Channel coding for polarization-mode dispersion limited optical fiber transmission

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Abstract: We investigate numerically the usefulness of Turbo and Reed-Solomon coding in the presence of Polarization-Mode Dispersion (PMD) using computer simulations. It is demonstrated that for a fixed level of PMD and a fixed data-rate, there is an optimal code overhead. This is in contrast to the case of negligible PMD, where high overhead codes perform best.

OCIS codes: (060.2330) Fiber Optics Communications, (060.4510) Optical Communications, (260.1440) Birefrigence.

References and links

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1. Introduction

Polarization-Mode Dispersion (PMD) is one of the obstacles preventing transmission rates greater than 10 Gb/s in fiber optic communications [1]. PMD occurs when the cylindrical symmetry in a fiber is broken due to a noncircular core or a noncircular symmetric stress [2]. This will cause different polarizations to traverse down the fiber at different speeds, causing inter-symbol-interference (ISI).

Deployed fiber optic communication systems have utilized some form of forward error correction (FEC) coding to detect and correct errors. Generally, FEC codes add redundant bits so that, when decoded, a number of errors can be detected and/or corrected. The current fiber optics industry standard uses low overhead Reed Solomon (RS) codes[3]. The RS codes employed are capable of correcting multiple bit errors in every codeword while only adding 7% parity bits to incorporate the redundant information needed to correct errors. The advantage of RS codes is that they are well known and easy to implement, and are thus very practical for real world systems. The weakness of currently employed RS codes is their apparent inability to cope with moderate or high PMD. There have been studies [5] into the possibility of combining different types of RS codes in an iterative fashion to raise the tolerance level of differential group delay (DGD), but few studies have been made on more effective FEC techniques such as Turbo codes.

The performance of several different FEC codes is studied here via Monte Carlo simulations employing a realistic PMD channel model. In conventional systems, the impact of PMD is normally negligible if the average DGD per bit period is less than 10%. In the simulations conducted, the DGD per uncoded bit is varied from 0 to 103% of the bit period.

With everything else equal, low overhead codes are clearly desirable since lower overhead represents a more efficient use of available bandwidth. However, high overhead codes may be powerful enough to perform better than a lower overhead code if the useful data-rate (rate without overhead of parity bits) are kept the same by increasing the bit-rate of the high overhead code. This paper investigates low and high overhead RS and Turbo codes in the presence of varying levels of PMD.

2. PMD model

Based on the PMD vector concatenation rule and the principle states model (PSP) [2][4][6], a PMD-limited optical channel can be modeled using a series of linear birefringent elements that are sandwiched between polarization adjustments. This model characterizes all orders of PMD, not only the first order [6]. Signal propagation along the polarization modes of the fiber is modeled in Jones space [2].

The transfer function of the PMD channel can be expressed by transmission matrix T(ω) as in (1).

$$T(\omega)=U(\alpha_N)\begin{bmatrix} e^{-j\tau_N\omega/2} & 0 \\ 0 & e^{j\tau_N\omega/2}\end{bmatrix}U(\alpha_{N-1})\begin{bmatrix} e^{-j\tau_{N-1}\omega/2} & 0 \\ 0 & e^{j\tau_{N-1}\omega/2}\end{bmatrix}U(\alpha_{N-2})..U(\alpha_1)\begin{bmatrix} e^{-j\tau_1\omega/2} & 0 \\ 0 & e^{j\tau_1\omega/2}\end{bmatrix}U(\alpha_0) \quad (1)$$

In eq. (1), U(α) is the rotation matrix, denoted as

$$\mathrm{U}(\alpha_i) = \begin{bmatrix} \cos(\alpha_i) & \sin(\alpha_i) \\ -\sin(\alpha_i) & \cos(\alpha_i) \end{bmatrix} \quad (2)$$

and τ_i is the delay of the i^{th} birefringence element while α_i is the rotation angle between two adjacent retarders. The worst case launching state of polarization is assumed for all transmissions, i.e. orthogonal to the PSPs. The receiver bandwidth is kept at a fixed 50% of the actual bit-rate including the overhead. The SNR is scaled accordingly based on the signal bandwidth assuming the noise is white.

The SNR is defined as the ratio of the squared detected signal current to the noise variance in absence of PMD. Accordingly, the formula for the uncoded bit error rate (BER) in the absence of PMD is

$$\mathrm{BER} = \frac{1}{2} erfc(\frac{\sqrt{SNR}}{2\sqrt{2}}) \quad (3)$$

The estimated BER values were calculated by simulating 900 randomly generated frames of length 4000 bits and averaging the results. Five RS codes with 61%, 33%, 14%, 10%, and 6% overhead as well as six Turbo codes with 100%, 67%, 43%, 25%, 11%, and 5% overhead were simulated. The Turbo codes used an iterative maximum a posteriori (MAP) decoder with 3 iterations with a (023,027) generator polynomial. The Turbo code frame length was 4000 bits.

The time duration of the bit period was adjusted for different overhead codes to maintain a fixed useful data-rate of 10Gb/s. DGD values of 0, 55.2, and 103ps (corresponding to 0, 55.2, and 103% of the uncoded data bit period) were considered. We present the results for the Turbo code with all DGD values and the RS code for DGD of 55.2ps.

A significant performance drawback of Turbo codes for optical communication can be the presence of the error floor. This could be resolved by using an outer low overhead RS code to correct the error floor and obtain even lower BER [7]. This paper assumes that this will not be an inhibition to the performance of Turbo codes.

3. Numerical results

The nature of our comparisons is somewhat different from what may be done in the communication theory community since we allow comparison of different code overheads when the useful data rates are kept constant. It is typically expected that higher overhead codes outperform lower overhead codes in the same channel. This is because higher overhead codes employ more parity bits and are capable of correcting more errors. However, to assure fairness in our tests the length of a bit period was decreased for the higher overhead codes to force all codes to transmit the same amount of information bits per time interval. The end result is that higher overhead codes see a larger amount of DGD per bit period than lower overhead codes, resulting in an optimal amount of overhead that is not necessarily high.

The BER performance of single-channel optical systems with various amount of static PMDs and noises were simulated. NRZ modulation was used in the simulations. The coding performance on time-varying PMD channels is beyond the scope of this paper. Fig. 1 contains the Monte Carlo BER simulation results for the set of four lowest overhead Turbo codes over the channel with a DGD of 55.2ps. The DGD divided by the length of a bit gives a measure of how much inter-symbol interference (ISI) the channel has. Both the 5% overhead and 25% overhead codes perform worse then the 11% overhead at a BER between 10^{-2} and 10^{-5}, indicating that the optimal overhead is close to 11% for this Turbo code and DGD level. The Turbo decoder was optimized for an AWGN channel; had it been optimized for a high PMD channel, we would expect better performance.

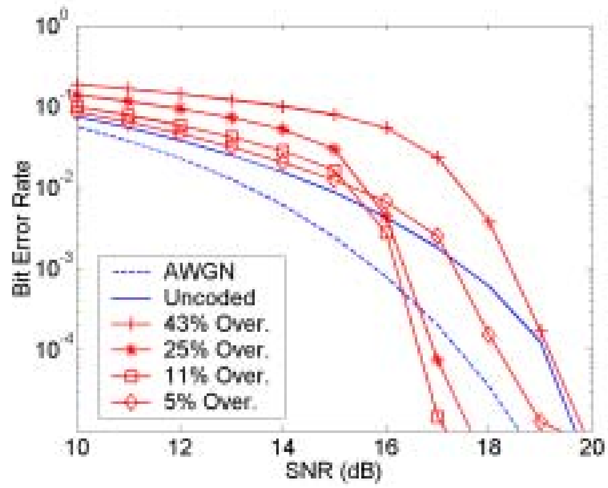

Fig. 1. BER vs. SNR in absence of PMD for Turbo codes in a PMD channel with DGD = 55.2ps

Fig. 2 shows the performance of the RS codes for similar case. In general, the RS codes perform worse than Turbo codes, which might be expected from similar comparisons for additive white Gaussian noise (AWGN) channels. However, the performance gain that Turbo codes show over RS codes in an AWGN channel is reduced to only a few dB following the introduction of DGD. It is apparent that the 5% overhead RS code is the best RS code for this channel by a very small margin. The 25% and 11% overhead RS codes are less than 0.5 dB below the optimal RS code. Comparing the optimal RS code to the optimal Turbo code in the previous Fig., the Turbo code is better by a margin of 0.6 dB at a BER of 10^{-5}.

Fig. 3 shows the performance of the set of four Turbo codes for a channel with 0ps DGD, which is equivalent to the AWGN case. As expected from traditional AWGN studies, higher overhead codes perform better. It is clear from Fig. 3 that for sufficiently small DGD, as

DGD is decreased the optimal overhead will increase. For small DGD values, we expect high overhead codes will be the optimal choice.

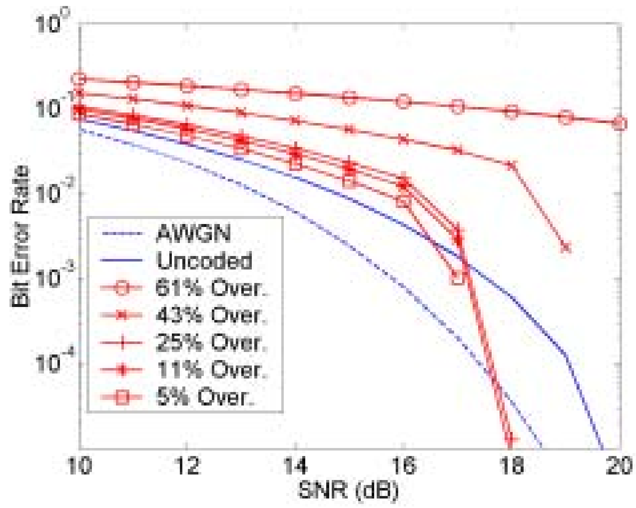

Fig. 2. BER vs. SNR in absence of PMD for RS codes in a PMD channel with DGD = 55.2ps

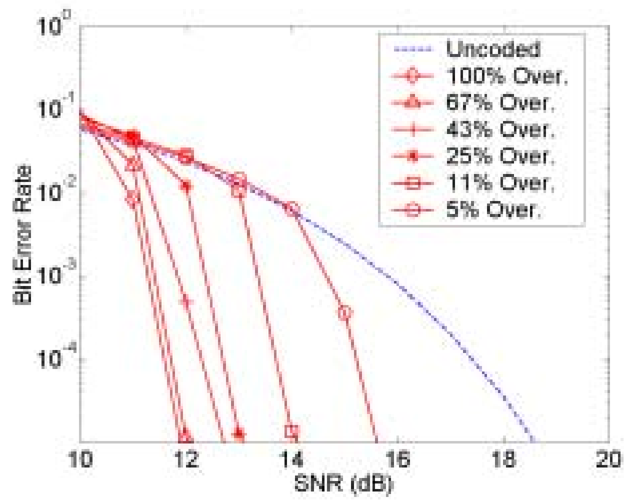

Fig. 3. BER vs. SNR in absence of PMD for Turbo codes in a PMD channel with DGD = 0ps

It is expected that at a certain level of sufficiently high DGD no amount of error correction will be capable of compensating for the ISI. To verify this point, simulations for a DGD of 103ps were conducted and the results are given in Fig. 4. The 5% and 11% overhead Turbo codes have merged with the uncoded data, offering no gain as expected. The high overhead codes offer some gain, but only at very high SNR values. The best code now uses an overhead of 67% for this DGD value. As DGD is increased even further, these higher overhead codes are also expected to merge with the uncoded data. The shift in the optimal code to higher overheads is similarly demonstrated in Fig. 5 with RS codes in the high DGD channel. The optimal RS code overhead is now 61%, the highest overhead simulated

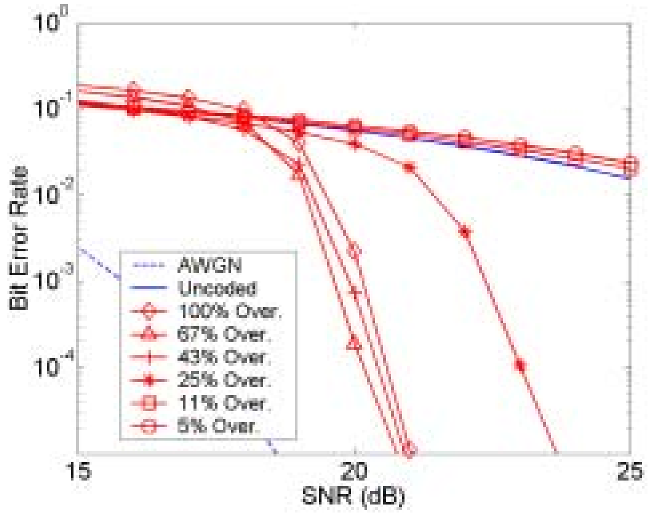

Fig. 4. BER vs. SNR in absence of PMD for Turbo codes in a PMD channel with DGD = 103ps

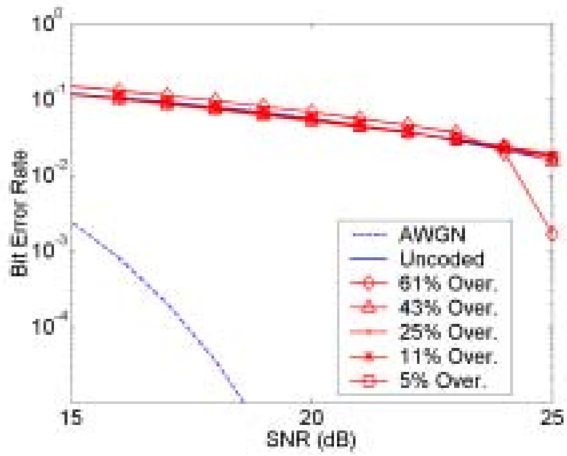

Fig. 5. BER vs. SNR in absence of PMD for RS codes in a PMD channel with DGD = 103ps

From these results it is clear that there is a predictable relationship between the amount of DGD in a channel and the optimal overhead for that channel. For any FEC in an entirely AWGN limited channel, higher overhead codes outperform lower overhead codes. For low DGD channels, we expect the same relationship between overhead and performance because this must converge to the AWGN case when there is no DGD. As DGD increases, the optimal overhead will shift from high overhead codes to lower overhead codes. When DGD is sufficiently high, the low overhead codes will no longer be able to compensate for the high ISI they experience, and the optimal overhead shifts back to higher overhead codes. As DGD reaches the upper limit, no overhead will be able to compensate for the DGD. These trends are illustrated in Fig. 6, which compares the SNR required to reach a BER of 10^{-4} at each overhead for the three different DGD cases.

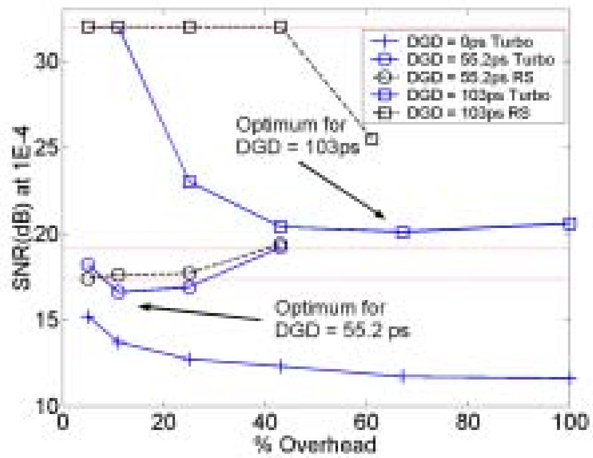

Fig. 6. Minimum required SNR to reach BER = 10^{-4} vs. overhead for different amounts of DGD. The dotted lines represent the uncoded SNR required for the three DGD levels, increasing from bottom to top.

The difference in complexity between RS and Turbo decoders is significant. The RS decoder uses hard decision and can be implemented in digital hardware very efficiently. The Turbo decoder manipulates soft decision data that naturally requires more memory and computation time. The Turbo decoder is also an iterative decoding scheme, again raising the complexity. There is potential for Turbo or Turbo-like codes to improve the performance of PMD impaired channels in the future as electronics become faster and smaller, when the additional complexity is not an issue.

4. Conclusion

We have studied several types of FEC codes for use on a PMD limited channel via BER simulations. Turbo codes offer a small amount of gain over RS codes in the presence of PMD at a BER above 10^{-5}. For medium levels of DGD, an optimal code overhead exists where the maximum performance can be obtained. As DGD is increased, higher overhead codes will offer more gain than lower overhead codes until the amount of DGD is such that no FEC can compensate the errors effectively. The simulations in this paper were completed with Turbo codes, but the same conclusions regarding choosing the optimal overhead code in a high DGD channel appear to apply to many types of FEC, including RS codes.

5. Acknowledgment

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