Routing Overhead as A Function of Node Mobility: Modeling Framework and Implications on Proactive Routing

[1]Xianren Wu, [1]Hamid R. Sadjadpour and [1,2]J.J. Garcia-Luna-Aceves
[1]University of California at Santa Cruz, Santa Cruz, CA 95064, USA, email: {wuxr,hamid}@soe.ucsc.edu
[2]Palo Alto Research Center, 3333 Coyote Hill Road, Palo Alto, CA 94304, USA, email: jj@soe.ucsc.edu

***Abstract*—The paper presents a mathematical framework for quantifying the overhead of proactive routing protocols in mobile ad hoc networks (MANETs). We focus on situations where the nodes are randomly moving around but the wireless transmissions can be decoded reliablely, when nodes are within communication range of each other. We explicitly present a framework to model the overhead as a function of stability of topology and analytically characterize the statistical distribution of topology evolutions. The OLSR protocol is further singled out for a detailed analysis, incorporating the proposed analytical model. Results are compared against *Qualnet* simulations for random movements, which corroborate the essential characteristics of the analytical results. The key insight that can be drawn from the analytical results of this paper is that nodal movements will drive up the overhead by a penalty factor, which is a function of the overall stability of the network.**

I. INTRODUCTION

Mobility brings fundamental challenges to the design of protocol stacks for mobile ad hoc networks (MANET). Because of nodes' movements, routing protocols (e.g OLSR [1], TORA [2]) of MANETs have to cope with frequent topology evolutions and ensure quick response and adaptation to topology changes. By continuously monitoring topology changes and disseminating such information over the whole network, proactive protocols provide fast response to topology change but at the price of increased overhead of control traffic. Increasing control traffic could further lead into less packet delivery ratio and increase in delay. Under the worst case, it could result in "broadcast-storm" [3] problem and the whole network will be congested. It is thus essential to understand the intricate relations between routing overhead and topology evolutions, for the design of routing protocols in MANETs.

Due to the inherent complexities, simulation-based approaches [4], [5], [6], [7], [8] have been the major tool to analyze the performance (routing overhead, packet delivery ratio, delays) of MANETs in terms of mobility, power and optimum transmission radios. Few analytical works have been pursued which bring deeper insights and complement simulation studies. Zhou et. al [9] gave an analytical view of routing overhead of reactive protocols, assuming static network (manhattan grid) with unreliable nodes and concludes the scalability of reactive protocols with localized traffic pattern. Topology evolutions from nodes' mobility was not considered in [9]. In [10], information theoretic analysis is pursued to bound the memory requirement and overhead for a hierarchical protocol in MANET upon entropy rate of topology evolutions. Such results provide good understanding of the scalability properties of routing overhead but are not sufficient to justify the practical impact from topology evolutions. However, to the best of our knowledge, there is no previous work that makes such analytical connection between routing overhead and topology evolutions. Moreover, none of the past work has analytically characterized topology evolutions as a function of node mobility, a solution of which is crucial to make the connection.

In this paper, we provide the first attempt to provide answers to these questions with a general analytical framework for proactive routing protocols, where the inter-dependence between topology evolutions and routing overhead is explored and quantitative measures are provided to justify the routing overhead as a function of node mobility. In the meanwhile, we also provide the answer to the characterization of topology evolution, by explicitly modeling it as a function of node mobility and deriving out analytical parameters. Clearly, such results will supplement the information theoretic analysis of [10] by providing entropy rate and model of topology evolutions.

We further take a practical view on optimized link state routing (OLSR) protocol [1] under the general framework, with the understanding of the significance and practicalness of OLSR protocol. Such an analysis not only gives us better insight to the operation of OLSR algorithm but also corroborates the effectiveness of the general modeling framework. Analytical results in this paper are compared against *Qualnet* simulations for random movements, which confirm the essential characteristics of the analytical results. The key insight that can be drawn from the results is that mobility will drive up the overhead by a penalty factor, which is a function of the overall stability of the network.

The rest of the paper is organized as follows. Section II breifly describes the network model and presents the problem of interest. Section III explains the general framework of modeling proactive routing overhead, followed by Section IV to discuss properties of topology and factors that affect the stability of topology. Section V presents a detailed proposition on exponentially modeling topology evolutions and demonstrates the mathematical evaluation of the model parameters. Section VI applies the analytical models for an in-depth analysis on routing overhead of OLSR protocol. Finally, we

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conclude this paper in Section VII.

II. System Model & Problem Statement

We consider a square network consistent with several prior analytical models of MANETs [11], [12], [13]. The entire network is of size $L \times L$ and there are n nodes initially randomly deployed in the square network. It should be noted that although we consider a square network in the paper, our analysis can be extended to networks of any shape in a straightforward way.

Nodes are mobile and initially randomly distributed over the network. The movement of each node is independent and unrestricted, i.e, the trajectories of nodes can be anywhere in the network. For node $i \in V = \{1, 2, \ldots, N\}$, let $\{T_i(t), t \geq 0\}$ be the random process representing its trajectory and take values in D, where D denotes the domain across which the node moves. To further impose the modelablility constraint, we have the following assumption on the trajectory processses.

Assumption 1: **[Stationarity]** Each of the trajectory process $T_i(t)$ is stationary, i.e., the spacial node distribution will reach its steady-state distribution irrespective of the initial location. And the N trajectory processes are *jointly stationary*, i.e., the whole network will eventually reach the same steady state from any initial node placements, within which the statistical spatial nodes' distribution of the network remain the same over time.

The assumption is quite fundamental in the sense that it lays the foundation of modeling nodes' movement. Most existing models, e.g. random direction mobility models [14], [15], [16], [17], [18], random waypoint mobility models [19], [20] as well as random trip mobility model [21], clearly satisfy the assumption. In other words, the assumption ensures that on the long run, the network will converge to its steady state and the stationary spatial nodes' distribution can be utilized in performance analysis of the network.

The availability of communication links, e.g. from node i to node j, is governed by the Signal-to-Interference-plus-Noise Ratio(SINR) protocol model as,

$$\frac{P_i(t)g_{ij}(t)}{N_0 + \sum_{k \in A_s(t), k \neq i} P_k(t)g_{kj}(t)} \geq \beta \tag{1}$$

where $P_i(t)$ denotes the transmitting power of node i at time t, $A_s(t)$ is the set of active nodes transmitting at time t, N_0 denotes the thermal noise and β is the minimum SINR for the receiver to successfully decode data packets. $g_{kl}(t)$ represents the channel gain from node k to node l at time t, capturing path loss, fading and shadowing effects in the wireless environment. Eq. (1) states the physical requirement of the existence of a directional link from node i to node j at time t. Since many routing algorithms necessitate the availability of bi-directional links, we should also expect the SINR law being satisfied for the reverse link, e.g. $j \rightarrow i$. We simply term bi-directional link as link throughout the paper.

The topology (or connectivity graph) $\mathcal{G}(t)$ of the network at time t can be obtained by replacing such available links with lines connecting nodes. In the paper, we use topology and connectivity graph interchangeably. Routing algorithms usually operate upon the connectivity graph and topology evolutions (or changes in the connectivity graph) could further trigger routing protocols to react to the change by disseminating control packets. As a result, distribution of topology evolutions is heavily related to control overhead of routing protocols.

However, there has been little work in the literature to analytically study such distribution of topology evolutions in MANETs. We attempt to provide the first (to the best of authors' knowledge) analytical work on modeling of topology evolutions in MANETs and make the practical connection between topology evolutions and proactive routing overhead. To summarize, we would like to seek answers for the following questions:

- *Does there exist an analytical model to statistically characterize the distribution of topology evolutions in MANETs? If so, are we able to derive the parameters analytically?*
- *If there is such a model, are we able to apply the model to analyze the effect of mobility on the control overhead of proactive routing protocols? Or mathematically, could we find the function $\mathcal{F}$ that projects the control overhead $\mathcal{O}_d$ in MANETs with the knowledge of mobility $\mathcal{V}$ and control overhead $\mathcal{O}_s$ of protocol at static scenarios?*

$$\mathcal{F} : \mathcal{O}_s \times \mathcal{V} \rightarrow \mathcal{O}_d \tag{2}$$

III. Proactive Routing Overhead of Dynamic Graph

We know that routing protocol operates on the connectivity graph (topology) $\mathcal{G}$. Let $\vec{\mathcal{G}} = \{\mathcal{G}_i\}$ be the set of all possible connectivity graphs. In the steady-state, the connectivity graph $\mathcal{G}(t)$ will travel across all such graphs with a stable distribution vector $\vec{p} = \{p_i\}$ derived from the stationary spatial nodes' distribution. Change that occurs in the connectivity signals the transition between connectivity graphs or topology evolution. However, if we look at the connectivity graph from a single node point of view, for an active node, it observes the local connectivity graph (local detailed topology) derived from neighbors that could be several hops away. Only changes in its local connectivity graph could trigger the node reacting to the change. Let's observe protocol behavior at a typical active node k. From $\vec{\mathcal{G}}$, we can derive $\vec{\mathcal{G}^k} = \{\mathcal{G}_i^k\}$ as the set of all possible local connectivity graphs with corresponding distribution vector $\vec{p^k} = \{p_i^k\}$.

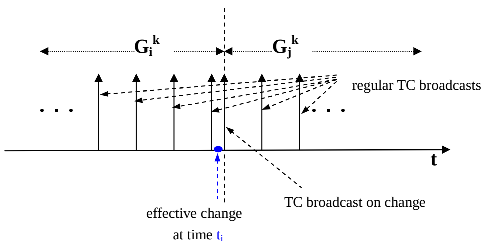

Fig. 1. Protocol behaviors with local connectivity graphs.

As illustrated in Fig. (1) and when there is no change in topology, we expect that the node will periodically broadcast topology control (TC) message at regular interval T_c. For this case, the average TC messages per active node at static scenarios $\mathcal{O}_s$ is simply

$$P(\mathcal{O}_s) = P(\mathcal{G}_i^k) = 1/T_c, \forall i \tag{3}$$

If we assume that a change happens at time $t_i, KT_c < t_i \leq (K+1)T_c$, signaling the transition of local connectivity graph from $\mathcal{G}_i^k$ to $\mathcal{G}_j^k$. The protocol usually responds to the change by advancing the TC message broadcast at some time $t_i^*, KT_c < t_i^* \leq (K+1)T_c$ rather than broadcast at the next planned time $(K+1)T_c$. And the subsequent TC message broadcast will perform regularly with graph $\mathcal{G}_j^k$. In this case, compared to the static scenario where no change occurs, the increase $\gamma_i(t)$ in generated TC message associated with $\mathcal{G}_i^k$ can be computed as

$$\gamma_i(t_i) = \frac{(K+1)}{t_i^*} / \frac{K+1}{(K+1)T_c} = \frac{\lceil t_i^*/T_c \rceil}{t_i^*/T_c} \tag{4}$$

where $\lceil \cdot \rceil$ is the ceiling operator. The average increase γ_i in geneated TC messages with the graph $\mathcal{G}_i^k$ can be computed as

$$\gamma_i = E_{t_i}(\frac{\lceil t_i^*/T_c \rceil}{t_i^*/T_c}) \tag{5}$$

Statistically, γ_i measures the normalized transition cost for $\mathcal{G}_i^k$ and t_i^* is determined by t_i that captures the stability of the local detailed topology $\mathcal{G}_i^k$. Summing over all possible topologies, we can estimate the average number of generated TC message per active node as

$$P = \sum_{\forall i} p_i^k P(\mathcal{G}_i^k) * \gamma_i \tag{6}$$

As we will see in Section V, if we are only concerned with nodal mobility and since nodes are moving independently and randomly, we could assume that link change arrives independently and $\{t_i\}$ are of identical statistical distributions, being a renewal process. We have

$$P = \gamma \times \sum_{\forall i} p_i^k P(\mathcal{G}_i^k) \tag{7}$$

$$\gamma = E(\frac{\lceil \zeta^*/T_c \rceil}{\zeta^*/T_c}) \tag{8}$$

where ζ^* is decided on ζ and ζ is the observed stability of the local connectivity graph per active node. γ is the *penalty factor* that measures the cost in graph transitions for an active node and as we will see later, it is a function of nodal mobility and stability of the local connectivity graph. Furthermore, a closer look at Eq. (7) reflects that the increased traffic overhead can be estimated from the average performance of static graphs, that is exactly the right term in the equation.

In a homogeneous network, every node in the network operate in a similar way. Therefore, we can expect similar results on the whole network. Eventually, we propose the following model that estimates the control traffic overhead from the knowledge of mean overhead $\mathcal{O}_s$ of static scenarios. Mathematically, we can write it as the tentative answer for the question raised in Section II as

We could have a function $\mathcal{F}$ that projects the control overhead $P(\mathcal{O}_d)$ in MANETs with the knowledge of mobility $\mathcal{V}$ and control overhead $P(\mathcal{O}_s)$ of protocol at static scenarios. And the function can be written as,

$$\mathcal{F} : P(\mathcal{O}_d) = \gamma(\mathcal{V}) * P(\mathcal{O}_s) \tag{9}$$

However, we need to know the distribution of topology evolutions (t_i in Eq. (4) for the computation of mobility effect on proactive routing overhead. To bring up such a model, we will first discuss factors that affect the stability of topology and then propose analytical model for topology evolution.

IV. Topology: Factors for Changes

A. Setup

Due to nodes' movements and surrounding parallel transmissions, links between nodes are set up and broken dynamically. We introduce a $\{0, 1\}$-valued on-off process $f_{ij}(t), t \geq 0$ to model such link changes as $f_{ij}(t) = 1$ (or $f_{ij}(t) = 0$) if the unidirectional link from node i to node j, is available (or unavailable) at time $t \geq 0$. Clearly, we have $f_{ij}(t) = f_{ji}(t)$ because we only consider bi-directional links.

If we map every active (on) link to a edge in a graph with N vertices where each vertices stands for a node in V, we can obtain the time-varing graph (topology) $\mathcal{G}(t)$ with a time-varying set $E(t)$ of edges as

$$E(t) := \{\{i, j\} \in V \times V, i \neq j; f_{ij}(t) = 1\} \tag{10}$$

It should be noted that $\mathcal{G}(t)$ is the connectivity graph of the network, which is an *undirected* graph as we consider bi-directional links. Let E be the complete set of possible links in the graph, i.e,

$$E := \{\{i, j\} \in V \times V, i \neq j\} \tag{11}$$

The complimentary set $E^c(t)$ of $E(t)$ can be computed as

$$E^c(t) = E - E(t) \tag{12}$$

For each link change such as new link formation or breakage of existing links, both will result in a change in the connectivity graph and could further result in a protocol event in the network to distribute such change.

Let τ be the moment that the connectivity graph $\mathcal{G}(t)$ changes at time $t+\tau$ from its last change at time t. Clearly, τ is the random variable describing the duration of stability of the connectivity graph $\mathcal{G}(t)$. In general, there are two different scenarios responsible for changes of $\mathcal{G}(t)$. One is the creation or arrival of new links, let τ_o be the random variable capturing the time duration of such new link arrivals or addition of new edge in $\mathcal{G}(t)$; Complementary, one will have another one random variable τ_f, characterizing the breakage of existing links or deletions of edges in $\mathcal{G}(t)$. We will have

$$\tau = \min\{\tau_o, \tau_f\} \tag{13}$$

Our objective is at first to identify the factors that affect the stability τ of the connectivity graph $\mathcal{G}(t)$ and then find the analytical model that characterizes the statistical distribution of τ.

B. Factors in Connectivity Graph

It can be observed that in Eq. (1), the availability of links not only depends on the wireless environment (captured in channel gain $g_{kl}(t)$ but also relies on the traffic and MAC schemes which together decide the active set of transmitting nodes $A_s(t)$. If we do not explicitly model the shadowing effect and short-term channel variations such as channel fading between nodes, it is reasonable to assume that the channel gain can be computed according to the exponential attenuation model as,

$$g = r^{-\alpha} \tag{14}$$

where r denotes the Euclidean distance between two communicating nodes and α is the exponential attenuation coefficient, normally ranging from 2 to 5 with various wireless environments.

By introducing a dynamic and sometimes intractable active set $A_s(t)$, the involvement of traffic and MAC schemes significantly complicates the problem with a dynamic varying interference term. Such a term, resulting from surrounding traffics and parallel transmissions, is called *environmental mobility* here.

When MAC is perfectly scheduled, the interference will become negligible compared to the noise and can be considered zeros, i.e, no environmental mobility. In such cases, the deciding factors for link availability lies in the transmission power and radio propagation loss and it can be expressed as,

$$\frac{P_i(t)g_{ij}(t)}{N_0} \geq \beta \quad and \quad \frac{P_j(t)g_{ji}(t)}{N_0} \geq \beta \tag{15}$$

When all nodes transmit at uniform power and together with Eq. (14), link between two nodes becomes available as soon as they become reachable, i.e., their Euclidean distance gets closer than some value R, that is the maximum radio coverage for a transmitting node. Obviously, the availability of such links is purely a function of their relative distances, that is governed by nodes' movements or *nodal mobility*.

Till now, we have identified two factors that affect the connectivity graph, i.e., *environmental mobility* and *nodal mobility*. The analytical models for the two factors are essential to the model of stability of the connectivity graph. However, the defining feature of MANETs is from *nodal mobility*, that is the natural result from nodes' dynamic movements. Furthermore, an analytical model of *nodal mobility* effect on the connectivity graph is by far not available. For the reason, we focus on analytical modeling of topology evolutions from *nodal mobility* in MANETs.

V. TOPOLOGY: MODEL OF NODAL MOBILITY

Nodes' motion will change the distance and therefore results in dynamic set-up and torn-down of links. When compared to the SINR law in Eq. (1), links defined in Eq. (15) are longer and stand for the maximum possible duration of link availability by solely considering mobility effect. In practice, the traffic and MAC schemes (*environmental mobility*) operate with scheduling and/or contention resolution schemes, which generally leads to less utilization of links. For example, a global time-division-multiple-access (TDMA) scheduling scheme together with Eq. (1) result the model in Eq. (15) but the utilizations of links are much less frequent due to the scheduling in channel access.

For each link in set $E(t)$, let $T^o_{ij}(t)$ denote the *residual* lifetime of the link after time t, i.e., $T^o_{ij}(t)$ is the amount of the time that elapses from time t onward until link is unavailable. Correspondingly, for each link in set $E^c(t)$, we have $T^f_{ij}(t)$ be the *residual* silence time of link after time t, i.e., $T^f_{ij}(t)$ is the amount of the time that elapses from time t onward until a link is available. Due the underlying stationarity implied from the joint stationarity of trajectory processes, it clearly suffices to consider only the case $t = 0$ and we can simply drop t here as we do from now on, e.g, T^o_{ij} instead of $T^o_{ij}(t)$. Clearly, we have

$$\tau_o = \min\{T^o_{ij} \text{ of link } \{\text{i,j}\}, \forall\{i,j\} \in E(t)\} \tag{16}$$

$$\tau_f = \min\{T^f_{ij} \text{ of link } \{\text{i,j}\}, \forall\{i,j\} \in E^c(t)\} \tag{17}$$

For each link $\{i,j\}$, the associated link availability process $f_{ij}(t); t \geq 0$ is simply an on-off process, with successive ups and downs with associated time durations, denoted by random variables $f_{ij}(k); k = 1, 2, \ldots$ and $f_{ji}(k); k = 1, 2, \ldots$, respectively. Such a processes can also be obtained from nodes' relative trajectories. When only nodal mobility is concerned, by Eq. (15), a link between nodes i and j in V is available at time $t \geq 0$ if and only if their distance is smaller than R. As a result, the link availability is defined as

$$f_{ij}(t) := 1[\|T_i(t) - T_j(t)\| \leq R]; t \geq 0, \tag{18}$$

where $\|\cdot\|$ denotes the Euclidean operator to compute the distance.

Let $Z(t) = \sum_{\forall\{i,j\}} f_{ij}(t)$ and it is clear that $Z(t)$ is a renewal process comprised from a total number of $|E|$ on-off link availability processes, where $|\cdot|$ is the cardinality operator. Clearly, τ describes the refreshing interval, τ_o specifies the interval between upward renewals and τ_f denotes the interval between downward renewals of the renewal process $Z(t)$. By applying the well-known results from renewal processes and independent on-off processes in equilibrium [22],we have following theorem on τ,

Theorem 1: **[Stability Model]** When both sets $E(t)$ and $E^c(t)$ involves sufficient number of links and all such links are assumed to be independent, the distribution of τ_o and τ_f can be approximated as exponentially distributed with parameter λ_o and λ_f. And the distribution of stability τ of the connectivity graph is also exponentially distributed with parameter $\lambda = \lambda_o + \lambda_f$. Mathematically, we can write is as

$$P(\tau_o \leq t) = 1 - e^{-\lambda_o t} \tag{19}$$

$$P(\tau_f \leq t) = 1 - e^{-\lambda_f t} \tag{20}$$

$$P(\tau \leq t) = 1 - e^{\lambda t} = 1 - e^{-(\lambda_o + \lambda_f)t} \tag{21}$$

It is also known as Palm's theorem [22] and in another words, it states that the distribution of a superposition of N_r i.i.d random variables will converge to the exponential distribution as N_r approaches infinite. The above results can be generalized to incorporate cases of independent but non-homogeneous motions, where some nodes may follow different mobility models from others. It might be worthy of noting that Palm's theorem has been applied to evaluate asymptotic distribution of route lifetime duration [23] in ad hoc networks, that are also empirically observed in [24]. However, to our knowledge, a model for topology evolutions is still not available both empirically and theoretically.

In MANETs, one might be wondering that since neighbor links share a common node, we cannot make the independent assumption on links and Palm's theorem cannot be applied. However, if the nodes' movements satisfy some *mixing condistions* or known as *m-dependence* [25], the statement in Theorem (1) still holds on such relaxed conditions. Such conditions introduce a form of asymptotic independence as the hop distance between links increases, while allowing dependence in neighborhoods. Specifically, *m-dependence* means that the correlation between links decreases as hop-distance between links increases and links can be considered as independent when the hop distance between links are greater than certain value of m. Fortunately, most of mobility models fall in this category, e.g. random waypoint mobility model, random direction mobility model and random trip mobility model and our results can be applied to a wide-variety of scenarios in MANETs.

A. *Relations between λ_o and λ_f*

Till now, we have learned that both the new link formation process and link breakage process can be approximated by Poisson process with parameters λ_f and λ_o, respectively. For the new link formation process (or the link breakage process), λ_f (or λ_o) characterizes the average number of new link arrivals (or link breakages). Let's consider a time window T and when T is sufficiently large, the number of new link arrivals N_a and link breakages N_b within the time window can be approximated by

$$N_a = \lambda_f * T \tag{22}$$

$$N_b = \lambda_o * T \tag{23}$$

For a network with finite number of nodes and when observed at an infinite length of time window, the difference of the number of new link arrivals and link breakages can be denoted by

$$\lim_{T\to\infty}(N_a - N_b) = \lim_{T\to\infty} T * (\lambda_f - \lambda_o). \tag{24}$$

Clearly, the only choice is

$$\lambda_f = \lambda_o. \tag{25}$$

It indicates that on the long run, the new link arrival process should be balanced off by the link breakage process. Otherwise, it contradicts with the fact that the network only involves finite number of nodes.

B. *Analytical Evaluation of λ_f or λ_o*

We understand that if we know the parameter for one of the two processes, we can infer the other one. The link breakage process is characterized by the distribution of residual link life time, a direct evaluation of which requires exact knowledge of the underlying mobility characteristics, and it is clearly not favorable. Luckily, we can have general statements on the underlying new link formation process, resorting to the exponential modeling with parameter λ_l of point-to-point link formation in [26].

For a particular connectivity graph $\mathcal{G}_i$ with associated sets E_i and E_i^c, there is a total number of $|E_i^c|$ potential point-to-point links to create. Since the time distribution of new link formation can be modeled as exponentially distributed with parameter λ_l, the stability for this particular connectivity graph can be measured with parameter

$$\lambda_f(\mathcal{G}_i) = |E_i^c| * \lambda_l \tag{26}$$

When network is running at steady-state and inferring from the joint stationarity assumption of underlying trajectory processes, $\mathcal{G}(t)$ is a stationary and ergodic process that will experience all possible connectivity graphs with associated probability vector derived from steady-state nodes' distribution. By averaging all possible graphs, we can compute the parameter λ_f as

$$\lambda_f = E(|E_i^c|) * \lambda_l \tag{27}$$

where $E(\cdot)$ stands for expected value.

A general model of MANETs in steady-state exists and known as *random geometric graph* [27], that has been widely adopted in analytical works of MANETs and considered as an improvement over the model of *random graph* in static networks. Using the model of *random geometric graph*, we can compute λ_f as

$$\lambda_f = \bar{N_f} * \lambda_l \tag{28}$$

where $\bar{N_f}$ is the average number of potential link pairs and it can be computed as [27]

$$\bar{N_f} = \frac{N*(N-1)}{2} * (1 - \frac{\pi R^2}{L^2}) \tag{29}$$

To summarize, we eventually arrive at the following theorem on the distribution of the stability τ of the connectivity graph

Theorem 2: **[Analytical Stability Model]** The distribution of stability τ of the connectivity graph in MANETs can be approximated as exponentially distributed with parameter λ and the parameter λ is given by

$$\begin{aligned}\lambda &= N*(N-1)*(1-\frac{\pi R^2}{L^2}) \\ &* 2E[V_*]R\int_0^L\int_0^L \pi^2(x,y)dxdy.\end{aligned} \tag{30}$$

where $\pi(x, y)$ denotes the steady-state spatial nodes' distribution and $E[V_*]$ is the average relative velocity.

C. *Model Validations*

There are a total of 100 nodes randomly placed for each $1000m \times 1000m$ square cell. Each node has the same transmit power and the radio transmission range considered is $250m$, that is the nominal coverage of IEEE 802.11 PHY layer. Four different speeds $\{5m/s, 10m/s, 15m/s, 20m/s\}$ are simulated for the random waypoint mobility model (RWMM).

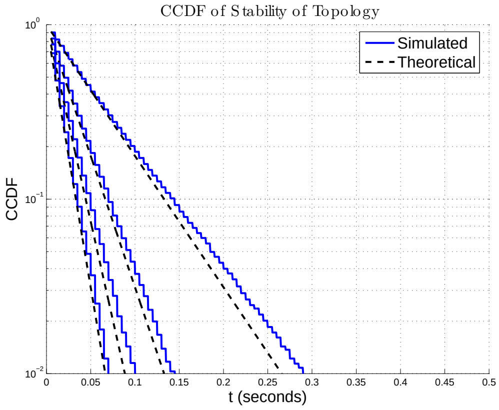

Fig. 2. Distribution of Stability of Topologies: RWMM, $R = 250m$.

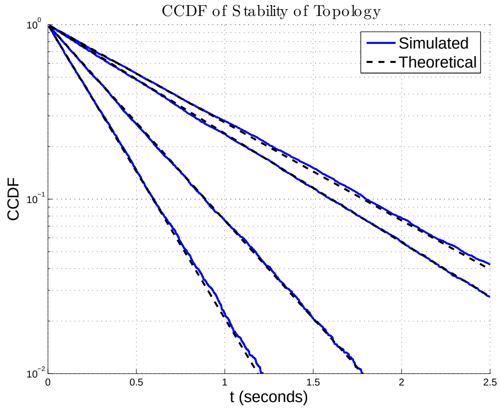

Fig. 3. Distribution of Stability of Topologies: RWMM, $R = 15m$.

Nodes are randomly activated to randomly choose destination node for data transmission. The traffic of activated nodes are supplied from a CBR source with a packet rate $0.5p/s$.

Figs. (2) present the results for RWMM and RDMM, respectively. It can be observed that for both cases, the exponential distribution model match pretty well with the simulation results and the analytical evaluation of the parameter also exhibits quite good approximation to the simulations. Some imperfections come in because of the edge effect that prevents random geometric graph model from accurately describing the steady-state of the network. We can minimize the edge effect in simulation by reducing the communication range, e.g. to $15m$. In that case, the simulations (Figs. (3) show almost perfect match with our analytical model and we predict that the analytical model is quite accurate for large networks.

VI. Practical Implication: Analyzing Control Traffic Overhead in OLSR protocol

As discussed, the control traffic overhead of a protocol is heavily related to the stability of the connectivity graph in MANETs. Solely concerned with nodal mobility, we already know that the distribution of stability of the connectivity graph can be approximated as exponentially distributed with parameter λ given in theorem 2. In this section, through utilizing the proposed model on proactive routing overhead and the connectivity graph, our target is to apply the model to project the control traffic overhead in order to better identify how nodes' mobility affect the control traffic overhead of OLSR protocol.

A. Brief Overview of OLSR protocol

In OLSR protocol, every nodes periodically send out HELLO messages for the purpose of neighbor sensing in the network. A HELLO message usually contain its one-hop neighbors and link status. Upon receiving and analyzing these consecutives HELLO messages, every nodes are able to keep track of up to two-hop neighbor links and use the information to compute its multipoint relays (MPR). The MPR set of the node is a subset of its neighbor nodes but maintains the coverage to its whole two-hop neighbors. The original node is called *MPR Selector*. And every node could have multiple nodes to select itself as a MPR node or in another words, can have multiple MPR selectors. Topology control (TC) messages are periodically generated from nodes with non-empty set of *MPR selectors* to disseminate {MPR selector, MPR} link information to the whole network. In case of nodes detecting changes in the set of *MPR selector*, TC message could be initiated earlier than the regular interval to respond to the change. Every nodes keep track the TC messages and use such link information for path selection and traffic routing.

The idea of multipoint relay (MPR) in OLSR is to minimize the flooding of broadcast packets and avoid the “broadcast storm” problem. For every node, its TC packets are retransmitted only by its MPR neighbor nodes and thus results in a saving of duplicate transmissions but still maintains satisfactory packet delivery. Clearly, the smaller the MPR set is, the more saving in the protocol. And the feature is particularly beneficial for deployment in dense network.

Link breakage is detected when a node fails to receive several consecutive HELLO messages from one of its neighbor node. And link addition is detected when a node starts to receive HELLO messages from a node not in its current one-hope neighbor set. Every change in the two-hop neighborhood link set will result in a protocol event of the node reacting to the change by recomputing its MPR set and could further result in MPR set. Therefore, it could lead to earlier TC message broadcast and the increase in the control traffic.

B. Parameterizing OLSR MPR Selection algorithm

In OLSR protocol, MPR scheme plays a critical role in reducing the flooding packet and maintain the whole network connectivity. By employing MPR, link changes will not necessarily result in a protocol event. However, the change that happens at *critical links* in OLSR protocol, i.e {MPR selector, MPR} pairs, will surely trigger a protocol event. For the reason, we need to find a parameter that characterizes the performance of MPR selection algorithm in OLSR protocol and further utilize it to derive the distribution of the connectivity graph. Before proceeding with choosing the appropriate performance metric, we need to first review the MPR selection algorithm. The MPR selection algorithm work as follows:

1) Select the node within the set of one-hop neighbor nodes as MPR node, if among the two-hop neighbor nodes, there are one or more than one nodes that are only covered by the node.
2) Choose a one-hop neighbor node as MPR node, if it covers the most of remaining two-hop neighbor nodes that are not covered by nodes in the MPR set. Repeat the step until all two-hop neighbor nodes are covered by the MPR set.

Clearly, the MPR selection algorithm is a greedy algorithm and its performance will vary with graphs on which it operates. The heuristic approach together with edge effect and graph dependent performance significantly complicates the problem and prevents an analytical modeling (if feasible) of the algorithm. For the reason, the parameter that we are looking for should reflect the statistical performance of the MPR algorithm and an evaluation of such parameter could be obtained by statistical evaluation with random geometric graph model.

A natural choice of the parameter should be the performance metric that answers the questions how much savings the MPR selection algorithm bring in reducing the duplicate flooding packet. Let's define $Neighbor\{i\}$ as the set of one-hop neighbor nodes and let $MPR\{i\}$ be the MPR set for node i. It is obvious that, $MPR\{i\} \subseteq Neighbor\{i\}$ Then the one-hop saving β_i from MPR selection can be evaluated as

$$\beta_i = \frac{|MPR\{i\}|}{|Neighbor\{i\}|} \tag{31}$$

Clearly, $0 < \beta_i \leq 1$. Eventually, we define a parameter β termed as *broadcast efficiency* to characterize the statistical performance of MPR selection algorithm. And it can be obtained through the statistical averaging over all possible nodes and graphs of the one-hop saving computed in Eq. (31).

$$\beta = E_{\mathcal{G},i}(\beta_i), 0 < \beta \leq 1 \tag{32}$$

The smaller β is, the more saving the MPR algorithm brings. β is also a statistical measure of the percentage of critical links ({MPR selector, MPR} pairs) out of total links in OLSR protocol. From Section V, we can infer that the distribution of link breakages of such links can also be approximated as exponentially distributed with parameter $\lambda_c = \beta * \lambda_o$.

C. Computation of Penalty Factor

The only remaining problem is to compute γ as a function of nodal mobility or the stability ζ of the local connectivity graph. First, we need to look at how ζ^* is determined from ζ, i.e., to understand how OLSR protocol reacts to an effective change. Effective change means that the node detect a change in the set of MPR selectors, since OLSR protocol operates on the sub-graph from critical links.

Fig. (4) illustrates how a protocol reacts to an effective change. Suppose that a change arrives at $KT_c < \zeta \leq (K+1)T_c$, then the scheduled next TC message will be advanced to be broadcasted at time ζ^*, the choice of which depends on when the change actually happened. If $KT_c < \zeta \leq KT_c + \Delta$,

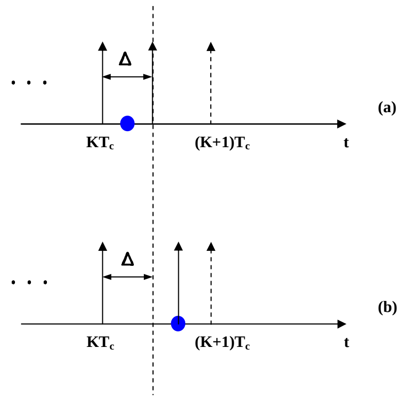

Fig. 4. Graphical Illustration on Change Response

then the TC message will be broadcasted at $\zeta^* = KT_c + \Delta$. For other cases $KT_c + \Delta < \zeta \leq (K+1)T_c$, TC message will be broadcasted immediately ($\zeta^* = \zeta$) when change is detected. The purpose of having Δ in protocol is to avoid the case where changes arrive too often and result in too much flooding from broadcasting TC messages. By aggregating such changes during Δ period in one TC message, the protocol can limit the maximum TC message broadcast rate but still achieve satisfactory performance. Summarizing the above analysis, one has

$$\zeta^* = \begin{cases} KT_c + \Delta, & KT_c < \zeta \leq KT_c + \Delta \\ \zeta, & KT_c + \Delta < \zeta \leq (K+1)T_c \end{cases} \tag{33}$$

As said, effective change is the change that results in a change in the set of MPR selectors. Such changes depend on the stability of local connectivity graph. Any changes in the local connectivity graph could lead to a re-computation of MPR set and further results in an effective change. We have the following itemized discussions on changes,

- A new link is detected in the local connectivity graph of node k. It will result in a MPR set recomputation of neighbors within two hop distance of the new link. Such link may or may not lead to a change in MPR selectors of node k.
- A link breakage is detected in the local connectivity graph but not in the critical links of node k. For such cases, it still leads to a recomputation of MPR set but not necessarily affect the operation of node k.
- A link breakage in critical links of node k is detected and as a result, node k will detect a change in the set of MPR selectors. Such change is surely an effective change on node k and node k needs to react to the change by earlier TC message broadcast.

Due to the heuristic characteristic of MPR selection algorithm, an analysis of the first two scenarios could be significantly complicated (if feasible at all). Taking a conservative approach, we only consider the last scenario where link breakage is detected in critical links. Since we know that the stability of overall critical links can be approximated as exponentially distributed with parameter λ_c, we can approximate the single-node stability ζ of critical links as also exponentially

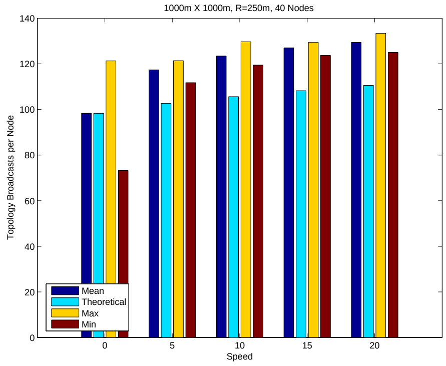

Fig. 5. perfectMac: N40

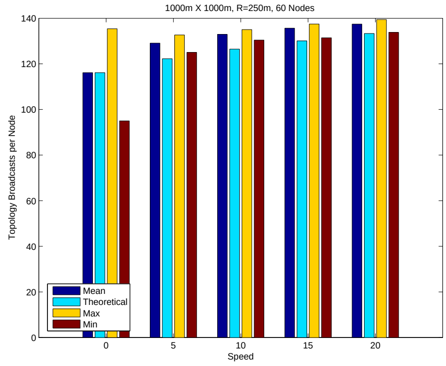

Fig. 6. perfectMac: N60

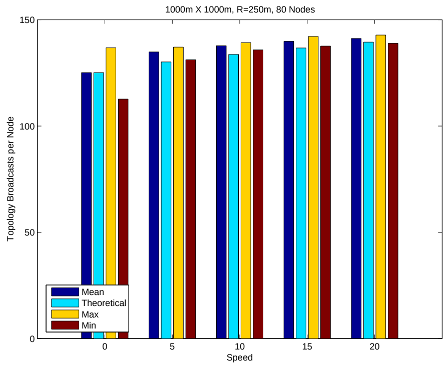

Fig. 7. perfectMac: N80

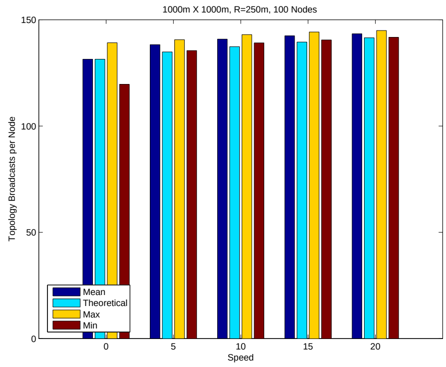

Fig. 8. perfectMac: N100

distributed with parameter $\lambda_s = N * \lambda_c$. Please be noted that such approximation becomes closer as node density increases, i.e., nodes associated with more critical links.

Till now, we can compute the penalty factor γ as a funciton mobility $\mathcal{V}$ as

$$\gamma(\mathcal{V}) = E(\frac{\lceil \zeta^*/T_c \rceil}{\zeta^*/T_c}) = f(\lambda_s) \quad (34)$$

where $f(\cdot)$ denotes mapping function and can be numerically computed after knowing the parameter λ_s of ζ (or ζ^*). It is also worthy of noting that the penalty factor is a direct function of local connectivity graph and suggests that the stability of connectivity graph can greatly affect the protocol performance.

D. Simulation Results

In the simulation, the network is a $1000m \times 1000m$ square cell. Each node has the same transmit power and the radio transmission range considered is $250m$. The number of nodes changes in the set $\{40, 60, 80, 100\}$ to simulate various node densities. The implementation of OLSR algorithm is the default implementation in *Qualnet 3.9.5*. Nodes are randomly activated to randomly choose destination node for data transmission. The traffic of activated nodes are supplied from a CBR source with a packet rate $0.5p/s$. And the movement follows the random waypoint model as the default setting in *Qualnet*. The maximum speeds considered are $\{0m/s, 5m/s, 10m/s, 15m/5, 20m/s\}$, ranging from static topologies, pedestrian speed to normal vehicle speed. And the MAC layer is set as the 802.11 MAC. Overall, we simulate a total of 20 different network configurations. For each configuration, 50 simulations with random generated seeds are conducted to capture the statistical performance.

To exclusively simulate the effect from nodal mobility, we modified the algorithm of *Qualnet* to exclusively consider such situations. Under such modifications, network layer will not experience packet loss from collisions i.e. due to environmental mobility— and we call it *perfect MAC*. Fig. (5) to (8) demonstrate the performance of analytical models versus simulative performance exclusively with *nodal mobility*. It can be observed that the analytical model provide good estimate to the simulations. Because we take a conservative approach in Section VI-C, the analytical model usually underestimates the overhead. And, as expected, the difference between the model and simulations decreases as node density increases, as critical links become more dominance in the local connectivity graph or link changes at non-critical links brings less effect on the sub-graph from critical links.

To evaluate the model in practical scenarios, we further turn back to the original setting of *Qaulnet* in interference computation. And in this case, the real 802.11 MAC works under collisions and back-offs. The simulation results are then illustrated in Fig. (9) and (10). In general, the model still provides a good approximation but the difference between the model and simulations are deeper due to additional effect from *environmental mobility*. Overall, we believe that our model provides satisfactory performance in estimating the routing overhead and brings deeper insight on how mobility affect the routing overhead.

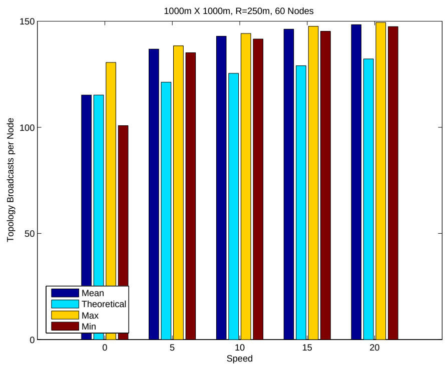

Fig. 9. Real Mac: N60

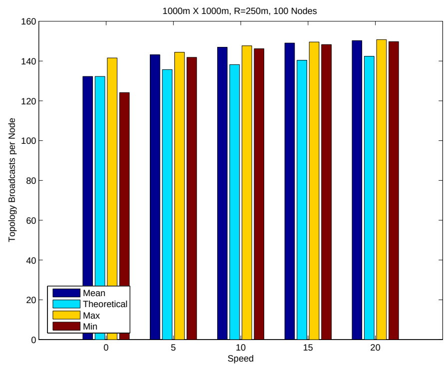

Fig. 10. Real Mac: N100

VII. Conclusion

In the paper, we analytically evaluated the inter-dependence between routing overhead and the stability of topologies, by characterizing the statistical distribution of topology evolutions. The stability of topology can be modeled as exponentially distributed with parameter computed from network configurations. Utilizing the proposed model, routing overhead of OLSR protocol is further analyzed and the results show that the proposed model gives good estimate of routing overhead and meanwhile provides good insight on how nodal mobility affect the routing overhead.

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