**36.** $$\frac{dP}{dx} = k(150,000 - 2P)$$

$$\int \frac{dP}{150,000 - 2P} = \int k\, dx$$

$$-\frac{1}{2}\ln[150,000 - 2P] = kx + C$$

Since $P(0) = 15,000$, we have

$-\frac{1}{2}\ln[150,000 - 30,000] = C$, so

$$-\frac{1}{2}\ln[150,000 - 2P] = kx - \frac{1}{2}\ln[120,000].$$

Since $P(1000) = 70,000$,

$$-\frac{1}{2}\ln[150,000 - 140,000] = 1000k - \frac{1}{2}\ln[120,000]$$

$$k = \frac{1}{2}\cdot\frac{\ln[120,000] - \ln[10,000]}{1000} = \frac{\ln 12}{2000}$$

Thus

$$-\frac{1}{2}\ln[150,000 - 2P] = \frac{\ln 12}{2000}x - \frac{1}{2}\ln[120,000]$$

$$\ln[150,000 - 2P] = -\frac{\ln 12}{1000}x + \ln[120,000]$$

$$150,000 - 2P = e^{-\frac{\ln 12}{1000}x}e^{\ln[120,000]}$$

$$150,000 - 2P = 120,000e^{-\frac{\ln 12}{1000}x}$$

$$P = \frac{1}{2}\left(150,000 - 120,000e^{-\frac{\ln 12}{1000}x}\right)$$

$$= 75,000 - 60,000\left(12^{-\frac{x}{1000}}\right)$$

If $x = 2000$, then

$P = 75,000 - 60,000(12^{-2}) \approx \$74,583.$

**37. a.** $$\frac{dV}{dt} = kV$$

$$\int \frac{1}{V}dV = \int k\ dt$$

$$\ln V = kt + C_1$$

$$V = e^{kt}e^{C_1}$$

or $V = Ce^{kt}$. Now $t = 0$ corresponds to July 1, 1996 where $V = 0.75 \cdot 80,000 = 60,000$, so $60,000 = C(1)$. Thus $V = 60,000e^{kt}$. Also $V = 38,900$ for $t = 9.5$, so

$$38,900 = 60,000e^{9.5k}$$

$$\frac{389}{600} = e^{9.5k}$$

$$9.5k = \ln\left(\frac{389}{600}\right)$$

$$k = \frac{1}{9.5}\ln\left(\frac{389}{600}\right)$$

Thus $V = 60,000e^{\frac{t}{9.5}\ln\left(\frac{389}{600}\right)}$.

**b.** $$14,000 = 60,000e^{\frac{t}{9.5}\ln\left(\frac{389}{600}\right)}$$

$$\frac{7}{30} = e^{\frac{t}{9.5}\ln\left(\frac{389}{600}\right)}$$

$$\ln\left(\frac{7}{30}\right) = \frac{t}{9.5}\ln\left(\frac{389}{600}\right)$$

$$t = \frac{9.5\ln\left(\frac{7}{30}\right)}{\ln\left(\frac{389}{600}\right)} \approx 31.903$$

This corresponds to about 31 years and 11 months after July 1, 1996 ⇒ June 2028.

## Problems 15.6

**1.** $$N = \frac{M}{1 + be^{-ct}}$$

$M = 100,000$

Since $N = 50,000$ at $t = 0$ (1995), we have

$50,000 = \frac{100,000}{1+b}$, so $1 + b = \frac{100,000}{50,000} = 2$, or $b = 1$.

Hence, $N = \frac{100,000}{1 + e^{-ct}}$. If $t = 5$, then $N = 60,000$, so

$$60,000 = \frac{100,000}{1 + e^{-5c}}$$

$$1 + e^{-5c} = \frac{100,000}{60,000} = \frac{5}{3}$$

$$e^{-5c} = \frac{5}{3} - 1 = \frac{2}{3}$$

$$e^{-c} = \left(\frac{2}{3}\right)^{1/5}$$

Hence, $N = \frac{100,000}{1 + \left(\frac{2}{3}\right)^{t/5}}$. In 2005, $t = 10$, so

$$N = \frac{100,000}{1 + \left(\frac{2}{3}\right)^2} \approx 69,200.$$

**2.** $N = \frac{M}{1+be^{-ct}}$

Since $M = 500$, and $N = 200$ when $t = 0$, we have

$200 = \frac{500}{1+b}$

$1+b = \frac{500}{200} = \frac{5}{2} \Rightarrow b = \frac{3}{2}$.

Hence $N = \frac{500}{1+\frac{3}{2}e^{-ct}}$. When $t = 1$ we are given $N = 300$. Thus

$300 = \frac{500}{1+\frac{3}{2}e^{-c}}$

$1+\frac{3}{2}e^{-c} = \frac{500}{300} = \frac{5}{3}$

$\frac{3}{2}e^{-c} = \frac{2}{3}$

$e^{-c} = \frac{4}{9}$

Hence $N = \frac{500}{1+\frac{3}{2}\left(\frac{4}{9}\right)^t}$. When $t = 2$, then

$N = \frac{500}{1+\frac{3}{2}\left(\frac{4}{9}\right)^2} \approx 386$.

**3.** $N = \frac{M}{1+be^{-ct}}$

$M = 40{,}000$, and $N = 20$ when $t = 0$, so

$20 = \frac{40{,}000}{1+b}$

$1+b = \frac{40{,}000}{20} = 2000$

$b = 1999$

Hence $N = \frac{40{,}000}{1+1999e^{-ct}}$.

Since $N = 100$ when $t = 1$, $100 = \frac{40{,}000}{1+1999e^{-c}}$,

$1+1999e^{-c} = \frac{40{,}000}{100} = 400$

$e^{-c} = \frac{399}{1999}$

Hence $N = \frac{40{,}000}{1+1999\left(\frac{399}{1999}\right)^t}$.

If $t = 2$, then $N = \frac{40{,}000}{1+1999\left(\frac{399}{1999}\right)^2} \approx 500$.

**4.** $N = \frac{M}{1+be^{-ct}}$

Since $M = 50{,}000$, and $N = 500$ when $t = 0$, we have

$500 = \frac{50{,}000}{1+b}$

$1+b = \frac{50{,}000}{500} = 100$

$b = 99$

Hence $N = \frac{50{,}000}{1+99e^{-ct}}$. If $t = 1$, then $N = 1500$. Thus

$1500 = \frac{50{,}000}{1+99e^{-c}}$

$1+99e^{-c} = \frac{50{,}000}{1500} = \frac{100}{3}$

$99e^{-c} = \frac{97}{3}$

$e^{-c} = \frac{97}{297}$

Hence $N = \frac{50{,}000}{1+99\left(\frac{97}{297}\right)^t}$.

**5.** $N = \frac{M}{1+be^{-ct}}$

$M = 100{,}000$, and since $N = 500$ when $t = 0$, we have

$500 = \frac{100{,}000}{1+b}$

$1+b = \frac{100{,}000}{500} = 200$

$b = 199$

Hence $N = \frac{100{,}000}{1+199e^{-ct}}$. If $t = 1$, then $N = 1000$. Thus

$$1000 = \frac{100,000}{1+199e^{-c}}$$

$$1+199e^{-c} = \frac{100,000}{1000} = 100$$

$$199e^{-c} = 99$$

$$e^{-c} = \frac{99}{199}$$

Hence $N = \frac{100,000}{1+199\left(\frac{99}{199}\right)^t}$. If $t = 2$, then

$$N = \frac{100,000}{1+199\left(\frac{99}{199}\right)^2} \approx 1990.$$

**6. a.**

$$\frac{dN}{dt} = N(1-N)$$

$$\frac{dN}{N(1-N)} = dt$$

$$\int \frac{dN}{N(1-N)} = \int dt$$

Using Formula 5 in the Table of Integrals, for $\int \frac{dN}{N(1-N)}$, we get $\ln\left|\frac{N}{1-N}\right| = t + C.$

Since $N(0) = \frac{1}{2}$, $\ln\left|\frac{\frac{1}{2}}{1-\frac{1}{2}}\right| = \ln 1 = 0 = C.$

Also, $N > 0$, and since $M = 1$, $N < 1$. Thus

$$\ln\left(\frac{N}{1-N}\right) = t.$$

$$\frac{N}{1-N} = e^t$$

$$N = (1-N)e^t$$

$$N(e^t + 1) = e^t$$

$$N = \frac{e^t}{e^t+1} = \frac{1}{1+e^{-t}}$$

**b.** $\frac{dN}{dt} = N(1-N) = N - N^2$

$$\frac{d^2N}{dt^2} = 1 - 2N$$

$\frac{d^2N}{dt^2} = 0$ when $N = \frac{1}{2}.$

$1 - 2N > 0$ when $N < \frac{1}{2}$ and $1 - 2N < 0$ when $N > \frac{1}{2}$, so there is an inflection point when $N = \frac{1}{2}.$

$$\frac{1}{2} = \frac{1}{1+e^{-t}}$$

$$1+e^{-t} = 2$$

$$e^{-t} = 1$$

$$t = 0$$

Thus the point $\left(0, \frac{1}{2}\right)$ is an inflection point on the graph.

**c.**

$$f(t) = \frac{1}{1+e^{-t}} - \frac{1}{2} = \frac{2-(1+e^{-t})}{2(1+e^{-t})} = \frac{1-e^{-t}}{2(1+e^{-t})} = \frac{e^t-1}{2(e^t+1)}$$

Replace $t$ by $-t$ then multiply numerator and denominator by $e^t$.

$$\frac{e^{-t}-1}{2(e^{-t}+1)} = \frac{1-e^t}{2(1+e^t)} = -\frac{e^t-1}{2(e^t+1)} = -f(t)$$

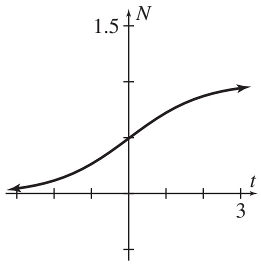
Thus, $f(t)$ is symmetric about the origin.

**d.** The graph of $N(t)$ is the graph of $f(t)$ shifted $\frac{1}{2}$ unit upward. Thus, since $f(t)$ is symmetric about (0, 0), $N(t)$ is symmetric about $\left(0, \frac{1}{2}\right)$.

$$N(t) = f(t) + \frac{1}{2}$$

$$N(-t) = f(-t) + \frac{1}{2} = -f(t) + \frac{1}{2} = -\left[f(t) + \frac{1}{2}\right] + 1 = 1 - N(t)$$

**e.**




**7. a.** $N = \dfrac{375}{1+e^{5.2-2.3t}} = \dfrac{375}{1+e^{5.2}e^{-2.3t}}$

$\approx \dfrac{375}{1+181.27e^{-2.3t}}$

**b.** $\lim\limits_{t\to\infty} N = \dfrac{375}{1+181.27(0)} = 375$

**8. a.** $N = \dfrac{0.2524}{e^{-2.128x}+0.005125}$

$= \dfrac{\frac{0.2524}{0.005125}}{\frac{e^{-2.128x}+0.005125}{0.005125}}$

$\approx \dfrac{49.25}{\frac{e^{-2.128x}}{0.005125}+1} \approx \dfrac{49.25}{1+195.1e^{-2.128x}}$

**b.** If $x = 0$, then $N \approx \dfrac{49.25}{1+195.1} \approx 0.2511\ \text{cm}^2$.

**9.** $\dfrac{dT}{dt} = k(T-a)$ where $a = -5$.

$\dfrac{dT}{T+5} = k\,dt$

$\int \dfrac{dT}{T+5} = \int k\,dt$

Thus $\ln(T + 5) = kt + C$. At $t = 0$, we have $T = 27$, so $\ln(27 + 5) = 0 + C$, $C = \ln 32$, and $\ln(T + 5) = kt + \ln 32$.

$\ln(T + 5) - \ln 32 = kt$

Hence $\ln\left(\dfrac{T+5}{32}\right) = kt.$

If $t = 1$, then $T = 19$. Thus $\ln\left(\dfrac{19+5}{32}\right) = k\cdot 1$, so

$k = \ln\dfrac{24}{32} = \ln\dfrac{3}{4}$. Hence $\ln\left(\dfrac{T+5}{32}\right) = \left(\ln\dfrac{3}{4}\right)t.$

If $T = 37$, then $\ln\left(\dfrac{42}{32}\right) = \left(\ln\dfrac{3}{4}\right)t$

$t = \dfrac{\ln\frac{42}{32}}{\ln\frac{3}{4}} \approx -0.945$ hr

which corresponds to 57 minutes. Time of murder: 3:17 A.M. − 57 min = 2:20 A.M.

**10.** $\dfrac{dp}{dt} = kp(I-p)$

This is logistic growth, so the maximum rate of formation (growth) occurs when $p = \dfrac{I}{2}$, which is when there are equal amounts of both enzymes.

**11.** $\dfrac{dx}{dt} = k(200{,}000 - x)$

$\int \dfrac{dx}{200{,}000-x} = \int k\ dt$

$-\ln(200{,}000 - x) = kt + C$

$\ln(200{,}000 - x) = -kt - C$

$200{,}000 - x = e^{-kt-C} = e^{-C}e^{-kt} = Ae^{-kt}$, where $A = e^{-C}$. Thus $x = 200{,}000 - Ae^{-kt}$. If $t = 0$, then $x = 50{,}000$, so

$50{,}000 = 200{,}000 - A \Rightarrow A = 150{,}000$. Thus $x = 200{,}000 - 150{,}000e^{-kt}$. If $t = 1$, then $x = 100{,}000$, so

$100{,}000 = 200{,}000 - 150{,}000e^{-k}$

$150{,}000e^{-k} = 100{,}000$

$e^{-k} = \dfrac{100{,}000}{150{,}000} = \dfrac{2}{3}$

Thus $x = 200{,}000 - 150{,}000\left(\dfrac{2}{3}\right)^t$. If $t = 3$, then

$x = 200{,}000 - 150{,}000\left(\dfrac{8}{27}\right) \approx \$155{,}555.56.$

**12.** $\dfrac{dN}{dt} = kN^2$

$\int \dfrac{dN}{N^2} = \int k\ dt$

$-\dfrac{1}{N} = kt + C$

If $t = 0$, then $N = N_0$. Thus $-\dfrac{1}{N_0} = C$, so

# Chapter 17

**Problems 17.1**

**1.** $f(x, y) = 2x^2 + 3xy + 4y^2 + 5x + 6y - 7$

$f_x(x, y) = 2(2x) + 3(1)y + 0 + 5(1) + 0 - 0$
$= 4x + 3y + 5$

$f_y(x, y) = 0 + 3x(1) + 4(2y) + 0 + 6(1) - 0$
$= 3x + 8y + 6$

**2.** $f(x, y) = 2x^2 + 3xy$

$f_x(x, y) = 2(2x) + 3(1)y = 4x + 3y$

$f_y(x, y) = 0 + 3x(1) = 3x$

**3.** $f(x, y) = 2y + 1$

$f_x(x, y) = 0 + 0 = 0$

$f_y(x, y) = 2(1) + 0 = 2$

**4.** $f(x, y) = \ln 2$

$f_x(x, y) = 0$

$f_y(x, y) = 0$

**5.** $g(x, y) = 3x^4 y + 2xy^2 - 5xy + 8x - 9y$

$g_x(x, y) = 3(4)x^3 y + 2(1)y^2 - 5(1)y + 8(1)$
$= 12x^3 y + 2y^2 - 5y + 8$

$g_y(x, y) = 3x^4(1) + 2x(2)y - 5x(1) - 9(1)$
$= 3x^4 + 4xy - 5x - 9$

**6.** $g(x, y) = (x^2 + 1)^2 + (y^3 - 3)^3 + 5xy^3 - 2x^2 y^2$

$g_x(x, y) = 2(x^2 + 1)(2x) + 0 + 5(1)y^3 - 2(2x)y^2$
$= 4x(x^2 + 1) + 5y^3 - 4xy^2$

$g_y(x, y)$
$= 0 + 3(y^3 - 3)^2(3y^2) + 5x(3y^2) - 2x^2(2y)$
$= 9y^2(y^3 - 3)^2 + 15xy^2 - 4x^2 y$

**7.** $g(p, q) = \sqrt{pq} = (pq)^{\frac{1}{2}}$

$$g_p(p, q) = \frac{1}{2}(pq)^{-\frac{1}{2}} \cdot q = \frac{q}{2\sqrt{pq}}$$

$$g_q(p, q) = \frac{1}{2}(pq)^{-\frac{1}{2}} \cdot p = \frac{p}{2\sqrt{pq}}$$

**8.** $g(w, z) = \sqrt[3]{w^2 + z^2} = \left(w^2 + z^2\right)^{\frac{1}{3}}$

$$g_w(w, z) = \frac{1}{3}\left(w^2 + z^2\right)^{-\frac{2}{3}}(2w) = \frac{2w}{3\left(w^2 + z^2\right)^{\frac{2}{3}}}$$

$$g_z(w, z) = \frac{1}{3}\left(w^2 + z^2\right)^{-\frac{2}{3}}(2z) = \frac{2z}{3\left(w^2 + z^2\right)^{\frac{2}{3}}}$$

**9.** $h(s, t) = \dfrac{s^2 + 4}{t - 3}$

$$h_s(s, t) = \frac{1}{t - 3}(2s) = \frac{2s}{t - 3}$$

Rewriting $h(s, t)$ as $\left(s^2 + 4\right)(t - 3)^{-1}$, we have

$$h_t(s, t) = \left(s^2 + 4\right)\left[(-1)(t - 3)^{-2}(1)\right] = -\frac{s^2 + 4}{(t - 3)^2}$$

**10.** $h(u, v) = \dfrac{8uv^2}{u^2 + v^2}$

$$h_u(u, v) = 8v^2 \frac{\left(u^2 + v^2\right)(1) - u(2u)}{\left(u^2 + v^2\right)^2} = \frac{8v^2\left(v^2 - u^2\right)}{\left(u^2 + v^2\right)^2}$$

$$h_v(u, v) = 8u \frac{\left(u^2 + v^2\right)(2v) - v^2(2v)}{\left(u^2 + v^2\right)^2} = \frac{16u^3 v}{\left(u^2 + v^2\right)^2}$$

**11.** $u(q_1, q_2) = \ln\sqrt{q_1 + 2} + \ln\sqrt[3]{q_2 + 5}$
$= \dfrac{1}{2}\ln(q_1 + 2) + \dfrac{1}{3}\ln(q_2 + 5)$

$$u_{q_1}(q_1, q_2) = \frac{1}{2} \cdot \frac{1}{q_1 + 2} + 0 = \frac{1}{2(q_1 + 2)}$$

$$u_{q_2}(q_1, q_2) = 0 + \frac{1}{3} \cdot \frac{1}{q_2 + 5} = \frac{1}{3(q_2 + 5)}$$

**12.** $Q(l, k) = 2l^{0.38}k^{1.79} - 3l^{1.03} + 2k^{0.13}$

$Q_l(l, k) = 2(0.38)l^{0.38-1}k^{1.79} - 3(1.03)l^{1.03-1} + 0 = 0.76l^{-0.62}k^{1.79} - 3.09l^{0.03}$

$Q_k(l, k) = 2l^{0.38}(1.79)k^{1.79-1} - 0 + 2(0.13)k^{0.13-1} = 3.58l^{0.38}k^{0.79} + 0.26k^{-0.87}$

**13.** $h(x, y) = \dfrac{x^2 + 3xy + y^2}{\sqrt{x^2 + y^2}}$

$$h_x(x, y) = \frac{\left(x^2 + y^2\right)^{\frac{1}{2}}[2x + 3y] - \left(x^2 + 3xy + y^2\right)\left[\frac{1}{2}\left(x^2 + y^2\right)^{-\frac{1}{2}}(2x)\right]}{\left[\left(x^2 + y^2\right)^{\frac{1}{2}}\right]^2}$$

$$= \frac{\left(x^2 + y^2\right)^{-\frac{1}{2}}\left[\left(x^2 + y^2\right)(2x + 3y) - \left(x^2 + 3xy + y^2\right)x\right]}{x^2 + y^2}$$

$$= \frac{2x^3 + 3x^2y + 2xy^2 + 3y^3 - x^3 - 3x^2y - xy^2}{\left(x^2 + y^2\right)^{\frac{3}{2}}} = \frac{x^3 + xy^2 + 3y^3}{\left(x^2 + y^2\right)^{\frac{3}{2}}}$$

$$h_y(x, y) = \frac{\left(x^2 + y^2\right)^{\frac{1}{2}}[3x + 2y] - \left(x^2 + 3xy + y^2\right)\left[\frac{1}{2}\left(x^2 + y^2\right)^{-\frac{1}{2}}(2y)\right]}{\left[\left(x^2 + y^2\right)^{\frac{1}{2}}\right]^2}$$

$$= \frac{\left(x^2 + y^2\right)^{-\frac{1}{2}}\left[\left(x^2 + y^2\right)(3x + 2y) - \left(x^2 + 3xy + y^2\right)y\right]}{x^2 + y^2}$$

$$= \frac{3x^3 + 2x^2y + 3xy^2 + 2y^3 - x^2y - 3xy^2 - y^3}{\left(x^2 + y^2\right)^{\frac{3}{2}}} = \frac{3x^3 + x^2y + y^3}{\left(x^2 + y^2\right)^{\frac{3}{2}}}$$

**14.** $h(x, y) = \dfrac{\sqrt{x+9}}{x^2y + y^2x}$

$$h_x(x, y) = \frac{\left(x^2y + y^2x\right)\frac{1}{2}(x+9)^{-\frac{1}{2}} - (x+9)^{\frac{1}{2}}\left(2xy + y^2\right)}{\left(x^2y + y^2x\right)^2}$$

$$= \frac{\frac{1}{2}(x+9)^{-\frac{1}{2}}\left[x^2y + y^2x - 2(x+9)\left(2xy + y^2\right)\right]}{\left(x^2y + y^2x\right)^2}$$

$$= \frac{y\left[x^2 + xy - 2(x+9)(2x + y)\right]}{2(x+9)^{\frac{1}{2}}[xy(x+y)]^2}$$

$$= \frac{y\left[x^2 + xy - 4x^2 - 36x - 2xy - 18y\right]}{2(x+9)^{\frac{1}{2}}x^2y^2(x+y)^2} = \frac{-\left(3x^2 + xy + 36x + 18y\right)}{2x^2y\sqrt{x+9}(x+y)^2}$$

Since $h(x, y) = \sqrt{x+9}\left(x^2y + y^2x\right)^{-1}$, then

$$h_y(x, y) = \sqrt{x+9}(-1)\left(x^2y+y^2x\right)^{-2}\left(x^2+2xy\right)$$

$$= \frac{-\sqrt{x+9}\left(x^2+2xy\right)}{\left(x^2y+y^2x\right)^2} = \frac{-x\sqrt{x+9}(x+2y)}{x^2y^2(x+y)^2} = \frac{-\sqrt{x+9}(x+2y)}{xy^2(x+y)^2}$$

**15.** $z = e^{5xy}$

$$\frac{\partial z}{\partial x} = e^{5xy}(5y) = 5ye^{5xy};\ \frac{\partial z}{\partial y} = e^{5xy}(5x) = 5xe^{5xy}$$

**16.** $z = (x^3 + y^3)e^{xy+3x+3y}$

$$\frac{\partial z}{\partial x} = (x^3+y^3)[e^{xy+3x+3y}(y+3)] + e^{xy+3x+3y}[3x^2]$$
$$= [3x^2 + (x^3+y^3)(y+3)]e^{xy+3x+3y}$$

$$\frac{\partial z}{\partial y} = (x^3+y^3)[e^{xy+3x+3y}(x+3)] + e^{xy+3x+3y}[3y^2]$$
$$= [3y^2 + (x^3+y^3)(x+3)]e^{xy+3x+3y}$$

**17.** $z = 5x\ln\left(x^2+y\right)$

$$\frac{\partial z}{\partial x} = 5\left\{x\left[\frac{1}{x^2+y}(2x)\right] + \ln\left(x^2+y\right)[1]\right\} = 5\left[\frac{2x^2}{x^2+y} + \ln\left(x^2+y\right)\right]$$

$$\frac{\partial z}{\partial y} = 5x\left(\frac{1}{x^2+y}[1]\right) = \frac{5x}{x^2+y}$$

**18.** $z = \ln(5x^3y^2 + 2y^4)^4 = 4\ln(5x^3y^2+2y^4)$

$$\frac{\partial z}{\partial x} = 4\cdot\frac{1}{5x^3y^2+2y^4}[5(3x^2)y^2+0] = \frac{60x^2y^2}{5x^3y^2+2y^4} = \frac{60x^2y^2}{y^2(5x^3+2y^2)} = \frac{60x^2}{5x^3+2y^2}$$

$$\frac{\partial z}{\partial y} = 4\cdot\frac{1}{5x^3y^2+2y^4}[5x^3(2y)+2(4y^3)] = \frac{4(10x^3y+8y^3)}{5x^3y^2+2y^4} = \frac{8y(5x^3+4y^2)}{y(5x^3y+2y^3)} = \frac{8(5x^3+4y^2)}{5x^3y+2y^3}$$

**19.** $f(r, s) = (r+2s)^{\frac{1}{2}}\left(r^3 - 2rs + s^2\right)$

$$f_r(r, s) = (r+2s)^{\frac{1}{2}}\left[3r^2-2s\right] + \left(r^3-2rs+s^2\right)\left[\frac{1}{2}(r+2s)^{-\frac{1}{2}}(1)\right]$$
$$= \sqrt{r+2s}\left(3r^2-2s\right) + \frac{r^3-2rs+s^2}{2\sqrt{r+2s}}$$

$$f_s(r, s) = (r+2s)^{\frac{1}{2}}[-2r+2s] + \left(r^3-2rs+s^2\right)\left[\frac{1}{2}(r+2s)^{-\frac{1}{2}}(2)\right]$$
$$= 2(s-r)\sqrt{r+2s} + \frac{r^3-2rs+s^2}{\sqrt{r+2s}}$$

**20.** $f(r,\ s) = (rs)^{\frac{1}{2}} e^{2+r}$

$f_r(r,\ s) = (rs)^{\frac{1}{2}}\left[e^{2+r}(1)\right] + e^{2+r}\left[\frac{1}{2}(rs)^{-\frac{1}{2}}(s)\right] = \left[\sqrt{rs} + \frac{s}{2\sqrt{rs}}\right]e^{2+r}$

$f_s(r,\ s) = e^{2+r}\left[\frac{1}{2}(rs)^{-\frac{1}{2}}(r)\right] = \frac{re^{2+r}}{2\sqrt{rs}}$

**21.** $f(r,\ s) = e^{3-r}\ln(7-s)$

$f_r(r,\ s) = \ln(7-s)\left[e^{3-r}(-1)\right] = -e^{3-r}\ln(7-s)$

$f_s(r,\ s) = e^{3-r}\left[\frac{1}{7-s}(-1)\right] = \frac{e^{3-r}}{s-7}$

**22.** $f(r,\ s) = \left(5r^2 + 3s^3\right)(2r-5s)$

$f_r(r,\ s) = \left(5r^2 + 3s^3\right)[2] + (2r-5s)[10r] = 2\left(5r^2 + 3s^3\right) + 10r(2r-5s)$

$f_s(r,\ s) = \left(5r^2 + 3s^3\right)[-5] + (2r-5s)\left[9s^2\right] = -5\left(5r^2 + 3s^3\right) + 9s^2(2r-5s)$

**23.** $g(x,\ y,\ z) = 2x^3y^2 + 2xy^3z + 4z^2$

$g_x(x,\ y,\ z) = 2y^2(3x^2) + 2y^3z(1) + 0 = 6x^2y^2 + 2y^3z$

$g_y(x,\ y,\ z) = 2x^3(2y) + 2xz(3y^2) + 0 = 4x^3y + 6xy^2z$

$g_z(x,\ y,\ z) = 0 + 2xy^3(1) + 4(2z) = 2xy^3 + 8z$

**24.** $g(x,\ y,\ z) = 2xy^2z^6 - 4x^2y^3z^2 + 3xyz$

$g_x(x,\ y,\ z) = 2(1)y^2z^6 - 4(2x)y^3z^2 + 3(1)yz$

$= 2y^2z^6 - 8xy^3z^2 + 3yz$

$g_y(x,\ y,\ z) = 2x(2y)z^6 - 4x^2(3y^2)z^2 + 3x(1)z$

$= 4xyz^6 - 12x^2y^2z^2 + 3xz$

$g_z(x,\ y,\ z) = 2xy^2(6z^5) - 4x^2y^3(2z) + 3xy(1)$

$= 12xy^2z^5 - 8x^2y^3z + 3xy$

**25.** $g(r,\ s,\ t) = e^{s+t}(r^2 + 7s^3)$

$g_r(r,\ s,\ t) = e^{s+t}[2r+0] = 2re^{s+t}$

$g_s(r,\ s,\ t) = e^{s+t}\left[0 + 21s^2\right] + \left(r^2 + 7s^3\right)\left[e^{s+t}(1)\right]$

$= \left(7s^3 + 21s^2 + r^2\right)e^{s+t}$

$g_t(r,\ s,\ t) = \left(r^2 + 7s^3\right)\left[e^{s+t}(1)\right] = e^{s+t}\left(r^2 + 7s^3\right)$

**26.** $g(r, s, t, u) = rs\ln(t)e^u$

$g_r(r, s, t, u) = s\ln(t)e^u$

$g_s(r, s, t, u) = r\ln(t)e^u$

$g_t(r, s, t, u) = rs\left(\frac{1}{t}\right)e^u = \frac{rse^u}{t}$

$g_u(r, s, t, u) = rs\ln(t)e^u$

**27.** $f(x, y) = x^3y + 7x^2y^2$

$f_x(x, y) = 3x^2y + 14xy^2$

$f_x(1, -2) = 3(1)^2(-2) + 14(1)(-2)^2 = 50$

**28.** $z = \sqrt{2x^3 + 5xy + 2y^2}$

$\frac{\partial z}{\partial x} = \frac{6x^2 + 5y}{2\sqrt{2x^3 + 5xy + 2y^2}}$

$\left.\frac{\partial z}{\partial x}\right|_{(0, 1)} = \frac{5}{2\sqrt{2}}$

**29.** $g(x, y, z) = e^x\sqrt{y + 2z}$

$g_z(x, y, z) = e^x\left[\frac{1}{2}(y + 2z)^{-\frac{1}{2}}(2)\right] = \frac{e^x}{\sqrt{y + 2z}}$

$g_z(0, 6, 4) = \frac{1}{\sqrt{6 + 8}} = \frac{1}{\sqrt{14}}$

**30.** $g(x, y, z) = \frac{3x^2y^2 + 2xy + x - y}{xy - yz + xz}$

$g_y(x, y, z) = \frac{(xy - yz + xz)(6x^2y + 2x - 1) - (3x^2y^2 + 2xy + x - y)(x - z)}{(xy - yz + xz)^2}$

$g_y(1, 1, 5) = \frac{(1 - 5 + 5)(6 + 2 - 1) - (3 + 2 + 1 - 1)(1 - 5)}{(1 - 5 + 5)^2} = 27$

**31.** $h(r, s, t, u) = (rst^2u)\ln(1 + rstu)$

$h_t(r, s, t, u) = [rs(2t)u]\ln(1 + rstu) + (rst^2u) \cdot \frac{rsu}{1 + rstu}$

$h_t(1, 1, 0, 1) = 0$

**32.** $h(r, s, t, u) = \frac{7r + 3s^2u^2}{s}$

$h_t(r, s, t, u) = 0$

$h_t(4, 3, 2, 1) = 0$

**33.** $f(r, s, t) = rst\left(r^2 + s^3 + t^4\right) = r^3st + rs^4t + rst^5$

$f_s(r, s, t) = r^3(1)t + r\left(4s^3\right)t + r(1)t^5 = r^3t + 4rs^3t + rt^5$

$f_s(1, -1, 2) = 2 + (-8) + 32 = 26$

**34.** $z = \frac{x^2 + y^2}{e^{x^2+y^2}} = (x^2 + y^2)e^{-(x^2+y^2)}$

$\frac{\partial z}{\partial x} = (2x)e^{-(x^2+y^2)} + (x^2 + y^2)e^{-(x^2+y^2)}(-2x)$

$= 2xe^{-(x^2+y^2)}[1 - (x^2 + y^2)]$

$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=0\\y=0}} = 2(0)e^0[1 - (0)] = 0$

By symmetry, $\frac{\partial z}{\partial y} = 2ye^{-(x^2+y^2)}[1 - (x^2 + y^2)]$.

$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=1\\y=1}} = 2(1)e^{-2}[1 - (2)] = -\frac{2}{e^2}$

**35.** $z = xe^{x-y} + ye^{y-x}$

$\frac{\partial z}{\partial x} = \left[xe^{x-y} + e^{x-y}\right] + \left[ye^{y-x}(-1)\right]$

$\frac{\partial z}{\partial y} = \left[xe^{x-y}(-1)\right] + \left[ye^{y-x} + e^{y-x}\right]$

Thus $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = e^{x-y} + e^{y-x}$, as was to be shown.

**36.** $u = f(t, r, z) = \frac{(1+r)^{1-z}\ln(1+r)}{(1+r)^{1-z} - t}$

$\frac{\partial u}{\partial z} = \ln(1+r)\frac{\partial}{\partial z}\left[\frac{(1+r)^{1-z}}{(1+r)^{1-z} - t}\right]$

$= \ln(1+r)\frac{\left[(1+r)^{1-z} - t\right]\frac{\partial}{\partial z}\left[(1+r)^{1-z}\right] - (1+r)^{1-z}\left\{\frac{\partial}{\partial z}\left[(1+r)^{1-z}\right] - 0\right\}}{\left[(1+r)^{1-z} - t\right]^2}$

$= \ln(1+r)\frac{-t\frac{\partial}{\partial z}\left[(1+r)^{1-z}\right]}{\left[(1+r)^{1-z} - t\right]^2}$

$= \ln(1+r)\frac{-t\left\{(1+r)^{1-z}\ln(1+r)[-1]\right\}}{\left[(1+r)^{1-z} - t\right]^2}$

$= \frac{t(1+r)^{1-z}\ln^2(1+r)}{\left[(1+r)^{1-z} - t\right]^2}$, as was to be shown.

**37.** $F(b, C, T, i) = \frac{bT}{C} + \frac{iC}{2}$

$\frac{\partial F}{\partial C} = \frac{\partial}{\partial C}\left[\frac{bT}{C}\right] + \frac{\partial}{\partial C}\left[\frac{iC}{2}\right] = -\frac{bT}{C^2} + \frac{i}{2}$

**38.** From $\eta = \frac{\frac{r}{D}}{\frac{\partial r}{\partial D}}$, we have $\frac{\partial r}{\partial D} = \frac{r}{D\eta}$. Substituting into Equation (3) gives

$r_L = r + D \cdot \frac{r}{D\eta} + \frac{dC}{dD}$

$r_L = r + \frac{r}{\eta} + \frac{dC}{dD}$

$r_L = r\left[1 + \frac{1}{\eta}\right] + \frac{dC}{dD}$

$r_L = r\left[\frac{\eta + 1}{\eta}\right] + \frac{dC}{dD}$

$r_L = r\left[\frac{1 + \eta}{\eta}\right] + \frac{dC}{dD}$

which is Equation (4).

**39.** $R = f(r, a, n) = \frac{r}{1 + a\left(\frac{n-1}{2}\right)} = r\left[1 + a\left(\frac{n-1}{2}\right)\right]^{-1}$

$\frac{\partial R}{\partial n} = r(-1)\left[1 + a\left(\frac{n-1}{2}\right)\right]^{-2} \cdot \frac{a}{2}$

$= -\frac{ra}{2\left[1 + a\left(\frac{n-1}{2}\right)\right]^2}$

**Problems 17.2**

**1.** $c = 7x + 0.3y^2 + 2y + 900$

$\frac{\partial c}{\partial y} = 0.6y + 2$

When $x = 20$ and $y = 30$, then

$\frac{\partial c}{\partial y} = 0.6(30) + 2 = 20$.

**2.** $c = 2x\sqrt{x + y} + 6000$

$\frac{\partial c}{\partial x} = \frac{x}{\sqrt{x + y}} + 2\sqrt{x + y}$

When $x = 70$ and $y = 74$, then

$\frac{\partial c}{\partial x} = \frac{70}{\sqrt{70 + 74}} + 2\sqrt{70 + 74} = \frac{179}{6}$.

**3.** $c = 0.03(x + y)^3 - 0.6(x + y)^2 + 9.5(x + y) + 7700$

$\frac{\partial c}{\partial x} = 0.09(x + y)^2 - 1.2(x + y) + 9.5$

When $x = 50$ and $y = 80$, then

$\frac{\partial c}{\partial x} = 0.09(130)^2 - 1.2(130) + 9.5 = 1374.5$.

**4.** $P = 15lk - 3l^2 + 5k^2 + 500$

$\frac{\partial P}{\partial k} = 15l + 10k$

$\frac{\partial P}{\partial l} = 15k - 6l$

**5.** $P = 2.314l^{0.357}k^{0.643}$

$\frac{\partial P}{\partial l} = 2.314(0.357)l^{-0.643}k^{0.643}$

$= 0.826098\left(\frac{k}{l}\right)^{0.643}$

$\frac{\partial P}{\partial k} = 2.314(0.643)l^{0.357}k^{-0.357}$

$= 1.487902\left(\frac{l}{k}\right)^{0.357}$

**6.** $P = Al^{\alpha}k^{\beta}$

**a.** $\frac{\partial P}{\partial l} = A\alpha l^{\alpha-1}k^{\beta} = \left(\frac{\alpha}{l}\right)Al^{\alpha}k^{\beta} = \frac{\alpha P}{l}$

**b.** $\frac{\partial P}{\partial k} = A\beta l^{\alpha}k^{\beta-1} = \left(\frac{\beta}{k}\right)Al^{\alpha}k^{\beta} = \frac{\beta P}{k}$

**c.** From parts (a) and (b),

$l\frac{\partial P}{\partial l} + k\frac{\partial P}{\partial k} = l\left(\frac{\alpha P}{l}\right) + k\left(\frac{\beta P}{k}\right)$

$= \alpha P + \beta P = P(\alpha + \beta) = P(1) = P$

**7.** $\frac{\partial q_A}{\partial p_A} = -40$, $\frac{\partial q_A}{\partial p_B} = 3$, $\frac{\partial q_B}{\partial p_A} = 5$, $\frac{\partial q_B}{\partial p_B} = -20$

Since $\frac{\partial q_A}{\partial p_B} > 0$ and $\frac{\partial q_B}{\partial p_A} > 0$ the products are competitive.