}} \mid A_i, h_{1,j}, 1 \le i \le n, 1 \le j \le m, 1 \le t \le l)$$
$$\le \prod_{j=1}^{m} \exp(-\mid h_{1,j} \mid^2 AB_s(\underline{\mathbf{C}}, \underline{\mathbf{E}}) A^* \frac{E_s}{4N_o}),$$
(8)

where $A = [A_1, A_2, ..., A_n]$ is the vector whose elements are the induced random variables.

Since $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$ is a Hermitian matrix [13], then it can be written as $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}}) = V^* D V$, where the diagonal matrix D has the real eigenvalues of matrix $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$ along its diagonal, i.e., $\lambda_i, 1 \leq i \leq n$.

Let $[T_1, \ldots, T_n] = AV^*$, then (8) can be written as

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}} \mid A_i, h_{1,j}, 1 \le i \le n, 1 \le j \le m) \le \prod_{j=1}^{m} \exp(-\mid h_{1,j} \mid^2 \sum_{i=1}^{n} \lambda_i \mid T_i^2 \mid^2 \frac{E_s}{4N_o}).$$
(9)

By the taking the average over (9) with respect to channel coefficients and induced random variables assuming Rayleigh fading, we arrive at

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}}) \le \left\langle \left(\frac{1}{\prod_{i=1}^{n} (1 + \lambda_i \mid T_i^2 \mid^2 \frac{E_s}{4N_o})} \right)^m \right\rangle, \quad (10)$$

where $\langle F(X_1, \ldots, X_n, Y) \rangle$ denotes the expected value of $F(X_1, \ldots, X_n, Y)$ with respect to all random variables. In (10), the expected value is with respect to T_1, \ldots, T_n .

IV. Optimum Solution for Slow Rayleigh Fading Channels

In this section, we first find the optimum solution for Rayleigh fading channels. To simplify matters we will search only for solutions with T_1, \ldots, T_n independent. Thus, for Rayleigh fading channels, we want to find the pdf of $|T_i|$ for all values of *i* such that the upper bound to the pairwise block error probability given in (10) is minimized. Mathematically, the optimization problem is defined as

$$\min_{\langle |T_i|^2 \rangle = 1} \left\langle \left(1 + R_i \mid T_i \mid^2 \right)^{-m} \right\rangle \tag{11}$$

where $R_i = \lambda_i \frac{E_s}{4N_o}$ and the condition $\langle | T_i |^2 \rangle = 1$ is set to normalize the power of the induced random variables to one (increasing power would clearly allow better performance so this is important). In order to solve this minimization problem, we use Jensen's inequality, i.e., if g(y) is a convex function $(g''(y) \geq 0)$, then $\langle g(Y) \rangle \geq g(\langle Y \rangle)$. The following theorem (without any proof) describes the optimum distribution for T_i such that (10) is minimized.

Theorem 4.1: The minimum of (10) is attained when the pdf of $|T_i|$ is $f_{|T_i|}(x) = \delta(x-1)$.

Theorem 4.1 indicates that the amplitude of T_i should be deterministic and equal to 1. However, the phase of T_i can be a random variable.

V. Selection of induced random variables

In general, it is not possible to select random variables A_1, \ldots, A_n to achieve $|T_i| = 1$ for all possible choices of v_i^* . Since v_i^* is eigenvector of $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$ which is constructed from the difference between the transmitted codeword and the decoded error data sequence, there are many values of v_i^* to consider. To address this difficulty, in the following we provide two alternate solutions, neither of which are optimum.

A. Design of induced random variables Based on Eigenvectors of $B_s(\underline{\mathbf{C}},\underline{\mathbf{E}})$

In the previous section, we showed that for Rayleigh fading channels, the optimum solution is $f_{|T_i|}(x) = \delta(x - 1)$. However, we do not have the freedom of selecting T_i . Instead, T_i depends on random variables A_1, \ldots, A_n and v_i^*

$$T_i = (A_1, \dots, A_n)v_i^*$$

In this section, we seek to minimize the average distance between $|T_i|$ and 1 for all choices of v_i^* . We use n = m = 2 to illustrate the derivation.

Because v_i^* is a unit vector, we can write it as

$$v_i^* = \begin{pmatrix} \sqrt{b} \exp(i\phi_1) \\ \sqrt{1-b} \exp(i\phi_2) \end{pmatrix}, \qquad i = \sqrt{-1}.$$

We can show that, in order to minimize the distance between $|T_i|$ and 1, we need to select $|A_1| = |A_2| = 1$. Thus, we write A_1 and A_2 as

$$A_1 = \exp(i\Theta_1), \qquad \qquad A_2 = \exp(i\Theta_2)$$

where Θ_1 and Θ_2 are random variables. Expanding $|T_i|$, we have

$$|T_i| = \left| A_1 \sqrt{b} \exp(i\phi_1) + A_2 \sqrt{1-b} \exp(i\phi_2) \right|$$

1494

$$=\sqrt{1+q\cos(\phi+\Theta)}$$

where $q = 2\sqrt{b(1-b)}$, $\phi = \phi_2 - \phi_1$ and $\Theta = \Theta_2 - \Theta_1$. We determine the optimum pdf of Θ by minimizing $\left\langle \left| |T_i| - 1 \right| \right\rangle$ for all values of ϕ . Mathematically, the optimization problem is

$$\min_{P_{\Theta}} \left(\max_{\phi} \left\langle \left| \sqrt{1 + q \cos(\phi + \Theta)} - 1 \right| \right\rangle_{P_{\Theta}} \right)$$
(12)

where P_{Θ} is the probability distribution function of Θ . The result is provided without any proof.

Theorem 5.1: The minimum of (12) is attained when Θ is uniformly distributed in $[0, 2\pi]$.

Note that we assumed in the proof of this theorem that the pdf of ϕ is uniform. However, the phase distributions of the eigenvectors of the $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$ matrix are not known and therefore, these results are suboptimal. Moreover, with the proposed scheme, the actual transmitted signal can have a different amplitude at each time interval. This poses some constraints on the complexity of the transmitter. Such an approach may not be very desirable from a practical point of view. Therefore, we propose to use random variables that are conditioned on the output of the STCC encoder such that, the final constellation has a finite number of points with possibly constant amplitude. The next section describes the criterion for designing these induced random variables.

B. Design of induced random variables that depend on information data

In the approaches discussed previously in this paper, the final transmitted symbols can have any continuous amplitude inside a circle of radius 2. It would be more desirable if the transmitted symbol employed only a finite number of constellation points.

Our objective in this section is to design a set of discrete random variables that statistically depend on the STCC encoder output. The objective is twofold, first, to reduce the number of points in the constellation for the transmit signal C_t that was defined in (6), and second to force the amplitude of the transmit signal to be constant. This approach will be more suitable for practical applications.

The conditional upper bound on the pairwise block error probability in this case can be derived as

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}} \mid h_{1,j}, 1 \le i \le n, 1 \le j \le m, 1 \le t \le l)$$
$$\le \prod_{j=1}^{m} \exp(-\mid h_{1,j} \mid^{2} \sum_{t=1}^{N} \mid C_{t} - E_{t} \mid^{2} \frac{E_{s}}{4N_{o}}), \quad (13)$$

where E_t is the error signal defined similar to (6). Note that in this case, the induced random variables, the A_i 's, are statistically dependent on the output of the STCC encoder and for that reason, it is not feasible to separate them in this equation. Averaging over (13) with respect to the channel coefficients, we arrive at

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}}) \le \left(\frac{1}{\prod_{i=1}^{n} (1 + \sum_{t=1}^{N} |C_t - E_t|^2 \frac{E_s}{4N_o})}\right)^m.$$
(14)

This equation suggests that in order to minimize the upper bound on the pairwise block error probability, we need to maximize the minimum Euclidean distance between the modified codeword $(C_t, t = 1, ..., N)$ and the codeword chosen in error $(E_t, t = 1, ..., N)$. Therefore, we need to design the induced random variables such that this minimum Euclidean distance is maximized.

VI. Simulation results

In the simulations presented in this section, coherent detection is assumed along with perfect knowledge of the channel coefficients at the receiver. We also assume that the induced random variables are generated in advance and known at the receiver. We apply this technique to MIMO systems where each transmit antenna can employ a STCC. Therefore, for a $n \times m$ system, we can model it as an equivalent group of n distinct $1 \times m$ systems, each one transmitting a STCC with small constellation size. Fig. 1c demonstrates this concept for a 2×2 system. We have used this approach to design a STCC with spectral efficiency of 4 bits/s/Hz, using a QPSK STCC for each antenna, and compare it with the 16-QAM STCC of [13]. Simulation results clearly show that our approach can perform better than that of [13] for 2×3 and 2×4 MIMO systems and block length 260 and 520 bits in Figures 2 and 3 respectively.

VII. Conclusion

In this paper, we explored the idea of exploiting the good performance of the space-time code designs from [13] for cases with a single transmit and multiple receive antennas. Using this idea, we can model an $n \times m$ system with n distinct $1 \times m$ systems. Therefore, each $1 \times m$ system can transmit a separate STCC. Simulation results show that this approach performs better than the 16-QAM STCC design when using two QPSK STCCs for 2×3 and 2×4 systems in Rayleigh fading channels.

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(b)

hinh

h33.h43

TxI

Fig. 2. Frame Error Rate Comparison Between 16-QAM STCC and the PA for 2×3 and 2×4 systems in Rayleigh fading Channels with 260 bits for each block.









(c)