AMS 212, Assignment #5

1. Use the method of multi-scales with $T_0 = t$ and $T_1 = \epsilon t$ to solve the IVP (initial value problem)
\[
\begin{cases}
y'' + \epsilon^2 y y' + y = 0 \\
y(0) = 1, \quad y'(0) = 0 \quad \epsilon \to 0^+
\end{cases}
\]
Find the leading term in the expansion.

2. Using the method of strained variable to solve the IVP (initial value problem).
\[
\begin{cases}
y'' + \left( 2 - \frac{1}{\sqrt{1 + \epsilon^2 y^2}} \right) y = 0 \\
y(0) = 1, \quad y'(0) = 0 \\
\end{cases}
\]
Find the first two terms (up to $\epsilon^2$ term) in the expansion.
Find the period of oscillation (find the first two terms, up to $\epsilon^2$ term).

3. Consider the IVP (initial value problem)
\[
\begin{cases}
y'' + \frac{1}{\epsilon} \sin(\epsilon y) = 0 \\
y(0) = 1, \quad y'(0) = 0 \quad \epsilon \to 0^+
\end{cases}
\]
Its regular expansion is
\[
y(t) \sim \cos(t) + \epsilon^2 \left[ \frac{t}{16} \sin t + \frac{1}{192} \left( \cos t - \cos 3t \right) \right]
\]
Use the renormalization technique to find the strained variable expansion.

4. Using the method of strained variable to solve the IVP (initial value problem).
\[
\begin{cases}
y'' + y + \left( e^{\epsilon y} - 1 \right) = 0 \\
y(0) = 1, \quad y'(0) = 0 \quad \epsilon \to 0^+
\end{cases}
\]
Find the first two terms (up to $\epsilon$ term) in the expansion.
Bonus Problem (optional):

5. In Problem 4 above, find the first three terms (up to $\epsilon^2$ term) in the expansion.