Capacity Scaling Laws of Information Dissemination Modalities in Wireless Ad Hoc Networks

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***Abstract*—We present capacity scaling laws for random wireless ad hoc networks under all information dissemination modalities (unicast, multicast, broadcast, anycast) when nodes are endowed with multi-packet transmission (MPT) or multi-packet reception (MPR) capabilities. Information dissemination modalities are modeled with an (n, m, k)-cast formulation, where n, m, and k denote the number of nodes in the network, the number of destinations for each communication group, and the actual number of communication group members that receive information (i. e., $k \leq m \leq n$), respectively. We show that $\Theta\left(T(n)\sqrt{m}/k\right)$, $\Theta\left(1/k\right)$, and $\Theta\left(T^2(n)\right)$ bits per second constitute a tight bound for the throughput capacity of random wireless ad hoc networks under the protocol model when $k \leq m \leq \Theta\left(T^{-2}(n)\right)$, $k \leq \Theta\left(T^{-2}(n)\right) \leq m$, and $\Theta\left(T^{-2}(n)\right) \leq k \leq m$, respectively. This result applies to both MPR and MPT, where $T(n)$ denotes the transceiver range, which depends on the encoding or decoding complexity of the nodes. For the minimum transceiver range of $\Theta\left(\sqrt{\log n/n}\right)$ to guarantee network connectivity, a gain of $\Theta(\log n)$ for (n, m, k)-casting is attained with either MPT or MPR compared to the capacity attained when transmitters and receivers can encode and decode at most one transmission at a time (i.e., point-to-point communication).**

I. Introduction

Gupta and Kumar [1] studied the capacity[1] of wireless ad hoc networks for the case of multi-pair unicasts in which the nodes are able to encode and decode at most one packet at a time. This work has motivated a large body of work over the past few years, which we summarize in Section II. One important area of the resulting research has focused on the study of different approaches to "embrace interference" in order to increase the capacity of wireless ad hoc networks. Embracing interference consists of increasing the concurrency with which the channel is accessed. We denote by multi-packet reception (MPR) [2] the ability of a receiver node to decode correctly multiple packets transmitted concurrently from different nodes, and by multi-packet transmission (MPT) the ability of a transmitter node to transmit concurrently multiple packets to different nodes. In practice, MPR and MPT can be achieved with a variety of techniques. For example, MPR can be implemented by allowing a node to decode multiple concurrent packets using multiuser detection (MUD); MPR or MPT capabilities can be implemented utilizing directional antennas [3], [4] or multiple input multiple output (MIMO) techniques.

A complementary approach to embracing interference consists of increasing the amount of information sent per transmission. Network coding (NC) [5] was introduced and shown to achieve the optimal capacity for single-source multicast in directed graphs corresponding to wired networks in which nodes are connected by point-to-point links. Since then, many attempts have been made to apply NC to wireless ad hoc networks, and recent work [6]–[10] has shown promising results on the application of NC in wireless ad hoc networks subject to multicast traffic. Interestingly, a careful review of these contributions reveals that analog network coding [6] and physical-layer network coding [7] implicitly require the integration of NC with a form of MPR, in that receivers must be allowed to decode successfully concurrent transmissions from multiple senders by taking advantage of the modulation scheme used at the physical layer (e.g., MSK modulation in ANC [6]). Similarly, [8]–[10] the other recent NC schemes discussed for wireless networks assume the integration of NC with MPR and MPT.

The work reported in this paper is motivated by three aspects of the prior work to date. First, while it is clear from recent work on NC that MPR and MPT may contribute to the capacity increase observed when NC is applied to wireless networks with multicast traffic, prior work does not decouple the performance gains due to NC (i.e., combining multiple packets into a single transmission) from those resulting from MPR or MPT (i.e., allowing multiple transmissions to be received or sent at the same time). Second, no capacity results have ever been reported on the benefits of MPT, which arguably may be easier to attain in practice than MPR. Third, although Garcia-Luna-Aceves et al. [11] have shown that the order capacity of wireless ad hoc networks subject to multi-pair unicast traffic is increased with MPR, no results have been reported on the order capacity of networks with MPR subject to broadcast, multicast or anycast traffic.

Section III presents the first contribution of this paper, which is a modeling framework for the computation of the throughput

[1]Throughput capacity was first introduced by Gupta and Kumar [1] for random networks. In this paper, we use throughput capacity or simply capacity interchangeably.

capacity of random wireless ad hoc networks with MPT or MPR subject to any type of information dissemination modality (unicast, multicast, broadcast, and anycast). We demonstrate that the throughput capacity of wireless ad hoc networks with MPT or MPR for any type of information dissemination can be derived using an (n, m, k)-cast formulation, where n, m, and k are defined earlier. For example, for the cases of $m = k = 1$, $m = k < n$, and $m = k = n$, the (n, m, k)-cast is equivalent to unicast, multicast, and broadcast communications, respectively. The (n, m, k)-cast can also represent different forms of anycasting.

Sections IV and V present the first results on the capacity of ad hoc networks with MPT under different forms of information dissemination, and the first results for the capacity of networks with MPR for dissemination modalities other than unicast traffic. In particular, we show that the per source-destination (n, m, k)-cast throughput capacity $C_{m,k}(n)$ of a wireless random ad hoc network with MPT or MPR is tight bounded (upper and lower bounds) by $\Theta\left(T(n)\sqrt{m}/k\right)$, $\Theta\left(1/k\right)$ and $\Theta\left(T^2(n)\right)$ w.h.p.[2] when $k \leq m \leq \Theta\left(T^{-2}(n)\right)$, $k \leq \Theta\left(T^{-2}(n)\right) \leq m$, and $\Theta\left(T^{-2}(n)\right) \leq k \leq m$, respectively. In these results, the transceiver range $T(n)$ in MPT or MPR is different from the transmission range $r(n)$ used in the capacity results for networks with point-to-point communication [1]. For comparison purposes, we also show the (n, m, k)-cast capacity result for point-to-point communication. Section VII discusses the behavior of the capacity of an ad hoc network with MPT, MPR, or point-to-point schemes as a function of the (n, m, k)-cast parameters and as a function of the transceiver range.

II. Related Work

Due to space limitations we only mention a few of the many prior contributions and focus on work addressing broadcasting and multicasting in static networks that we did not mention in the prior section.

Many papers have extended the results by Gupta and Kumar [1], which showed a gap between the upper and lower bounds on capacity under the physical model. Franceschetti et al. [12] closed this gap using percolation theory, and Zhang et al. [13] extended this work to networks with unrestricted bandwidth. It has also been shown that, if bandwidth is allowed to increase proportionally to the number of nodes in the network [13], [14], higher transport capacities can be attained for static wireless networks. Other works demonstrated that changing physical layer assumptions such as using multiple channels [15] or MIMO cooperation [16] can change the capacity of wireless networks. Recently, Ozgur et al. [16] proposed a hierarchical cooperation technique based on virtual MIMO to achieve linear capacity.

Tavli [17] was the first to show that $\Theta(n^{-1})$ is an upper bound on the per-node broadcast capacity of arbitrary networks. Zheng [18] derived the broadcast capacity of power-constrained networks, together with another quantity called "information diffusion rate". Keshavarz et al. [19] compute the broadcast capacity of a network for any number of sources. We use a number of techniques from this work in the derivation of our results for MPT or MPR. Jacquet and Rodolakis [20] proved that the scaling of multicast capacity is decreased by a factor of $\Theta(\sqrt{n})$ compared to the unicast capacity result by Gupta and Kumar [1]. The work by Shakkottai et al. [21] is an extension of the work by Gupta and Kumar when there are n^{ϵ} multicast sources and $n^{1-\epsilon}$ destinations per flow for some $\epsilon > 0$. These results are limited in scope, because of the constraints on the number of sources and destinations. Li et al. [22] compute the capacity of wireless ad hoc networks for unicast, multicast, and broadcast applications for point-to-point communications.

III. Network Model and Preliminaries

We assume a random wireless ad hoc network with n nodes distributed uniformly in a network of unit square area. Our analysis is based on dense networks, where the area of the network is a square of unit value[3]. Hence, in our model, as n goes to infinity, the density of the network also goes to infinity. Our capacity analysis is based on the protocol model for dense networks introduced by Gupta and Kumar [1]. Gupta and Kumar defined the protocol model for point-to-point communications. In that model, a common transmission range $r(n)$ for all nodes is defined. Node X_i can successfully transmit to node X_j if for any node $X_k, k \neq i$, that transmits at the same time as X_i, then $|X_i - X_j| \leq r(n)$ and $|X_k - X_j| \geq (1 + \Delta)r(n)$, where X_i, X_j and X_k are the cartesian position in the unit square network for these nodes. We need to define the protocol model for both MPR or MPT, and in doing so we extend the MPR protocol model in [11].

In wireless ad hoc networks with MPT (or MPR), the protocol model assumption allows MPT (or MPR) capability at nodes as long as they are within a radius of $T(n)$ from the transmitter (or receiver) and all other receiving (or transmitting) nodes have a distance larger than $(1 + \Delta)T(n)$. The difference is that we allow the transmitter (or receiver) node to transmit (or receive) multiple packets to (or from) different nodes within its disk of radius $T(n)$ simultaneously in MPT (or MPR) scheme. Note that $r(n)$ in Gupta and Kumar's model is a random variable while $T(n)$ in MPT (or MPR) is a predefined value which depends on the complexity of the nodes. We assume that nodes cannot transmit and receive at the same time, which is equivalent to half duplex communications [1]. The data rate for each transmitter-receiver pair is a constant value of W bits/second and does not depend on n. Given that W does not change the order capacity of the network, we normalize its value to one. The relationship between transceiver range $T(n)$ of MPT and MPR throughout this paper and transmission range in point-to-point communication is defined as

$$T(n) = r(n) \geq \Theta\left(\sqrt{(\log n)/n}\right). \quad (1)$$

The MPT protocol model is shown in Fig. 1. For the case of MPR, the only difference is the fact that the center node

[2] An event occurs with high probability (w.h.p.) if its probability tends to one as n goes to infinity. Θ, Ω and O are the standard order bounds.

[3] The unit square of the network simplifies the analysis. For different shape of the network area, the result can be extended similarly.

in each circle receives packets from all the nodes within its communication range. In general, it is easy to see that MPT and MPR are dual of each other leading to the same throughput capacity. However, it may be easier to implement MPT in practice using directional antennas [3], [4].

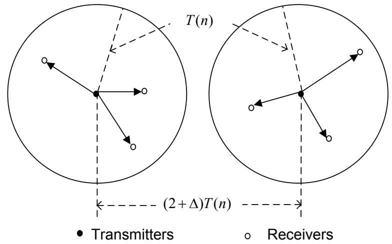

Fig. 1. MPT protocol model

In this paper, we study the case in which each of n nodes in a network acts as a source with a group of m receivers (with $m \leq n$), with k (where $k \leq m$) of those receivers being randomly selected to obtain the information from the source. We call this characterization of information dissemination from sources to receivers (n, m, k)-casting. The throughput capacity for an (n, m, k)-cast simply extrapolates the original definition of feasible throughput capacity for unicasting given by Gupta and Kumar [1].

Definition 3.1: Feasible Throughput capacity of (n, m, k)-cast: In a wireless ad hoc network with n nodes in which each source node transmits its packets to k out of m destinations, a throughput of $C_{m,k}(n)$ bits per second for each node is feasible if there is a spatial and temporal scheme for scheduling transmissions, such that by operating the network in a multi-hop fashion and buffering at intermediate nodes when awaiting transmission, every node can send $C_{m,k}(n)$ bits per second on average to its k out of its m chosen destination nodes. That is, there is a $T < \infty$ such that in every time interval $[(i-1)T, iT]$, every node can send $T \times C_{m,k}(n)$ bits to its corresponding destination nodes.

Definition 3.2: Order of throughput capacity: $C_{m,k}(n)$ is said to be of order $\Theta(f(n))$ bits per second if there exist deterministic positive constants c and c' such that

$$\begin{cases} \lim\limits_{n\to\infty} \text{ Prob } (C_{m,k}(n) = cf(n) \text{ is feasible}) = 1 \\ \lim\limits_{n\to\infty} \text{ Prob } (C_{m,k}(n) = c'f(n) \text{ is feasible}) < 1. \end{cases} \tag{2}$$

Definition 3.3: Euclidean Minimum Spanning Tree (EMST): Consider a connected undirected graph $G = (V, E)$, where V and E are sets of vertices and edges in the graph G, respectively. The EMST of G is a spanning tree of G with the total minimum Euclidean distance between connected vertices of this tree.

Definition 3.4: (n, m, k)-cast tree: An (n, m, k)-cast tree is a set of nodes that connects a source node of an (n, m, k)-cast with all its intended k receivers out of m, in order for the source to send information to k of those receivers.

By construction, it can be shown that (n, m, k)-cast represents all forms of communications in wireless ad hoc networks, i.e., unicast ($k = m = 1$), broadcast ($k = m = n$), multicast ($k = m < n$), anycast ($k = 1 < m \leq n$), and all forms of "manycast" ($k \leq m \leq n$). We use the term *manycast* as a generalization of anycast. In particular, it may suffice to send information (e.g., a request for data) from a source to the k members of a group of $m > k$ members, or to receive information from the k members from a group of m members. The group of m nodes may be a type of servers, or nodes that maintain copies of specific information objects.

The total Euclidean length of an (n, m, k)-cast tree is a function of the transceiver range $T(n)$. Therefore, the optimum (n, m, k)-cast tree with minimum Euclidean distance is a function of $T(n)$.

Definition 3.5: Maximum Independent Set (MIS $(\Delta, r(n))$): An independent set $\text{IS}(\Delta, r(n))$ of a graph G is a set of vertices in G such that the distance between any two elements of this set is greater than $r(n)$. The $\text{MIS}(\Delta, r(n))$ of G is an $\text{IS}(\Delta, r(n))$ such that, by adding any vertex from G to this set, there is at least one edge shorter than or equal to $r(n)$.

We note that $\text{MIS}(\Delta, r(n))$ is unique. Finding such a set in a general graph G is called the MIS problem and is an NP-hard problem. Keshavarz et al. [19] use $\text{MIS}(\Delta, r(n))$ to describe the maximum number of simultaneous transmitters when nodes use point-to-point communication. For the same purpose and to account for the use of MPT or MPR, we define the Maximum MPT or MPR Independent Set (MMIS) as follows.

Definition 3.6: Maximum MPT (or MPR) Independent Set (MMIS $(\Delta, T(n))$): An MPT (or MPR) independent set is a set of nodes in G that contains one transmitter (or receiver) node and all receiving (or transmitting) nodes are within a distance of $T(n)$ from this transmitter (or receiver) node. A Maximum MPT (or MPR) Independent Set $(\text{MMIS}\,(\Delta, T(n)))$ consists of the maximum number nodes of MPT (or MPR) sets that simultaneously transmit (or receive) their packets while MPT (or MPR) protocol model is satisfied for all these MPT (or MPR) sets. If we add any receiver (or transmitter) node from G to $\text{MMIS}(\Delta, T(n))$, there is at least one MPT (or MPR) set that violates the MPT (or MPR) protocol model.

Definition 3.7: Minimum Connected Dominating Set (MCDS $(r(n))$): A dominating set $(\text{DS}\,(r(n)))$ of a graph G is defined as a set of nodes such that every node in the network either belongs to this set or it is within a transmission range of $r(n)$ of one of the elements of $\text{DS}(r(n))$. A Connected Dominating Set $(\text{CDS}\,(r(n)))$ is a dominating set such that the subgraph induced by its nodes is connected. A Minimum Connected Dominating Set $(\text{MCDS}\,(r(n)))$ is a $\text{CDS}(r(n))$ of G with the minimum number of nodes.

Keshavarz et al. [19] use $\text{MCDS}(r(n))$ to describe the minimum rebroadcasting times required to reach the destinations in a network in which broadcast is assumed. Similarly, to account for the use of (n, m, k)-cast, we define the Minimum Euclidean (n, m, k)-cast Tree $(\text{MEMKT}\,(T(n)))$ as follows.

Definition 3.8: Minimum Euclidean (n, m, k)-cast Tree (MEMKT $(T(n))$): The $\text{MEMKT}(T(n))$ is an (n, m, k)-cast tree in which the k destinations out of m nodes receive information from the source and this (n, m, k)-cast tree has the minimum total Euclidean distance. For example, when $k = m$, $\text{MEMKT}(T(n))$ denotes the minimum Euclidean multicast tree and it is the same as $(\text{MEMT}\,(T(n)))$ defined in graph theory.

In the rest of this paper, $\|S\|$ denotes the total Euclidean distance of a tree S; $\#S$ is used to denote either the total number of vertices (nodes) in a set or the total number of relays for a tree depending on whether S is a set or a tree; and $\overline{\|S\|}$ is used for the statistical average of the total Euclidean distance of a tree. Note when S is a tree, $\#S$ is equivalent with the channel usage in that tree to transmit information which is the same order as the number of the relays instead of the total number of the nodes in that tree (include the source and the destinations).

To compute the multicast capacity in networks with point-to-point communication, Li et al. [22] used the total Euclidean distance of MEMT and its relationship with EMST. We use a similar approach for networks with MPT or MPR. Steele [23] determined a tight bound for $\overline{\|\text{EMST}\|}$ for large values of m, and for a two-dimensional imples that

$$\overline{\|\text{EMST}\|} = \Theta\left(\sqrt{m}\right) \tag{3}$$

Given that the distribution of nodes in a random network is uniform, if there are n nodes in a unit square, then the density of nodes equals n. Hence, if $|S|$ denotes the area of space region S, the expected number of the nodes, $E(N_S)$, in this area is given by $E(N_S) = n|S|$. Let N_j be a random variable defining the number of nodes in S_j . Then, for the family of variables N_j, we have the following standard results known as the Chernoff bounds [24]:

Lemma 3.9: Chernoff bound

For any $0 < \delta < 1$, we have

$$P[|N_j - n|S_j|| > \delta n|S_j|] < e^{-\theta n|S_j|}, \tag{4}$$

where θ is a variable function of δ.

Therefore, for any $\theta > 0$, there exist constants such that deviations from the mean by more than these constants occur with probability approaching zero as $n \to \infty$. It follows that, w.h.p., we can get a very sharp concentration on the number of nodes in an area, so we can find the achievable lower bound w.h.p., provided that the upper bound (mean) is given. In the next section, we first derive the upper bound, and then use the Chernoff bound to prove the achievable lower bound w.h.p..

IV. Upper Bound on the Throughput Capacity of (n,m,k)-Cast with MPT or MPR

Keshavarz et al. [19] used $\overline{\#\text{MIS}(\Delta, r(n))}$ to express the maximum possible number of simultaneous transmissions (i.e. channel usage) for the case of point-to-point communication. We adopt a similar approach to obtain the maximum number of concurrent transmissions for the case of MPT or MPR by using $\overline{\#\text{MMIS}(\Delta, T(n))}$. The following Lemma provides an upper bound capacity as the ratio of $\overline{\#\text{MIS}(\Delta, r(n))}$ to $\overline{\#\text{MEMKT}(T(n))}$. Note that $\overline{\#\text{MEMKT}(T(n))}$ equals the minimum number of transmissions required to (n,m,k)-cast a packet to k destinations out of m when MPT or MPR is used.

Lemma 4.1: In random dense wireless ad hoc networks, the per-node throughput capacity of (n,m,k)-cast with MPT or MPR is given by $O\left(\frac{1}{n} \times \frac{\overline{\#\text{MMIS}(\Delta,T(n))}}{\overline{\#\text{MEMKT}(T(n))}}\right)$.

Proof: We observe that $\overline{\#\text{MEMKT}(T(n))}$ represents the total number of channel usage required to transmit information from a (n,m,k)-cast source to all its k destinations for a single (n,m,k)-cast group of m nodes using MPT (or MPR). By definition, the total (n,m,k)-cast throughput capacity in the network is equal to $nC_{m,k}(n)$. Denote by N_T the total number of generated (n,m,k)-cast bits in $[0,T]$, then

$$nC_{m,k}(n) = \lim_{T\to\infty} \frac{N_T}{T}. \tag{5}$$

Note that for each bit, we require $\overline{\#\text{MEMKT}(T(n))}$ channel usage to (n,m,k)-cast one bit to all destinations. Clearly for N_T bits, we need to use the channel $N_T \times \overline{\#\text{MEMKT}(T(n))}$ times. Since all (n,m,k)-cast bits are received within a finite time $T_{\max}$, at time $T + T_{\max}$ all transmissions of N_T bits are finished. Therefore, with the definition of $\overline{\#\text{MMIS}(\Delta, T(n))}$, we have

$$\overline{\#\text{MMIS}(\Delta, T(n))}(T + T_{\max}) \geq N_T \times \overline{\#\text{MEMKT}(T(n))}. \tag{6}$$

By combining the two previous equations we obtain

$$C_{m,k}(n) = \frac{1}{n} \times \lim_{T\to\infty} \frac{N_T}{T} \leq \frac{1}{n} \times \frac{\overline{\#\text{MMIS}(\Delta, T(n))}}{\overline{\#\text{MEMKT}(T(n))}},$$

which proves the lemma. ■

Lemma 4.1 provides the upper bound for the (n,m,k)-cast throughput capacity with MPT or MPR as a function of $\overline{\#\text{MMIS}(\Delta, T(n))}$ and $\overline{\#\text{MEMKT}(T(n))}$. We next compute the upper bound of $\overline{\#\text{MMIS}(\Delta, T(n))}$ and the lower bound for $\overline{\#\text{MEMKT}(T(n))}$. Combining these results provide an upper bound for the (n,m,k)-cast throughput capacity with MPT or MPR.

Lemma 3 gives the average total Euclidean distance for EMST. To compute the lower bound for $\overline{\#\text{MEMKT}(T(n))}$, we find the relationship between $\overline{\#\text{MEMKT}(T(n))}$ and $\overline{\|\text{EMST}\|}$.

Lemma 4.2: In (n,m,k)-cast applications, the average number of nodes in MEMKT$(T(n))$ has the following lower bound as a function of the transceiver range $T(n)$:

$$\overline{\#\text{MEMKT}(T(n))} \geq \begin{cases} \Theta\left(k(\sqrt{m}T(n))^{-1}\right) & \text{for } m \leq \Theta\left(m_b\right) \\ \Theta\left(k\right) & \text{for } k \leq \Theta\left(m_b\right) \leq m \\ \Theta\left(T^{-2}(n)\right) & \text{for } \Theta\left(m_b\right) \leq k \leq m \end{cases} \tag{7}$$

where $m_b = T^{-2}(n)$. Clearly when $m = k$, the lower bound for (n,m,m)-cast is given by

$$\overline{\#\text{MEMT}(T(n))} \geq \begin{cases} \Theta(\sqrt{m}T^{-1}(n)) \text{ for } m < \Theta(T^{-2}(n)) \\ \Theta(T^{-2}(n)) \text{ for } m \geq \Theta\left(T^{-2}(n)\right). \end{cases} \tag{8}$$

Proof: we note here that $\overline{\#\text{MEMKT}(T(n))}$ denotes the number of the relays in the (n,m,k)-cast tree, which is equivalent to the channel usage. It means that $\overline{\#\text{MEMKT}(T(n))}$ can be smaller than k, although the (n,m,k)-cast tree should connect all k destinations. A similar case holds for $\overline{\#\text{MEMT}(T(n))}$, which can be smaller than m.

Let $m = k$ (i.e., assume (n,m,m)-casting). From Eq. (3) and assuming that the (n,m,m)-cast tree has $m+1$ nodes, then $\overline{\|\text{EMST}\|}$ is equal to $\Theta\left(\sqrt{m}\right)$ when $m >> 1$. If the transceiver range $T(n)$ is arbitrarily large, then for any node in this tree, all adjacent nodes in the (n, m, m)-cast tree are connected in one hop. In this case, $\overline{\#\text{MEMT}(T(n))}$ is equal to $\Theta(m_b)$, where m_b is the threshold that makes this set a $\text{CDS}(T(n))$. Now, if the transceiver range is not large enough to connect any two adjacent nodes in the (n, m, m)-cast tree in one hop, then there are some nodes from the other $n - m$ nodes in the network that must be used to create a connected (n, m, m)-cast tree. In this case, $\overline{|\text{MEMT}\,(T(n))\,|}$ is greater than $\Theta\left(\sqrt{m}\right)$, which is derived by connecting all the nodes directly to each other in an (n, m, m)-cast tree (see Fig. 2). Under this condition, $\overline{\#\text{MEMT}(T(n))}$ is at least $\Theta\left((\sqrt{m})/T(n)\right)$.

Now the question is what the threshold for m is between these two limits. This threshold is derived by computing the number of destinations in (n, m, m)-cast, m_b, such that the two limits are equal, i.e., $\Theta\left((\sqrt{m_b})/T(n)\right) = \Theta(m_b)$. This equality holds when $m_b = \Theta\left(T^{-2}(n)\right)$. The result implies that, when $m < m_b$ or $m \geq m_b$, then the lower bound of $\overline{\#\text{MEMT}\,(T(n))}$ is $\Theta\left(\sqrt{m}T^{-1}(n)\right)$ or $\Theta\left(T^{-2}(n)\right)$, respectively, which proves Eq. (8) for the (n, m, m)-cast case.

Let $k < m$. Given that there are m destinations for each tree for the case of an (n, m, k)-cast, the average Euclidean distance between any two nodes for this tree is $\frac{\Theta(\sqrt{m})}{m}$. Because the assignment of source-destinations groups is completely random, we do not know where the k destinations are in advance. However, we have to connect all of the k destinations. We first construct the multicast tree for m receivers and then find the k destinations. Then we can say that, on average, the total Euclidean distance for k destinations is equal to $\frac{k\Theta(\sqrt{m})}{m}$ because of the random distribution of k destinations. Using a similar argument as before, we can say that when the transceiver range is not a very large value, and so the number of relays in such a tree is lower bounded by $\Theta\left(\sqrt{m}\frac{k}{mT(n)}\right)$. This is the top lower bound in Eq. (7). When the transceiver range is very long, all m destinations in the (n, m, k)-cast tree are connected. Hence, given that we only need the closest k nodes in the set, then the number of nodes is $\Theta(k)$. This is the second lower bound in Eq. (7). In a similar fashion to the proof for (n, m, m)-cast, the threshold for $T(n)$ is derived when the first lower bound in Eq. (7) is equal to the number of nodes in broadcast when all the nodes are reachable in one hop, i.e., $\overline{\#\text{MEMKT}(T(n))} = k$. Therefore, it is true that $\Theta\left((\sqrt{m}k/mT(n)\right) = \Theta(k)$, and the solution to this equality is $m_b = \Theta\left(T^{-2}(n)\right)$. This means that, when $m \leq m_b$ or $m > m_b$, the lower bound of $\overline{\#\text{MEMKT}(T(n))}$ is $\Theta\left(\frac{k}{\sqrt{m}T(n)}\right)$ or $\Theta(k)$, respectively. Once $k \geq \Theta\left(T^{-2}(n)\right)$, then the transceiver range is so large that we can use $\Theta\left(T^{-2}(n)\right)$ as the lower bound similar to the second lower bound in (8), which is the last lower bound in Eq. (7). This proves the lemma. ■

Note that $\overline{\#\text{MEMKT}(T(n))}$ and $\overline{\#\text{MEMT}(T(n))}$ are the same value for MPR, MPT, or point-to-point communication and they only depend on the communication range in the network. The next lemma states the upper bound for $\overline{\#\text{MMIS}(\Delta, T(n))}$ for a network using MPT or MPR.

Lemma 4.3: The average number of simultaneous transmissions, $\overline{\#\text{MMIS}(\Delta, T(n))}$, has the following upper bound in networks with MPT or MPR.

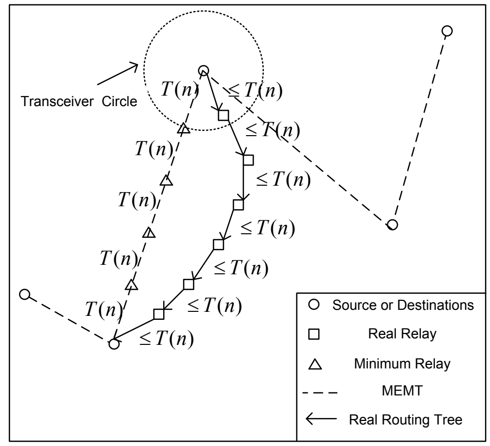

Fig. 2. The direct line between any two adjacent nodes in an (n, m, m)-cast tree is equal to or smaller than the total Euclidean distance in the tree through multiple relays.

$$\overline{\#\text{MMIS}(\Delta, T(n))} \leq \Theta(n) \tag{9}$$

Proof: We want to find out the maximum number of simultaneous transmissions in these dense networks. From the protocol model for MPT shown in Fig. 1, the disk with radius $T(n)$ centered at any transmitter or receiver should be disjoint from other disks centered at other receivers. For example, in the case of MPT, if the disks are not disjoint, there exist a receiver node that is located within the transmission range of two nodes. Based on the assumption of MPT that each node can only receive a single packet at a time (no MPR capability), these nodes in the overlapping areas have to receive two different packets at any time from different receivers, which is in contradiction with the assumption that each node can only receive one packet at a time. This means that the disk with radius $T(n)$ centered at any transmitter should be disjoint. Similar argument can be given for networks with MPR capability.

Thus, it is clear that, on average, there are $\pi T^2(n)n$ transmissions in one transceiver range $T(n)$ consuming an area of at least $\pi\left(T(n) + \frac{\Delta T(n)}{2}\right)^2$ in a dense network. Using this argument, it follows that the upper bound of $\overline{\#\text{MMIS}(\Delta, T(n))}$ is $\frac{n}{\left(1+\frac{\Delta}{2}\right)^2}$, which proves the lemma. ■

Lemma 4.3 implies that the number of simultaneous transmissions with MPT or MPR is upper bounded by $\Theta(n)$. By contrast, for the case of point-to-point communication, Keshavarz et al. [19] proved that $\#\text{MIS}(\Delta, r(n)) = \Theta(r^{-2}(n))$. Therefore, the maximum number of simultaneous transmissions cannot scale when neither MPT nor MPR is used.

Combining Lemmas 4.1, 4.2, and 4.3, we can compute the upper bound of (n, m, k)-cast capacity for MPT or MPR in the following theorem.

Theorem 4.4: In dense random wireless ad hoc networks with MPT or MPR, the upper bound per-node throughput capacity of (n,m,k)-cast is

$$C_{m,k}(n)=\begin{cases}O\left(k^{-1}\sqrt{m}T(n)\right) & \text{for } m\le\Theta\left(m_b\right)\\ O\left(k^{-1}\right) & \text{for } k\le\Theta\left(m_b\right)\le m\\ O\left(T^2(n)\right) & \text{for } \Theta\left(m_b\right)\le k\le m\end{cases}, \quad (10)$$

where $m_b=T^{-2}(n)$. When $k=m$, the per node throughput capacity is upper bounded as

$$C_{m,m}(n)=\begin{cases}O(T(n)/\sqrt{m}) & \text{for } m\le\Theta(T^{-2}(n))\\ O(T^2(n)) & \text{for } m\ge\Theta(T^{-2}(n))\end{cases}. \quad (11)$$

V. Lower Bound on The Throughput Capacity of (n,m,k)-Cast with MPT or MPR

To derive an achievable lower bound, we use a TDMA scheme for random dense wireless ad hoc networks similar to the approach used in [25], [26].

We first divide the network area into square cells. Each square cell has an area of $T^2(n)/2$, which makes the diagonal length of the square equal to $T(n)$, as shown in Fig. 3. Under this condition, connectivity inside all cells is guaranteed and all nodes inside a cell are within transceiver range $T(n)$ of each other. We build a cell graph over the cells that are occupied with at least one vertex (node). Two cells are connected if there exist a pair of nodes, one in each cell, that are less than or equal to $T(n)$ distance apart. Because the whole network is connected when $T(n)=r(n)\ge\Theta\left(\sqrt{\log n/n}\right)$, it follows that the cell graph is connected [25], [26].

To satisfy the MPT or MPR protocol model, we organize cells in groups so that simultaneous transmissions within each group does not violate the conditions for successful communication in the MPT or MPR protocol model. Let L represent the minimum number of cell separations in each group of cells that communicate simultaneously. Utilizing the protocol model, L satisfies the following condition:

$$L=\left\lceil 1+\frac{T(n)+(1+\Delta)T(n)}{T(n)/\sqrt{2}}\right\rceil=\lceil 1+\sqrt{2}(2+\Delta)\rceil \quad (12)$$

If we divide time into L^2 time slots and assign each time slot to a single group of cells, interference is avoided and the protocol model is satisfied. The separation example can be shown for the upper two transmitter (MPT case) or receiver (MPR case) circles in Fig. 3. For the MPT or MPR protocol model, the distance between two adjacent transmitting (MPT protocol model) or receiving (MPR protocol model) nodes is $(2+\Delta)T(n)$. Because this distance is smaller than $(L-1)T(n)$, this organization of cells guarantees that the MPT or MPR protocol model is satisfied. Fig. 3 represents one of these groups with a cross sign inside those cells for $L=4$.

We can derive an achievable (n,m,k)-cast capacity for MPT or MPR by taking advantage of this cell arrangement and the following property of the TDMA scheme with parameter L.

Lemma 5.1: The capacity reduction caused by the TDMA scheme is a constant factor and does not change the order capacity of the network.

Proof: The TDMA scheme introduced above requires cells to be divided into L^2 groups, such that only nodes in each group can transmit or receiver simultaneously. Eq. (12) demonstrates that the upper bound of L is not a function of n and is only a constant factor. Because the proposed TDMA scheme requires L^2 channel uses, it follows that this TDMA scheme reduces the capacity by a constant factor. ∎

Next we prove that, when n nodes are distributed uniformly over a unit square area, no matter in MPR or MPR scheme, we have simultaneously at least $\frac{1}{(LT(n)/\sqrt{2})^2}$ circular regions (see Fig. 3), each one containing $\Theta(nT^2(n))$ nodes w.h.p.. The objective is to find an achievable lower bound using the Chernoff bound, such that the distribution of the number of edges in this unit space is sharply concentrated around its mean, and hence the actual number of simultaneous transmissions occurring in the unit space in a randomly chosen network is indeed $\Theta(n)$ w.h.p..

Lemma 5.2: The circular area of radius $T(n)$ corresponding to the transceiver range of a transceiver (transmitter in MPT or receiver in MPR) j contains $\Theta(nT^2(n))$ nodes w.h.p., and is uniformly distributed for all values of j, $1\le j\le\frac{1}{(LT(n)/\sqrt{2})^2}$.

Proof: The statement of this lemma can be expressed as

$$\lim_{n\to\infty}P\left[\bigcap_{j=1}^{\frac{1}{(LT(n)/\sqrt{2})^2}}|N_j-E(N_j)|<\delta E(N_j)\right]=1, \quad (13)$$

where N_j and $E\left(N_j\right)$ are the random variables that represent the number of nodes in the transceiver circle of radius $T(n)$ centered around node j and the expected value of this random variable respectively, and δ is a positive arbitrarily small value close to zero.

¿From the Chernoff bound in Eq. (4), for any given $0<\delta<1$, we can find $\theta>0$ such that $P\left[|N_j-E(N_j)|>\delta E(N_j)\right]<e^{-\theta E(N_j)}$. Thus, we can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n\to\infty$. This is a key step in showing that when all the events $\bigcap_{j=1}^{\frac{1}{(LT(n)/\sqrt{2})^2}}|N_j-E(N_j)|<\delta E(N_j)$ occur simultaneously, then all N_j's converge uniformly to their expected values. Utilizing the union bound, we arrive at

$$\begin{aligned}&P\left[\bigcap_{j=1}^{\frac{1}{(LT(n)/\sqrt{2})^2}}|N_j-E(N_j)|<\delta E(N_j)\right]\\ =\;& 1-P\left[\bigcup_{j=1}^{\frac{1}{(LT(n)/\sqrt{2})^2}}|N_j-E(N_j)|>\delta E(N_j)\right]\\ \ge\;& 1-\sum_{j=1}^{\frac{1}{(LT(n)/\sqrt{2})^2}}P\left[|N_j-E(N_j)|>\delta E(N_j)\right]\\ >\;& 1-\frac{1}{(LT(n)/\sqrt{2})^2}e^{-\theta E(N_j)}. \end{aligned} \quad (14)$$

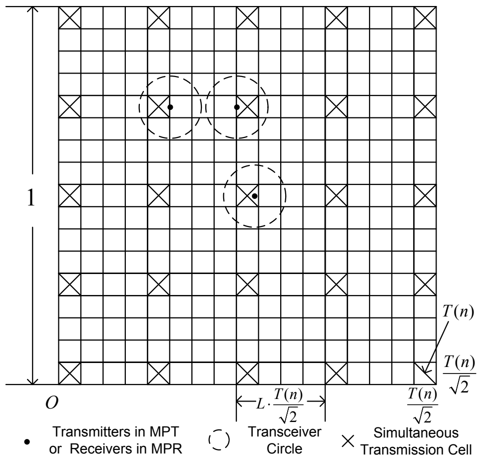

Fig. 3. Cell construction used to derive a lower bound on capacity

Given that $E(N_j) = \pi n T^2(n)$, then we have

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\frac{1}{(LT(n)/\sqrt{2})^2}} |N_j - E(N_j)| < \delta E(N_j)\right]$$
$$\geq 1 - \lim_{n\to\infty} \frac{1}{(LT(n)/\sqrt{2})^2} e^{-\theta\pi n T^2(n)} \tag{15}$$

Utilizing the connectivity criterion in Eq. (1), $\lim_{n\to\infty} \frac{e^{-\theta\pi n T^2(n)}}{T^2(n)} \to 0$, which completes the proof. ■

The previous lemma proves that, w.h.p., there are indeed $\Theta(n)$ simultaneous transmissions (receivers in MPT case or transmitters in MPR case) which are in $\frac{1}{(LT(n)/\sqrt{2})^2}$ circles of radius $T(n)$ around the transceivers (transmitters in MPT case or receivers in MPR case), who can transmit or receive simultaneously, as shown in Fig. 3. With Lemmas 5.1 and 5.2, we have done the preparation for the following achievable lower bound.

Let us define $\overline{\#\text{MEMKTC}(T(n))}$ as the total number of cells that contain all the nodes in an (n, m, k)-cast group. Also, $\overline{\#\text{MEMTC}(T(n))}$ is defined as the total number of cells that contain all the nodes in an (n, m, m)-cast group. The following lemma establishes the achievable lower bound for the (n, m, k)-cast throughput capacity of MPR or MPR as a function of $\overline{\#\text{MEMKTC}(T(n))}$. Note that $\overline{\#\text{MEMKTC}(T(n))}$ only depends on the (n, m, k)-cast network parameters regardless of using MPR or MPT techniques.

Lemma 5.3: The achievable lower bound for the (n, m, k)-cast capacity is given by

$$C_{m,k}(n) = \Omega\left(\left(\overline{\#\text{MEMKTC}(T(n))}\right)^{-1}\right). \tag{16}$$

Proof: There are $(T(n)/\sqrt{2})^{-2}$ cells in the unit square network area. From the definition of $\overline{\#\text{MEMKTC}(T(n))}$ and the fact that our TDMA scheme does not change the order capacity (Lemma 5.1), it is clear that there are at most in the order of $\overline{\#\text{MEMKTC}(T(n))}$ interfering cells for any (n, m, k)-cast communication. For each cell, the order of nodes in each cell is $\Theta\left(\pi T^2(n) n\right)$. Accordingly, the total lower bound capacity is given by $\Omega\left((T(n)/\sqrt{2})^{-2} \times \left(\pi T^2(n) n\right) \times \left(\overline{\#\text{MEMKTC}(T(n))}\right)^{-1}\right)$. Normalizing this value by total number of nodes in the network, n, proves the lemma. ■

Given the above lemma, to express the achievable lower bound of $C_{m,k}(n)$ as a function of network parameters, we need to compute the order of $\overline{\#\text{MEMKTC}(T(n))}$, which we do next.

Lemma 5.4: The average number of cells covered by the nodes in MEMKTC$(T(n))$, is tight bounded w.h.p. as follows:

$$\overline{\#\text{MEMKTC}(T(n))} = \begin{cases} \Theta\left(k\left(\sqrt{m}T(n)\right)^{-1}\right) & \text{for } m \leq \Theta(m_b) \\ \Theta(k) & \text{for } k \leq \Theta(m_b) \leq m \\ \Theta\left(T^{-2}(n)\right) & \text{for } \Theta(m_b) \leq k \leq m \end{cases} \tag{17}$$

where $m_b = T^{-2}(n)$. When $m = k$ for (n, m, m)-cast,

$$\overline{\#\text{MEMTC}(T(n))} = \begin{cases} \Theta\left(\sqrt{m}(T(n))^{-1}\right) & \text{for } m < \Theta(m_b) \\ \Theta\left(T^{-2}(n)\right) & \text{for } m \geq \Theta(m_b) \end{cases} \tag{18}$$

Proof: We first prove Eq. (18) for the case of (n, m, m)-casting. Because the total number of cells in this network is equal to $\Theta\left(T^{-2}(n)\right)$, it is clear that one bound for $\overline{\#\text{MEMTC}(T(n))}$ is this value. That is, $\overline{\#\text{MEMTC}(T(n))}$ cannot exceed the total number of cells in the network. On the other hand, the total Euclidean distance of the (n, m, m)-cast tree was shown earlier to be $\Theta(\sqrt{m})$. Because $T(n)$ is the transceiver range of the network, the maximum number of cells for this (n, m, m)-cast tree can be $\Theta\left(\sqrt{m}T^{-1}(n)\right)$, i.e., $\overline{\#\text{MEMTC}(T(n))} = \Theta\left(\sqrt{m}T^{-1}(n)\right)$. This bound can be achieved at the worst case when every two adjacent nodes in the (n, m, m)-cast tree belong to two different cells in the network. However, in practice, it is possible that some adjacent nodes in (n, m, m)-cast tree locate in a single cell which means this bound can be achieved for sure. The actual achievable bound is clearly the minimum of these two extreme values in the network, which is a function of the topology of the network, and this proves Eq. (18).

The proof of Eq. (17) can be derived from the proof of Eq. (18) and similar steps as those taken to prove Lemma 4.2 straightforwardly. ■

Combining Lemmas 5.3 and 5.4, we arrive at the achievable lower bound of the (n, m, k)-cast throughput capacity in dense random wireless ad hoc networks with MPT or MPR.

Theorem 5.5: The achievable lower bound of the (n, m, k)-cast throughput capacity with MPT or MPR is

$$C_{m,k}(n) = \begin{cases} \Omega\left(k^{-1}\sqrt{m}T(n)\right) & \text{for } m \leq \Theta(m_b) \\ \Omega\left(k^{-1}\right) & \text{for } k \leq \Theta(m_b) \leq m \\ \Omega\left(T^2(n)\right) & \text{for } \Theta(m_b) \leq k \leq m \end{cases}, \tag{19}$$

Clearly when $m = k$, we have

$$C_{m,m}(n) = \begin{cases} \Theta\left(T(n)/\sqrt{m}\right) & \text{for } m < \Theta\left(T^{-2}(n)\right) \\ \Theta\left(T^2(n)\right) & \text{for } m \geq \Theta\left(T^{-2}(n)\right). \end{cases} \tag{20}$$

VI. Capacity with MPT, MPR or Point-to-Point Communication

From Theorems 4.4 and 5.5, we can provide the tight bound throughput capacity for the (n, m, k)-cast when the nodes have MPT or MPR capability in dense random wireless ad hoc networks as follows.

Theorem 6.1: The throughput capacity of (n, m, k)-cast in a random dense wireless ad hoc network with MPT or MPR is

$$C_{m,k}(n) = \begin{cases} \Theta\left(k^{-1}\sqrt{m}T(n)\right) & \text{for } m \leq \Theta\left(m_b\right) \\ \Theta\left(k^{-1}\right) & \text{for } k \leq \Theta\left(m_b\right) \leq m \\ \Theta\left(T^2(n)\right) & \text{for } \Theta\left(m_b\right) \leq k \leq m \end{cases}. \quad (21)$$

The transceiver range of MPT or MPR should satisfy $T(n) \geq \Theta\left(\sqrt{\log n/n}\right)$. Note that the thresholds for different values for m and k provide various capacities for (n, m, k)-cast in MPT or MPR. Clearly when $k = m$, then

$$C_{m,m}(n) = \begin{cases} \Theta\left(T(n)/\sqrt{m}\right) & \text{for } \ m < \Theta\left(T^{-2}(n)\right) \\ \Theta\left(T^2(n)\right) & \text{for } \ m \geq \Theta\left(T^{-2}(n)\right) \end{cases}. \quad (22)$$

The (n, m, k)-cast throughput capacity of MPT and MPR can be extended to 3-D easily.

The throughput capacity for networks using point-to-point communication is given in [22] for the case of multicasting (i.e., $(n,\ m,\ m)$-cast). However, the results we just derived for the capacity of (n, m, k)-cast with MPT or MPR can be extended to address point-to-point communication as stated in the following theorem. The proof is presented in [27].

Theorem 6.2: The throughput capacity of (n, m, k)-cast in a random dense wireless ad hoc network with point-to-point communication is

$$C_{m,k}(n) = \begin{cases} \Theta\left(\sqrt{m}\left(nkr(n)\right)^{-1}\right) & \text{for } m \leq \Theta\left(m_b\right) \\ \Theta\left(\left(nkr^2(n)\right)^{-1}\right) & \text{for } k \leq \Theta\left(m_b\right) \leq m \\ \Theta\left(n^{-1}\right) & \text{for } \Theta\left(m_b\right) \leq k \leq m \end{cases}. \quad (23)$$

Summary of proof: The proof follows the same approach used for the case of MPT (or MPR) with two key differences. First, for point-to-point communication, the transceiver range $T(n)$ must be changed into the transmission range $r(n)$. Second, in point-to-point communication, there can be at most a single successful transmission inside a circle of radius of $r(n)$ centered around each receiver node.

VII. Discussion of Results

Our (n, m, k)-cast framework allows us to analyze the throughput capacity $C_{m,k}(n)$ in dense random wireless ad hoc networks using MPT, MPR or point-to-point communication under the protocol model as a function of the number of receivers k, the group size m of each (n, m, k)-cast, and the transceiver range $T(n)$ for MPT or MPR. In the following, we compare the order capacity attained when MPT or MPR is used at each transceiver. Our results clearly indicate that both MPT and MPR can provide order capacity gains for *all* modalities of information dissemination compared to the order capacity attained with point-to-point communication.

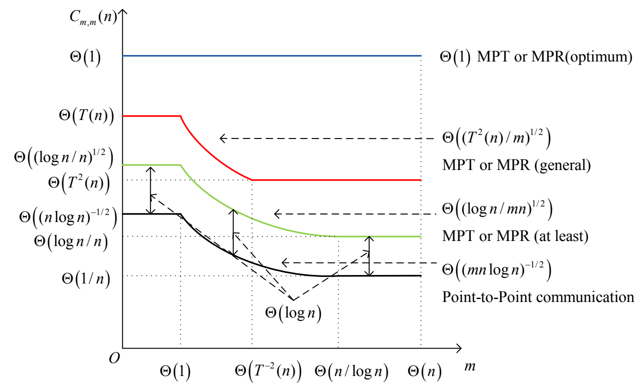

Fig. 4. Order throughput capacity of $(n,\ m,\ m)$-cast with MPT or MPR and point-to-point communication as a function of number of destinations m and transceiver range $T(n)$

A. $C_{m,m}(n)$ as a Function of Group Size (m)

Fig. 4 shows the throughput capacity of MPT or MPR in a wireless ad hoc network obtained from Eq. (22) as a function of the number of destinations for each source node with different transceiver range $T(n)$ when $m = k$. As the number of destinations per source (m) is varied from one to n, the throughput capacity becomes that of unicast, multicast, and broadcast, correspondingly. The figure clearly shows that there are two threshold values for m (denoted by m_u and m_b) that are critical to the throughput capacity.

If the number of destinations $m = k$ is not a function of n, then the order of capacity does not change, i.e., $m_u = \Theta(1)$. This result implies that the order capacity for both unicast and multicast with limited number of destinations is the same! This is very relevant for large wireless networks in practice, because the constituency of a multicast group in a large network is likely much smaller than and independent of the total number of nodes. The main reason for this result is the fact that, when the number of destinations is constant, the order of the total Euclidean distance of a multicast tree does not change.

The second threshold for the values of m is $m_b = \Theta(T^{-2}(n))$. If $m \geq m_b$, then the capacity of the wireless ad hoc network with MPT or MPR converges to the broadcast capacity of MPT or MPR, regardless of the number of destinations in the network. This is the lowest capacity that can be attained by the network.

When $m_u < m < m_b$, then the capacity of the network with MPT or MPR decreases as the number of destinations per communication group increases (see Fig. 4). Note that the decrease in throughput capacity is by a factor of $\sqrt{m}$. The main reason behind this behavior is the fact that when the number of receiver nodes increases in a two dimensional space, the size of the Euclidean distance of the (n, m, m)-cast tree increases by a factor of $\sqrt{m}$ instead of m.

For the case of point-to-point communication, our result for $m = k = \Theta(1)$ equals the well-known capacity result for multi-pair unicast introduced by Gupta and Kumar [1]. $\Theta(1/\sqrt{mn\log n})$ bits per second constitutes a tight bound for the capacity of multicast communication (i.e., $m = k < n$) when $m \leq \Theta\left(n/\log n\right)$ and $r(n)$ is chosen as the minimum value to guarantee the connectivity criterion. The multicast order capacity of a wireless network equals its capacity for multi-pair unicast when the number of destinations per multicast source is not a function of n. It has been shown [22] that the multicast capacity of a random wireless ad hoc network is $\Theta(1/n)$, which is the broadcast capacity of the network [19] when $m \geq \Theta(n/\log n)$. From these results, it is clear that our model incorporates and agrees with all prior results on the capacity of wireless networks for unicast, multicast, and broadcast when point-to-point communication is assumed.

B. $C_{m,k}(n)$ as a Function of Group Size (m)

Fig. 5 compares the throughput capacity of MPT or MPR to that of point-to-point communication when $k < m$. Comparing the results for both cases when the number of nodes is smaller than $\Theta(m_b)$, it appears that they both have the same term as $\sqrt{m}/k$. However, for MPT or MPR this term is multiplied by $T(n)$, while for point-to-point communication this term is divided by $r(n)$. If we assume $T(n) = r(n)$, it appears that increasing the transceiver range increases the capacity for the MPT or MPR scheme, while it decreases the capacity for point-to-point communication. This fundamental difference is due to the fact that the MPT or MPR scheme embraces interference, while point-to-point communication is based on avoiding it by limiting transmissions around receivers.

Comparing the capacities attained with MPT or MPR and point-to-point communication for unicast traffic (see Fig. 4), the ratio is equal to $\Theta(T(n)\sqrt{n \log n})$. The same ratio is equal to $\Theta(T^2(n)n)$ for the case of broadcasting. If we choose a larger value for the communication range for MPT and MPR, i.e., $T(n) > \Theta\left(\sqrt{\log n/n}\right)$, then it is easy to show that the capacity gain for MPT or MPR compared to point-to-point communication is larger in broadcast communication than for unicasting. The larger gains attained with MPT or MPR for broadcast communication are a consequence of the fact that, as the number of broadcast destinations increases, more copies of the same packets must be sent to a larger number of nodes. In a network using MPT or MPR, concurrent broadcast transmissions can be decoded by the receivers while at most one broadcast transmission can succeed at a time when point-to-point communication is assumed.

We note that the capacity of anycast or manycast is greater than the capacity of unicast if $k < \Theta(\sqrt{m})$, even if each node requires to transmit its packets to more than one destination. This result shows that, as long as $k < \Theta(\sqrt{m})$, the total number of hops required to transmit packet to k destinations is always, on average, less than sending the packet from the same source to a single randomly selected destination in unicast communications. Equivalently, the total Euclidean distance for a manycast tree is on average less than the Euclidean distance between any randomly selected source and destination in unicast communication. However, these Euclidean distances become the same, on average, when $k = \Theta(\sqrt{m})$. As it can be predicted from this figure, the total Euclidean distance in a manycast tree increases as k increase and for $k > \Theta(\sqrt{m})$, the capacity of manycast becomes less than unicast because of the total Euclidean distance in the manycast tree.

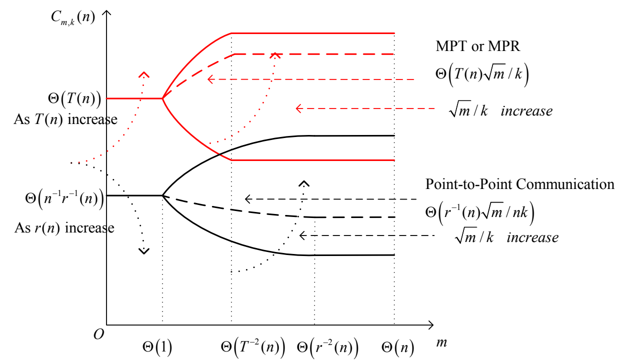

Fig. 5. Order throughput capacity of (n, m, k)-cast with MPT or MPR and point-to-point communication

C. $C_{m,k}(n)$ as a Function of Transceiver Range ($T(n)$)

Equations (22) and (21) show that the throughput capacity of wireless ad hoc networks do increase with the increase in the transceiver range $T(n)$ when the transceivers encode (MPT case) or decode (MPR case) more than one packet at a time. Similar result for MPR was shown in [11] for the case of unicasting. This result is in sharp contrast to results attained with point-to-point communication, with which increasing the communication range decreases the capacity. In networks with MPT or MPR, by increasing the transceiver range in the network we actually increase the total number of simultaneous transmissions at a given time! In contrast, for networks with point-to-point communication, a larger transmission range leads to increased interference at larger number of nodes, which forces these nodes to be silent during a communication session.

Clearly, the capacity of the network is maximized if we maximize the number of simultaneous transmissions in the network. Ideally, if the transceiver range can be made $\Theta(1)$, then a network using MPT or MPR can scale linearly with n. Obviously, the transceiver range is restricted in practice by the complexity of the nodes. However, even the transceiver range is assumed to have the minimum value, which is the connectivity criterion in Eq. (1), MPT or MPR still renders a capacity gain compared to point-to-point communication. Furthermore, this gain is still an order gain equal to $\Theta(\log n)$ compared to the capacity attained with point-to-point communication for (n, m, k)-casting. Our result agrees and extends the capacity gain result reported in [11] for unicast.

D. Duality Between MPT and MPR

From the analysis above, it is clear that MPT and MPR are two cooperative techniques that are equivalent in terms of capacity scaling laws. MPT concentrates on increasing the encoding complexity at the transmitter, while MPR requires more decoding complexity at the receiver side. The results in this paper provide new directions and opportunities for future research activities in wireless ad hoc networks.

The fact that MPR and MPT are equivalent to each other in terms of capacity and delay scaling laws is important, because MPT may be a more practical approach to embracing interference than implementing MPR (e.g., by means of directional antennas or beam forming). Our work shows that addressing the practical implications of MPR and MPT schemes should be the subject of future studies.

VIII. Conclusion

We showed that the throughput capacity of (n, m, k)-cast with multi-packet reception/transmission is $\Theta\left(T(n)\sqrt{m}/k\right)$ when $k \leq m < \Theta\left(T^{-2}(n)\right)$, $\Theta\left(1/k\right)$ when $k \leq \Theta\left(T^{-2}(n)\right) < m$ and $\Theta\left(T^{2}(n)\right)$ when $\Theta\left(T^{-2}(n)\right) < k \leq m$. When $T(n) \geq \Theta\left(\sqrt{\log n/n}\right)$ to satisfy the connectivity criterion, MPT (MPR) leads to the minimum throughput capacity gain of at least $\Theta(\log n)$ compared to the (n, m, k)-cast throughput capacity with point-to-point communication. When $T(n) = \Theta(1)$, which is the maximum transceiver range for MPT (MPR), the network is linearly scalable. However, this case is not practical in real systems, and simply provides the guideline for designing networks. It suggests that, in order to increase the capacity of wireless ad hoc networks, we must embrace interference by using MPT (MPR). This result is in sharp contrast with traditional interference dominated networks based on point-to-point communication. Finally, when the number of destinations is greater than $\Theta\left(T^{-2}(n)\right)$ or equivalently the transceiver range is larger than $\Theta\left(\sqrt{\log n/n}\right)$, there are higher throughput capacity gains with MPT (MPR). This is the case in broadcasting or multicasting with larger numbers of destinations, because MPT (MPR) schemes can inhibit the negative effects of interference compared to point-to-point communication.

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