AMS 212, Assignment #2

1. Use a perturbation method to solve the IVP (initial value problem)
   \[
   \begin{aligned}
   y' - \varepsilon y + 1 &= 0 \\
   y(0) &= \varepsilon
   \end{aligned}
   \]
   Find the first two terms in the expansion.

2. Use a perturbation method to solve the IVP (initial value problem)
   \[
   \begin{aligned}
   y'' &= 2y - \frac{1}{(1 + \varepsilon y)^2} \\
   y(0) &= 0, \quad y'(0) = 0
   \end{aligned}
   \]
   Find the first two terms in the expansion of \( y \).
   Find the first two terms in the expansion of \( T \), the period of oscillation.

3. Use a perturbation method to solve the BVP (boundary value problem)
   \[
   \begin{aligned}
   y'' - 2y' + \varepsilon y &= 0 \\
   y(0) &= 0, \quad y(1) = 1
   \end{aligned}
   \]
   Find the first two terms in the expansion.

4. (Optional) Solve numerically the IVP
   \[
   \begin{aligned}
   y'' &= \frac{1}{\varepsilon} \sin(\varepsilon y) \\
   y(0) &= 1, \quad y'(0) = 0
   \end{aligned}
   \]
   Compute \( T(\varepsilon) \), the period of oscillation as a function of \( \varepsilon \), for \( \varepsilon \) in \([0.01:0.01:1]\).
   Plot \( T(\varepsilon) \) as a function of \( \varepsilon \) and compare with the asymptotic expansion
   \[
   T(\varepsilon) \sim 2\pi \left( 1 + \frac{\varepsilon^2}{16} \right)
   \]
Plot \( \frac{1}{\varepsilon^4} \left( \frac{T(\varepsilon)}{2\pi} - 1 - \frac{\varepsilon^2}{16} \right) \) as a function of \( \varepsilon \) to numerically predict the next coefficient.