37. $F(b, C, T, i) = \frac{bT}{C} + \frac{iC}{2}$

$\frac{\partial F}{\partial C} = \frac{\partial}{\partial C}\left[\frac{bT}{C}\right] + \frac{\partial}{\partial C}\left[\frac{iC}{2}\right] = -\frac{bT}{C^2} + \frac{i}{2}$

38. From $\eta = \frac{\frac{r}{D}}{\frac{\partial r}{\partial D}}$, we have $\frac{\partial r}{\partial D} = \frac{r}{D\eta}$. Substituting into Equation (3) gives

$r_L = r + D \cdot \frac{r}{D\eta} + \frac{dC}{dD}$

$r_L = r + \frac{r}{\eta} + \frac{dC}{dD}$

$r_L = r\left[1 + \frac{1}{\eta}\right] + \frac{dC}{dD}$

$r_L = r\left[\frac{\eta + 1}{\eta}\right] + \frac{dC}{dD}$

$r_L = r\left[\frac{1 + \eta}{\eta}\right] + \frac{dC}{dD}$

which is Equation (4).

39. $R = f(r, a, n) = \frac{r}{1 + a\left(\frac{n-1}{2}\right)} = r\left[1 + a\left(\frac{n-1}{2}\right)\right]^{-1}$

$\frac{\partial R}{\partial n} = r(-1)\left[1 + a\left(\frac{n-1}{2}\right)\right]^{-2} \cdot \frac{a}{2}$

$= -\frac{ra}{2\left[1 + a\left(\frac{n-1}{2}\right)\right]^2}$

Problems 17.2

1. $c = 7x + 0.3y^2 + 2y + 900$

$\frac{\partial c}{\partial y} = 0.6y + 2$

When $x = 20$ and $y = 30$, then

$\frac{\partial c}{\partial y} = 0.6(30) + 2 = 20$.

2. $c = 2x\sqrt{x + y} + 6000$

$\frac{\partial c}{\partial x} = \frac{x}{\sqrt{x + y}} + 2\sqrt{x + y}$

When $x = 70$ and $y = 74$, then

$\frac{\partial c}{\partial x} = \frac{70}{\sqrt{70 + 74}} + 2\sqrt{70 + 74} = \frac{179}{6}$.

3. $c = 0.03(x + y)^3 - 0.6(x + y)^2 + 9.5(x + y) + 7700$

$\frac{\partial c}{\partial x} = 0.09(x + y)^2 - 1.2(x + y) + 9.5$

When $x = 50$ and $y = 80$, then

$\frac{\partial c}{\partial x} = 0.09(130)^2 - 1.2(130) + 9.5 = 1374.5$.

4. $P = 15lk - 3l^2 + 5k^2 + 500$

$\frac{\partial P}{\partial k} = 15l + 10k$

$\frac{\partial P}{\partial l} = 15k - 6l$

5. $P = 2.314 l^{0.357} k^{0.643}$

$\frac{\partial P}{\partial l} = 2.314(0.357) l^{-0.643} k^{0.643}$

$= 0.826098\left(\frac{k}{l}\right)^{0.643}$

$\frac{\partial P}{\partial k} = 2.314(0.643) l^{0.357} k^{-0.357}$

$= 1.487902\left(\frac{l}{k}\right)^{0.357}$

6. $P = Al^{\alpha}k^{\beta}$

a. $\frac{\partial P}{\partial l} = A\alpha l^{\alpha-1}k^{\beta} = \left(\frac{\alpha}{l}\right)Al^{\alpha}k^{\beta} = \frac{\alpha P}{l}$

b. $\frac{\partial P}{\partial k} = A\beta l^{\alpha}k^{\beta-1} = \left(\frac{\beta}{k}\right)Al^{\alpha}k^{\beta} = \frac{\beta P}{k}$

c. From parts (a) and (b),

$l\frac{\partial P}{\partial l} + k\frac{\partial P}{\partial k} = l\left(\frac{\alpha P}{l}\right) + k\left(\frac{\beta P}{k}\right)$

$= \alpha P + \beta P = P(\alpha + \beta) = P(1) = P$

7. $\frac{\partial q_A}{\partial p_A} = -40$, $\frac{\partial q_A}{\partial p_B} = 3$, $\frac{\partial q_B}{\partial p_A} = 5$, $\frac{\partial q_B}{\partial p_B} = -20$

Since $\frac{\partial q_A}{\partial p_B} > 0$ and $\frac{\partial q_B}{\partial p_A} > 0$ the products are competitive.

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8. $\frac{\partial q_A}{\partial p_A} = -1$, $\frac{\partial q_A}{\partial p_B} = -2$, $\frac{\partial q_B}{\partial p_A} = -2$, $\frac{\partial q_B}{\partial p_B} = -3$

Since $\frac{\partial q_A}{\partial p_B} < 0$ and $\frac{\partial q_B}{\partial p_A} < 0$ the products are complementary.

9. $q_A = 100 p_A^{-1} p_B^{-\frac{1}{2}}$

$q_B = 500 p_B^{-1} p_A^{-\frac{1}{3}}$

$$\frac{\partial q_A}{\partial p_A} = 100(-1) p_A^{-2} p_B^{-\frac{1}{2}} = \frac{-100}{p_A^2 p_B^{\frac{1}{2}}}$$

$$\frac{\partial q_A}{\partial p_B} = 100\left(-\frac{1}{2}\right) p_A^{-1} p_B^{-\frac{3}{2}} = \frac{-50}{p_A p_B^{\frac{3}{2}}}$$

$$\frac{\partial q_B}{\partial p_A} = 500\left(-\frac{1}{3}\right) p_B^{-1} p_A^{-\frac{4}{3}} = \frac{-500}{3 p_B p_A^{\frac{4}{3}}}$$

$$\frac{\partial q_B}{\partial p_B} = 500(-1) p_B^{-2} p_A^{-\frac{1}{3}} = -\frac{500}{p_B^2 p_A^{\frac{1}{3}}}$$

Since $\frac{\partial q_A}{\partial p_B} < 0$ and $\frac{\partial q_B}{\partial p_A} < 0$, the products are complementary.

10. $\frac{\partial P}{\partial l} = 15.18 l^{-0.54} k^{0.52}$

$\frac{\partial P}{\partial k} = 17.16 l^{0.46} k^{-0.48}$

If $l = 1$ and $k = 1$, then $\frac{\partial P}{\partial l} = 15.18$ and

$\frac{\partial P}{\partial k} = 17.16$

11. $\frac{\partial P}{\partial B} = 0.01 A^{0.27} B^{-0.99} C^{0.01} D^{0.23} E^{0.09} F^{0.27}$

$\frac{\partial P}{\partial C} = 0.01 A^{0.27} B^{0.01} C^{-0.99} D^{0.23} E^{0.09} F^{0.27}$

12. $P = \frac{kl}{3k + 5l}$

a. $$\frac{\partial P}{\partial k} = \frac{l(3k+5l) - kl(3)}{(3k+5l)^2} = \frac{5l^2}{(3k+5l)^2}$$

$$\frac{\partial P}{\partial l} = \frac{k(3k+5l) - kl(5)}{(3k+5l)^2} = \frac{3k^2}{(3k+5l)^2}$$

b. When $k = l$, then

$$\frac{\partial P}{\partial k} + \frac{\partial P}{\partial l} = \frac{5l^2}{(3l+5l)^2} + \frac{3l^2}{(3l+5l)^2} = \frac{8l^2}{64l^2} = \frac{1}{8}$$

13. $\frac{\partial z}{\partial x} = 4480$. If a staff manager with an M.B.A. degree had an extra year of work experience before the degree, the manager would receive \$4480 more per year in extra compensation.

14. $S_g = 7 S_e^{\frac{1}{3}} S_i^{\frac{1}{2}}$

$$\frac{\partial S_g}{\partial S_e} = 7\left(\frac{1}{3}\right) S_e^{-\frac{2}{3}} S_i^{\frac{1}{2}} = \left(\frac{7}{3}\right) \frac{\sqrt{S_i}}{\sqrt[3]{S_e^2}}$$

$$\frac{\partial S_g}{\partial S_i} = 7\left(\frac{1}{2}\right) S_e^{\frac{1}{3}} S_i^{-\frac{1}{2}} = \left(\frac{7}{2}\right) \frac{\sqrt[3]{S_e}}{\sqrt{S_i}}$$

If $S_e = 125$ and $S_i = 100$, then

$\frac{\partial S_g}{\partial S_e} = \left(\frac{7}{3}\right)\frac{10}{5^2} = \frac{14}{15}$ and $\frac{\partial S_g}{\partial S_i} = \left(\frac{7}{2}\right)\frac{5}{10} = \frac{7}{4}$.

Thus if S_e increases to 126 and S_i remains at 100, then S_g increases by approximately $\frac{14}{15}$; if S_i increases to 101 and S_e remains at 125, then S_g increases by approximately $\frac{7}{4}$.

15. a. $\frac{\partial R}{\partial w} = -1.015$; $\frac{\partial R}{\partial s} = -0.846$

b. One for which $w = w_0$ and $s = s_0$ since increasing w by 1 while holding s fixed decreases the reading ease score.

16. $\omega = b^{-1} L^{-1} \sqrt{\frac{\tau}{\pi\rho}} = \frac{1}{bL}\sqrt{\frac{\tau}{\pi}}\rho^{-\frac{1}{2}} = \frac{1}{bL}\sqrt{\frac{1}{\pi\rho}}\tau^{\frac{1}{2}}$

$$\frac{\partial \omega}{\partial b} = (-1) b^{-2} L^{-1} \sqrt{\frac{\tau}{\pi\rho}} = -\frac{1}{b^2 L}\sqrt{\frac{\tau}{\pi\rho}}$$

$$\frac{\partial \omega}{\partial L} = b^{-1}(-1) L^{-2} \sqrt{\frac{\tau}{\pi\rho}} = -\frac{1}{bL^2}\sqrt{\frac{\tau}{\pi\rho}}$$

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$$\frac{\partial \omega}{\partial \rho} = \frac{1}{bL}\sqrt{\frac{\tau}{\pi}}\left(-\frac{1}{2}\right)\rho^{-\frac{3}{2}} = -\frac{1}{2bL\rho^{\frac{3}{2}}}\sqrt{\frac{\tau}{\pi}}$$

$$\frac{\partial \omega}{\partial \tau} = \frac{1}{bL}\sqrt{\frac{1}{\pi\rho}}\left(\frac{1}{2}\right)\tau^{-\frac{1}{2}} = \frac{1}{2bL}\sqrt{\frac{1}{\pi\rho\tau}}$$

17. $\frac{\partial g}{\partial x} = \frac{1}{V_F} > 0$ for $V_F > 0$. Thus if *x* increases and V_F and V_S are fixed, then *g* increases.

18. $q_A = e^{-(p_A + p_B)}$ and $q_B = \frac{16}{p_A^2 p_B^2} = 16 p_A^{-2} p_B^{-2}$

a. $\frac{\partial q_A}{\partial p_B} = -e^{-(p_A + p_B)} < 0$

$\frac{\partial q_B}{\partial p_A} = -32 p_A^{-3} p_B^{-2} < 0$

Since both are < 0, A and B are complementary.

b. Note that p_A and p_B are in units of thousands of dollars. When $p_A = 1$ and $p_A = 2$, then

$\frac{\partial q_A}{\partial p_B} = -e^{-3} = -\frac{1}{e^3}.$

A decrease in the price of B of \$20 is a decrease in p_B of $\frac{20}{2000} = 0.01.$ Thus the change in q_B is approximately $-\frac{1}{e^3}(-0.01) = \frac{0.01}{e^3}.$ So demand increases by approximately $\frac{0.01}{e^3}$ unit.

19. a. $\frac{\partial q_A}{\partial p_A} = 10\sqrt{p_B}\left(-\frac{1}{2}p_A^{-\frac{3}{2}}\right)$

$\frac{\partial q_A}{\partial p_B} = \frac{10}{\sqrt{p_A}}\left(\frac{1}{2}p_B^{-\frac{1}{2}}\right)$

When $p_A = 9$ and $p_B = 16,$ then $\frac{\partial q_A}{\partial p_A} = 10(4)\left(-\frac{1}{2}\cdot\frac{1}{27}\right) = -\frac{20}{27}$ and $\frac{\partial q_A}{\partial p_B} = \frac{10}{3}\left(\frac{1}{2}\cdot\frac{1}{4}\right) = \frac{5}{12}.$

b. From (a), when $p_A = 9$ and $p_B = 16,$ then $\frac{\partial q_A}{\partial p_B} = \frac{5}{12}.$ Hence each \$1 reduction in p_B decreases q_A by approximately $\frac{5}{12}$ unit. Thus a \$2 reduction in p_B (from \$16 to \$14) decreases the demand for A by approximately $\frac{5}{12}(2) = \frac{5}{6}$ unit.

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20. $c = \dfrac{q_A^2\left(q_B^3 + q_A\right)^{\frac{1}{2}}}{17} + q_A q_B^{\frac{1}{3}} + 600$

a. $$\frac{\partial c}{\partial q_A} = \frac{1}{17}\left[q_A^2 \cdot \frac{1}{2}\left(q_B^3 + q_A\right)^{-\frac{1}{2}} + \left(q_B^3 + q_A\right)^{\frac{1}{2}}(2q_A)\right] + q_B^{\frac{1}{3}}$$

$$= \frac{1}{17}\left[\frac{1}{2}q_A^2\left(q_B^3 + q_A\right)^{-\frac{1}{2}} + 2q_A\left(q_B^3 + q_A\right)^{\frac{1}{2}}\right] + q_B^{\frac{1}{3}}$$

$$\frac{\partial c}{\partial q_B} = \frac{1}{17}\left[q_A^2 \cdot \frac{1}{2}\left(q_B^3 + q_A\right)^{-\frac{1}{2}}\left(3q_B^2\right)\right] + q_A \cdot \frac{1}{3}q_B^{-\frac{2}{3}}$$

$$= \frac{1}{17}\left[\frac{3}{2}q_A^2 q_B^2\left(q_B^3 + q_A\right)^{-\frac{1}{2}}\right] + \frac{1}{3}q_A q_B^{-\frac{2}{3}}$$

b. When $q_A = 17$ and $q_B = 8$, then

$$\frac{\partial c}{\partial q_A} = \frac{1}{17}\left[\frac{1}{2}(17)^2\left(\frac{1}{23}\right) + 2(17)(23)\right] + 2 = \left[\frac{1}{2}(17)\frac{1}{23} + 2(23)\right] + 2 \approx 48.37.$$

c. From (b), if q_A is reduced by one unit (from 17 to 16) while q_B remains at 8, then the cost will decrease by approximately $48.37.

21. a. $$\frac{\partial R}{\partial E_r} = 2.5945 - 0.1608E_r - 0.0277I_r$$

If $E_r = 18.8$ and $I_r = 10$, then $\dfrac{\partial R}{\partial E_r} = -0.70564$. Since $\dfrac{\partial R}{\partial E_r} < 0$, such a candidate should not be so advised.

b. $$\frac{\partial R}{\partial N} = 0.8579 - 0.0122N$$

If $\dfrac{\partial R}{\partial N} < 0$, then $N > 70.3 \approx 70\%$

22. $S = \dfrac{AT + 450}{\sqrt{A + T^2}}$. Note: A is expressed in hundreds of dollars.

a. $$\frac{\partial S}{\partial T} = \frac{\left(A + T^2\right)^{\frac{1}{2}}(A) - (AT + 450)\left[\frac{1}{2}\left(A + T^2\right)^{-\frac{1}{2}}(2T)\right]}{\left(\sqrt{A + T^2}\right)^2}$$

$$= \frac{\left(A + T^2\right)^{-\frac{1}{2}}\left[\left(A + T^2\right)A - (AT + 450)T\right]}{A + T^2} = \frac{A^2 - 450T}{\left(A + T^2\right)^{\frac{3}{2}}}$$

as was to be shown.

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b. We want to find when $\frac{\partial S}{\partial T} < 0$ and $A = \frac{9000}{100} = 90$. First we find when $\frac{\partial S}{\partial T} = 0$ and $A = 90$:

$$\frac{90^2 - 450T}{\left(90 + T^2\right)^{\frac{3}{2}}} = 0 \Rightarrow 90^2 - 450T = 0$$

$$\Rightarrow T = \frac{90^2}{450} = 18.$$

$\frac{\partial S}{\partial T} > 0$ for $T < 18$, and $\frac{\partial S}{\partial T} < 0$ for $T > 18$.
Thus 18 months elapse before the sales volume begins to decrease.

23. $q_A = 1000 - 50p_A + 2p_B$

$$\eta_{p_A} = \left(\frac{p_A}{q_A}\right)\frac{\partial q_A}{\partial p_A} = \left(\frac{p_A}{q_A}\right)(-50)$$

$$\eta_{p_B} = \left(\frac{p_B}{q_A}\right)\frac{\partial q_A}{\partial p_B} = \left(\frac{p_B}{q_A}\right)(2)$$

When $p_A = 2$ and $p_B = 10$, then $q_A = 920$, from which $\eta_{p_A} = -\frac{5}{46}$ and $\eta_{p_B} = \frac{1}{46}$

24. $q_A = 60 - 3p_A - 2p_B$

$$\eta_{p_A} = \left(\frac{p_A}{q_A}\right)\frac{\partial q_A}{\partial p_A} = \left(\frac{p_A}{q_A}\right)(-3)$$

$$\eta_{p_B} = \left(\frac{p_B}{q_A}\right)\frac{\partial q_A}{\partial p_B} = \left(\frac{p_B}{q_A}\right)(-2)$$

When $p_A = 5$ and $p_B = 3$, then $q_A = 39$, from which $\eta_{p_A} = -\frac{5}{13}$ and $\eta_{p_B} = -\frac{2}{13}$.

25. $q_A = \frac{100}{p_A\sqrt{p_B}}$

$$\eta_{p_A} = \left(\frac{p_A}{q_A}\right)\frac{\partial q_A}{\partial p_A} = \left(\frac{p_A}{q_A}\right)\left(\frac{-100}{p_A^2\sqrt{p_B}}\right)$$

$$\eta_{p_B} = \left(\frac{p_B}{q_A}\right)\frac{\partial q_A}{\partial p_B} = \left(\frac{p_B}{q_A}\right)\left(\frac{-50}{p_A\sqrt{p_B^3}}\right)$$

When $p_A = 1$ and $p_B = 4$, then $q_A = 50$. This gives $\eta_{p_A} = -1$ and $\eta_{p_B} = -\frac{1}{2}$.

Problems 17.3

1. $4x + 0 + 10z\frac{\partial z}{\partial x} = 0$

$$\frac{\partial z}{\partial x} = -\frac{4x}{10z} = -\frac{2x}{5z}$$

2. $2z\frac{\partial z}{\partial x} - 10x + 0 = 0$

$$\frac{\partial z}{\partial x} = \frac{10x}{2z} = \frac{5x}{z}$$

3. $6z\frac{\partial z}{\partial y} - 0 - 14y = 0$

$$\frac{\partial z}{\partial y} = \frac{14y}{6z} = \frac{7y}{3z}$$

4. $0 + 2y + 6z^2\frac{\partial z}{\partial y} = 0$

$$\frac{\partial z}{\partial y} = \frac{-2y}{6z^2} = -\frac{y}{3z^2}$$

5. $x^2 - 2y - z^2 + y\left(x^2z^2\right) = 20$

$$2x - 0 - 2z\frac{\partial z}{\partial x} + y\left[x^2 \cdot 2z\frac{\partial z}{\partial x} + z^2 \cdot 2x\right] = 0$$

$$\left(2x^2yz - 2z\right)\frac{\partial z}{\partial x} = -2x - 2xyz^2$$

$$\frac{\partial z}{\partial x} = \frac{-2x\left(1 + yz^2\right)}{2z\left(x^2y - 1\right)} = \frac{x\left(yz^2 + 1\right)}{z\left(1 - x^2y\right)}$$

6. $3z^2\frac{\partial z}{\partial x} + 2x^2\left(2z\frac{\partial z}{\partial x}\right) + 2z^2(2x) - y = 0$

$$(3z^2 + 4x^2z)\frac{\partial z}{\partial x} = y - 4xz^2$$

$$\frac{\partial z}{\partial x} = \frac{y - 4xz^2}{3z^2 + 4x^2z}$$

7. $0 + e^y + e^z\frac{\partial z}{\partial y} = 0$

$$\frac{\partial z}{\partial y} = -\frac{e^y}{e^z} = -e^{y-z}$$

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