Fundamental Limits of Information Dissemination in Wireless Ad Hoc Networks—Part I: Single-Packet Reception

Zheng Wang, *Student Member, IEEE,* Hamid R. Sadjadpour, *Senior Member, IEEE,* J. J. Garcia-Luna-Aceves, *Fellow, IEEE*, and Shirish S. Karande, *Member, IEEE*

***Abstract*—We present the first unified modeling framework for the computation of the capacity-delay tradeoff of random wireless ad hoc networks. This framework considers information dissemination by means of unicast routing, multicast routing, broadcasting, or different forms of anycasting. We introduce (n, m, k)-casting as a generalization of all forms of one-to-one, one-to-many, and many-to-many information dissemination in wireless networks. In this context, n, m, and k denote the total number of nodes in the network, the number of destinations for each communication group, and the actual number of communication-group members that receive information ($k \leq m$), respectively. We describe the capacity-delay tradeoff for (n, m, k)-casting in wireless ad hoc networks in which receivers perform single-packet reception (SPR). Our results are consistent with prior results in wireless networks and extend them to the general (n, m, k)-cast case.**

***Index Terms*—Capacity, delay, scaling law, wireless ad hoc networks, (n, m, k)-cast.**

I. Introduction

THE seminal work by Gupta and Kumar [1] on the capacity of wireless networks has sparked a growing amount of interest in the understanding of the fundamental capacity limits of wireless ad hoc networks. Many researchers focused on improving the capacity of wireless networks with unicast communications [2]–[4]. A number of studies on the capacity of ad hoc networks concentrated on broadcasting (e.g., [5], [6]) and multicasting (e.g., [7], [8]). The work presented in this paper is motivated by the fact that, to date, there has been no unified treatment on the capacity and delay scaling laws of wireless networks subject to different types of forwarding disciplines.

We present the first unified modeling framework for the computation of fundamental limits of capacity-delay tradeoffs in wireless ad hoc networks in which information is disseminated by means of unicast routing, multicast routing, broadcasting, or different forms of anycasting, and in which receivers are capable of single-packet reception (SPR). Our conclusions in this paper are consistent with the results by Gupta and Kumar [1]. In the second part of this paper, we analyze the scaling laws for multi-packet reception (MPR), which allows a single node to receive and decode multiple simultaneous transmissions.

We define (n, m, k)-casting as a generalization of all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks. In the context of (n, m, k)-casting, n, m, and k denote the number of nodes in the network, the number of destinations for each communication group, and the actual number of communication-group members that receive information optimally[1], respectively. Section III describes the network model and necessary concepts for the development of our framework.

We address the capacity and delay scaling laws of (n, m, k)-casting under the protocol model in Section IV. Section V discusses several possible implications of our new model and concludes the paper.

II. Related Work

Many contributions have been made on the capacity of wireless networks subject to unicasting, and we mention only a few of them. A number of papers have extended the results by Gupta and Kumar. Franceschetti et al. [2] used percolation theory to close the gap between the upper and lower bounds of the unicast capacity under the physical model reported by Gupta and Kumar [1]. Recently, Ozgur et al. [3] proposed a hierarchical cooperation technique based on virtual MIMO to achieve linear per source-destination capacity. Cooperation can be extended to the simultaneous transmission and reception at the various nodes in the network, which can result in significant improvement in capacity [4].

Considerable prior work has focused on the capacity of broadcasting in wireless networks. Tavli [5] was the first to show that $\Theta\left(n^{-1}\right)$ is a bound on the per-node broadcast capacity of arbitrary networks. Keshavarz et al. [6] presented the most general work on the computation of the broadcast capacity for any number of sources in the network. Our work in this paper was inspired by some of the contributions in [6].

There are prior contributions on the multicast capacity of wireless networks [7], [8]. Jacquet and Rodolakis [7] proved that the scaling of multicast capacity is decreased by a factor of $O(\sqrt{m})$ compared to the unicast capacity result by Gupta and Kumar [1] where m is the number of destinations for each

[1]Optimality is defined as the k closest (in terms of Euclidean distance of the tree) destinations to the source in an (n, m, k)-cast group.

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Z. Wang and H. R. Sadjadpour are with the Department of Electrical Engineering, University of California, Santa Cruz, 1156 High Street, Santa Cruz, CA 95064, USA (e-mail:{wzgold, hamid}@soe.ucsc.edu).

J. J. Garcia-Luna-Aceves is with the Department of Computer Engineering, University of California, Santa Cruz, 1156 High Street, Santa Cruz, CA 95064, USA, and the Palo Alto Research Center (PARC), 3333 Coyote Hill Road, Palo Alto, CA 94304, USA (e-mail:jj@soe.ucsc.edu).

S. S. Karande is with Philips Research Bangalore, No.1, Murphy Road, Ulsoor, Bangalore, 560 008, India (e-mail:shirish.karande@philips.com).

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source. Li et al. [8] compute the capacity of wireless ad hoc networks for unicast, multicast, and broadcast applications. While the results by Li et al [8] are equivalent to our capacity results for $(n, m, k = m)$-casting, it is worth noting that our derivation of the capacity is different than the work by Li et al [8]. In addition, our derivation is for the case of (n, m, k)-cast which is more general than only unicast, multicast and broadcast communications. More importantly, this is the first paper to present a capacity-delay tradeoff study for all kinds of information dissemination as a general function of the transmission range, $r(n)$.

III. Network Model and Preliminaries

We assume a random wireless network with n *static* nodes distributed uniformly in the network area. Our analysis is based on *dense* networks, where the area of the network is a square of unit value and the density of the network goes to infinity as n goes to infinity. Our capacity analysis is based on the protocol model for dense networks. To simplify our analysis, the network area is assumed to be unit-square area but similar results can be achieved with torus or sphere shape area. All nodes use a common transmission range $r(n)$ for all their communication.

Definition 3.1: The Protocol Model: Node i at location X_i can successfully transmit to node j at location X_j if, for any node $X_k, k \neq i$ that transmits at the same time as X_i, we have $|X_i - X_j| \leq r(n)$ and $|X_k - X_j| \geq (1 + \Delta)r(n)$.

Lemma 3.2: Connectivity criterion for protocol model in dense networks [1]: To ensure that there is no isolated node in the network, the transmission range $r(n)$ in random dense networks satisfies [2].

$$r(n) = \Omega\left(\sqrt{\log n/n}\right). \tag{1}$$

In this paper, we study the case in which all n nodes in the network act as sources that communicates with a group of m receivers (with $m < n$) and that k of those receivers obtain the information reliably. We call this characterization of information dissemination from sources to receivers (n, m, k)-casting. This characterization is useful because it can model all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks.

Definition 3.3: Feasible throughput capacity of (n, m, k)-cast: A throughput of $\lambda(n)$ bits per second for each node is feasible if we can define a scheduling transmission scheme that allows each node in the network to transmit $\lambda(n)$ bits per second on average to its k out of m destinations.

The per-node throughput capacity of the network is defined as the number of bits per second in Definition 3.3 that every node can transmit to its destination.

Definition 3.4: Order of throughput capacity: $\lambda(n)$ is said to be of order $\Theta(f(n))$ bits per second if there exist deterministic positive constants c and c' such that

$$\begin{cases} \lim_{n\to\infty} \text{Prob } (\lambda(n) = cf(n) \text{ is feasible}) = 1 \\ \liminf_{n\to\infty} \text{Prob } (\lambda(n) = c'f(n) \text{ is feasible}) < 1. \end{cases} \tag{2}$$

[2]Given two functions $f(n)$ and $g(n)$. We say that $f = O(g(n))$ if $\sup_n(f(n)/g(n)) < \infty$. We say that $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$. If both $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$, then we say $f(n) = \Theta(g(n))$.

Computing the throughput capacity of a network of n nodes requires us to consider the minimum (n, m, k)-cast trees between sources and their intended receivers. Furthermore, computing this capacity requires that the selection of k out of m receivers be optimum. The rest of this section introduces additional concepts necessary for the computation of the throughput capacity of a random ad hoc network.

Definition 3.5: (n, m, k)-Cast Tree: An (n, m, k)-cast tree is a minimum set of nodes that connect a source node of an (n, m, k)-cast with all its intended m receivers in order for the source to send information to k of those receivers.

The construction of (n, m, k)-cast tree starts with connecting the source to m destinations using minimum number of relays or hops. After constructing this tree, we pick k out of m nodes in this tree that have minimum total Euclidean distance to the source. We refer to this selection of k nodes as "optimum" because it results in maximum throughput capacity for the network. Note that there are $\binom{m}{k}$ choices for selecting k nodes and in this paper, we have selected the above criterion for this selection.

When communicating over a broadcast channel, a transmission from a source or relay in an (n, m, k)-cast may interfere with other transmissions in the same or different (n, m, k)-casts. For a given (n, m, k)-cast to succeed, the packet from the source must reach k of the m receivers in the group reliably at least once. Furthermore, any given relay forwards a packet only once. Accordingly, one or multiple (n, m, k)-cast trees can be defined by the set of transmissions that reach each relay and destination of a given (n, m, k)-cast for the first time. When $m = k$, the resulting (n, m, m)-cast tree is also called a multicast tree. For the case in which $k \leq m$, the selection of the subset of k receivers that correctly receive the packet from the source is such that each of them is reached through a branch of the (n, m, k)-cast tree.

Given the distribution of nodes in the plane and the protocol model we assume, the possible (n, m, k)-cast trees we need to consider include only those that render the minimum number of transmissions for a packet from the source to reach all the intended receivers (k or m) at least once. Because transmissions occur over a common broadcast channel, this implies that the (n, m, k)-cast trees in which we are interested are those that involve the minimum number of relay nodes needed to connect the source and intended receivers of an (n, m, k)-cast. That is, we focus on (n, m, k)-cast trees built by the aggregation of shortest paths (minimum-hop paths) between a source and all of its intended destinations. Accordingly, we adopt the following definition for (n, m, k)-cast trees in the rest of this paper.

Definition 3.6: Euclidean Minimum Spanning Tree (EMST): [9] Consider a connected undirected graph $G = (V, E)$, where V and E are the sets of vertices and edges in the graph G, respectively. The EMST of G is a spanning tree of G with the total minimum Euclidean distance of the edges of the tree.

An (n, m, k)-cast tree is a function of the transmission range $r(n)$. Therefore, the optimum tree that has the minimum Euclidean distance is a function of $r(n)$. For this reason, changing the transmission range will change the optimum (n, m, k)-cast tree.

Definition 3.7: Minimum Euclidean (n, m, k)-Cast Tree (MEMKT($r(n)$)): The MEMKT($r(n)$) of an (n, m, k)-cast is an (n, m, k)-cast tree in which the k destinations that receive information from the source among the m receivers of the (n, m, k)-cast have the minimum total Euclidean distance.

Definition 3.8: Minimum Area (n, m, k)-cast Tree (MAMKT($r(n)$)): The MAMKT($r(n)$) in a (n, m, k)-cast tree with k out of m destinations for each source is a (n, m, k)-cast tree that has minimum total area. The area of a (n, m, k)-cast tree is defined as the total area covered by the circles centered around each source or relay with radius $r(n)$.

Note that EMST is spanning tree that consider only the source and destinations, while MEMKT and MAMKT are related to a real routing tree that includes the relays needed to connect the source with the destinations.

In our delay analysis, we assume that the delay associated with packet transmission is negligible and the delay is essentially proportional to the number of hops from source to destination. When the packet size is large, then the transmission delay is considerable and we no longer can ignore this delay. Our analysis does not consider this case and this is the subject of future study.

Definition 3.9: Delay of an (n, m, k)-Cast: In an (n, m, k)-cast, the delay of a packet in a network is the time it takes the packet to reach *all* k destinations after it leaves the source.

We do not take queuing delay at the source into account, because our interest is in the network delay. The average packet delay for a network with n nodes, $D_{m,k}(n)$, is obtained by averaging over all packets, all source-destination pairs, and all random network configurations.

In the rest of this paper, $\|T\|$ denotes the total Euclidean distance of a tree T, $\#T$ is used to denote the total number of vertices (nodes) in a tree T, $S(T)$ denotes the area of tree T covered and $\overline{\#T}$ denotes the total average number of vertices (nodes) in a tree T.

To compute the (n, m, k)-cast capacity, we use the relationship between the total Euclidean distance of MEMKT and EMST. Steele [9] determined a tight bound for $\overline{\|\text{EMST}\|}(m)$ of a group of m nodes and for large values of m, which we restate in the following lemma.

Lemma 3.10: Let $f(x)$ denote the node probability distribution function in the network area. Then, for large values of m and $d > 1$, the $\overline{\|\text{EMST}\|}(m)$ is tight bounded as

$$\overline{\|\text{EMST}\|}(m) = \Theta\left(c(d)m^{\frac{d-1}{d}}\int_{R^d} f(x)^{\frac{d-1}{d}}dx\right), \quad (3)$$

where d is the dimension of the network. Note that both $c(d)$ and the integral are constant values and not functions of m. When $d = 2$, then $\overline{\|\text{EMST}\|}(m) = \Theta\left(\sqrt{m}\right)$.

Table I summarizes key terms we use in the computation of scaling laws for (n, m, k)-casting.

TABLE I
ABBREVIATION TABLE

EMST	Euclidean Minimum Spanning Tree
MEMKT	Minimum Euclidean (n, m, k)-cast Tree
MEMKTC	Minimum Euclidean (n, m, k)-cast Tree Cells
MAMKT	Minimum Area (n, m, k)-cast Tree
$r(n)$	Transmission Range in Protocol Model

IV. THE (n, m, k)-CAST CAPACITY

A. Upper Bound

Note that MEMKT includes intermediate relays while EMST(m) only includes m destinations. Lemma 3.10 computes the average total Euclidean distance for EMST(m). To compute the upper bound for (n, m, k)-cast, we will first demonstrate the relationship between $\overline{S(\text{MAMKT})}$ and $\overline{\|\text{EMST}\|}(m)$.

Lemma 4.1: The average area for MEMKT($r(n)$) has the following lower bound.

$$\overline{S\left(\text{MAMKT}(r(n)\right)} = \begin{cases} \Omega\left(kr(n)/\sqrt{m}\right), m = O\left(r^{-2}(n)\right) \\ \Omega\left(kr^2(n)\right), \Omega(k) = r^{-2}(n) = O(m) \\ \Omega\left(1\right), \quad k = \Omega\left(r^{-2}(n)\right) \end{cases} \quad (4)$$

Proof: From Lemma 3.10, if we select only m destinations ($m + 1$ nodes including source and m destinations) out of n nodes to construct an EMST(m), then the total average Euclidean distance of the EMST(m) is at least $\Theta(\sqrt{m})$. Given that there are m destinations for the tree, then the average Euclidean distance between any two nodes for this tree is $\Theta\left(\sqrt{m}/m\right)$, so the k closest destinations and the source construct a tree with average length of $\Theta\left(\sqrt{m}k/m\right)$. If we just select the k destinations randomly, then the problem is an (n, k, k)-cast in our formulation and then the distance of that tree is $\Theta(\sqrt{k})$ based on Lemma 3.10. Here, we assume to construct m multicast tree first, and then choose the optimal (smallest length of the tree) ones as the real destinations.

It has been proved [8] that the average area of a tree T with transmission range $r(n)$ is lower bounded by the multiplication between the length of the tree and transmission range $r(n)$ when the number of the actual destinations satisfies $m = O\left(r^{-2}(n)\right)$. Thus, when the transmission range is not a large value, then the total area in such a tree is lower bounded by $\Omega\left(kr(n)/\sqrt{m}\right)$. This is the top lower bound in Eq. (4). When the transmission range is larger, given that we only need the closest k nodes in the set, then the area of that tree is lower bounded by $\Omega\left(kr^2(n)\right)$ ($\pi r^2(n)$ is the area covered by one node). This is the second lower bound in Eq. (4). Once $k = \Omega\left(r^{-2}(n)\right)$, then the MAMKT($r(n)$) covers the entire network and we can use $\Omega(1)$ as the lower bound, which is the last value in Eq. (4). The threshold for $r(n)$ is derived when the first two lower bounds are equal, i.e., $\Theta\left(kr(n)/\sqrt{m}\right) = \Theta\left(kr^2(n)\right)$. The solution to the value of m_b is $m_b = \Theta\left(r^{-2}(n)\right)$. This result means that, when $m = O(m_b)$ or $m = \Omega(m_b)$, the lower bound of $\overline{S\left(\text{MAMKT}\right)}$ is $\Omega\left(kr(n)/\sqrt{m}\right)$ or $\Theta\left(kr^2(n)\right)$, respectively. ■

Theorem 4.2: The upper bound of the per-node (n, m, k)-cast throughput capacity in dense wireless ad hoc networks is

$$C_{m,k}(n) = \begin{cases} O\left(\sqrt{m}(nkr(n))^{-1}\right), \ m = O\left(r^{-2}(n)\right) \\ O\left(\left(nkr^2(n)\right)^{-1}\right), \ \Omega(k) = r^{-2}(n) = O(m) \\ O\left(n^{-1}\right), \ k = \Omega\left(r^{-2}(n)\right) \end{cases} \quad (5)$$

Proof: The proof is immediate by combining Lemma 4.1 with the fact that total area for a unit network was "1". Using the same argument in [1], the total throughput capacity is the total area divided by the consumed area for one (n, m, k)-cast tree. The result is immediate by normalizing the result by n.

Note that $\overline{S\,(\text{MAMKT})}$ can have some overlap for different (n, m, k)-cast sessions. The exclusive area for each multicast session is in the same order as the $\overline{S\,(\text{MAMKT})}$.

In [1], disks of radius $\Delta r(n)/2$ centered at each receiver are disjoint in order to guarantee the protocol model. Therefore, the actual minimum exclusive area for each (n, m, k)-cast session is at least

$$\overline{S\,(\text{MAMKT})} \times \pi \left(\frac{\Delta r(n)}{2}\right)^2 \times \frac{1}{\pi r^2(n)} = \frac{\Delta^2}{4}\overline{S\,(\text{MAMKT})}. \quad (6)$$

The difference is at most $\Delta^2/4$ which does not change the order. Hence, the capacity is the network area divided by the total occupied area of one (n, m, k)-cast tree normalized by n, which leads to the per-node capacity. ■

B. Lower Bound

To derive the achievable lower bound, we use a TDMA scheme for random dense networks similar to the approach used in [10]. We first divide the network area into square cells. Each square cell has an area of $r^2(n)/2$, which makes the diagonal length of square equal to $r(n)$. Under this condition, connectivity inside all cells is guaranteed and all nodes inside a cell are within transmission range of each other. We build a cell graph over the cells that are occupied with at least one vertex (node). Two cells are connected if there exist a pair of nodes, one in each cell, which are less than or equal to $r(n)$ distance apart. Because the whole network is connected when Eq. (1) is satisfied, it follows that the cell graph is connected.

To satisfy the protocol model, we should design cells in groups such that simultaneous transmissions within each group do not violate the condition for successful communication in the protocol model. Let L represent the minimum number of cell separations in each group of cells that communicate simultaneously. Utilizing the protocol model, L is given as $L = \left\lfloor 1 + \frac{r(n)+(1+\Delta)r(n)}{r(n)/\sqrt{2}} \right\rfloor = \lfloor 1 + \sqrt{2}(2+\Delta) \rfloor$ If we divide time into L^2 time slots and assign each time slot to a single group of cells, interference is avoided and the protocol model is satisfied.Given that the parameter L is not a function of n, the TDMA scheme does not change the order capacity of the network.

Definition 4.3: Minimum Euclidean (n, m, k)-Cast Tree Cells(MEMKTC$(r(n))$): The MEMKTC$(r(n))$ of an (n, m, k)-cast tree is the total cells containing all the nodes in the (n, m, k)-cast tree.

The following lemma establishes the achievable lower bound for the (n, m, k)-cast capacity as a function of $\overline{\#\text{MEMKTC}(r(n))}$, the total number of cells that contain all the nodes in an (n, m, k)-cast group.

Lemma 4.4: The achievable lower bound of the per-node (n, m, k)-cast throughput capacity in dense wireless ad hoc networks is given by

$$C_{m,k}(n) = \Omega\left(1/\overline{\#\text{MEMKTC}(r(n))} \times 1/nr^2(n)\right), \quad (7)$$

Proof: The proof is presented as lemma 4.6 in [11]. ■

Lemma 4.5: The average number of cells in MEMKT$(r(n))$ tree is tight bounded as

$$\overline{\#\text{MEMKTC}(r(n))} = \begin{cases} \Theta\left(k(\sqrt{m}r(n))^{-1}\right), \ m = O\left(r^{-2}(n)\right), \\ \Theta\left(k\right), \ \Omega(k) = r^{-2}(n) = O(m), \\ \Theta\left(r^{-2}(n)\right), \ k = \Omega\left(r^{-2}(n)\right). \end{cases} \quad (8)$$

Proof: The proof is given in lemmas 4.7 and 5.5 in [11]. ■

Theorem 4.6: The achievable lower bound of the (n, m, k)-cast throughput capacity in dense wireless ad hoc networks is

$$C_{m,k}(n) = \begin{cases} \Omega\left(\sqrt{m}(nkr(n))^{-1}\right), \ m = O\left(r^{-2}(n)\right) \\ \Omega\left(\left(nkr^2(n)\right)^{-1}\right), \ \Omega(k) = r^{-2}(n) = O(m) \\ \Omega\left(n^{-1}\right), \ k = \Omega\left(r^{-2}(n)\right) \end{cases} \quad (9)$$

Proof: The proof is immediate by combining Lemmas 4.4 and 4.5. ■

It is clear that by combining Theorems 4.2 and 4.6, a tight bound for the capacity of (n, m, k)-cast can be derived. We notice from the above results that there are three distinct capacity regions for (n, m, k)-casting. These three different regions are achieved based on different values of transmission range $r(n)$, m, and k. In the first region, the order capacity of wireless ad hoc networks is similar to that of unicast communication. Therefore, we refer to this first capacity region as unicast region. This unicast capacity region also includes the capacity for multicasting or any type of anycast communication. Once the number of receiver nodes is smaller than $\Theta\left(r^{-2}(n)\right)$, then we enter into a second capacity region, which we call the multicast capacity region. The last region is defined for the case when both m and k are larger than $\Theta\left(r^{-2}(n)\right)$. The network capacity in this region is equivalent to the broadcast capacity of the network, and hence we call this region the broadcast capacity region. The (n, m, k)-cast tree associated to this region spans all the elements of the graph and it is equivalent to a connected dominating set for the entire network. Therefore, regardless of having multicast, broadcast, or any type of anycast communications, the capacity reaches its minimum possible value for a given transmission range which is the same as broadcast capacity.

C. Delay Analysis of (n, m, k)-Cast

In this section, we present the delay of (n, m, k)-casting and its tradeoff with capacity. As Definition 3.9 states, the packet delay is proportional to the total number of hops required from each source to its destinations. In order to compute this delay, we first prove the following lemma.

Lemma 4.7: The delay of (n, m, k)-cast in a random dense wireless ad hoc network with SPR is

$$D_{m,k}(n) = \Theta\left(\overline{\#\text{MEMKTC}(r(n))}\right). \quad (10)$$

Proof: From the definition of $\overline{\#\text{MEMKTC}(r(n))}$ and Lemma 4.5, we conclude that $\overline{\#\text{MEMKTC}(r(n))}$ is proportional to the minimum number of hops in which the information is routed from source to all its destinations. Because we are assuming a TDMA scheme to achieve the lower bound for the capacity, it is clear that, to transport the information from one cell to the next adjacent cell, one to two hops are required. Therefore, $\overline{\#\text{MEMKTC}(r(n))}$ is also in the same order as the total number of hops needed. Based on the definition of delay, it is clear that $\overline{\#\text{MEMKTC}(r(n))}$ is also the same order bound as the total delay, which proves the Lemma. ■

It is clear that we can compute a tight bound for delay in (n, m, k)-cast as a function of $r(n)$ by combining lemmas 4.5 and 4.7.

Theorem 4.8: The relationship between capacity and delay for (n, m, k)-cast is given by

$$C_{m,k}(n)D_{m,k}(n) = \Theta\left(\left(nr^2(n)\right)^{-1}\right). \quad (11)$$

Proof: The results can be easily derived by comparing Theorems 4.6, 4.2 with lemmas 4.5 and 4.7. ■

V. Discussion of Results and Conclusion

There is much valuable insight to be gained from modeling the capacity of unicasting, multicasting, broadcasting and anycasting using the same framework. Our (n, m, k)-cast framework allows us to analyze the throughput capacity of wireless networks as a function of the number of receivers of a communication group, which can range from 1 up to the number of nodes in the network, as well as a function of the transmission range. Accordingly, the results obtained in all prior work can be derived from our model by selecting the appropriate values for $r(n)$ and m in the capacity results obtained in Sections IV. In addition, our framework also provides new insight on the capacity of information dissemination techniques that are becoming more prevalent with the availability of in-network storage, namely anycasting, and allows us to reason about the nature that route signaling should be rendered more scalable wireless networks.

A. $C_{m,k}(n)$ as a Function of Transmission Range ($r(n)$) and Group Size (m)

The relationship between $C_{m,k}(n)$ and the transmission range $r(n)$ can be seen in Fig. 1. From this figure, we see that maximum capacity can be attained when the transmission range has its minimum value, i.e., $r(n) = \Omega\left(\sqrt{\log n/n}\right)$. We can conclude the throughput capacity of dense wireless ad hoc networks is proportional with the $\Theta\left(\sqrt{m}/k\right)$ and inversely proportional with the transmission range $r(n)$. Besides, the broadcast threshold m_b will be decreased when $r(n)$ increases.

Fig. 2 shows $C_{m,k}(n)$ as a function of m. As it was the case for $C_{m,m}(n)$, if m varies from 1 to $m_u = \Theta(1)$, the capacity of the network does not change and equals $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$. For values of m larger than m_u, the (n, m, k)-cast order capacity can increase or decrease depending on the value of k. The smallest order capacity corresponds to the case when $k = m$, i.e., multicasting ($m < n$) or broadcasting ($m = n$), and the largest order capacity is attained for anycasting ($k = 1$). The shaded area in the figure shows the achievable capacity for manycasting ($1 < k < m$) for different values of m and k.

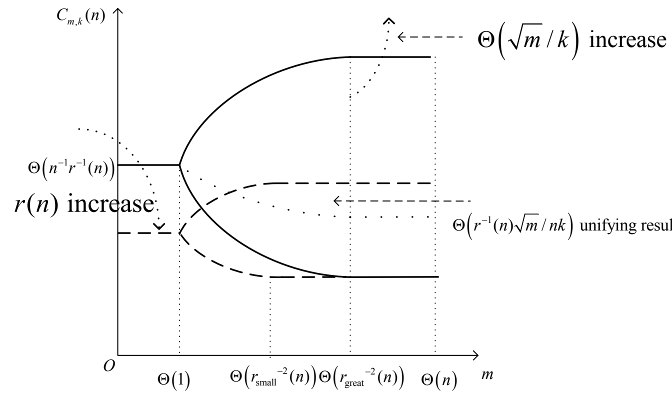


Fig. 1. $C_{m,k}(n)$ as a function of transmission range $r(n)$, real number of destinations k, and the number of destination group choices m.

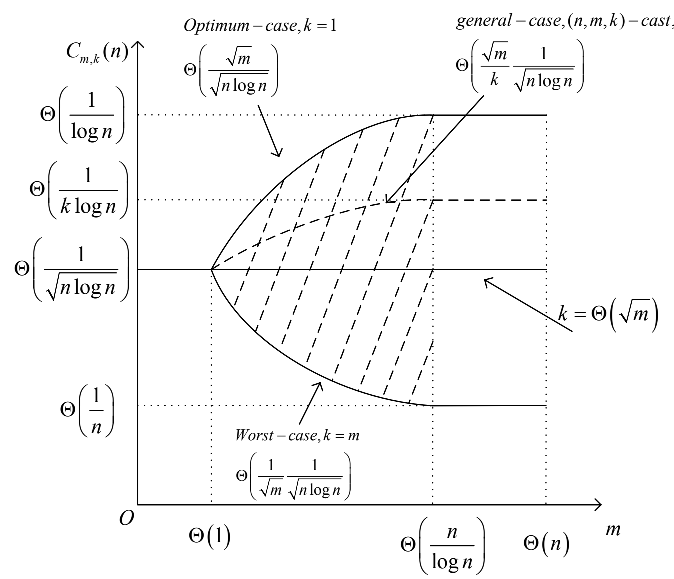
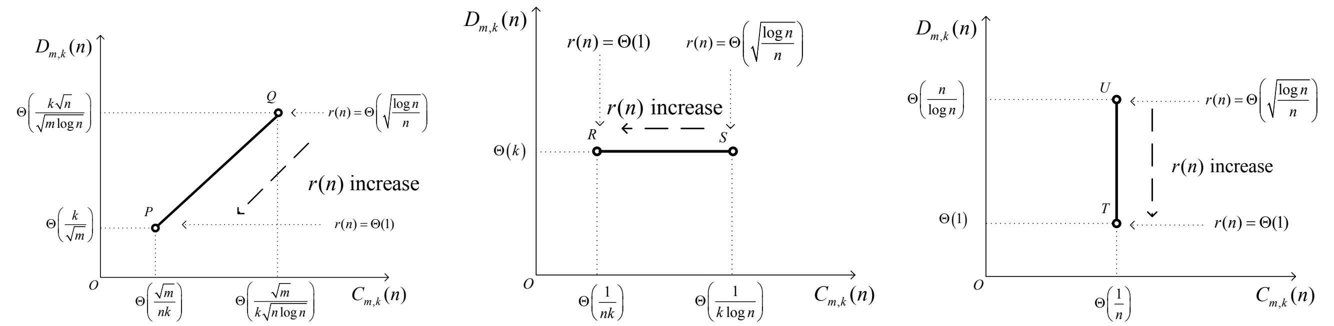


Fig. 2. Unifying view of throughput capacity.

We observe that, regardless of the value of k, the capacity of wireless ad hoc networks becomes constant when $m = \Omega(n/\log n)$ and an increase in the value of m does not change the throughput capacity. This result can be understood by the fact that, when the number of destinations reaches $\Theta(n/\log n)$, this set becomes the connected dominating set ($\text{CDS}(r(n))$) of the entire network as long as the transmission range $r(n)$ is chosen such that the network is a connected network. Equivalently, if a broadcast is made to the entire network, the capacity does not change because all the nodes in the network are either inside this set or within one hop from an element in this set.

We note that the capacity of anycast or manycast is greater than the capacity of unicast if $k = O\left(\sqrt{m}\right)$, even if each node



(a) $m = O\left(r^{-2}(n)\right)$ (b) $\Omega(k) = r^{-2}(n) = O(m)$ (c) $k = \Omega\left(r^{-2}(n)\right)$

Fig. 3. The relationship between delay and capacity.

requires to transmit its packets to more than one destination. This result shows that as long as $k = O\left(\sqrt{m}\right)$, the total number of hops required to transmit packet to k destinations is always, on average, less than sending the packet from the same source to a single randomly selected destination in unicast communications. Equivalently, the total Euclidean distance for a manycast tree is on average less than the Euclidean distance between any randomly selected source and destination in unicast communication. However, this Euclidean distances become the same, on average, when $k = \Theta\left(\sqrt{m}\right)$. As it can be predicted from this figure, the total Euclidean distance in a manycast tree increases as k increases and for $k = \Omega\left(\sqrt{m}\right)$, the capacity of manycast becomes less than unicast because of the total Euclidean distance in the manycast tree.

B. $D_{m,k}(n)$ as a Function of Transmission Range ($r(n)$) and Tradeoff between $D_{m,k}(n)$ and $C_{m,k}(n)$

Figs. 3(a), 3(b), and 3(c) depict the relationship between $D_{m,k}(n)$ and $C_{m,k}(n)$ when SPR is used in a wireless ad hoc network. With our model, we can generalize the unifying relationship between capacity and delay into multicast and broadcast, as shown in Figs. 3(b) and 3(c). In the unicast capacity region, the transmission range $r(n)$ should be made as small as possible to increase the capacity of the network and to avoid interference, with the corresponding cost of increasing delay. To decrease the delay, the transmission range $r(n)$ should be increased, so that the number of hops required to disseminate information is reduced; however, doing so decreases the capacity of the network by increasing multiple access interference (MAI). In the multicast capacity region (see Fig. 3(b)), we observe that the transmission range should be made as small as possible to increase the capacity with no penalty of delay increases. However for the broadcast capacity region (see Fig. 3(c)), increasing the transmission range decreases the delay in the network with no penalty for capacity. In this region, maximizing the transmission range should be the strategy.

The above results indicate that there are different tradeoffs between the capacity $C_{m,k}(n)$ and the delay $D_{m,k}(n)$ in terms of transmission range $r(n)$ for the three capacity regions of wireless ad hoc networks.

We introduced a unifying framework for the modeling of the order capacity of wireless networks subject to different types of information dissemination. To do so, we defined (n, m, k)-casting as a generalization of all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks. Our modeling framework provides a unique perspective to the understanding of the capacity of wireless ad hoc networks. Our approach unifies existing results on the order capacity of wireless networks subject to unicasting , multicasting, or broadcasting and provides new capacity and delay results for anycasting and manycasting.

The multicast throughput and delay has been investigated recently for mobile environments [12]. Future studies should investigate the (n, m, k)-cast for mobile ad hoc networks.

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REFERENCES

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, no. 2, pp. 388–404, 2000.
[2] M. Franceschetti, O. Dousse, D. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Trans. Inform. Theory*, vol. 53, no. 3, pp. 1009–1018, 2007.
[3] A. Ozgur, O. Leveque, and D. Tse, "Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 2549–3572, 2007.
[4] R. M. de Moraes, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "Many-to-many communication: a new approach for collaboration in manets," in *Proc. IEEE INFOCOM 2007*, Anchorage, Alaska, USA, May 2007.
[5] B. Tavli, "Broadcast capacity of wireless networks," *IEEE Commun. Lett.*, vol. 10, no. 2, pp. 68–69, 2006.
[6] A. Keshavarz, V. Ribeiro, and R. Riedi, "Broadcast capacity in multihop wireless networks," in *Proc. ACM MobiCom 2006*, Los Angeles, CA, USA., Sept. 2006.
[7] P. Jacquet and G. Rodolakis, "Multicast scaling properties in massively dense ad hoc networks," in *Proc. IEEE ICPADS 2005*, Fukuoka, Japan, July 2005.
[8] X.-Y. Li, S.-J. Tang, and O. Frieder, "Multicast capacity for large scale wireless ad hoc networks," in *Proc. ACM MobiCom 2007*, Montreal, Canada, Sept. 2007.
[9] M. Steele, "Growth rates of euclidean minimal spanning trees with power weighted edges," *Annals of Probability*, vol. 16, no. 4, pp. 1767–1787, 1988.
[10] A. Giridhar and P. R. Kumar, "Computing and communicating functions over sensor networks," *IEEE J. Select. Areas Commun.*, vol. 23, no. 4, pp. 755–764, 2005.
[11] Z. Wang, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "A unifying perspective on the capacity of wireless ad hoc networks," in *Proc. IEEE INFOCOM 2008*, Phoenix, Arizona, USA, Apr. 2008.
[12] C. Hu, X. Wang, and F. Wu, "Motioncast: on the capacity and delay tradeoffs," in *Proc. ACM MobiHoc 2009*, New Orleans, LA, May 2009.