1. Find the leading term in the asymptotic expansion of
\[
\int_0^1 \sin(\lambda \cos(t)) \, dt, \quad \lambda \to +\infty
\]

2. Find the leading term in the asymptotic expansion of
\[
\int_{-1}^1 \sqrt{1 + t^2} \cos \left[ \lambda \left( \frac{e^t + e^{-t}}{2} - \cos(t) \right) \right] \, dt, \quad \lambda \to +\infty
\]

3. Consider the 2 \times 2 nonlinear system below for \((R, \theta)\) where \(a\) and \(b\) are parameters.
\[
\begin{align*}
R^3 - R^0 + a \cdot \sin \theta & = 0 \\
b \cdot R + \cos \theta & = 0
\end{align*}
\]
Write a code to scan the parameter space \((a, b)\) = \([0.1 : 0.05 : 1] \times [0.1 : 0.05 : 2]\).
For each point \((a, b)\) in the parameter space, find ALL solutions of the non-linear system.
Select ONLY solutions with \(R > 0\).
Plot the region of parameter space where the non-linear system has exact \(k\) solutions with \(R > 0\), respectively for \(k = 0, 1, 2, 3, 4, \ldots\)
Plot all regions in one figure.

4. Consider the function
\[
F(x) = \exp(-x^2) \int_0^x \exp(t^2) \, dt
\]
Find the first 2 terms of the expansion of \(F(x)\) as \(x \to +\infty\).
\textbf{Hint:} Use change of variable \(t^2 = x^2(1 - s)\)