2. Let $u = 2x$, $a^2 = 25$. Then $du = 2dx$.

$$\int \frac{dx}{\left(25-4x^2\right)^{\frac{3}{2}}} = \frac{1}{2}\int \frac{(2dx)}{\left[25-(2x)^2\right]^{\frac{3}{2}}}$$

$$= \frac{1}{2}\left[\frac{(2x)}{25\sqrt{25-(2x)^2}}\right]+C$$

$$= \frac{x}{25\sqrt{25-4x^2}}+C$$

3. Let $u = 4x$, $a^2 = 3$. Then $du = 4\ dx$.

$$\int \frac{dx}{x^2\sqrt{16x^2+3}} = 4\int \frac{(4\ dx)}{(4x)^2\sqrt{(4x)^2+3}}$$

$$= 4\left[-\frac{\sqrt{(4x)^2+3}}{3(4x)}\right]+C$$

$$= -\frac{\sqrt{16x^2+3}}{3x}+C$$

4. Let $u = x^2$, $a^2 = 9$. Then $du = 2x\ dx$.

$$\int \frac{3\,dx}{x^3\sqrt{x^4-9}} = \frac{3}{2}\int \frac{(2x\,dx)}{(x^2)^2\sqrt{(x^2)^2-9}}$$

$$= \frac{3}{2}\left[-\frac{-\sqrt{(x^2)^2-9}}{9x^2}+C\right]$$

$$= \frac{\sqrt{x^4-9}}{6x^2}+C$$

5. Formula 5 with $u = x$, $a = 6$, $b = 7$. Then $du = dx$.

$$\int \frac{dx}{x(6+7x)} = \frac{1}{6}\ln\left|\frac{x}{6+7x}\right|+C$$

6. Formula 8 with $u = x$, $a = 2$, $b = 3$. Then $du = dx$.

$$\int \frac{5x^2\,dx}{(2+3x)^2}$$

$$= 5\left[\int \frac{x^2\,dx}{(2+3x)^2}\right]$$

$$= 5\left[\frac{x}{9}-\frac{4}{27(2+3x)}-\frac{4}{27}\ln|2+3x|\right]+C$$

7. Formula 28 with $u = x$, $a = 3$. Then $du = dx$.

$$\int \frac{dx}{x\sqrt{x^2+9}} = \frac{1}{3}\ln\left|\frac{\sqrt{x^2+9}-3}{x}\right|+C$$

8. Formula 32 with $u = x$, $a^2 = 7$. Then $du = dx$.

$$\int \frac{dx}{(x^2+7)^{3/2}} = \frac{x}{7\sqrt{x^2+7}}+C$$

9. Formula 12 with $u = x$, $a = 2$, $b = 3$, $c = 4$, $k = 5$. Then $du = dx$.

$$\int \frac{x\,dx}{(2+3x)(4+5x)}$$

$$= \frac{1}{2}\left[\frac{4}{5}\ln|4+5x|-\frac{2}{3}\ln|2+3x|\right]+C$$

10. Formula 37 with $u = 5x$, $a = 2$. Then $du = 5\ dx$.

$$\int 2^{5x}\,dx = \frac{1}{5}\int 2^{5x}(5\ dx) = \frac{1}{5}\cdot\frac{2^{5x}}{\ln 2}+C$$

11. Formula 45 with $u = x$, $a = 1$, $b = 2$, $c = 3$. Then $du = dx$. $\int \frac{dx}{1+2e^{3x}} = \frac{1}{3}\left(3x-\ln\left|1+2e^{3x}\right|\right)+C$

12. Formula 14 with $u = x$, $a = 1$, $b = 1$. Then $du = dx$.

$$\int x^2\sqrt{1+x}\,dx = \frac{2\left(8-12x+15x^2\right)(1+x)^{\frac{3}{2}}}{105}+C$$

13. Formula 9 with $u = x$, $a = 5$, $b = 2$. Then $du = dx$.

$$\int \frac{7\,dx}{x(5+2x)^2} = 7\left[\int \frac{dx}{x(5+2x)^2}\right]$$

$$= 7\left[\frac{1}{5(5+2x)}+\frac{1}{25}\ln\left|\frac{x}{5+2x}\right|\right]+C$$

14. Formula 20 with $u = \sqrt{11}x$, $a = \sqrt{5}$. Then $du = \sqrt{11}\ dx$.

$$\int \frac{dx}{x\sqrt{5-11x^2}} = \int \frac{\sqrt{11}dx}{\left(\sqrt{11}x\right)\sqrt{\left(\sqrt{5}\right)^2-\left(\sqrt{11}x\right)^2}}$$

$$= -\frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5}+\sqrt{5-11x^2}}{\sqrt{11}x}\right|+C$$

15. Formula 3 with $u = x$, $a = 2$, $b = 1$. Then $du = dx$.

$$\int_0^1 \frac{x\,dx}{2+x} = \left(x-2\ln|2+x|\right)\Big|_0^1 = 1-2\ln 3+2\ln 2$$

$$= 1 - \ln 9 + \ln 4 = 1+\ln\left(\frac{4}{9}\right)$$

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16. Formula 4 with $u = x$, $a = 2$, $b = -5$. Then $du = dx$.

$$\int \frac{-3x^2 dx}{2-5x} = -3\int \frac{x^2 dx}{2-5x}$$
$$= -3\left(\frac{x^2}{-10} - \frac{2x}{25} + \frac{4}{-125}\ln|2-5x|\right) + C$$
$$= 3\left(\frac{x^2}{10} + \frac{2x}{25} + \frac{4}{125}\ln|2-5x|\right) + C$$

17. Formula 23 with $u = x$, $a^2 = 3$. Then $du = dx$.

$$\int \sqrt{x^2-3}\,dx = \frac{1}{2}\left(x\sqrt{x^2-3} - 3\ln\left|x + \sqrt{x^2-3}\right|\right) + C$$

18. Formula 11 with $u = x$, $a = 1$, $b = 5$, $c = 3$, $k = 2$. Then $du = dx$.

$$\int \frac{dx}{(1+5x)(2x+3)} = \frac{1}{13}\ln\left|\frac{1+5x}{2x+3}\right| + C$$

19. Formula 38 with $u = x$, $a = 12$. Then $du = dx$.

$$\int_0^{1/12} xe^{12x}dx = \frac{e^{12x}}{144}(12x-1)\Bigg|_0^{1/12} = \frac{1}{144}[e(0) - 1(-1)] = \frac{1}{144}$$

20. Formula 46 with $u = 3x$, $a = 2$, $b = 5$.
Then $du = 3\,dx$.

$$\int \sqrt{\frac{2+3x}{5+3x}}dx = \frac{1}{3}\int \sqrt{\frac{2+3x}{5+3x}}(3\,dx) = \frac{1}{3}\left[\sqrt{(2+3x)(5+3x)} - 3\ln\left(\sqrt{2+3x} + \sqrt{5+3x}\right)\right] + C$$

21. Formula 39 with $u = x$, $n = 3$, $a = 1$. Then $du = dx$.

$$\int x^3 e^x dx = x^3 e^x - 3\int x^2 e^x dx$$

Applying Formula 39 to $\int x^2 e^x dx$ with $u = x$, $n = 2$, and $a = 1$ (so $du = dx$) gives $\int x^2 e^x dx = x^2 e^x - 2\int xe^x dx$.

Applying Formula 38 to $\int xe^x dx$ with $u = x$, $a = 1$ (so $du = dx$) gives $\int xe^x dx = e^x(x-1) + C_1$. Thus

$$\int x^3 e^x dx = x^3 e^x - 3x^2 e^x + 6e^x(x-1) + C.$$

22. Formula 6 with $u = x$, $a = 1$, $b = 1$. Then $du = dx$.

$$\int_1^2 \frac{4\,dx}{x^2(1+x)} = 4\int_1^2 \frac{dx}{x^2(1+x)} = 4\left(-\frac{1}{x} + \ln\left|\frac{1+x}{x}\right|\right)\Bigg|_1^2 = 4\left(-\frac{1}{2} + \ln\frac{3}{2}\right) - 4(-1 + \ln 2) = 2 + 4\ln\frac{3}{4}$$

23. Formula 26 with $u = \sqrt{5}x$, $a^2 = 1$. Then $du = \sqrt{5}\,dx$.

$$\int \frac{\sqrt{5x^2+1}}{2x^2}dx = \frac{5}{2\sqrt{5}}\int \frac{\sqrt{5x^2+1}}{5x^2}\left(\sqrt{5}\,dx\right)$$
$$= \frac{\sqrt{5}}{2}\left(-\frac{\sqrt{5x^2+1}}{\sqrt{5}x} + \ln\left|\sqrt{5}x + \sqrt{5x^2+1}\right|\right) + C$$

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24. Formula 17 with $u = x$, $a = 2$, $b = -1$. Then $du = dx$.

$$\int \frac{dx}{x\sqrt{2-x}} = \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{2-x}-\sqrt{2}}{\sqrt{2-x}+\sqrt{2}}\right| + C$$

25. Formula 7 with $u = x$, $a = 1$, $b = 3$. Then $du = dx$.

$$\int \frac{x\,dx}{(1+3x)^2} = \frac{1}{9}\left(\ln|1+3x| + \frac{1}{1+3x}\right) + C$$

26. Formula 47 with $u = 2x$, $a = 1$, $b = 3$. Then $du = 2\,dx$.

$$\int \frac{2\,dx}{\sqrt{(1+2x)(3+2x)}} = \ln\left|2+2x+\sqrt{(1+2x)(3+2x)}\right| + C$$

27. Formula 34 with $u = \sqrt{5}x$, $a = \sqrt{7}$. Then $du = \sqrt{5}dx$

$$\int \frac{dx}{7-5x^2} = \frac{1}{\sqrt{5}} \int \frac{1}{\left(\sqrt{7}\right)^2 - \left(\sqrt{5}x\right)^2}\left(\sqrt{5}dx\right) = \frac{1}{\sqrt{5}}\left(\frac{1}{2\sqrt{7}} \ln\left|\frac{\sqrt{7}+\sqrt{5}x}{\sqrt{7}-\sqrt{5}x}\right|\right) + C$$

28. Formula 24 with $u = \sqrt{3}x$, $a^2 = 6$. Then $du = \sqrt{3}dx$.

$$\int 7x^2\sqrt{3x^2-6}\,dx = \frac{7}{\left(\sqrt{3}\right)^3} \int \left(\sqrt{3}x\right)^2 \sqrt{\left(\sqrt{3}x\right)^2 - 6}\left(\sqrt{3}\,dx\right)$$

$$= \frac{7}{3\sqrt{3}}\left[\frac{\sqrt{3}x}{8}(6x^2-6)\sqrt{3x^2-6} - \frac{36}{8}\ln\left|\sqrt{3}x+\sqrt{3x^2-6}\right|\right] + C$$

29. Formula 42 with $u = 3x$, $n = 5$. Then $du = 3\,dx$.

$$\int 36x^5 \ln(3x)dx = 36\int x^5 \ln(3x)dx = \frac{36}{3^6}\int (3x)^5 \ln(3x)(3\,dx)$$

$$= \frac{4}{81}\left[\frac{(3x)^6 \ln(3x)}{6} - \frac{(3x)^6}{36}\right] + C = x^6[6\ln(3x)-1] + C$$

30. Formula 10 with $u = x$, $a = 3$, $b = 2$. Then $du = dx$.

$$\int \frac{5\,dx}{x^2(3+2x)^2} = 5\left[\int \frac{dx}{x^2(3+2x)^2}\right]$$

$$= 5\left[-\frac{3+4x}{9x(3+2x)} + \frac{4}{27}\ln\left|\frac{3+2x}{x}\right|\right] + C$$

31. Formula 13 with $u = x$, $a = 1$, $b = 2$. Then $du = dx$.

$$\int 5x\sqrt{1+2x}dx = 5\int x\sqrt{1+2x}\,dx = 5\left[\frac{2(6x-2)(1+2x)^{\frac{3}{2}}}{15 \cdot 4}\right] + C$$

$$= \frac{1}{3}(3x-1)(1+2x)^{3/2} + C$$

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32. Formula 42 with $u = x$, $n = 2$. Then $du = dx$.

$$\int 9x^2 \ln x\, dx = 9\int x^2 \ln x\, dx$$

$$= 9\left(\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right) + C = 3x^3(\ln x) - x^3 + C$$

33. Formula 27 with $u = 2x$, $a^2 = 13$. Then $du = 2\, dx$.

$$\int \frac{dx}{\sqrt{4x^2 - 13}} = \frac{1}{2}\int \frac{1}{\sqrt{(2x)^2 - 13}}(2\, dx)$$

$$= \frac{1}{2}\ln\left|2x + \sqrt{4x^2 - 13}\right| + C$$

34. Formula 44 with $u = 2x$. Then $du = 2\, dx$.

$$\int \frac{dx}{x\ln(2x)} = \int \frac{(2\, dx)}{(2x)\ln(2x)}$$

$$= \ln\left|\ln(2x)\right| + C$$

35. Formula 21 with $u = 3x$, $a^2 = 16$. Then $du = 3\, dx$.

$$\int \frac{2\,dx}{x^2\sqrt{16 - 9x^2}} = 2(3)\int \frac{(3\,dx)}{(3x)^2\sqrt{16 - (3x)^2}}$$

$$= 6\left(-\frac{\sqrt{16 - 9x^2}}{16(3x)}\right) + C$$

$$= -\frac{\sqrt{16 - 9x^2}}{8x} + C$$

36. Formula 22 with $u = x$, $a = \sqrt{3}$. Then $du = dx$.

$$\int \frac{\sqrt{3 - x^2}}{x}\, dx$$

$$= \int \frac{\sqrt{\left(\sqrt{3}\right)^2 - (x)^2}}{x}\, dx$$

$$= \sqrt{3 - x^2} - \sqrt{3}\ln\left|\frac{\sqrt{3} + \sqrt{3 - x^2}}{x}\right| + C$$

37. Formula 45 with $u = \sqrt{x}$, $a = \pi$, $b = 7$, $c = 4$.

Then $du = \frac{1}{2\sqrt{x}}\, dx$

$$\int \frac{dx}{\sqrt{x}\left(\pi + 7e^{4\sqrt{x}}\right)} = 2\int \frac{1}{\pi + 7e^{4\sqrt{x}}}\left(\frac{1}{2\sqrt{x}}\, dx\right)$$

$$= 2\left[\frac{1}{4\pi}\left(4\sqrt{x} - \ln\left|\pi + 7e^{4\sqrt{x}}\right|\right)\right] + C$$

$$= \frac{1}{2\pi}\left(4\sqrt{x} - \ln\left|\pi + 7e^{4\sqrt{x}}\right|\right) + C$$

38. Formula 2 with $u = x^3$, $a = 1$, $b = 2$. Then $du = 3x^2\, dx$.

$$\int_0^1 \frac{3x^2\, dx}{1 + 2x^3} = \frac{1}{2}\ln\left|1 + 2x^3\right|\Big|_0^1$$

$$= \frac{1}{2}\ln|3| - \frac{1}{2}\ln|1| = \ln\sqrt{3}$$

39. Can be put in the form $\int \frac{1}{u}\, du$.

$$\int \frac{x\, dx}{x^2 + 1} = \frac{1}{2}\int \frac{1}{x^2 + 1}(2x\, dx)$$

$$= \frac{1}{2}\ln\left(x^2 + 1\right) + C$$

40. Can be put in the form $\int e^u\, du$.

$$\int 3x\sqrt{x}e^{x^{5/2}}\, dx = 3\cdot\frac{2}{5}\int e^{x^{5/2}}\left[\frac{5}{2}x^{3/2}dx\right]$$

$$= \frac{6}{5}e^{x^{5/2}} + C$$

41. Can be put in the form $\int u^n\, du$.

$$\int \frac{(\ln x)^3}{x}\, dx = \int (\ln x)^3\left[\frac{1}{x}\, dx\right] = \frac{1}{4}(\ln x)^4 + C$$

42. $$\int \frac{5x^3 - \sqrt{x}}{2x}\, dx = \int\left(\frac{5}{2}x^2 - \frac{1}{2}x^{-\frac{1}{2}}\right)dx$$

$$= \frac{5}{6}x^3 - \sqrt{x} + C$$

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43. $\int \frac{1}{x^2-5x+6}dx = \int \frac{1}{(x-3)(x-2)}dx$

Formula 11 with $u = x$, $a = -3$, $b = 1$, $c = -2$, and $k = 1$. Then $du = dx$.

$$\int \frac{1}{x^2-5x+6}dx = \int \frac{1}{(x-3)(x-2)}dx$$
$$= \ln\left|\frac{x-3}{x-2}\right| + C$$

44. Can be put in the form $\int u^n du$.

$$\int \frac{e^{2x}}{\sqrt{e^{2x}+3}}dx = \frac{1}{2}\int \left(e^{2x}+3\right)^{-\frac{1}{2}}(2e^{2x}dx)$$
$$= \sqrt{e^{2x}+3} + C$$

45. Formula 42 with $u = x$ and $n = 3$. Then $du = dx$.

$$\int x^3 \ln x\, dx = \frac{x^4}{4}\left[\ln(x) - \frac{1}{4}\right] + C$$

46. Formula 38 with $u = 3x - 2$, $a = -10$. Then $du = 3\ dx$.

$$\int (9x-6)e^{-30x+20}dx$$
$$= \int 3(3x-2)e^{-10(3x-2)}dx$$
$$= \frac{e^{-30x+20}}{100}[-10(3x-2)-1] + C$$
$$= \frac{1}{100}e^{-30x+20}(-30x+19) + C$$

47. Formula 38 with $u = x^2$ and $a = 3$. Then $du = 2x\ dx$.

$$\int 4x^3 e^{3x^2}dx = 2\int x^2 e^{3x^2}[2x\,dx]$$
$$= 2\left[\frac{e^{3x^2}}{9}(3x^2-1)\right] + C$$
$$= \frac{2}{9}e^{3x^2}(3x^2-1) + C$$

48. Formula 14 with $u = x$, $a = 3$ and $b = 2$. Then $du = dx$.

$$\int_1^2 35x^2\sqrt{3+2x}dx = 35\int_1^2 x^2\sqrt{3+2x}dx$$
$$= 35 \cdot \left.\frac{2\left(72-72x+60x^2\right)(3+2x)^{\frac{3}{2}}}{840}\right|_1^2$$
$$= 98\sqrt{7} - 25\sqrt{5}$$

49. Formula 43 and then Formula 41. For Formula 43, let $u = x$, $n = 0$, and $m = 2$. Then $du = dx$.

$$\int \ln^2 x\ dx = x\ln^2 x - 2\int \ln x\ dx$$

Now we apply Formula 41 to the last integral with $u = x$ (so $du = dx$).

$$\int \ln^2 x\, dx = x(\ln x)^2 - 2x(\ln x) + 2x + C$$

50. Formula 41 with $u = x^2$. Then $du = 2x\ dx$.

$$\int_1^3 3x\ln x^2\ dx$$
$$= \frac{3}{2}\int_1^e \ln(x^2)[2x\,dx] = \left.\frac{3}{2}[x^2\ln(x^2) - x^2]\right|_1^e$$
$$= \frac{3}{2}[(e^2\ln(e^2) - e^2) - (1\cdot\ln 1 - 1)]$$
$$= \frac{3}{2}(e^2+1)$$

51. Formula 15 with $u = x$, $a = 3$, and $b = 1$. Then $du = dx$.

$$\int_{-2}^1 \frac{x\,dx}{\sqrt{3+x}} = \left.\frac{2(x-6)\sqrt{3+x}}{3}\right|_{-2}^1$$
$$= -\frac{10}{3}\sqrt{4} + \frac{16}{3}\sqrt{1}$$
$$= -\frac{4}{3}$$

52. Formula 13 with $u = x$, $a = 2$, and $b = 3$. Then $du = dx$.

$$\int_2^3 x\sqrt{2+3x}\,dx = \left.\frac{2(9x-4)(2+3x)^{3/2}}{135}\right|_2^3$$
$$= \frac{2}{135}[23(11)^{3/2} - 14(8)^{3/2}]$$
$$= \frac{2}{135}\left(253\sqrt{11} - 224\sqrt{2}\right)$$

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53. Can be put in the form $\int u^n du$.

$$\int_0^1 \frac{2x\,dx}{\sqrt{8-x^2}} = -\int_0^1 \left(8-x^2\right)^{-\frac{1}{2}}(-2x\,dx)$$

$$= -\left.\frac{\left(8-x^2\right)^{\frac{1}{2}}}{\frac{1}{2}}\right|_0^1$$

$$= -2\left(8-x^2\right)^{\frac{1}{2}}\Big|_0^1 = -2\left(\sqrt{7}-\sqrt{8}\right)$$

$$= -2\left(\sqrt{7}-2\sqrt{2}\right)$$

$$= 2\left(2\sqrt{2}-\sqrt{7}\right)$$

54. Formula 39 with $u = x$, $n = 2$, $a = 3$. Then $du = dx$.

$$\int x^2 e^{3x}\,dx = \frac{x^2 e^{3x}}{3} - \frac{2}{3}\int xe^{3x}\,dx$$

For $\int xe^{3x}\,dx$, use Formula 38 with $u = x$ and $a = 3$. Then $du = dx$.

$$\int x^2 e^{3x}\,dx = \frac{x^2 e^{3x}}{3} - \frac{2}{3}\left[\frac{e^{3x}}{9}(3x-1)\right]$$

$$= \frac{e^{3x}}{27}[9x^2-6x+2]$$

$$\int_0^{\ln 2} x^2 e^{3x}\,dx = \left.\left(\frac{e^{3x}}{27}[9x^2-6x+2]\right)\right|_0^{\ln 2}$$

$$= \frac{8}{27}[9(\ln 2)^2 - 6\ln 2 + 2] - \frac{1}{27}[2]$$

$$= \frac{2}{27}[36(\ln 2)^2 - 24\ln 2 + 7]$$

55. Integration by parts or Formula 42. For Formula 42, let $u = 2x$, $n = 1$. Then $du = 2\,dx$.

$$\int_1^2 x\ln(2x)dx = \frac{1}{4}\int_1^2 (2x)\ln(2x)[2\,dx]$$

$$= \frac{1}{4}\left[\frac{(2x)^2\ln(2x)}{2} - \frac{(2x)^2}{4}\right]\Bigg|_1^2$$

$$= 2\ln(4) - 1 - \frac{1}{2}\ln(2) + \frac{1}{4}$$

$$= 2\ln\left(2^2\right) - \frac{1}{2}\ln(2) - \frac{3}{4}$$

$$= 4\ln(2) - \frac{1}{2}\ln(2) - \frac{3}{4}$$

$$= \frac{7}{2}(\ln 2) - \frac{3}{4}$$

56. Can be put in the form $\int k\,dx$.

$$\int_3^5 dA = \int_3^5 1\,dA = A\Big|_3^5 = 5-3 = 2$$

57. Formula 5 with $u = q$, $a = 1$, and $b = -1$. Then $du = dq$.

$$\int_{q_0}^{q_n} \frac{dq}{q(1-q)} = \ln\left|\frac{q}{1-q}\right|\Bigg|_{q_0}^{q_n} = \ln\left|\frac{q_n}{1-q_n}\right| - \ln\left|\frac{q_0}{1-q_0}\right|$$

$$= \ln\left|\frac{q_n(1-q_0)}{q_0(1-q_n)}\right|$$

58. Formula 6 with $u = q$, $a = 1$ and $b = -1$. Then $du = dq$.

$$n = -\frac{1}{0.4}\int_{0.3}^{0.1} \frac{dq}{q^2(1-q)}$$

$$= -\frac{1}{0.4}\left[-\frac{1}{q} - \ln\left|\frac{1-q}{q}\right|\right]\Bigg|_{0.3}^{0.1}$$

$$= -\frac{1}{0.4}\left\{[-10-\ln 9] - \left[-\frac{10}{3} - \ln\frac{7}{3}\right]\right\}$$

$$= -\frac{1}{0.4}\left(-\frac{20}{3} - \ln 9 + \ln\frac{7}{3}\right) \approx 20$$

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59. a. For $\int_0^9 1000e^{-0.04t}dt$, the form $\int e^u du$ can be applied.

$$\int_0^9 1000e^{-0.04t}dt$$
$$= \frac{1000}{-0.04}\int_0^9 e^{-0.04t}(-0.04\,dt)$$
$$= -\frac{1000}{0.04}e^{-0.04t}\Big|_0^9$$
$$= -\frac{1000}{0.04}(e^{-0.36}-1)$$
$$\approx \$7558.09$$

b. For $\int_0^{10} 500te^{-0.06t}dt$ use Formula 38 with $t = u$ and $a = -0.06$, so $du = dt$.

$$\int_0^{10} 500te^{-0.06t}dt$$
$$= 500\int_0^{10} te^{-0.06t}dt$$
$$= 500\left[\frac{e^{-0.06t}}{0.0036}(-0.06t-1)\right]\Bigg|_0^{10}$$
$$= \frac{500}{0.0036}[e^{-0.6}(-1.6)-(-1)]$$
$$\approx \$16,930.75$$

60. $$\int_0^T ke^{-rt}dt = k\left(-\frac{1}{r}\right)\int_0^T e^{-rt}(-r\ dt) = \frac{-ke^{-rt}}{r}\Bigg|_0^T$$
$$= -\frac{ke^{-rT}}{r} + \frac{k}{r} = k\left(\frac{1-e^{-rT}}{r}\right)$$

61. a. $$\int_0^{10} 100e^{0.02(10-t)}dt = 100\int_0^{10} e^{0.2-0.02t}dt$$
$$= 100\int_0^{10} e^{0.2}e^{-0.02t}dt$$
$$= 100e^{0.2}\int_0^{10} e^{-0.02t}dt$$
$$= 100e^{0.2}\left(\frac{1}{-0.02}\right)\int_0^{10} e^{-0.02t}(-0.02\ dt)$$
$$= -5000e^{0.2}e^{-0.02t}\Big|_0^{10}$$
$$= -5000e^{0.2}\left[e^{-0.2}-1\right]$$
$$\approx \$1107.01$$

b. $$\int_0^{10} 200e^{0.01(10-t)}dt$$
$$= 200\int_0^{10} e^{0.1-0.01t}dt$$
$$= 200e^{0.1}\int_0^{10} e^{-0.01t}dt$$
$$= 200e^{0.1}\left(\frac{1}{-0.01}\right)\int_0^{10} e^{-0.01t}(-0.01\,dt)$$
$$= -20,000e^{0.1}\cdot e^{-0.01t}\Big|_0^{10}$$
$$= -20,000e^{0.1}(e^{-0.1}-1)$$
$$\approx \$2103.42$$

62. Use Formula 38 with $u = t$ and $a = -0.07$, so $du = dt$.

$$\int_0^5 50,000te^{-0.07t}\ dt = 50,000\int_0^5 te^{-0.07t}\ dt$$
$$= 50,000\left[\frac{e^{-0.07t}}{0.0049}(-0.07t-1)\right]\Bigg|_0^5$$
$$= \frac{50,000}{0.0049}[e^{-0.35}(-1.35)-1(-1)]$$
$$= \$496,640$$

Problems 15.4

1. $$\bar{f} = \frac{1}{3-(-1)}\int_{-1}^3 x^2dx = \frac{1}{4}\cdot\frac{x^3}{3}\Bigg|_{-1}^3 = \frac{1}{4}\left(9+\frac{1}{3}\right) = \frac{7}{3}$$

2. $$\bar{f} = \frac{1}{1-0}\int_0^1 (2x+1)dx = (x^2+x)\Big|_0^1 = 2-0 = 2$$

3. $$\bar{f} = \frac{1}{2-(-1)}\int_{-1}^2\left(2-3x^2\right)dx$$
$$= \frac{1}{3}\left(2x-x^3\right)\Big|_{-1}^2 = -1$$

4. $$\bar{f} = \frac{1}{3-1}\int_1^3\left(x^2+x+1\right)dx$$
$$= \frac{1}{2}\left(\frac{x^3}{3}+\frac{x^2}{2}+x\right)\Bigg|_1^3 = \frac{22}{3}$$

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5. $\bar{f} = \frac{1}{3-(-3)}\int_{-3}^{3} 2t^5\,dt$
$= \frac{1}{6}\cdot\frac{t^6}{3}\Big|_{-3}^{3}$
$= \frac{1}{18}[3^6 - (-3)^6]$
$= 0$

6. $\bar{f} = \frac{1}{4-0}\int_0^4 t\sqrt{t^2+9}\,dt$
$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\int_0^4 \sqrt{t^2+9}[2t\ dt]$
$= \frac{1}{8}\left[\frac{2\left(t^2+9\right)^{\frac{3}{2}}}{3}\right]\Bigg|_0^4 = \frac{49}{6}$

7. $\bar{f} = \frac{1}{1-0}\int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^1 = \frac{2}{3}$

8. $\bar{f} = \frac{1}{3-1}\int_1^3 \frac{5}{x^2}\,dx = \frac{1}{2}\cdot -\frac{5}{x}\Big|_1^3 = \frac{1}{2}\left(-\frac{5}{3}+5\right)$
$= \frac{5}{3}$

9. $\bar{P} = \frac{1}{100-0}\int_0^{100}\left(369q - 2.1q^2 - 400\right)dq$
$= \frac{1}{100}\left(184.5q^2 - 0.7q^3 - 400q\right)\Big|_0^{100}$
$= \frac{1}{100}(1{,}845{,}000 - 700{,}000 - 40{,}000) - 0$
= 11,050
Answer: \$11,050

10. $\bar{c} = \frac{1}{500-100}\int_{100}^{500}\left(4000+10q+0.1q^2\right)dq$
$= \frac{1}{400}\left(4000q + 5q^2 + \frac{0.1q^3}{3}\right)\Bigg|_{100}^{500} \approx 17{,}333.33$
Answer: \$17,333.33

11. $\frac{1}{2-0}\int_0^2 3000e^{0.05t}\,dt$
$= \frac{3000}{2}\cdot\frac{1}{0.05}\int_0^2 e^{0.05t}[0.05\ dt]$
$= 30{,}000e^{0.05t}\Big|_0^2 = 30{,}000\left(e^{0.1}-1\right) \approx 3155.13$
Answer: \$3155.13

12. $\bar{C} = \frac{1}{T-0}\int_0^T \frac{R}{F(t)}\,dt = \frac{1}{T}\int_0^T \frac{R(1+\alpha t)^2}{F_1}\,dt$
$\frac{R}{TF_1}\cdot\frac{1}{\alpha}\int_0^T (1+\alpha t)^2[\alpha\ dt] = \frac{R}{\alpha TF_1}\left[\frac{(1+\alpha t)^3}{3}\right]\Bigg|_0^T$
$= \frac{R}{\alpha TF_1}\left[\frac{(1+\alpha T)^3}{3} - \frac{1}{3}\right]$
$= \frac{R}{3\alpha TF_1}\left[1 + 3\alpha T + 3\alpha^2T^2 + \alpha^3T^3 - 1\right]$
$= \frac{R}{3\alpha TF_1}(3\alpha T)\left(1 + \alpha T + \frac{1}{3}\alpha^2T^2\right)$
$= \frac{R\left(1+\alpha T + \frac{1}{3}\alpha^2T^2\right)}{F_1}$

13. Average value $= \frac{1}{q_0 - 0}\int \frac{dr}{dq}dq$.
$= \frac{1}{q_0}\left[r(q_0) - r(0)\right]$
But *r*(0) = 0, so avg. value $= \frac{r(q_0)}{q_0}$. Since
$r(q_0)$
= [price per unit when q_0 units are sold] $\cdot q_0$,
we have
avg. value $= \frac{\left[\text{price per unit when } q_0 \text{ units are sold}\right]\cdot q_0}{q_0}$
= price per unit when $\cdot q_0$ units are sold.

14. $\bar{f} = \frac{1}{1-0}\int_0^1 \frac{1}{x^2-4x+5}\,dx \approx 0.32$

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Apply It 15.5

5. Separating variables, we have

$$\frac{dI}{dx} = -0.0085I$$

$$\frac{dI}{I} = -0.0085dx$$

$$\int \frac{1}{I}\,dI = -\int 0.0085\ dx$$

$$\ln|I| = -0.0085x + C_1$$

To solve for I, we convert to exponential Formula

$I = e^{-0.0085x+C_1} = Ce^{-0.0085x}$. Since $I = I_0$ when $x = 0$, $I_0 = Ce^0 = C$, so

$$I(x) = I_0e^{-0.0085x}.$$

Problems 15.5

1. $y' = 2xy^2$

$$\frac{dy}{dx} = 2xy^2$$

$$\frac{dy}{y^2} = 2x\ dx$$

$$\int y^{-2}dy = \int 2x\ dx$$

$$-\frac{1}{y} = x^2 + C$$

$$y = -\frac{1}{x^2 + C}$$

2. $y' = x^2y^2$

$$\frac{dy}{dx} = x^2y^2$$

$$\frac{dy}{y^2} = x^2\ dx$$

$$\int \frac{dy}{y^2} = \int x^2\ dx$$

$$-\frac{1}{y} = \frac{x^3}{3} + C_1$$

$$-\frac{1}{y} = \frac{1}{3}(x^3 + 3C_1)$$

$$\frac{1}{y} = -\frac{1}{3}(x^3 + C)$$

$$y = -\frac{3}{x^3 + C}$$

3. $\frac{dy}{dx} - 2x\ln(x^2+1) = 0$

$$dy = 2x\ln(x^2+1)dx$$

$$\int dy = \int 2x\ln(x^2+1)dx$$

$$\int dy = \int \ln(x^2+1)[2x\,dx]$$

Using Formula 41 gives

$y = (x^2+1)\ln(x^2+1) - (x^2+1) + C.$

4. $\frac{dy}{dx} = \frac{x}{y}$

$$y\ dy = x\ dx$$

$$\int y\ dy = \int x\ dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C_1$$

$$y^2 = x^2 + 2C_1$$

$$y^2 = x^2 + C$$

5. $\frac{dy}{dx} = y$, where $y > 0$.

$$\frac{dy}{y} = dx$$

$$\int \frac{dy}{y} = \int dx$$

$$\ln y = x + C_1$$

$y = e^{x+C_1} = e^{C_1}e^x = Ce^x$, where $C = e^{C_1}$. Thus

$y = Ce^x$, where $C > 0$.

6. $y' = e^xy^3$

$$\frac{dy}{dx} = e^xy^3$$

$$\frac{dy}{y^3} = e^xdx$$

$$\int \frac{dy}{y^3} = \int e^xdx$$

$$-\frac{1}{2y^2} = e^x + C$$

$$y^2 = -\frac{1}{2(e^x + C)}$$

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7. $y' = \frac{y}{x}$, where $x, y > 0$.

$\frac{dy}{dx} = \frac{y}{x}$

$\frac{dy}{y} = \frac{dx}{x}$

$\int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln y = \ln x + C_1$

$\ln y = \ln x + \ln C$, where $C > 0$.

$\ln y = \ln(Cx) \Rightarrow y = Cx$, where $C > 0$.

8. $\frac{dy}{dx} - x \ln x = 0$

$dy = x \ln x \, dx$

$\int dy = \int x \ln x \, dx$

Using Formula 42 gives

$y = \frac{x^2 \ln x}{2} - \frac{x^2}{2^2} + C$

$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C.$

9. $y' = \frac{1}{y^2}$ where $y(1) = 1$.

$\frac{dy}{dx} = \frac{1}{y^2}$

$y^2 dy = dx$

$\int y^2 \, dy = \int dx$

$\frac{y^3}{3} = x + C$

Given $y(1) = 1$, we obtain $\frac{1^3}{3} = 1 + C$, so

$C = -\frac{2}{3}$. Thus $y^3 = 3\left(x - \frac{2}{3}\right) = 3x - 2$,

$y = \sqrt[3]{3x-2}$.

10. $y' = e^{x-y}$, where $y(0) = 0$

$\frac{dy}{dx} = \frac{e^x}{e^y}$

$e^y dy = e^x dx$

$\int e^y dy = \int e^x dx$

$e^y = e^x + C$

Since $y(0) = 0$, we have $e^0 = e^0 + C$, $1 = 1 + C$, $C = 0$. Thus $e^y = e^x$, so $y = x$.

11. $e^y y' - x^2 = 0$, where $y = 0$ when $x = 0$.

$e^y \frac{dy}{dx} = x^2$

$e^y dy = x^2 dx$

$\int e^y dy = \int x^2 dx$

$e^y = \frac{x^3}{3} + C$

Given that $y(0) = 0$, we have $e^0 = 0 + C$, so $1 = C \Rightarrow e^y = \frac{x^3}{3} + 1$, $e^y = \frac{x^3+3}{3}$, so

$y = \ln \frac{x^3+3}{3}$.

12. $x^2 y' + \frac{1}{y^2} = 0$, where $y(1) = 2$

$x^2 \frac{dy}{dx} = -\frac{1}{y^2}$

$y^2 dy = -\frac{dx}{x^2}$

$\int y^2 dy = -\int \frac{dx}{x^2}$

$\frac{y^3}{3} = \frac{1}{x} + C$

Now, $y(1) = 2$ implies $C = \frac{5}{3}$. Thus

$\frac{y^3}{3} = \frac{1}{x} + \frac{5}{3}$, $y^3 = \frac{3}{x} + 5$, $y = \sqrt[3]{\frac{3}{x} + 5}$.

13. $(3x^2+2)^3 y' - xy^2 = 0$, where $y(0) = 2$.

$(3x^2+2)^3 \frac{dy}{dx} = xy^2$

$\frac{dy}{y^2} = \frac{x}{(3x^2+2)^3} dx$

$\int \frac{dy}{y^2} = \int \frac{x}{(3x^2+2)^3} dx$

$\int y^{-2} dy = \frac{1}{6} \int (3x^2+2)^{-3} [6x \, dx]$

$-\frac{1}{y} = -\frac{1}{12(3x^2+2)^2} + C$

Given that $y(0) = 2$ we have

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$-\frac{1}{2}=-\frac{1}{12(0+2)^2}+C=-\frac{1}{48}+C$, so $C=-\frac{23}{48}$.

Thus,

$-\frac{1}{y}=-\frac{1}{12(3x^2+2)^2}-\frac{23}{48}=-\frac{4+23(3x^2+2)^2}{48(3x^2+2)^2}$.

Hence, $y=\frac{48(3x^2+2)^2}{4+23(3x^2+2)^2}$.

14. $y'+x^3y=0$ and $y=e$ when $x=0$.

$\frac{dy}{dx}=-x^3y$

$\frac{dy}{y}=-x^3\,dx$

$\int\frac{dy}{y}=-\int x^3\,dx$

$\ln|y|=-\frac{x^4}{4}+C$

Given $y(0)=e$, $\ln e=0+C$, so $C=1$.

Thus $\ln y=-\frac{x^4}{4}+1$, so $y=e^{-\frac{x^4}{4}+1}$.

15. $\frac{dy}{dx}=\frac{3x\sqrt{1+y^2}}{y}$, where $y>0$ and $y(1)=\sqrt{8}$.

$\frac{y\,dy}{\sqrt{1+y^2}}=3x\,dx$

$\frac{1}{2}\int\left(1+y^2\right)^{-\frac{1}{2}}[2y\,dy]=3\int x\,dx$

$\left(1+y^2\right)^{\frac{1}{2}}=\frac{3x^2}{2}+C$

$y(1)=\sqrt{8}\Rightarrow(1+8)^{\frac{1}{2}}=\frac{3}{2}+C$

$C=\frac{3}{2}$

Thus

$\left(1+y^2\right)^{\frac{1}{2}}=\frac{3x^2}{2}+\frac{3}{2}$

$1+y^2=\left[\frac{3x^2}{2}+\frac{3}{2}\right]^2$

$y^2=\left[\frac{3x^2}{2}+\frac{3}{2}\right]^2-1$

Since $y>0$, $y=\sqrt{\left[\frac{3x^2}{2}+\frac{3}{2}\right]^2-1}$.

16. $2y\left(x^3+2x+1\right)\frac{dy}{dx}=\frac{3x^2+2}{\sqrt{y^2+9}}$, where $y(0)=0$.

$\int 2y\sqrt{y^2+9}dy=\int\frac{3x^2+2}{x^3+2x+1}dx$

$\int\left(y^2+9\right)^{\frac{1}{2}}[2y\,dy]=\int\frac{1}{x^3+2x+1}\left[\left(3x^2+2\right)dx\right]$

$\frac{2}{3}\left(y^2+9\right)^{\frac{3}{2}}=\ln\left|x^3+2x+1\right|+C$

Now $y(0)=0$ implies that $\frac{2}{3}(27)=\ln(1)+C$, so $C=18$. Thus

$\frac{2}{3}\left(y^2+9\right)^{\frac{3}{2}}=\ln\left|x^3+2x+1\right|+18$.

17. $2\frac{dy}{dx}=\frac{xe^{-y}}{\sqrt{x^2+3}}$, where $y(1)=0$.

$e^y\,dy=\frac{1}{2}x\left(x^2+3\right)^{-\frac{1}{2}}\,dx$

$\int e^y dy=\frac{1}{2}\cdot\frac{1}{2}\int\left(x^2+3\right)^{-\frac{1}{2}}[2x\,dx]$

$e^y=\frac{1}{2}\left(x^2+3\right)^{\frac{1}{2}}+C$

Now, $y(1)=0\Rightarrow e^0=\frac{1}{2}(2)+C$, so $C=0$. Thus

$e^y=\frac{1}{2}\left(x^2+3\right)^{\frac{1}{2}}\Rightarrow y=\ln\left(\frac{1}{2}\sqrt{x^2+3}\right)$.

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18. $dy = 2xye^{x^2}dx$, where $y > 0$ and $y(0) = e$.

$$\frac{dy}{y} = 2xe^{x^2}dx$$

$$\int\frac{dy}{y} = \int 2xe^{x^2}dx$$

$$\int\frac{dy}{y} = \int e^{x^2}[2x\,dx]$$

$$\ln y = e^{x^2} + C$$

Now $y(0) = e$ gives $\ln e = 1 = e^0 + C = 1 + C$, so $C = 0$. Thus $\ln y = e^{x^2}$, or $e^{\ln y} = y = e^{e^{x^2}}$.

19. $(q+1)^2\frac{dc}{dq} = cq$

$$\int\frac{1}{c}dc = \int\frac{q}{(q+1)^2}dq$$

Using partial fractions or Formula 7 for $\int\frac{q}{(q+1)^2}dq$, we obtain

$\ln c = \ln(q+1) + \frac{1}{q+1} + C$. Now, fixed cost is given to be e, which means that $c = e$ when $q = 0$. This implies $1 = 0 + 1 + C$, so $C = 0$. Thus

$\ln c = \ln(q+1) + \frac{1}{q+1} \Rightarrow c = e^{\ln(q+1)+\frac{1}{q+1}}$,

$c = e^{\ln(q+1)}e^{\frac{1}{q+1}}$, or $c = (q+1)e^{\frac{1}{q+1}}$.

20. $\frac{dy}{dx} = xe^{x-y} = \frac{xe^x}{e^y}$

$$\int e^y dy = \int xe^x dx$$

Using integration by parts or formula 38 gives $e^y = e^x(x-1) + C$. Now,

$f(1) = 0 \Rightarrow 1 = e(0) + C$, $1 = C$, so

$e^y = e^x(x-1) + 1$, $y = \ln\left[e^x(x-1)+1\right]$. Thus

$f(2) = \ln\left(e^2 + 1\right)$.

21. $\frac{dy}{dt} = -0.025y$

$$\int\frac{1}{y}dy = -0.025\int dt$$

$$\ln|y| = -0.025t + C$$

Given that $y = 1000$ when $t = 0$, we have $\ln 1000 = -0 + C = C$. Thus

$\ln|y| = -0.025t + \ln 1000$. To find t when money is 95% new, we note that y would be 5%(1000) = 50. Solving $\ln 50 = -0.025t + \ln 1000$ gives

$t = \frac{\ln 1000 - \ln 50}{0.025} \approx 120$ weeks.

22. $\frac{dr}{dq} = (50-4q)e^{-\frac{r}{5}}$

$$\int e^{\frac{r}{5}}dr = \int(50-4q)dq$$

$$5e^{\frac{r}{5}} = 50q - 2q^2 + C$$

Since $r = 0$ when $q = 0$, we have $5(1) = C$, $C = 5$.

$$5e^{\frac{r}{5}} = 50q - 2q^2 + 5$$

$$e^{\frac{r}{5}} = 10q - \frac{2}{5}q^2 + 1$$

$$\frac{r}{5} = \ln\left|10q - \frac{2}{5}q^2 + 1\right|$$

$$r = 5\ln\left|10q - \frac{2}{5}q^2 + 1\right|$$

Since $r = pq$, $p = \frac{1}{q}r = \frac{5}{q}\ln\left|10q - \frac{2}{5}q^2 + 1\right|$.

23. Let N be the population at time t, where $t = 0$ corresponds to 1990. Since N follows exponential growth, $N = N_0e^{kt}$. Now, $N = 60{,}000$ when $t = 0$, so $N_0 = 60{,}000$. Therefore $N = 60{,}000e^{kt}$. Since $N = 64{,}000$ when $t = 10$, we have $64{,}000 = 60{,}000e^{10k}$,

$\frac{16}{15} = e^{10k}$, $\ln\frac{16}{15} = 10k$, $k = \frac{\ln\frac{16}{15}}{10}$

Thus $N = 60{,}000e^{\left(\ln\frac{16}{15}\right)\left(\frac{t}{10}\right)}$

$$= 60{,}000\left(e^{\ln\frac{16}{15}}\right)^{t/10}$$

$$= 60{,}000\left(\frac{16}{15}\right)^{t/10}.$$

$$N(2010) = 60{,}000\left(\frac{16}{15}\right)^{20/10} \approx 68{,}267$$

24. Exponential growth applies, so $N = N_0e^{kt}$. When $t = 0$, then $N = 50{,}000$, So $N_0 = 50{,}000$. Thus $N = 50{,}000e^{kt}$. When $t = 50$, then

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$N = 100{,}000$, or $100{,}000 = 50{,}000e^{50k}$ or

$k = \frac{\ln 2}{50}$. Thus

$N = 50{,}000e^{\frac{t \ln 2}{50}}$ (*)

$N = 50{,}000e^{\left(\frac{0.69}{50}\right)t}$

$N = 50{,}000e^{0.0138t}$ (First form)

From (*), $N = 50{,}000\left[e^{\ln 2}\right]^{\frac{t}{50}}$, so

$N = 50{,}000(2)^{\frac{t}{50}}$. (Second form)

When $t = 100$, then

$N = 50{,}000(2)^{\frac{100}{50}} = 50{,}000(2)^2 = 200{,}000$

25. Let N be the population (in billions) at time t, where t is the number of years past 1930.
N follows exponential growth, so $N = N_0e^{kt}$.
When $t = 0$, then $N = 2$, so $N_0 = 2$. Thus
$N = 2e^{kt}$. Since $N = 3$ when $t = 30$, then

$3 = 2e^{30k}$

$\frac{3}{2} = e^{30k}$

$30k = \ln\frac{3}{2}$

$k = \frac{\ln\frac{3}{2}}{30}$

Thus $N = 2e^{\frac{t}{30}\ln\frac{3}{2}}$.

In 2015, $t = 85$ and so

$N = 2e^{\frac{85}{30}\ln\frac{3}{2}} \approx 2e^{1.14882}$ billion.

26. Let N = population at time t and N_0 = population at $t = 0$. Then $N = N_0e^{kt}$.
When $t = 100$, then $N = 3N_0$, so

$3N_0 = N_0e^{100k}$ or $k = \frac{\ln 3}{100}$.

Setting $N = 2N_0$ and solving for t gives

$2N_0 = N_0e^{\frac{t \ln 3}{100}}$

$2 = e^{\frac{t \ln 3}{100}}$

$\ln 2 = \frac{t \ln 3}{100}$

$t = \frac{100 \ln 2}{\ln 3} \approx 63.$

The population will double in approximately 63 years.

27. Let N be amount of sample that remains after t seconds. Then $N = N_0e^{-\lambda t}$, where N_0 is the initial amount present. When $t = 100$, then $N = 0.3N_0$. Thus

$0.3N_0 = N_0e^{-100\lambda}$

$0.3 = e^{-100\lambda}$

$-100\lambda = \ln 0.3$

$\lambda = -\frac{\ln 0.3}{100}$

Thus $\lambda \approx 0.01204$. The half-life is

$\frac{\ln 2}{\lambda} = \frac{\ln 2}{-\frac{\ln 0.3}{100}} = -100\frac{\ln 2}{\ln 0.3} \approx 57.57$ s.

28. $N = N_0e^{-\lambda t}$

After 100 s, 80% remains.

$0.8N_0 = N_0e^{-100\lambda}$

$0.8 = e^{-100\lambda}$

$-100\lambda = \ln 0.8$

$\lambda = -\frac{\ln 0.8}{100}$

$\lambda \approx 0.0022314$

The half-life is

$\frac{\ln 2}{\lambda} = \frac{\ln 2}{-\frac{\ln 0.8}{100}} = -100\frac{\ln 2}{\ln 0.8} \approx 310.63$ s.

29. Let N be the amount of ${}^{14}C$ present in the scroll t years after it was made. Then $N = N_0e^{-\lambda t}$, where N_0 is amount of ${}^{14}C$ present when $t = 0$.
We must find t when $N = 0.7N_0$.

$0.7N_0 = N_0e^{-\lambda t}$

$0.7 = e^{-\lambda t}$

$-\lambda t = \ln 0.7$

so $t = -\frac{\ln 0.7}{\lambda}$. By Equation 15 in the text,

$\lambda = \frac{\ln 2}{5730}$, so

$t = -\frac{\ln 0.7}{\frac{\ln 2}{5730}} = -\frac{5730 \ln 0.7}{\ln 2} \approx 2900$ years.

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