A Unifying Perspective on the Capacity of Wireless Ad Hoc Networks

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***Abstract*—We present the first unified modeling framework for the computation of the throughput capacity of random wireless ad hoc networks in which information is disseminated by means of unicast routing, multicast routing, broadcasting, or different forms of anycasting. We introduce (n, m, k)-casting as a generalization of all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks. In this context, n, m, and k denote the total number of nodes in the network, the number of destinations for each communication group, and the actual number of communication-group members that receive information (i.e., $k \leq m$), respectively.**

We compute upper and lower bounds for the (n, m, k)-cast throughput capacity in random wireless networks. When $m = k = \Theta(1)$, the resulting capacity equals the well-known capacity result for multi-pair unicasting by Gupta and Kumar. We demonstrate that $\Theta(1/\sqrt{mn \log n})$ bits per second constitutes a tight bound for the capacity of multicasting (i.e., $m = k < n$) when $m \leq \Theta(n/(\log n))$. We show that the multicast capacity of a wireless network equals its capacity for multi-pair unicasting when the number of destinations per multicast source is not a function of n. We also show that the multicast capacity of a random wireless ad hoc network is $\Theta(1/n)$, which is the broadcast capacity of the network, when $m \geq \Theta(n/\log n)$. Furthermore, we show that $\Theta(\sqrt{m}/(k\sqrt{n \log n}))$, $\Theta(1/(k \log n))$ and $\Theta(1/n)$ bits per second constitutes a tight bound for the throughput capacity of multicasting (i.e., $k < m < n$) when $\Theta(1) \leq m \leq \Theta(n/\log n)$, $k \leq \Theta(n/\log n) \leq m \leq n$ and $\Theta(n/\log n) \leq k \leq m \leq n$ respectively.

I. INTRODUCTION

The seminal work by Gupta and Kumar [1] on the capacity of wireless networks [1] has sparked a growing amount of interest in the understanding of the fundamental capacity limits of wireless ad hoc networks. The prior work that we summarize in Section II falls into three main areas. The first research area has focused on extending the results by Gupta and Kumar (e.g., [2], [3]). The second area consists of developing and analyzing schemes capable of increasing the capacity of wireless networks for unicast applications (e.g., [4]–[9]). The third area of research has addressed the fact that Gupta and Kumar's results apply only to wireless networks subject to multi-pair unicasts, and includes a number of studies on the capacity of ad hoc networks subject to broadcasting (e.g., [10]–[12]) and multicasting (e.g., [13]–[15]).

The work presented in this paper is motivated by the fact that, to date, there has been no unified treatment on the capacity of wireless networks subject to different types of forwarding disciplines. The main contribution of this paper consists of presenting the first unified modeling framework for the computation of the throughput capacity of random wireless ad hoc networks in which information is disseminated by means of unicast routing, multicast routing, broadcasting, or different forms of anycasting. We define (n, m, k)-casting as a generalization of all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks. In the context of (n, m, k)-casting, n, m, and k denote the number of nodes in the network, the number of destinations for each communication group, and the actual number of communication-group members that receive information (i.e., $k \leq m$), respectively. Section III describes the network model and necessary concepts for the development of our framework.

Section IV presents the capacity of (n, m, m)-casting, which corresponds to broadcasting or multicasting, and should be familiar to the reader. We compute the upper and lower bounds for the capacity of (n, m, m)-casting, and show that[2]: (a) $\Theta\left(1/(\sqrt{mn \log n})\right)$ bits per second is a tight bound for the capacity of multicasting (i.e., $m = k < n$) when $m \leq \Theta(n/(\log n))$; (b) the multicast capacity of a wireless network equals its capacity for multi-pair unicasting when the number of destinations for each multicast source is not a function of n; and (c) and the multicast capacity of a random wireless ad hoc network is $(\Theta(1/n))$, which is the broadcast capacity of the network, when $m \geq \Theta(n/(\log n))$.

Section V addresses the capacity of (n, m, k)-casting. We show that $\Theta\left(\frac{\sqrt{m}}{k\sqrt{n \log n}}\right)$, $\Theta\left(\frac{1}{k \log n}\right)$, and $\Theta\left(\frac{1}{n}\right)$ are the tight bounds for the three capacity regions of (n, m, k)-casting, respectively. This result generalizes prior results on the throughput capacity of ad hoc networks for unicasting, multicasting and broadcasting. When $m = k = \Theta(1)$, the resulting capacity equals the well-known capacity result for multi-pair unicasting by Gupta and Kumar. For $k = 1$, this constitutes the first capacity result for anycasting. Furthermore, this is the first result for the capacity of "manycasting" $(1 < k < m)$ in wireless ad hoc networks, where k out of m members of a communication group receive information

[1] In this paper, capacity is used to denote throughput capacity as was originally used by Gupta and Kumar [1]. We use the two terms interchangeably.

[2] Θ, Ω and O are the standard order bounds.

from a source.

Section VI discusses several possible implications of our new model. We show that the capacity of wireless ad hoc networks for any type of information dissemination can be derived with our unified approach using the (n, m, k)-cast formulation. Our results on the (n, m, k)-cast capacity of a random wireless network subsume the results from prior work on the capacity of unicasting, multicasting, and broadcasting in random wireless ad hoc networks. Furthermore, our modeling framework opens up new areas of research related to anycasting (routing to or from any one of the nodes in a communication group) "manycasting" (routing to or from a subset of the nodes in a communication group), and the effect of route control signaling on the scaling properties of wireless networks.

II. Related Work

Many contributions have been made on the capacity of wireless networks subject to unicasting, and due to space limitations we can mention only a few of them. A number of papers have extended the results by Gupta and Kumar. Gupta and Kumar's original work [1] showed a gap between the upper and lower bounds on capacity under the physical model. Franceschetti et al. [2] closed this gap using percolation theory, and Zhang et al. extended this work to networks with unrestricted bandwidth [3].

Several techniques have been developed aimed at improving the capacity of wireless networks. It has also been shown that changing physical layer assumptions such as using multiple channels [6] or MIMO cooperation [7] can change the capacity of wireless networks. Recently, Ozgur et al. [7] proposed a hierarchical cooperation technique based on virtual MIMO to achieve linear per source-destination capacity. Cooperation can be extended to the simultaneous transmission and reception at the various nodes in the network, which can result in significant improvement in capacity [8]. El Gamal et al. [16] characterized the fundamental throughput-delay tradeoff for both static and mobile networks. We have shown [9] that using multi-packet reception (MPR) at the receivers can increase the order capacity of wireless networks.

Considerable prior work has focused on the capacity of broadcasting and multicasting in wireless networks. Tavli [10] was first to show that $\Theta\left(n^{-1}\right)$ is a bound on the per-node broadcast capacity of arbitrary networks. Zheng [11] derived the broadcast capacity of power-constrained networks, together with another quantity called "information diffusion rate." The work by Keshavarz et al. [12] is perhaps the most general case of computing broadcast capacity for any number of sources in the network. Our work in this paper was inspired by some of the contributions in this work.

To the best of our knowledge, there are only three prior contributions on the multicast capacity of wireless networks [14], [14], [15]. Jacquet and Rodolakis [14] proved that the scaling of multicast capacity is decreased by a factor of $O(\sqrt{n})$ compared to the unicast capacity result by Gupta and Kumar [1]. The work by Shakkottai et al [14] is an extension of the work by Gupta and Kumar when there are n^{ϵ} multicast sources and $n^{1-\epsilon}$ destinations per flow for some $\epsilon > 0$. The results from this work are limited in scope, because of its constraints on the number of sources and destinations. Li et al. [15] compute the capacity of wireless ad hoc networks for unicast, multicast, and broadcast applications. While these results are equivalent to the capacity results we present for (n, m, m)-casting, it is worth noting that our work was done concurrent with and independent of the work in [15], our derivation of the capacity of (n, m, m)-casting is different than this work, and the results in this paper are more general.

III. Network Model and Preliminaries

We assume a random wireless network with n nodes distributed uniformly in the network area. Our analysis is based on *dense networks*, where the area of the network is a square of unit value. Hence, in our model, as n goes to infinity, the density of the network also goes to infinity. Our capacity analysis is based on the protocol model for dense networks introduced by Gupta and Kumar [1].

Definition 3.1: The Protocol Model: All nodes use a common transmission range $r(n)$ for all their communication. The network area is assumed to be a unit square area. Node X_i can successfully transmit to node X_j if for any node $X_k, k \neq i$, that transmits at the same time as X_i it is true that $|X_i - X_j| \leq r(n)$ and $|X_k - X_j| \geq (1 + \Delta)r(n)$.

The data rate for each transmitter-receiver pair is W bits/second, which is a constant value and does not depend on n. Given that W does not change the order capacity of the network, we normalize its value to 1.

Definition 3.2: Connectivity criterium in dense networks [1]: The transmission range $r(n)$ in random dense networks satisfies $r(n) \geq \Theta\left(\sqrt{\log n/n}\right)$.

Definition 3.3: Feasible Throughput capacity of (n, m, k)-casting: In a wireless ad hoc network of n nodes in which each source node transmits its packets to k out of m destinations, a throughput of $\lambda_{m,k}(n)$ bits per second for each node is feasible if there is a spatial and temporal scheme for scheduling transmissions, such that by operating the network in a multi-hop fashion and buffering at intermediate nodes when awaiting transmission, every node can send $\lambda_{m,k}(n)$ bits per second on average to its k out of its m chosen destination nodes. That is, there is a $T < \infty$ such that in every time interval $[(i-1)T, iT]$ every node can send $T\lambda_{m,k}(n)$ bits to its corresponding destination node.

Definition 3.4: Order of throughput capacity: $C_{m,k}(n)$ is said to be of order $\Theta(f(n))$ bits per second if there exist deterministic positive constants c and c' such that

$$\begin{cases} \lim\limits_{n\to\infty} \text{Prob } (C_{m,k}(n) = cf(n) \text{ is feasible}) = 1 \\ \lim\limits_{n\to\infty} \text{Prob } (C_{m,k}(n) = c'f(n) \text{ is feasible}) < 1. \end{cases} \quad (1)$$

Definition 3.5: Euclidean Minimum Spanning Tree (EMST): Consider a connected undirected graph $G = (V, E)$, where V and E are sets of vertices and edges in the graph G, respectively. The EMST of G is a spanning tree of G with the total minimum Euclidean distance between connected vertices of this tree.

Definition 3.6: Independent Set (IS($r(n)$)): An IS($r(n)$) of a graph G is a set of vertices in G such that the distance between any two elements of this set is greater than $r(n)$.

Definition 3.7: Maximum Independent Set (MIS($r(n)$)): The MIS($r(n)$) of G is an IS($r(n)$) such that, by adding any vertex from G to this set, there is at least one edge shorter than or equal to $r(n)$.

We note that MIS($r(n)$) is a unique and largest independent set for a given graph. Finding such a set in a general graph G is called the MIS problem and is an NP-hard problem [17].

Definition 3.8: Minimum Connected Dominating Set (MCDS($r(n)$)): A dominating set (DS($r(n)$)) of a graph G is defined as a set of nodes such that every node in the network either belongs to this set or it is within a range of $r(n)$ of one of the elements of DS($r(n)$). A Connected Dominating Set (CDS($r(n)$)) is a dominating set such that the subgraph induced by its nodes is connected. A Minimum Connected Dominating Set (MCDS($r(n)$)) is a CDS of G with the minimum number of nodes.

Definition 3.9: (n,m,k)-cast tree: An (n,m,k)-cast tree is a minimum set of nodes that connect a source node of an (n,m,k)-cast with all its intended m receivers, in order for the source to send information to k of those receivers.

An (n,m,k)-cast tree is a function of transmission range $r(n)$. Therefore, the optimum m-cast tree that has the minimum Euclidean distance is a function of $r(n)$. For this reason, changing the transmission range will change the optimum m-cast tree.

For simplicity, we also denote by m-cast tree an (n,m,m)-cast tree (i.e., when $m=k$).

Definition 3.10: Minimum Euclidean (n,m,k)-cast Tree (MEMKT($r(n)$)): The MEMKT($r(n)$) of an (n,m,k)-cast is an (n,m,k)-cast tree in which the k destinations that receive information from the source among the m receivers of the (n,m,k)-cast have the minimum total Euclidean distance. When $k=m$, we denote by minimum Euclidean m-cast tree (MEMT($r(n)$)) an (n,m,m)-cast tree with a total minimum Euclidean distance.

In the rest of this paper, $\|T\|$ denotes the total Euclidean distance of a tree T, and $\#T$ is used to denote the total number of vertices (nodes) in a tree T, then $\overline{\#T}$ denotes the total average number of vertices (nodes) in a tree T.

To compute the (n,m,k)-cast capacity, we will use the total Euclidean distance of MEMT and its relationship with EMST. Steele [18] determined a tight bound for $\overline{\|\text{EMST}\|}$ for large values of n, which we re-state in the following lemma.

Lemma 3.11: Let $f(x)$ denote the node probability distribution function of the network area. Then, for large values of n and $d>1$, the $\overline{\|\text{EMST}\|}$ is tight bounded as

$$\overline{\|\text{EMST}\|} = \Theta\left(c(d)n^{\frac{d-1}{d}}\int_{R^d} f(x)^{\frac{d-1}{d}}dx\right), \quad (2)$$

where d is the dimension of the network. Note that both $c(d)$ and the integral are constants and not functions of n. When $d=2$, then $\overline{\|\text{EMST}\|}=\Theta\left(\sqrt{n}\right)$.

IV. The Capacity of (n,m,m)-Casting

In this section, we compute the capacity of (n,m,k)-casting when k is set to m, which corresponds to the case of multicasting, and also applies to broadcasting when $m=n$.

A. Upper Bound

Lemma 3.11 computes the average total Euclidean distance for EMST. To compute the upper bound for (n,m,m)-casting, we first demonstrate the relationship between $\overline{\#\text{MEMT(r(n))}}$ and $\overline{\|\text{EMST}\|}$.

Theorem 4.1: The average number of nodes in an (n,m,m)-cast tree with transmission range $r(n)$ has the following lower bound:

$$\overline{\#\text{MEMT}(r(n))} \geq \begin{cases} \Theta\left(\sqrt{m}\left(r^{-1}(n)\right)\right), m \leq \Theta\left(r^{-2}(n)\right) \\ \Theta\left(r^{-2}(n)\right), m \geq \Theta\left(r^{-2}(n)\right) \end{cases} \quad (3)$$

Proof: From Eq. (2) and assuming that a network has $m+1$ nodes, then $\overline{\|\text{EMST}\|}$ is equal to $\Theta\left(\sqrt{m}\right)$. If the transmission range is arbitrarily large, then all the adjacent nodes in the (n,m,m)-cast tree are connected in one hop. In this case, $\overline{\#\text{MEMT}(r(n))}$ is equal to $\Theta(m_b)$ where m_b is the threshold that makes this set a CDS($r(n)$). Now, if the transmission range is not large enough to connect any two adjacent nodes in the (n,m,m)-cast tree in one hop, then there are some nodes from the n nodes in the network that will be used to create a connected (n,m,m)-cast tree. In this case, clearly $\overline{\|\text{MEMT}(r(n))\|}$ is greater than $\Theta\left(\sqrt{m}\right)$, which is derived by connecting all the nodes directly to each other in an (n,m,m)-cast tree (see Fig. 1). Under this condition, $\overline{\#\text{MEMT}(r(n))}$ is at least $\Theta\left(\sqrt{m}/r(n)\right)$.

Now the question is what threshold value exists for m between these two limits. This threshold is derived by computing the number of destinations in an (n,m,m)-cast, m_b, such that the two limits are equal, i.e., $\Theta\left(\frac{\sqrt{m_b}}{r(n)}\right)=\Theta(m_b)$. This equality is true for $m_b=\Theta\left(\frac{1}{r^2(n)}\right)$. This result implies that, when $m \leq m_b$ or $m \geq m_b$, the lower bound of $\overline{\#\text{MEMT}(r(n))}$ is $\Theta\left(\sqrt{\frac{m}{r(n)}}\right)$ or $\Theta\left(\frac{1}{r^2(n)}\right)$, respectively. ■

Theorem 4.1 implies that, when $m \geq m_b$, the source and all the destinations in the (n,m,m)-cast tree are connected without any need to use extra nodes outside of the multicast group. We will use Theorem 4.1 subsequently to prove the upper bound of the (n,m,m)-cast throughput capacity.

Lemma 4.2: The per-node capacity of (n,m,m)-casting is upper bounded by $O\left(\frac{1}{n}\times\frac{\#\text{MIS}(\Delta r(n))}{\#\text{MEMT}(r(n))}\right)$.

Proof: MIS($\Delta r(n)$) is the upper bound on the number of simultaneous transmissions in the network. Suppose X_i (with destination X_j) and X_k are two nodes transmitting simultaneously. From the definition of the protocol model and by utilizing the triangle inequality, we have

$$|X_k - X_i| \geq |X_i - X_j| - |X_k - X_j| = \Delta r(n). \quad (4)$$

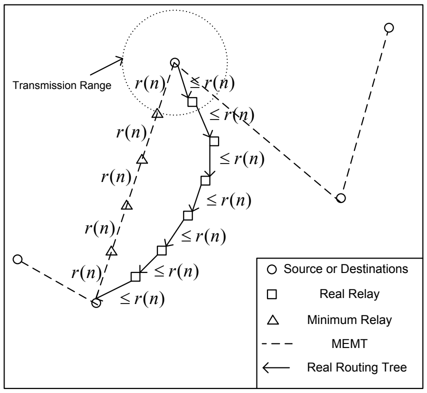

Fig. 1. The direct line between any two adjacent nodes in an (n, m, m)-cast tree is always equal or smaller than the total Euclidean distance in the tree through multiple relays.

This result implies that the minimum distance between any two simultaneous transmitters is $\Delta r(n)$. Therefore, the maximum number of simultaneous transmissions in a network is upper bounded by $\overline{\#\text{MIS}(\Delta r(n))}$.

We observe that $\overline{\#\text{MEMT}(r(n))}$ represents the total number of channel usage required to transmit information from an (n, m, m)-cast source to all its destinations for a single (n, m, m)-cast group. By definition, the total (n, m, m)-cast capacity in the network is equal to $nC_{m,m}(n) = \sum_{i=1}^{n} \lambda_{m,m}^{i}(n)$. To achieve a $\lambda_{m,m}^{i}(n)$ throughput for all n sources in the network, it is obvious that we require $nC_{m,m}(n) \times \overline{\#\text{MEMT}(r(n))} \leq \overline{\#\text{MIS}(\Delta r(n))}$. The upper bound on the per-node upper bound capacity for (n, m, m)-casting can be derived directly from this last inequality, which proves the lemma. ■

The following lemma states the upper bound of $\overline{\#\text{MIS}(\Delta r(n))}$, which was first proven in [12].

Lemma 4.3: The average number of nodes in the maximum independent set $\overline{\#\text{MIS}(\Delta r)}$ has the following upper bound:

$$\overline{\#\text{MIS}(\Delta r(n))} \leq \frac{1}{\pi \Delta^2 r^2(n)/16} = \frac{16}{\pi \Delta^2} \frac{1}{r^2(n)}. \tag{5}$$

Proof: From Eq. (4), it is clear that the minimum distance between any two transmitters in a dense network is $\Delta r(n)$. This minimum separation distance requires that each transmitter covers an area of at least $\pi \left(\frac{\Delta r(n)}{2}\right)^2$. Using this argument, it is shown in [12] that the upper bound of $\overline{\#\text{MIS}(\Delta r(n))}$ is given by Eq. (5). ■

We are now ready to derive an upper bound for $C_{m,m}(n)$, which is stated in the following theorem.

Theorem 4.4: The per-node upper bound on the throughput capacity of (n, m, m)-casting is given by

$$C_{m,m}(n) = \begin{cases} O\left(\frac{1}{nr(n)\sqrt{m}}\right), \Theta(1) \leq m \leq \Theta\left(r^{-2}(n)\right) \\ O\left(n^{-1}\right), \quad \Theta\left(r^{-2}(n)\right) \leq m \leq n \end{cases} \tag{6}$$

The transmission range $r(n)$ should satisfy the connectivity criteria in the network, i.e., $r(n) \geq \Theta\left(\sqrt{\log n/n}\right)$ which leads to

$$C_{m,m}(n) = \begin{cases} O\left(\frac{1}{\sqrt{nm\log n}}\right), \Theta(1) \leq m \leq \Theta\left(\frac{n}{\log n}\right) \\ O\left(n^{-1}\right), \quad \Theta\left(\frac{n}{\log n}\right) \leq m \leq n \end{cases} \tag{7}$$

Proof: The proof is immediate by combining Theorem 4.1 with Lemmas 4.2 and 4.3. ■

B. Lower Bound

To derive the achievable lower bound, we use a TDMA scheme for random dense networks similar to the approach used in [12], [19], [20].

We first divide the network area into square cells. Each square cell has an area of $\frac{r^2(n)}{2}$ which makes the diagonal length of square equal to $r(n)$ as shown in Fig. 2. Under this condition, connectivity inside all cells is guaranteed and all nodes inside a cell are within transmission range of each other. We build a cell graph over the cells that are occupied with at least one vertex (node). Two cells are connected if there exist a pair of nodes, one in each cell, that are less than or equal to $r(n)$ distance apart. Because the whole network is connected when $r(n) \geq \Theta\left(\sqrt{\log n/n}\right)$, it follows that the cell graph is connected.

To satisfy the protocol model, we should design cells in groups such that simultaneous transmissions within each group do not violate the protocol model for successful communication. Let L represent the minimum number of cell separations in each group of cells that communicate simultaneously. Utilizing the protocol model, L satisfies the following condition:

$$L = \left\lceil 1 + \frac{r(n) + (1 + \Delta)r(n)}{r(n)/\sqrt{2}} \right\rceil = \lceil 1 + \sqrt{2}(2 + \Delta) \rceil \tag{8}$$

If we divide time into L^2 time slots and assign each time slot to a single group of cells, interference is avoided and the protocol model is satisfied. Fig. 2 represents one of these groups with a cross sign inside those cells for $L = 4$.

We can derive an achievable capacity for (n, m, m)-casting taking advantage of this cell arrangement and the following property of the TDMA scheme with L parameter that we have introduced.

Lemma 4.5: The capacity reduction caused by the TDMA scheme is a constant factor and does not change the order capacity of the network.

Proof: The TDMA scheme introduced above requires cells to be divided into L^2 groups, such that only nodes in each group can transmit simultaneously. Eq. (8) demonstrates that the upper bound of L is not a function of n and is only a constant factor. Because the proposed TDMA scheme requires L^2 channel uses, it follows that this TDMA scheme reduces the capacity by a constant factor. ■

The following lemma establishes the achievable lower bound for the (n, m, m)-cast capacity as a function of $\overline{\#\text{MEMTC}(r(n))}$, the total number of cells that contains all the nodes in an (n, m, m)-cast group.

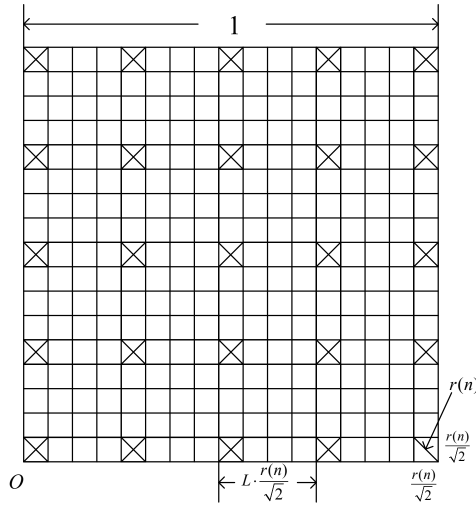

Fig. 2. Cell construction used to derive a lower bound on capacity

Lemma 4.6: The achievable lower bound of the (n,m,m)-cast capacity is

$$C_{m,m}(n)=\Omega\left(\frac{1}{\overline{\#\text{MEMTC}(r(n))}}\times\frac{1}{nr^2(n)}\right). \quad (9)$$

Proof: There are $\frac{1}{(r(n)/\sqrt{2})^2}$ cells in the unit square network area. From the definition of $\overline{\#\text{MEMTC}(r(n))}$ and the fact that our TDMA scheme does not change the order capacity (Lemma 4.5), it is clear that there are at most $\overline{\#\text{MEMTC}(r(n))}$ interfering cells for any (n,m,m)-cast communications. Therefore, at any given time we can have at least $\Omega(\frac{1}{\overline{\#\text{MEMTC}(r(n))}}\frac{1}{r^2(n)})$ simultaneous communications in the network. Accordingly, the per-node lower bound capacity is given by $\Omega(\frac{1}{\overline{\#\text{MEMTC}(r(n))}}\frac{1}{nr^2(n)})$, which proves the lemma. ■

Given the above lemma, to express the lower bound of $C_{m,m}(n)$ as a function of network parameters, we need to compute the upper bound of $\overline{\#\text{MEMTC}(r(n))}$, which we do next.

Lemma 4.7: The average number of the cells that belongs to an (n,m,m)-cast tree satisfy the following upper bound.

$$\overline{\#\text{MEMTC}(r(n))}\leq\min\left(\Theta\left(\frac{\sqrt{m}}{r(n)}\right),\Theta\left(\frac{1}{r^2(n)}\right)\right) \quad (10)$$

Proof: Because the maximum number of cells in this network is equal to $\Theta\left(\frac{1}{r^2(n)}\right)$, it is clear that one upper bound for $\overline{\#\text{MEMTC}(r(n))}$ is this value. That is, $\overline{\#\text{MEMTC}(r(n))}$ cannot exceed the total number of cells in the network. On the other hand, the total Euclidean distance of the (n,m,m)-cast tree was shown earlier to be $\Theta(\sqrt{m})$. Because $r(n)$ is the transmission range of the network, the maximum number of cells for this (n,m,m)-cast tree must be at most $\Theta\left(\frac{\sqrt{m}}{r(n)}\right)$, i.e., $\overline{\#\text{MEMTC}(r(n))}\leq\Theta\left(\frac{\sqrt{m}}{r(n)}\right)$. This upper bound can be achieved only if every two adjacent nodes in the (n,m,m)-cast tree belong to two different cells in the network. However, in practice, it is possible that some adjacent nodes in the (n,m,m)-cast tree are located in a single cell. Consequently, this value is the upper bound. The actual upper bound clearly is the minimum of these two extreme values in the network, which is a function of the topology of the network and this proves the lemma. ■

The achievable lower bound when the transmission range is fixed to its minimum value for connectivity (i.e., $r(n)=\Theta\left(\sqrt{\frac{\log n}{n}}\right)$) is given in the following theorem.

Theorem 4.8: The achievable lower bound of the (n,m,m)-cast capacity is

$$C_{m,m}(n)=\begin{cases}\Omega\left(\frac{1}{nr(n)\sqrt{m}}\right),\Theta(1)\leq m\leq\Theta\left(r^{-2}(n)\right)\\ \Omega\left(n^{-1}\right),\quad\Theta\left(r^{-2}(n)\right)\leq m\leq n\end{cases} \quad (11)$$

When $r(n)=\Theta\left(\sqrt{\log n/n}\right)$, we can get

$$C_{m,m}(n)=\begin{cases}\Omega\left(\frac{1}{\sqrt{n\log n}\sqrt{m}}\right),\quad\Theta(1)\leq m\leq\Theta\left(\frac{n}{\log n}\right)\\ \Omega\left(\frac{1}{n}\right),\quad\Theta\left(\frac{n}{\log n}\right)\leq m\leq n\end{cases} \quad (12)$$

Proof: Combining Lemmas 4.6 and 4.7, we arrive this theorem

$$\begin{aligned}C_{m,m}(n)&=\Omega\left(\frac{1}{\overline{\#\text{MEMTC}(r(n))}}\frac{1}{nr^2(n)}\right)\\&=\Omega\left(\max\left[\left(\frac{1}{\sqrt{m}nr(n)}\right),\left(n^{-1}\right)\right]\right)\end{aligned} \quad (13)$$

In order to compute the threshold for m to achieve either of these capacities, we compute the values of m that make one of these capacities larger than the other one.

$$\begin{cases}\left(\frac{1}{\sqrt{m}nr(n)}\right)\geq\left(n^{-1}\right)\quad\text{when}\quad m\leq\Theta\left(r^{-2}(n)\right)\\ \left(n^{-1}\right)\geq\left(\frac{1}{\sqrt{m}nr(n)}\right)\quad\text{when}\ m\geq\Theta\left(r^{-2}(n)\right),\end{cases} \quad (14)$$

Combining (14) and (13) proves the theorem. The second equation of this theorem is derived using $r(n)=\Theta\left(\sqrt{\log n/n}\right)$. ■

From theorems 4.8 and 4.4, we can provide a tight bound for the (n,m,m)-cast capacity.

Theorem 4.9: The (n,m,m)-cast capacity is tightly bounded as follows

$$C_{m,m}(n)=\begin{cases}\Theta\left(\frac{1}{nr(n)\sqrt{m}}\right),\Theta(1)\leq m\leq\Theta\left(r^{-2}(n)\right)\\ \Theta\left(n^{-1}\right),\quad\Theta\left(r^{-2}(n)\right)\leq m\leq n\end{cases} \quad (15)$$

When transmission range equals to the minimum one as

$\Theta\left(\sqrt{\log n/n}\right)$, the tight bound is

$$C_{m,m}(n)=\begin{cases}\Theta\left(\frac{1}{\sqrt{n\log n}\sqrt{m}}\right), \Theta(1)\le m\le\Theta\left(\frac{n}{\log n}\right)\\ \Theta\left(\frac{1}{n}\right), \Theta\left(\frac{n}{\log n}\right)\le m\le n.\end{cases} \quad (16)$$

V. The Capacity of (n,m,k)-Casting

This section provides the throughput capacity of (n,m,k)-casting in random wireless networks. The proofs of many theorems and lemmas for the case of (n,m,k)-cast capacity are much the same as those presented for the (n,m,m)-cast capacity in the previous section. Accordingly, we only include those proofs that have important differences.

A. Upper Bound

As we did for the proof of Theorem 4.1, we will first compute the lower bound on the number of nodes in a MEMKT($r(n)$) tree.

Theorem 5.1: The average number of nodes in MEMKT($r(n)$) has the following lower bound as a function of the transmission range $r(n)$.

$$\overline{\#\text{MEMKT}(r(n))}\ge\begin{cases}\Theta\left(\frac{k}{\sqrt{m}r(n)}\right), m\le\Theta\left(\frac{1}{r^2(n)}\right)\\ \Theta\left(k\right), m\ge\Theta\left(\frac{1}{r^2(n)}\right)\ge k\\ \Theta\left(\frac{1}{r^2(n)}\right), m\ge k\ge\Theta\left(\frac{1}{r^2(n)}\right)\end{cases} \quad (17)$$

Proof: From Lemma 3.11, if we select only m destinations of n nodes to construct an EMST, then the total average Euclidean distance of the EMST is at least $\Theta(\sqrt{m})$. Given that there are m destinations for the tree, then the average Euclidean distance between any two nodes for this tree is $\frac{\sqrt{m}}{m}$. Because the assignment of source-destinations groups is completely random, we can say that, on average, the total Euclidean distance for k destinations is equal to $\frac{\sqrt{m}k}{m}$. Using a similar argument to that used in the proof of Theorem 4.1, we can say that when the transmission range is not a very large value, then the number of nodes in such tree is lower bounded by $\sqrt{m}\frac{k}{mr(n)}$. This is the top lower bound in Eq. (17). When the transmission range is very long, all the m destinations in the (n,m,k)-cast tree are connected and given that we only need the closest k nodes in the set, then the number of nodes is $\Theta(k)$. This is the second lower bound in Eq. (17). Once $k\ge\Theta\left(\frac{1}{r^2(n)}\right)$, then the transmission range is so large that we can use $\Theta\left(\frac{1}{r^2(n)}\right)$ as the lower bound, which is the last lower bound in Eq. (17). In a similar fashion to the proof of Theorem 4.1, the threshold for $r(n)$ is derived when the first lower bound in Eq. (17) is equal to the number of nodes in broadcast when all the nodes are reachable in one hop, i.e., $\overline{\#\text{MEMKT}(r(n))}=k$. Therefore, it is true that $\Theta\left(\frac{\frac{\sqrt{m}}{m}k}{r(n)}\right)=\Theta(k)$, and the solution to this last equality is $m_b=\Theta\left(\frac{1}{r^2(n)}\right)$. This means that, when $m\le m_b$ or $m\ge m_b$, the lower bound of $\overline{\#\text{MEMKT}(r(n))}$ is $\Theta\left(\frac{k}{\sqrt{m}r(n)}\right)$ or $\Theta\left(k\right)$, respectively. This proves the lemma. ∎

Theorem 5.1 implies that, when $m\ge m_b$, all of the source and destinations are connected by themselves without any need to additional nodes in the network.

Lemma 5.2: The upper bound of the per-node (n, m, k)-cast capacity is $O\left(\frac{1}{n}\times\frac{\overline{\#\text{MIS}(\Delta r(n))}}{\overline{\#\text{MEMKT}(r(n))}}\right)$.

Proof: The proof is similar to the proof of lemma 4.2, with the only difference that we must use $\overline{\#\text{MEMKT}(r(n))}$ instead of $\overline{\#\text{MEMT}(r(n))}$. ∎

Theorem 5.3: The upper bound on the per-node (n,m,k)-cast capacity is

$$C_{m,k}(n)\le\begin{cases}O\left(\sqrt{m}(nkr(n))^{-1}\right),\ \Theta(1)\le m\le\Theta\left(r^{-2}(n)\right)\\ O\left(\left(nkr^2(n)\right)^{-1}\right),\ k\le\Theta\left(r^{-2}(n)\right)\le m\le n\\ O\left(n^{-1}\right),\ \Theta\left(r^{-2}(n)\right)\le k\le m\le n\end{cases} \quad (18)$$

We omit the proof of this theorem, because it is essentially the same as the proof of Theorem 4.4. The following section describes a design for achieving a lower bound that matches our upper bound.

B. Lower Bound

We obtain an achievable lower bound following the same approach we presented in Section IV-B.

Lemma 5.4: The achievable lower bound of the (n,m,k)-cast capacity is given by

$$C_{m,k}(n)=\Omega\left(\frac{1}{\overline{\#\text{MEMKTC}(r(n))}}\times\frac{1}{nr^2(n)}\right), \quad (19)$$

where $\overline{\#\text{MEMKTC}(r(n))}$ is the mean number of nodes in MEMKTC($r(n)$).

The proof is essentially the same as for Lemma 4.6, and we omit it for brevity.

Lemma 5.5: The average number of cells in MEMKT($r(n)$) tree, with high probability, is upper bounded as follows:

$$\overline{\#\text{MEMKTC}(r(n))}=\begin{cases}O\left(k(\sqrt{m}r(n))^{-1}\right),\ m\le\Theta\left(r^{-2}(n)\right)\\ O\left(k\right),\ k\le\Theta\left(r^{-2}(n)\right)\le m\le n\\ O\left(r^{-2}(n)\right),\ \Theta\left(r^{-2}(n)\right)\le k\le m\end{cases} \quad (20)$$

The proof of this lemma is very similar to the proof of lemma 4.7 in the previous section, and is therefore omitted.

Combining these results, we can compute the achievable lower bound capacity when the transmission range is set to $r(n)=\Theta\left(\sqrt{\log n/n}\right)$.

Theorem 5.6: The achievable lower bound of the (n,m,k)-cast capacity is

$$C_{m,k}(n)=\begin{cases}\Omega\left(\sqrt{m}(nkr(n))^{-1}\right),\ \Theta(1)\le m\le\Theta\left(r^{-2}(n)\right)\\ \Omega\left(\left(nkr^2(n)\right)^{-1}\right),\ k\le\Theta\left(r^{-2}(n)\right)\le m\le n\\ \Omega\left(n^{-1}\right),\ \Theta\left(r^{-2}(n)\right)\le k\le m\le n\end{cases} \quad (21)$$

Proof: From lemmas 5.4 and 5.5, we can complete the proof which is similar as Theorem 4.8. ■

Combining theorems 5.6 and 5.3, a tight bound for the capacity of (n, m, k)-cast can be derived.

Theorem 5.7: The capacity of (n, m, k)-cast in a random wireless network is

$$C_{m,k}(n) = \begin{cases} \Theta\left(\sqrt{m}(nkr(n))^{-1}\right), \ \Theta(1) \leq m \leq \Theta\left(r^{-2}(n)\right) \\ \Theta\left(\left(nkr^2(n)\right)^{-1}\right), \ k \leq \Theta\left(r^{-2}(n)\right) \leq m \leq n \\ \Theta\left(n^{-1}\right), \ \Theta\left(r^{-2}(n)\right) \leq k \leq m \leq n \end{cases} \tag{22}$$

When $r(n) = \Theta(\sqrt{\log n/n})$,

$$C_{m,k}(n) = \begin{cases} \Theta\left(\frac{\sqrt{m}}{k\sqrt{n\log n}}\right) \text{ for } \Theta(1) \leq m \leq \Theta\left(\frac{n}{\log n}\right) \\ \Theta\left(\frac{1}{k\log n}\right) \text{ for } k < \Theta\left(\frac{n}{\log n}\right) \leq m \leq n \\ \Theta\left(\frac{1}{n}\right) \text{ for } \Theta\left(\frac{n}{\log n}\right) \leq k < m \leq n \end{cases} \tag{23}$$

Note that the thresholds for different values for m and k provide various capacities for (n, m, k)-casting.

VI. Discussion of Results and Their Implications

There is much valuable insight to be gained from modeling the capacity of unicasting, multicasting, broadcasting and anycasting using the same framework. Our (n, m, k)-cast framework allows us to analyze the throughput capacity of wireless networks as a function of the number of receivers of a communication group, which can set from 1 up to the number of nodes in the network, as well as a function of the transmission range. Accordingly, the results obtained in all prior work can be derived from our model by selecting the appropriate values for $r(n)$ and m in the capacity results obtained in Sections IV and V. For example when $m = 1$ and $r(n) = \Theta\left(\sqrt{\log n/n}\right)$, the result by Gupta and Kumar for unicast capacity follows from Eq. (16). In addition, our framework also provides new insight on the capacity of information dissemination techniques that are becoming more prevalent with the availability of in-network storage, namely anycasting, and allows us to reason about the nature that route signaling should be to render more scalable wireless networks. In the following, we first address a number of implications of our results as they relate to the capacity of multicasting and broadcasting, and then address the more general case of the capacity of (n, m, k)-cast.

A. $C_{m,m}(n)$ as a Function of Group Size (m)

Fig. 3 shows the throughput capacity of a wireless network obtained from Eq. (16) as a function of the number of destinations of each source node. As the number of destinations per source m is varied from 1 to n, the capacity of (n, m, m)-cast becomes that of unicast, multicast, and broadcast. The figure clearly shows that there are two threshold values for m (denoted by m_u and m_b) that are critical to the throughput capacity of (n, m, m)-cast.

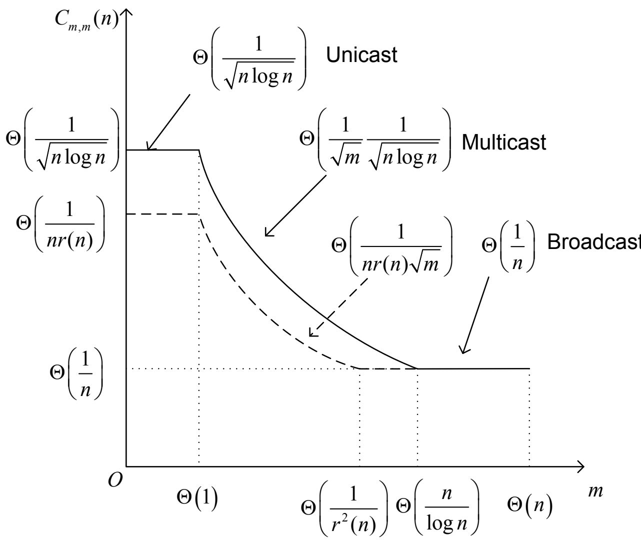

Fig. 3. Order throughput capacity as a function of m

First, if the number of destinations m is not a function of n, then the order of capacity does not change. More explicitly, if m varies from 1 to $m_u = \Theta(1)$, then the capacity of the network is $\Theta\left(\frac{1}{\sqrt{n\log n}}\right)$, which is the well known results computed originally by Gupta and Kumar for multiple unicasts [1]. This result implies that the order capacity for both unicast and multicast with limited number of destinations is the same! This is very relevant for real networks, where the constituency of any given multicast group is much smaller than the total number of nodes, and is independent of the size of the network. The main reason for this result is the fact that, when the number of destinations is constant, the order of total Euclidean distance of multicast tree does not change. Consequently, the order of channel uses for the multicast tree which is inversely proportional to the network capacity does not change. For this reason, the order capacity is the same $\Theta\left(\frac{1}{\sqrt{n\log n}}\right)$ for unicast and multicast whose number of destinations is order $\Theta(1)$.

The second threshold for the values of m is $m_b = \Theta\left(n/\log n\right)$. If $m \geq m_b$, then the capacity of the wireless ad hoc network converges to the broadcast capacity, regardless of the number of destinations in the network, as long as this number is greater than m_b. This is the lowest capacity that can be attained by the network utilizing multihop communications.

When $m_u \leq m \leq m_b$, then the capacity of the network decreases as the number of destinations per communication group increases (see fig. 3). Note that the decrease in capacity is by a factor of $\Theta(\sqrt{m})$ instead of $\Theta(m)$. The reason behind this behavior of the network capacity is the fact that the destinations in a plane are spread within a total Euclidean distance of $\Theta(\sqrt{m})$ from the source in the multicast tree. Because we use multihop communication to reach all the destinations, it follows that the number of hops is proportional to this total Euclidean distance $\Theta(\sqrt{m})$ and therefore, because of channel reuse, the capacity decreases inversely to this value.

B. $C_{m,m}(n)$ as a Function of Transmission Range ($r(n)$)

Eq. (6) shows that the (n, m, m)-cast order capacity of wireless ad hoc networks decreases when the transmission range $r(n)$ increases and receivers cannot decode more than one packet at a time. This is equivalent to the well-known fact that multihop communications is the optimal technique for interference-dominant wireless ad hoc networks.

From Definition 3.2, a minimum transmission range is needed to maintain connectivity. Based on Eq. (6) and the second line of Eq. (11), it follows that increasing transmission range will decrease the capacity. Therefore, the optimum choice for transmission range is the minimum value such that the network remains connected. The reason behind this behavior of the network is the negative effect of interference. By increasing the transmission range in the network, we actually increase the interference to more adjacent nodes and force them to be silent during a communication session. Clearly, the capacity is maximized if we maximize the number of simultaneous transmissions in the network. Ideally, if connectivity for all the nodes were not a concern, then one could reduce the transmission range further than the minimum value. This observation suggests that, if we use power control for those nodes that require longer range for connection, one can potentially obtain higher capacities in the network (e.g., see [21]).

The results we have derived are based on the assumption that nodes try to avoid interference. However, a different emerging viewpoint in wireless ad hoc network is based on *embracing* interference [8], [9]. We have shown [9] that with this new paradigm, which we call many-to-many communication [8], increasing the transmission range actually does not decrease the capacity of wireless ad hoc networks. This approach requires nodes to have more complex receivers in order to be able to decode multiple transmitters simultaneously. Determining $C_{m,m}(n)$ under many-to-many communication is an open problem.

C. $C_{m,k}(n)$ as a Function of Group Size (m)

Figure 4 shows $C_{m,k}(n)$ as a function of m. As it was the case for $C_{m,m}(n)$, if m varies from 1 to $m_u = \Theta(1)$, the capacity of the network does not change and equals $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$. For values of m larger than m_u, the (n, m, k)-cast order capacity can increase or decrease depending on the value of k. The smallest order capacity corresponds to the case when $k = m$, i.e., multicasting ($m < n$) or broadcasting ($m = n$), and the largest order capacity is attained for anycasting ($k = 1$). The shaded area in the figure shows the achievable capacity for manycasting ($1 < k < m$) for different values of m and k.

We observe that, regardless of the value of k, the capacity of wireless ad hoc networks becomes constant when $m \geq \Theta(n/\log n)$ and an increase in the value of m does not change the transport capacity. This result can be understood by the fact that, when the number of destinations reaches $\Theta(n/\log n)$, this set becomes the connected dominating set

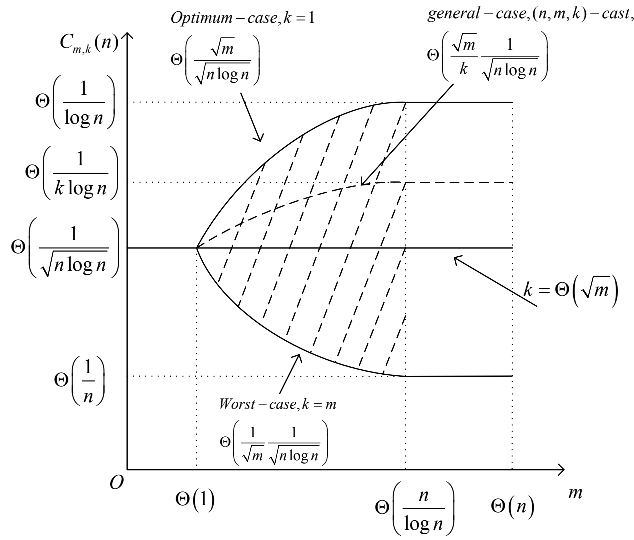

Fig. 4. Unifying view of throughput capacity

(CDS($r(n)$)) of the entire network as long as the transmission range $r(n)$ is chosen such that the network is a connected network. Equivalently, if a broadcast is made to the entire network, the capacity does not change because all the nodes in the network are either inside this set or within one hop from an element in this set.

We note that the capacity of anycast or manycast is greater than the capacity of unicast if $k < \Theta(\sqrt{m})$, even if each node requires to transmit its packets to more than one destination. This result shows that as long as $k < \Theta(\sqrt{m})$, the total number of hops required to transmit packet to k destinations is always, on average, less than sending the packet from the same source to a single randomly selected destination in unicast communications. Equivalently, the total Euclidean distance for a manycast tree is on average less than the Euclidean distance between any randomly selected source and destination in unicast communication. However, this Euclidean distances become the same, on average, when $k = \Theta(\sqrt{m})$. As it can be predicted from this figure, the total Euclidean distance in a manycast tree increases as k increase and for $k > \Theta(\sqrt{m})$, the capacity of manycast becomes less than unicast because of the total Euclidean distance in the manycast tree.

D. New Implications on Scaling Laws

An important observation to be made regarding the behavior of $C_{m,k}(n)$ as a function of m is that, in a real wireless network, data dissemination occurs together with route signaling. The issue is that, in practice, all unicast and multicast routing protocols involve some form of broadcasting (e.g., flooding of link-state updates, propagation of route requests or join requests, or diffusion of distance updates). Hence, the scaling properties of an ad hoc wireless network is really determined by the dominating form of information dissemination, which may occur for control signaling or data. Accordingly, in order for future wireless ad hoc networks to have the best possible scaling properties, it is clear from our results that the number of nodes impacted by any one route signaling packet on the average should be bounded by $\Theta(1)$. One possible approach for scale-free route signaling is to ensure that only those nodes who actually have an interest in the routes for which a signaling packet is needed receive the signaling packet. Clearly, this poses a challenge, because no distributed oracle exists with instantaneous knowledge of what control information each node needs. However, such an interest-driven signaling can be an important ingredient for scalable wireless ad hoc networks.

Another result derived from the behavior of $C_{m,k}(n)$ as a function of m is that anycasting and manycasting render a higher order capacity than unicasting when the number of destinations in the communication group of a source that actually receive the information from the source is $k < \Theta\left(\sqrt{m}\right)$. This indicates that capacity increase can be attained by an appropriate use of in-network storage and information dissemination from the random site(s) of a communication group to the group source (the node with interest in the information), rather than from pre-defined origins hosting the content. If the communication group is the entire network ($m = n$), information flows from the closest neighbor(s) to each node and the maximum capacity gain is attained. If the group size is independent of the size of the network ($m = \Theta(1)$), the order capacity is the same as for unicasting.

VII. Conclusion

We introduced a unifying framework for the modeling of the order capacity of wireless networks subject to different types of information dissemination. To do so, we defined (n, m, k)-casting as a generalization of all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks. Our modeling framework provides a unique perspective on the understanding of the capacity of wireless ad hoc networks. Our approach unifies existing results on the order capacity of wireless networks subject to unicasting [1], multicasting, or broadcasting [10]–[12], provides new capacity results for anycasting and manycasting, and helps to develop new insight on the role of route signaling and in-network storage on the capacity of wireless ad hoc networks.

We showed that the capacity of wireless ad hoc networks depends greatly on the number of destinations m. We introduced two important thresholds, namely $m_u = \Theta(1)$ and $m_b = \Theta\left(\frac{n}{\log n}\right)$, and used them to characterize the capacity of a wireless ad hoc network when each source communicates with a receiver group of size m. When $m \leq m_u$, the capacity of a wireless network is similar to its unicast capacity; when $m_u < m < m_b$, then the network capacity is similar to its multicast capacity; and when $m \geq m_b$, the capacity of the network is equivalent to its broadcast capacity. We also showed that anycasting and manycasting provide higher order capacity than unicasting when $k < \Theta\left(\sqrt{m}\right)$.

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References

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, 2000.

[2] M. Franceschetti, O. Dousse, D. N. C. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Transactions on Information Theory*, vol. 53, no. 3, pp. 1009–1018, 2007.

[3] H. Zhang and J. Hou, "Capacity of wireless ad-hoc networks under ultra wide band with power constraint," in *Proc. of IEEE INFOCOM 2005*, Miami, Florida, USA., March 13-17 2005.

[4] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Transactions on Networking*, vol. 10, no. 4, pp. 477–486, 2002.

[5] R. Negi and A. Rajeswaran, "Capacity of power constrained ad-hoc networks," in *Proc. of IEEE INFOCOM 2004*, Hong Kong, March 7-11 2004.

[6] P. Kyasanur and N. H. Vaidya, "Capacity of multi-channel wireless networks: Impact of number of channels and interfaces," in *Proc. of ACM MobiCom 2005*, Cologne, Germany, August 28-September 2 2005.

[7] A. Ozgur, O. Leveque, and D. Tse, "Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks," *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 2549–3572, 2007.

[8] R. M. de Moraes, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "Many-to-many communication: A new approach for collaboration in manets," in *Proc. of IEEE INFOCOM 2007*, Anchorage, Alaska, USA., May 6-12 2007.

[9] J. J. Garcia-Luna-Aceves, H. R. Sadjadpour, and Z. Wang, "Challenges: Towards truly scalable ad hoc networks," in *Proc. of ACM MobiCom 2007*, Montreal, Quebec, Canada, September 9-14 2007.

[10] B. Tavli, "Broadcast capacity of wireless networks," *IEEE Communications Letters*, vol. 10, no. 2, pp. 68–69, 2006.

[11] R. Zheng, "Information dissemination in power-constrained wireless networks," in *Proc. of IEEE INFOCOM 2006*, Barcelona, Catalunya, Spain, April 23-29 2006.

[12] A. Keshavarz, V. Ribeiro, and R. Riedi, "Broadcast capacity in multihop wireless networks," in *Proc. of ACM MobiCom 2006*, Los Angeles, California, USA., September 23-29 2006.

[13] P. Jacquet and G. Rodolakis, "Multicast scaling properties in massively dense ad hoc networks," in *Proc. of IEEE ICPADS 2005*, Fukuoka, Japan, July 20-22 2005.

[14] S. Shakkottai, X. Liu, and R. Srikant, "The multicast capacity of wireless ad-hoc networks," in *Proc. of ACM MobiHoc 2007*, Montreal, Canada, September 9-14 2007.

[15] X.-Y. Li, S.-J. Tang, and O. Frieder, "Multicast capacity for large scale wireless ad hoc networks," in *Proc. of ACM MobiCom 2007*, Montreal, Canada, September 9-14 2007.

[16] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-delay trade-off in wireless networks," in *Proc. of IEEE INFOCOM 2004*, Hong Kong, March 7-11 2004.

[17] N. Alon, L. Babai, and A. Itai, "A fast and simple randomized parallel algorithm for the maximal independent set problem," *Journal of Algorithms*, vol. 7, no. 4, pp. 567–583, 1986.

[18] M. Steele, "Growth rates of euclidean minimal spanning trees with power weighted edges," *The Annals of Probability*, vol. 16, no. 4, pp. 1767–1787, 1988.

[19] A. Giridhar and P. R. Kumar, "Computing and communicating functions over sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 4, pp. 755–764, 2005.

[20] S. R. Kulkarni and P. Viswanath, "A deterministic approach to throughput scaling wireless networks," *IEEE Transactions on Information Theory*, vol. 50, no. 6, pp. 1041–1049, 2004.

[21] O. Dousse and P. Thiran, "Connectivity vs capacity in dense ad hoc networks," in *Proc. of IEEE INFOCOM 2004*, Hong Kong, March 7-11 2004.