Chapter 14

Problems 14.1

1. $y = ax + b$

$$dy = \frac{d}{dx}(ax+b)dx = a\ dx$$

2. $dy = y'dx = 0\ dx = 0$

3. $d[f(x)] = f'(x)dx = \frac{1}{2}(x^4-9)^{-\frac{1}{2}}(4x^3)dx$

$$= \frac{2x^3}{\sqrt{x^4-9}}dx$$

4. $d[f(x)] = f'(x)dx$

$$= 3(8x-5)\left(4x^2-5x+2\right)^2 dx$$

5. $u = x^{-2}$

$$du = \frac{d}{dx}\left(x^{-2}\right)dx = -2x^{-3}dx = -\frac{2}{x^3}dx$$

6. $u = \sqrt{x}$

$$du = \frac{d}{dx}(x^{1/2})dx = \frac{1}{2}x^{-1/2}dx = \frac{1}{2\sqrt{x}}dx$$

7. $dp = \frac{d}{dx}\left[\ln\left(x^2+7\right)\right]dx = \frac{1}{x^2+7}(2x)dx$

$$= \frac{2x}{x^2+7}dx$$

8. $dp = \frac{d}{dx}\left(e^{x^3+2x-5}\right)dx = (3x^2+2)e^{x^3+2x-5}dx$

9. $dy = y'dx$

$$= \left[(9x+3)e^{2x^2+3}(4x) + e^{2x^2+3}(9)\right]dx$$

$$= 3e^{2x^2+3}[(3x+1)(4x)+3]dx$$

$$= 3e^{2x^2+3}\left(12x^2+4x+3\right)dx$$

10. $y = \ln\sqrt{x^2+12} = \frac{1}{2}\ln(x^2+12)$

$$dy = \frac{1}{2}\cdot\frac{1}{x^2+12}(2x)dx = \frac{x}{x^2+12}dx$$

11. $y = ax + b$; $dy = adx$

$\Delta y = adx = dy$

12. $\Delta y = \left[5(-1.02)^2 - 5(-1)^2\right] = 0.202$

$dy = 10x\ dx = 10(-1)(-0.02) = 0.2$

13. Δy

$$= [2(-1.9)^2 + 5(-1.9) - 7] - [2(-2)^2 + 5(-2) - 7]$$
$$= -0.28$$

$dy = (4x + 5)dx = [4(-2) + 5](0.1) = -0.3$

14. $\Delta y = [3(-1.03)+2]^2 - [3(-1)+2]^2 = 0.1881$

$dy = 6(3x + 2)\ dx = 6[3(-1) + 2](-0.03) = 0.18$

15. $\Delta y = \sqrt{32-(3.95)^2} - \sqrt{32-(4^2)} \approx 0.049$

$$dy = \frac{-x}{\sqrt{32-x^2}}dx = \frac{-4}{\sqrt{16}}(-0.05) = 0.050$$

16. $\Delta y = \ln 1.01 - \ln 1 \approx 0.00995$

$$dy = \frac{1}{x}dx = \frac{1}{1}(0.01) = 0.01$$

17. **a.** $f(x) = \frac{x+5}{x+1}$

$$f'(x) = \frac{(x+1)(1)-(x+5)(1)}{(x+1)^2} = \frac{-4}{(x+1)^2}$$

$$f'(1) = \frac{-4}{4} = -1$$

b. We use $f(x + dx) \approx f(x) + dy$ with $x = 1$, $dx = 0.1$.

$f(1.1) = f(1+0.1) \approx f(1) + f'(1)dx$

$$= \frac{6}{2} + (-1)(0.1) = 2.9$$

18. **a.** $y = f(x) = x^{3x}$

Using logarithmic differentiation,

$\ln y = 3x \ln x$,

$$\frac{1}{y}\cdot\frac{dy}{dx} = 3x\left(\frac{1}{x}\right) + (\ln x)(3) = 3(1+\ln x)$$

$$\frac{dy}{dx} = y[3(1+\ln x)] = 3x^{3x}(1+\ln x)$$

$$f'(1) = 3(1)(1+0) = 3$$

b. We use $f(x + dx) \approx f(x) + dy$ with $x = 1$, $dx = -0.02$

$f(0.98) = f(1-0.02) \approx f(1) + f'(1)dx$

$= 1^3 + (3)(-0.02) = 0.94$

19. Let $y = f(x) = \sqrt{x}$

$f(x+dx) \approx f(x)+dy = \sqrt{x} + \frac{1}{2\sqrt{x}}dx$

If $x = 289$ and $dx = -1$, then

$\sqrt{288} = f(289-1)$

$\approx \sqrt{289} + \frac{1}{2\sqrt{289}}(-1)$

$= \frac{577}{34}$

≈ 16.97

20. Let $y = f(x) = \sqrt{x}$

$f(x+dx) \approx f(x)+dy = \sqrt{x} + \frac{1}{2\sqrt{x}}dx$

If $x = 121$ and $dx = 1$, then

$\sqrt{122} = f(121+1) \approx \sqrt{121} + \frac{1}{2\sqrt{121}}(1)$

$= 11\frac{1}{22}$.

21. Let $y = f(x) = \sqrt[3]{x}$

$f(x+dx) \approx f(x)+dy = \sqrt[3]{x} + \frac{1}{3x^{\frac{2}{3}}}dx$

If $x = 8$ and $dx = 1$, then

$\sqrt[3]{9} = f(8+1) \approx \sqrt[3]{8} + \frac{1}{3\left(\sqrt[3]{8}\right)^2}(1)$

$= 2 + \frac{1}{3 \cdot 2^2} = 2 + \frac{1}{12} = \frac{25}{12}$

22. Let $y = f(x) = \sqrt[4]{x}$.

$f(x+dx) = f(x)+dy = \sqrt[4]{x} + \frac{1}{4x^{\frac{3}{4}}}dx$

If $x = 16$ and $dx = 0.3$, then

$\sqrt[4]{16.3} = f(16+0.3) \approx \sqrt[4]{16} + \frac{1}{4\left(\sqrt[4]{16}\right)^3}(0.3)$

$= 2 + \frac{0.3}{2^3} = 2\frac{3}{320}$

23. Let $y = f(x) = \ln x$

$f(x+dx) \approx f(x)+dy = \ln(x) + \frac{1}{x}dx$

If $x = 1$ and $dx = -0.03$, then

$\ln(0.97) = f(1+(-0.03))$

$\approx \ln(1) + \frac{1}{1}(-0.03) = -0.03$

24. Let $y = f(x) = \ln x$

$f(x+dx) \approx f(x)+dy = \ln(x) + \frac{1}{x}dx$

If $x = 1$ and $dx = 0.01$, then

$\ln 1.01 = f(1+0.01) \approx \ln(1) + \frac{1}{1}(0.01) = 0.01$

25. Let $y = f(x) = e^x$

$f(x+dx) \approx f(x)+dy = e^x + e^x dx$

If $x = 0$ and $dx = 0.001$, then

$e^{0.001} = f(0+0.001) \approx e^0 + e^0(0.001) = 1.001$

26. Let $y = f(x) = e^x$

$f(x+dx) \approx f(x)+dy = e^x + e^x dx$

If $x = 0$ and $dx = -0.002$, then

$e^{-0.002} = f(0+(-0.002))$

$\approx e^0 + e^0(-0.002) = 0.998$

27. $\frac{dy}{dx} = 2$, so $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{2}$

28. $\frac{dy}{dx} = 10x+3$, so $\frac{dx}{dy} = \frac{1}{10x+3}$

29. $\frac{dq}{dp} = 6p\left(p^2+5\right)^2$, so $\frac{dp}{dq} = \frac{1}{6p\left(p^2+5\right)^2}$

30. $\frac{dq}{dp} = \frac{1}{2\sqrt{p+5}}$, so $\frac{dp}{dq} = 2\sqrt{p+5}$

31. $q = p^{-2}$, $\frac{dq}{dp} = -2p^{-3} = \frac{-2}{p^3}$, so $\frac{dp}{dq} = -\frac{p^3}{2}$

32. $\frac{dq}{dp} = -2e^{4-2p}$, so $\frac{dp}{dq} = -\frac{1}{2e^{4-2p}} = -\frac{1}{2}e^{2p-4}$

33. $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{14x-6}$

If $x = 3$, $\frac{dx}{dy} = \frac{1}{36}$

34. $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{\frac{2}{x}} = \frac{x}{2}$

If $x = 3$, $\frac{dx}{dy} = \frac{3}{2}$.

35. $p = \frac{500}{q+2}$

$\frac{dp}{dq} = \frac{-500}{(q+2)^2}$

$\frac{dq}{dp} = -\frac{(q+2)^2}{500}$

$\left.\frac{dq}{dp}\right|_{q=18} = \left.-\frac{(q+2)^2}{500}\right|_{q=18} = -\frac{4}{5}$

36. $p = 60 - \sqrt{2q}$

$\frac{dp}{dq} = -\frac{1}{\sqrt{2q}}$

$\frac{dq}{dp} = -\sqrt{2q}$

$\left.\frac{dq}{dp}\right|_{q=50} = \left.-\sqrt{2q}\right|_{q=50} = -10$

37. $P = 397q - 2.3q^2 - 400$, q changes from 90 to 91.

$\Delta P \approx dP = P'dq = (397 - 4.6q)dq$

Choosing $q = 90$ and $dq = 1$,

$\Delta P \approx [397 - 4.6(90)](1) = -17$.

True change is

$P(91) - P(90) = 16{,}680.7 - 16{,}700 = -19.3$.

38. $r = 250q + 45q^2 - q^3$, q increases from 40 to 41.

$\Delta r \approx dr = r'dq = \left(250 + 90q - 3q^2\right)dq$

Choosing $q = 40$ and $dq = 1$,

$\Delta r \approx (-950)(1) = -950$

True change is

$r(41) - r(40) = 16{,}974 - 18{,}000 = -1026$

39. $p = \frac{10}{\sqrt{q}}$. We approximate p when $q = 24$.

$p(q+dq) \approx p + dp = \frac{10}{\sqrt{q}} - \frac{5}{\sqrt{q^3}}dq$

If $q = 25$ and $dq = -1$, then

$p(24) = p(25 + (-1)) \approx \frac{10}{\sqrt{25}} - \frac{5}{\sqrt{(25)^3}}(-1)$

$= 2 + \frac{1}{25} = \frac{51}{25} = 2.04$

40. $p = \frac{200}{\sqrt{q+8}}$

We approximate p when $q = 40$.

$p(q+dq) \approx p + dp = \frac{200}{\sqrt{q+8}} - \frac{100}{(q+8)^{\frac{3}{2}}}dq$

If $q = 41$ and $dq = 1$, then

$p(40) = p(41-1) \approx \frac{200}{\sqrt{49}} - \frac{100}{(49)^{\frac{3}{2}}}(1)$

$= \frac{200}{7} - \frac{100}{343} = \frac{9700}{343} \approx 28.28$

41. $c = f(q) = \frac{q^2}{2} + 5q + 300$

If $q = 10$ and $dq = 2$,

$\frac{\Delta c}{c} \approx \frac{dc}{c} = \frac{(q+5)dq}{\frac{q^2}{2} + 5q + 300}$

$= \frac{15(2)}{50 + 50 + 300}$

$= \frac{3}{40}$

$= 0.075 \approx 0.1$

42. $S = 20\sqrt{I}$, I decreases from 45 to $44\frac{1}{2}$.

$\Delta S \approx dS = S'dI = \frac{10}{\sqrt{I}}dI$

Choosing $I = 45$ and $dI = -\frac{1}{2}$, then

$\Delta S \approx \frac{10}{\sqrt{45}}\left(-\frac{1}{2}\right) \approx -0.745$.

43. $V = \frac{4}{3}\pi r^3$

$\Delta V \approx dV = V'dr = 4\pi r^2 dr$

$dr = \left(6.6\times10^{-4}\right) - \left(6.5\times10^{-4}\right)$

$= 0.1\times10^{-4} = 10^{-5}$

$\Delta V \approx 4\pi\left(6.5\times10^{-4}\right)^2\left(10^{-5}\right) = \left(1.69\times10^{-11}\right)\pi \text{ cm}^3$.

44. $(P + a)(v + b) = k$

$P = \frac{k}{v+b} - a$

$dP = -k(v+b)^{-2}\,dv$

45. a. We substitute $q = 40$ and $p = 20$

$$2 + \frac{40^2}{200} = \frac{4000}{20^2}$$
$$2 + 8 = 10$$
$$10 = 10$$

b. We differentiate implicitly with respect to p.

$$0 + \frac{1}{200}\left(2q\frac{dq}{dp}\right) = -\frac{8000}{p^3}$$

From part (a) $q = 40$ when $p = 20$. Substituting gives

$$\frac{1}{200}\left(2\cdot 40\frac{dq}{dp}\right) = -\frac{8000}{20^3}$$

$$\frac{dq}{dp} = -2.5$$

c. $q(p+dp) \approx q(p) + dq = q(p) + q'(p)dp$

$q(19.20) = q(20 + (-0.8))$

$\approx q(20) + q'(20)dp$

$= 40 + (-2.5)(-0.8)$

$= 42$ units

46. a. Profit $= TR - TC = pq - \bar{c}q$

$$P = \frac{1}{2}q^3 - 66q^2 + 7000q - \left(500q - q^2 + \frac{80{,}000}{2}\right) = \frac{1}{2}q^3 - 65q^2 + 6500q - 40{,}000$$

If $q = 100$, then $P = \frac{1}{2}(100)^3 - 65(100)^2 + 6500(100) - 40{,}000 = 460{,}000$

b. We use $P(q + dq) \approx P(q) + dP$ with $q = 100$ and $dq = 1$.

$$
\begin{aligned}
P(101) &= P(100+1) \\
&\approx P(100) + \left(\frac{3}{2}q^2 - 130q + 6500\right)dq \\
&= 460{,}000 + \left[\frac{3}{2}(100)^2 - 130(100) + 6500\right](1) \\
&= \$468{,}500
\end{aligned}
$$

Apply It 14.2

1. $\int 28.3\, dq = 28.3q + C$

The form of the cost function is $28.3q + C$.

2. $\int 0.12t^2 dt = 0.12\frac{t^3}{3} + C = 0.04t^3 + C$

The form of the revenue function is $R(t) = 0.04t^3 + C$.

3. Let $S(t)$ = the number of subscribers t months after the competition entered the market, then $S'(t) = -\frac{480}{t^3}$.

$$S(t) = \int -\frac{480}{t^3}dt = -480\int t^{-3}dt$$

$$= -480\left(\frac{t^{-2}}{-2}\right) + C = 240t^{-2} + C = \frac{240}{t^2} + C$$

The number of subscribers is $S(t) = \frac{240}{t^2} + C$.

4. $\int \left(500 + 300\sqrt{t}\right)dt = \int \left(500 + 300t^{\frac{1}{2}}\right)dt$

$$= 500t + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = 500t + \frac{2}{3}t^{\frac{3}{2}} + C$$

The population is $N(t) = 500t + \frac{2}{3}t^{\frac{3}{2}} + C$

5. The amount of money saved is $\int \frac{dS}{dt}dt$.

$$\int \left(2.1t^2 - 65.4t + 491.6\right)dt$$

$$= 2.1\left(\frac{t^3}{3}\right) - 65.4\left(\frac{t^2}{2}\right) + 491.6t + C$$

$$= 0.7t^3 - 32.7t^2 + 491.6t + C$$

The amount of money saved is $S(t) = 0.7t^3 - 32.7t^2 + 491.6t + C$

Problems 14.2

1. $\int 7\,dx = 7x + C$

2. $\int \frac{1}{x}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$

3. $\int x^8 dx = \frac{x^{8+1}}{8+1} + C = \frac{x^9}{9} + C$

4. $\int 5x^{24}\,dx = 5\int x^{24}\,dx = 5\cdot\frac{x^{24+1}}{24+1} + C$
$= 5\cdot\frac{x^{25}}{25} + C = \frac{x^{25}}{5} + C$

5. $\int 5x^{-7}\,dx = 5\int x^{-7}\,dx = 5\cdot\frac{x^{-7+1}}{-7+1} + C$
$= 5\cdot\frac{x^{-6}}{-6} + C = -\frac{5}{6x^6} + C$

6. $\int \frac{z^{-3}}{3}\,dz = \frac{1}{3}\int z^{-3}dz = \frac{1}{3}\cdot\frac{z^{-3+1}}{-3+1} + C$
$= \frac{1}{3}\cdot\frac{z^{-2}}{-2} + C = -\frac{1}{6z^2} + C$

7. $\int \frac{5}{x^7}\,dx = 5\int x^{-7}dx = 5\cdot\frac{x^{-7+1}}{-7+1} + C$
$= \frac{5x^{-6}}{-6} + C = -\frac{5}{6x^6} + C$

8. $\int \frac{7}{x^4}\,dx = 7\int x^{-4}dx = 7\cdot\frac{x^{-4+1}}{-4+1} + C = \frac{7x^{-3}}{-3} + C$
$= -\frac{7}{3x^3} + C$

9. $\int \frac{1}{t^{7/4}}\,dt = \int t^{-7/4}dt = \frac{t^{-7/4+1}}{-\frac{7}{4}+1} + C = \frac{t^{-3/4}}{-\frac{3}{4}} + C$
$= -\frac{4}{3t^{3/4}} + C$

10. $\int \frac{7}{2x^{\frac{9}{4}}}\,dx = \frac{7}{2}\int x^{-\frac{9}{4}}dx = \frac{7}{2}\cdot\frac{x^{\frac{-9}{4}+1}}{\frac{-9}{4}+1} + C$
$= \frac{7}{2}\cdot\frac{x^{-\frac{5}{4}}}{-\frac{5}{4}} + C$
$= -\frac{14}{5x^{\frac{5}{4}}} + C$

11. $\int (4+t)dt = \int 4\,dt + \int t\,dt = 4t + \frac{t^{t+1}}{1+1} + C$
$= 4t + \frac{t^2}{2} + C$

12. $\int (7r^5 + 4r^2 + 1)dr = 7\int r^5 dr + 4\int r^2 dr + \int dr$
$= 7\cdot\frac{r^{5+1}}{5+1} + 4\cdot\frac{r^{2+1}}{2+1} + r + C$
$= \frac{7r^6}{6} + \frac{4r^3}{3} + r + C$

13. $\int \left(y^5 - 5y\right)dy = \int y^5 dy - \int 5y\ dy$
$= \frac{y^{5+1}}{5+1} - 5\cdot\frac{y^{1+1}}{1+1} + C$
$= \frac{y^6}{6} - 5\cdot\frac{y^2}{2} + C = \frac{y^6}{6} - \frac{5y^2}{2} + C$

14. $\int \left(5 - 2w - 6w^2\right)dw$
$= \int 5\ dw - 2\int w\ dw - 6\int w^2 dw$
$= 5w - 2\cdot\frac{w^2}{2} - 6\cdot\frac{w^3}{3} + C$
$= 5w - w^2 - 2w^3 + C$

15. $\int \left(3t^2 - 4t + 5\right)dt = 3\int t^2 dt - 4\int t\ dt + \int 5\ dt$
$= 3\cdot\frac{t^3}{3} - 4\cdot\frac{t^2}{2} + 5t + C = t^3 - 2t^2 + 5t + C$

16. $\int (1 + t^2 + t^4 + t^6)dt$
$= \int 1\,dt + \int t^2\,dt + \int t^4\,dt + \int t^6\,dt$
$= t + \frac{t^3}{3} + \frac{t^5}{5} + \frac{t^7}{7} + C$

17. Since $\sqrt{2}+e$ is a constant,
$\int\left(\sqrt{2}+e\right)dx=\left(\sqrt{2}+e\right)x+C$.

18. $\int\left(5-2^{-1}\right)dx=\int\left(5-\frac{1}{2}\right)dx=\int\frac{9}{2}dx=\frac{9}{2}x+C$

19. $\int\left(\frac{x}{7}-\frac{3}{4}x^4\right)dx=\frac{1}{7}\int x\,dx-\frac{3}{4}\int x^4dx$
$=\frac{1}{7}\cdot\frac{x^2}{2}-\frac{3}{4}\cdot\frac{x^5}{5}+C$
$=\frac{x^2}{14}-\frac{3x^5}{20}+C$

20. $\int\left(\frac{2x^2}{7}-\frac{8}{3}x^4\right)dx=\frac{2}{7}\int x^2dx-\frac{8}{3}\int x^4dx$
$=\frac{2}{7}\cdot\frac{x^3}{3}-\frac{8}{3}\cdot\frac{x^5}{5}+C$
$=\frac{2x^3}{21}-\frac{8x^5}{15}+C$

21. $\int\pi e^xdx=\pi\int e^xdx=\pi e^x+C$

22. $\int(e^x+3x^2+2x)dx=\int e^xdx+3\int x^2dx+2\int x\,dx$
$=e^x+3\cdot\frac{x^{2+1}}{2+1}+2\cdot\frac{x^{1+1}}{1+1}+C$
$=e^x+x^3+x^2+C$

23. $\int\left(x^{8.3}-9x^6+3x^{-4}+x^{-3}\right)dx$
$=\frac{x^{9.3}}{9.3}-9\cdot\frac{x^7}{7}+3\cdot\frac{x^{-3}}{-3}+\frac{x^{-2}}{-2}+C$
$=\frac{x^{9.3}}{9.3}-\frac{9x^7}{7}-\frac{1}{x^3}-\frac{1}{2x^2}+C$

24. $\int(0.7y^3+10+2y^{-3})dy$
$=0.7\cdot\frac{y^4}{4}+10y+2\cdot\frac{y^{-2}}{-2}+C$
$=0.175y^4+10y-\frac{1}{y^2}+C$

25. $\int\frac{-2\sqrt{x}}{3}dx=-\frac{2}{3}\int x^{\frac{1}{2}}dx=-\frac{2}{3}\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$
$=-\frac{2}{3}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C=-\frac{4x^{\frac{3}{2}}}{9}+C$

26. $\int dz=\int 1\,dz=1\cdot z+C=z+C$

27. $\int\frac{5}{3\sqrt[3]{x^2}}dx=\frac{5}{3}\int x^{-2/3}dx$
$=\frac{5}{3}\cdot\frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}+C$
$=5x^{1/3}+C$

28. $\int\frac{-4}{(3x)^3}dx=\int\frac{-4}{27x^3}dx=-\frac{4}{27}\int x^{-3}\,dx$
$=-\frac{4}{27}\cdot\frac{x^{-3+1}}{-3+1}+C$
$=-\frac{4}{27}\cdot\frac{x^{-2}}{-2}+C=\frac{2}{27x^2}+C$

29. $\int\left(\frac{x^3}{3}-\frac{3}{x^3}\right)dx=\frac{1}{3}\int x^3dx-3\int x^{-3}dx$
$=\frac{1}{3}\cdot\frac{x^{3+1}}{3+1}-3\cdot\frac{x^{-3+1}}{-3+1}+C$
$=\frac{1}{3}\cdot\frac{x^4}{4}-3\cdot\frac{x^{-2}}{-2}+C=\frac{x^4}{12}+\frac{3}{2x^2}+C$

30. $\int\left(\frac{1}{2x^3}-\frac{1}{x^4}\right)dx=\frac{1}{2}\int x^{-3}dx-\int x^{-4}dx$
$=\frac{1}{2}\cdot\frac{x^{-2}}{-2}-\frac{x^{-3}}{-3}+C$
$=-\frac{1}{4x^2}+\frac{1}{3x^3}+C$

31. $\int\left(\frac{3w^2}{2}-\frac{2}{3w^2}\right)dw=\frac{3}{2}\int w^2dw-\frac{2}{3}\int w^{-2}dw$
$=\frac{3}{2}\cdot\frac{w^3}{3}-\frac{2}{3}\cdot\frac{w^{-1}}{-1}+C=\frac{w^3}{2}+\frac{2}{3w}+C$

32. $\int 7e^{-s}\,ds = 7\int e^{-s}\,ds$
$= 7\cdot e^{-s}(-1) + C$
$= -7e^{-s} + C$

33. $\int \frac{3u-4}{5}\,du = \frac{1}{5}\int (3u-4)du = \frac{1}{5}\left(3\int u\ du - 4\int du\right)$
$= \frac{1}{5}\left(3\frac{u^2}{2} - 4u\right) + C = \frac{3}{10}u^2 - \frac{4}{5}u + C$
$= \frac{1}{7}\left(2\int z\ dz - \int 5\ dz\right)$
$= \frac{1}{7}\left(2\cdot\frac{z^2}{2} - 5z\right) + C = \frac{1}{7}\left(z^2 - 5z\right) + C$

34. $\int \frac{1}{12}\left(\frac{1}{3}e^x\right)dx = \int \frac{1}{36}e^x dx$
$= \frac{1}{36}\int e^x dx = \frac{1}{36}e^x + C$

35. $\int (u^e + e^u)du = \int u^e\ du + \int e^u\ du$
$= \frac{u^{e+1}}{e+1} + e^u + C$

36. $\int \left(3y^3 - 2y^2 + \frac{e^y}{6}\right)dy$
$= 3\int y^3 dy - 2\int y^2 dy + \frac{1}{6}\int e^y dy$
$= 3\cdot\frac{y^4}{4} - 2\cdot\frac{y^3}{3} + \frac{1}{6}\cdot e^y + C$
$= \frac{3y^4}{4} - \frac{2y^3}{3} + \frac{e^y}{6} + C$

37. $\int \left(\frac{3}{\sqrt{x}} - 12\sqrt[3]{x}\right)dx = \int (3x^{-1/2} - 12x^{1/3})dx$
$= 3\int x^{-1/2}dx - 12\int x^{1/3}dx$
$= 3\cdot\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - 12\cdot\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C$
$= 6x^{1/2} - 9x^{4/3} + C$
$= 6\sqrt{x} - 9\sqrt[3]{x^4} + C$

38. $\int 0\ dt = 0\cdot t + C = C$

39. $\int \left(-\frac{\sqrt[3]{x^2}}{5} - \frac{7}{2\sqrt{x}} + 6x\right)dx$
$= \int \left(-\frac{x^{\frac{2}{3}}}{5} - \frac{7x^{-\frac{1}{2}}}{2} + 6x\right)dx$
$= -\frac{1}{5}\int x^{\frac{2}{3}}dx - \frac{7}{2}\int x^{-\frac{1}{2}}dx + 6\int x\ dx$
$= -\frac{1}{5}\cdot\frac{x^{\frac{5}{3}}}{\frac{5}{3}} - \frac{7}{2}\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 6\cdot\frac{x^2}{2} + C$
$= -\frac{3x^{\frac{5}{3}}}{25} - 7x^{\frac{1}{2}} + 3x^2 + C$

40. $\int \left(\sqrt[3]{u} + \frac{1}{\sqrt{u}}\right)du = \int \left(u^{\frac{1}{3}} + u^{-\frac{1}{2}}\right)du$
$= \int u^{\frac{1}{3}}du + \int u^{-\frac{1}{2}}du$
$= \frac{u^{\frac{4}{3}}}{\frac{4}{3}} + \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$
$= \frac{3u^{\frac{4}{3}}}{4} + 2u^{\frac{1}{2}} + C$

41. $\int \left(x^2+5\right)(x-3)dx = \int \left(x^3 - 3x^2 + 5x - 15\right)dx$
$= \frac{x^4}{4} - 3\cdot\frac{x^3}{3} + 5\cdot\frac{x^2}{2} - 15x + C$
$= \frac{x^4}{4} - x^3 + \frac{5x^2}{2} - 15x + C$

42. $\int x^3(x^2+5x+2)dx = \int (x^5 + 5x^4 + 2x^3)dx$
$= \frac{x^6}{6} + 5\cdot\frac{x^5}{5} + 2\cdot\frac{x^4}{4} + C$
$= \frac{x^6}{6} + x^5 + \frac{x^4}{2} + C$

43. $\int \sqrt{x}(x+3)dx = \int \left(x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right)dx$
$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$
$= \frac{2x^{\frac{5}{2}}}{5} + 2x^{\frac{3}{2}} + C$

44. $\int (z+2)^2\,dz = \int \left(z^2+4z+4\right) dz$

$= \frac{z^3}{3} + 4\cdot\frac{z^2}{2} + 4z + C$

$= \frac{z^3}{3} + 2z^2 + 4z + C$

45. $\int (3u+2)^3\,du = \int (27u^3+54u^2+36u+8)\,du$

$= 27\cdot\frac{u^4}{4} + 54\cdot\frac{u^3}{3} + 36\cdot\frac{u^2}{2} + 8u + C$

$= \frac{27}{4}u^4 + 18u^3 + 18u^2 + 8u + C$

46. $\int \left(\frac{2}{\sqrt[5]{x}} - 1\right)^2 dx = \int \left(2x^{-\frac{1}{5}} - 1\right)^2 dx$

$= \int \left(4x^{-\frac{2}{5}} - 4x^{-\frac{1}{5}} + 1\right) dx$

$= 4\cdot\frac{x^{\frac{3}{5}}}{\frac{3}{5}} - 4\cdot\frac{x^{\frac{4}{5}}}{\frac{4}{5}} + x + C$

$= \frac{20x^{\frac{3}{5}}}{3} - 5x^{\frac{4}{5}} + x + C$

47. $\int x^{-2}(3x^4+4x^2-5)\,dx = \int (3x^2+4-5x^{-2})\,dx$

$= 3\cdot\frac{x^3}{3} + 4x - 5\cdot\frac{x^{-1}}{-1} + C$

$= x^3 + 4x + \frac{5}{x} + C$

48. $\int \left[6e^u - u^3\left(\sqrt{u}+1\right)\right] du = \int \left[6e^u - u^{\frac{7}{2}} - u^3\right] du$

$= 6\cdot e^u - \frac{u^{\frac{9}{2}}}{\frac{9}{2}} - \frac{u^4}{4} + C$

$= 6e^u - \frac{2u^{\frac{9}{2}}}{9} - \frac{u^4}{4} + C$

49. $\int \frac{z^4+10z^3}{2z^2}\,dz = \frac{1}{2}\int \left(\frac{z^4}{z^2} + \frac{10z^3}{z^2}\right) dz$

$= \frac{1}{2}\int \left(z^2+10z\right) dz$

$= \frac{1}{2}\left(\frac{z^3}{3} + 10\cdot\frac{z^2}{2}\right) + C$

$= \frac{z^3}{6} + \frac{5z^2}{2} + C$

50. $\int \frac{x^4-5x^2+2x}{5x^2}\,dx = \frac{1}{5}\int \left(x^2 - 5 + \frac{2}{x}\right) dx$

$= \frac{1}{5}\left(\frac{x^3}{3} - 5x + 2\ln|x|\right) + C$

51. $\int \frac{e^x+e^{2x}}{e^x}\,dx = \int \left(\frac{e^x}{e^x} + \frac{e^{2x}}{e^x}\right) dx$

$= \int \left(1+e^x\right) dx$

$= x + e^x + C$

52. $\int \frac{(x^2+1)^3}{x}\,dx = \int \frac{x^6+3x^4+3x^2+1}{x}\,dx$

$= \int (x^5+3x^3+3x+x^{-1})\,dx$

$= \frac{x^6}{6} + 3\cdot\frac{x^4}{4} + 3\cdot\frac{x^2}{2} + \ln|x| + C$

$= \frac{x^6}{6} + \frac{3x^4}{4} + \frac{3x^2}{2} + \ln|x| + C$

53. No, $F(x) - G(x)$ might be a nonzero constant.

54. a. $F(x) = \frac{d}{dx}\left(xe^x\right) = xe^x + e^x(1) = e^x(x+1)$

b. There is only one.

55. Because an antiderivative of the derivative of a function is the function itself, we have

$$\int \frac{d}{dx}\left[\frac{1}{\sqrt{x^2+1}}\right] dx = \frac{1}{\sqrt{x^2+1}} + C.$$

Apply It 14.3

6. $N(t) = \int \frac{dN}{dt}\,dt = \int \left(800 + 200e^t\right) dt$

$= 800t + 200e^t + C$

Since $N(5) = 40{,}000$, we have

$40{,}000 = 800(5) + 200e^5 + C$, so

$C = 40{,}000 - \left(4000 + 200e^5\right)$

$= 36{,}000 - 200e^5 \approx 6317.37$

$N(t) = 800t + 200e^t + 6317.37$

7. Since $y'' = \frac{d}{dt}(y') = 84t + 24$

$y' = \int (84t + 24)dt = 84\left(\frac{t^2}{2}\right) + 24t + C_1$

$= 42t^2 + 24t + C_1$

Since $y'(8) = 2891$, we have

$2891 = 42(8)^2 + 24(8) + C_1 = 2880 + C_1$, so

$C_1 = 2891 - 2880 = 11$, and $y' = 42t^2 + 24t + 11$.

$y(t) = \int y'dt = \int \left(42t^2 + 24t + 11\right) dt$

$= 42\left(\frac{t^3}{3}\right) + 24\left(\frac{t^2}{2}\right) + 11t + C_2$

$= 14t^3 + 12t^2 + 11t + C_2$

Since $y(2) = 185$, we have

$185 = 14(2)^3 + 12(2)^2 + 11(2) + C_2$

$= 182 + C_2$, so $C_2 = 185 - 182 = 3$.

$y(t) = 14t^3 + 12t^2 + 11t + 3$

Problems 14.3

1. $\frac{dy}{dx} = 3x - 4$

$y = \int (3x - 4)dx = \frac{3x^2}{2} - 4x + C$

Using $y(-1) = \frac{13}{2}$ gives

$\frac{13}{2} = \frac{3(-1)^2}{2} - 4(-1) + C$

$\frac{13}{2} = \frac{11}{2} + C$

Thus $C = 1$, so $y = \frac{3x^2}{2} - 4x + 1$.

2. $\frac{dy}{dx} = x^2 - x$

$y = \int \left(x^2 - x\right) dx = \frac{x^3}{3} - \frac{x^2}{2} + C$

Using $y(3) = \frac{19}{2}$ gives $\frac{19}{2} = \frac{3^3}{3} - \frac{3^2}{2} + C$

$\frac{19}{2} = \frac{9}{2} + C$

Thus, $C = 5$, so $y = \frac{x^3}{3} - \frac{x^2}{2} + 5$.

3. $y' = \frac{9}{8\sqrt{x}}$

$y = \int \frac{9}{8\sqrt{x}}\,dx$

$= \frac{9}{8}\int x^{-1/2}dx$

$= \frac{9}{8} \cdot \frac{x^{1/2}}{\frac{1}{2}} + C$

$= \frac{9\sqrt{x}}{4} + C$

$y(16) = 10$ implies $10 = \frac{9\sqrt{16}}{4} + C$, $10 = 9 + C$,

$C = 1$. Thus $y = \frac{9\sqrt{x}}{4} + 1$.

$y(9) = \frac{9\sqrt{9}}{4} + 1 = \frac{9 \cdot 3}{4} + 1 = \frac{31}{4}$

4. $y' = -x^2 + 2x$

$y = \int \left(-x^2 + 2x\right) dx = -\frac{x^3}{3} + x^2 + C$

$y(2) = 1$ implies $1 = -\frac{8}{3} + 4 + C$, so $C = -\frac{1}{3}$.

Thus $y = -\frac{x^3}{3} + x^2 - \frac{1}{3}$.

$y(1) = -\frac{1}{3} + 1 - \frac{1}{3} = \frac{1}{3}$

5. $y'' = -3x^2 + 4x$

$y' = \int(-3x^2 + 4x)dx = -x^3 + 2x^2 + C_1$

$y'(1) = 2$ implies $2 = -1 + 2 + C_1$, so $C_1 = 1$.

$y = \int(-x^3 + 2x^2 + 1)dx = -\frac{x^4}{4} + \frac{2x^3}{3} + x + C_2$

$y(1) = 3$ implies $3 = -\frac{1}{4} + \frac{2}{3} + 1 + C_2$, so

$C_2 = \frac{19}{12}$. Thus $y = -\frac{x^4}{4} + \frac{2x^3}{3} + x + \frac{19}{12}$.

6. $y'' = x + 1$

$y' = \int(x+1)dx = \frac{x^2}{2} + x + C_1$

$y'(0) = 0$ implies $0 = 0 + 0 + C_1$, so $C_1 = 0$.

$y = \int\left[\frac{x^2}{2} + x\right]dx = \frac{x^3}{6} + \frac{x^2}{2} + C_2$.

$y(0) = 5$ implies $5 = 0 + 0 + C_2$, so $C_2 = 5$. Thus

$y = \frac{x^3}{6} + \frac{x^2}{2} + 5$.

7. $y''' = 2x$

$y'' = \int 2x\, dx = x^2 + C_1$

$y''(-1) = 3$ implies that $3 = 1 + C_1$, so $C_1 = 2$.

$y' = \int\left(x^2 + 2\right)dx = \frac{x^3}{3} + 2x + C_2$

$y'(3) = 10$ implies $10 = 9 + 6 + C_2$, so $C_2 = -5$.

$y = \int\left(\frac{x^3}{3} + 2x - 5\right)dx = \frac{x^4}{12} + x^2 - 5x + C_3$.

$y(0) = 13$ implies that $13 = 0 + 0 - 0 + C_3$, so

$C_3 = 13$. Therefore $y = \frac{x^4}{12} + x^2 - 5x + 13$.

8. $y''' = 2e^{-x} + 3$

$y'' = \int(2e^{-x} + 3)dx = -2e^{-x} + 3x + C_1$

$y''(0) = 7$ implies $7 = -2 + C_1$, so $C_1 = 9$.

$y' = \int(-2e^{-x} + 3x + 9)dx = 2e^{-x} + \frac{3x^2}{2} + 9x + C_2$

$y'(0) = 5$ implies $5 = 2 + C_2$, so $C_2 = 3$.

$y = \int\left(2e^{-x} + \frac{3x^2}{2} + 9x + 3\right)dx$

$= -2e^{-x} + \frac{x^3}{2} + \frac{9x^2}{2} + 3x + C_3$

$y(0) = 1$ implies $1 = -2 + C_3$, so $C_3 = 3$.

Thus $y = -2e^{-x} + \frac{x^3}{2} + \frac{9x^2}{2} + 3x + 3$.

9. $\frac{dr}{dq} = 0.7$

$r = \int 0.7dq = 0.7q + C$

If $q = 0$, r must be 0, so $0 = 0 + C$, $C = 0$. Thus $r = 0.7q$. Since $r = pq$, we have

$p = \frac{r}{q} = \frac{0.7q}{q} = 0.7$. The demand function is

$p = 0.7$.

10. $\frac{dr}{dq} = 10 - \frac{1}{16}q$

$r = \int\left[10 - \frac{1}{16}q\right]dq = 10q - \frac{1}{32}q^2 + C$

When $q = 0$, then $r = 0$, so $C = 0$ and

$r = 10q - \frac{1}{32}q^2$. Since $r = pq$, then

$p = \frac{r}{q} = 10 - \frac{1}{32}q$. The demand function is

$p = 10 - \frac{1}{32}q$.

11. $\frac{dr}{dq} = 275 - q - 0.3q^2$

Thus $r = \int\left(275 - q - 0.3q^2\right)dq$

$= 275q - 0.5q^2 - 0.1q^3 + C$. When $q = 0$, r must be 0, so $C = 0$ and $r = 275q - 0.5q^2 - 0.1q^3$.

Since $r = pq$, then $p = \frac{r}{q} = 275 - 0.5q - 0.1q^2$.

Thus the demand function is

$p = 275 - 0.5q - 0.1q^2$.

12. $\frac{dr}{dq} = 5000 - 3(2q + 2q^3)$, so

$r = \int (5000 - 6q - 6q^3)dq$

$= 5000q - 3q^2 - \frac{3q^4}{2} + C$

When $q = 0$, then $r = 0$, so $C = 0$ and

$r = 5000q - 3q^2 - \frac{3q^4}{2}$. Since $r = pq$, then

$p = \frac{r}{q} = 5000 - 3q - \frac{3q^3}{2}$. Therefore the demand function is $p = 5000 - 3q - \frac{3q^3}{2}$.

13. $\frac{dc}{dq} = 2.47$

$c = \int 2.47\,dq = 2.47q + C$

When $q = 0$, then $c = 159$, so $159 = 0 + C$, or $C = 159$. Thus $c = 2.47q + 159$.

14. $\frac{dc}{dq} = 2q + 75$

$c = \int (2q + 75)dq = q^2 + 75q + C$

When $q = 0$, then $c = 2000$, so $C = 2000$. Thus the cost function is $c = q^2 + 75q + 2000$.

15. $\frac{dc}{dq} = 0.08q^2 - 1.6q + 6.5$

$c = \int \left(0.08q^2 - 1.6q + 6.5\right)dq$

$\frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + C$. If $q = 0$, then $c = 8000$, from which $C = 8000$. Hence

$c = \frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + 8000$. If $q = 25$, substituting gives $c(25) = 8079\frac{1}{6}$ or \$8079.17.

16. $\frac{dc}{dq} = 0.000204q^2 - 0.046q + 6$

$c = \int (0.000204q^2 - 0.046q + 6)dq$

$= 0.000068q^3 - 0.023q^2 + 6q + C$

When $q = 0$, then $c = 15{,}000$, from which $C = 15{,}000$. The cost function is

$c = 0.000068q^3 - 0.023q^2 + 6q + 15{,}000$. When $q = 200$, substitution gives $c(200) = 15{,}824$.

17. $G = \int \left[-\frac{P}{25} + 2\right]dP = -\frac{P^2}{50} + 2P + C$

When $P = 10$, then $G = 38$, so $38 = -2 + 20 + C$, from which $C = 20$. Thus

$G = -\frac{1}{50}P^2 + 2P + 20$.

18. $\frac{dy}{dx} = -1.5 - x$

$y = \int (-1.5 - x)dx = -1.5x - \frac{x^2}{2} + C$

When $x = 1$, then $y = 59.6$, so $59.6 = -1.5 - 0.5 + C$, or $C = 61.6$. Thus

$y = -1.5x - 0.5x^2 + 61.6$ for $1 \le x \le 9$.

19. $v = \int -\frac{(P_1 - P_2)r}{2l\eta}dr = -\frac{(P_1 - P_2)r^2}{4l\eta} + C$

Since $v = 0$ when $r = R$, then

$0 = -\frac{(P_1 - P_2)R^2}{4l\eta} + C$, so $C = \frac{(P_1 - P_2)R^2}{4l\eta}$.

Thus

$v = -\frac{(P_1 - P_2)r^2}{4l\eta} + \frac{(P_1 - P_2)R^2}{4l\eta}$

$= \frac{(P_1 - P_2)\left(R^2 - r^2\right)}{4l\eta}$.

20. $\frac{dr}{dq} = 100 - 3q^2$

$r = \int \left(100 - 3q^2\right)dq = 100q - q^3 + C$

When $q = 0$, then $r = 0$, so $C = 0$ and $r = 100q - q^3$. Since $r = pq$, then

$p = \frac{r}{q} = 100 - q^2$.

$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{\frac{p}{q}}{-2q} = -\frac{p}{2q^2}$

When $q = 5$, then $p = 75$, so $\eta = \frac{-75}{2(25)} = -\frac{3}{2}$.

21. $\frac{dc}{dq} = 0.003q^2 - 0.4q + 40$

$c = \int \left(0.003q^2 - 0.4q + 40\right)dq$

$= 0.001q^3 - 0.2q^2 + 40q + C$

When $q = 0$, then $c = 5000$, so
$5000 = 0 - 0 + 0 + C$, or $C = 5000$. Thus $c = 0.001q^3 - 0.2q^2 + 40q + 5000$. When $q = 100$, then $c = 8000$. Since Avg. Cost $= \overline{c} = \frac{\text{Total Cost}}{\text{Quantity}} = \frac{c}{q}$, when $q = 100$, we have $\overline{c} = \frac{8000}{100} = \$80$. (Observe that knowing $\frac{dc}{dq} = 27.50$ when $q = 50$ is not relevant to the problem.)

22. $f''(x) = 30x^4 + 12x$

$f'(x) = \int (30x^4 + 12x)dx = 6x^5 + 6x^2 + C_1$

$f'(1) = 10,$ so $10 = 6 + 6 + C_1$ and $C_1 = -2.$

$f'(x) = 6x^5 + 6x^2 - 2$

$f(x) = \int (6x^5 + 6x^2 - 2)dx = x^6 + 2x^3 - 2x + C_2$

Thus

$$f(965.335245) - f(-965.335245)$$
$$= [(965.335245)^6 + 2(965.335245)^3 - 2(965.335245) + C_2]$$
$$-[(-965.335245)^6 + 2(-965.335245)^3 - 2(-965.335245) + C_2]$$
$$= 3{,}598{,}280{,}000$$

Apply It 14.4

8. Using the values given, $\frac{dT}{dt} = -0.5(70 - 60)e^{-0.5t} = -5e^{-0.5t}$

$$T(t) = \int \frac{dT}{dt}dt = \int -5e^{-0.5t}dt = 10e^{-0.5t} + C$$

9. The number of words memorized is $v(t)$.

$$v(t) = \int \frac{35}{t+1}dt = 35\ln|t+1| + C.$$

Problems 14.4

1. Let $u = x + 5 \Rightarrow du = 1dx = dx$

$$\int (x+5)^7[dx] = \int u^7 du = \frac{u^8}{8} + C = \frac{(x+5)^8}{8} + C$$

2. $\int 15(x+2)^4 dx = 15\int (x+2)^4[dx] = 15 \cdot \frac{(x+2)^5}{5} + C = 3(x+2)^5 + C$

3. Let $u = x^2 + 3 \Rightarrow du = 2x\ dx$

$$\int 2x\left(x^2+3\right)^5 dx = \int \left(x^2+3\right)^5 [2x\ dx] = \int u^5 du = \frac{u^6}{6} + C$$
$$= \frac{\left(x^2+3\right)^6}{6} + C$$