Capacity of Composite Networks: Combining Social and Wireless Ad Hoc Networks

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***Abstract*—We define composite networks when nodes communicate only with their long-range social contacts and there is no direct link between a node and its long-range contact. Each node has a single long-range contact and all nodes within its transmission range are local contacts for the node. The long-range contact is the destination for each node in the network and since there is no direct link from source to its destination, nodes communicate using multi-hop communications. This is an extension of the famous work by Kleinberg [3] to random wireless ad hoc networks. The throughput capacity of such networks are studied. The routing is based on each node sending the packets to one of its local contact until the packets reach the destination. The long-range contact distance from a source follows power law distribution with parameter α which is a characteristic of social networks. A tight bound of throughput capacity for different values of α is derived. The results demonstrate that when α increases or equivalently the distance between source and destination decreases, the throughput capacity increases. For $\alpha > 3$, throughput capacity of $\Theta(1/\log n)$ is achieved by utilizing simple point-to-point communications where n is the total number of nodes in the network. This is the maximum feasible throughput that can be achieved in point-to-point communications. The result demonstrates the effect of social groups on wireless ad hoc networks. A new parameter called degradation factor is defined which illustrates the asymptotic behavior of networks for large values of n[1].**

I. INTRODUCTION

Gupta and Kumar [1] computed the achievable throughput capacity of wireless ad hoc networks. They derived the capacity in a dense network with randomly distributed nodes, in which the source-destination pair is selected randomly, and the routing algorithm transports information through the shortest path to the destination. However, in most practical networks the source-destination association does not have uniform and random distribution. Each source belongs to a social group and it only communicates to the members of social group. Therefore, a wireless network at least consists of two underlying networks, namely, wireless communication and social networks. These networks are called composite networks. To the best of our knowledge, there is no single publication in literature on the study of interaction between wireless communication and social networks.

Social networks have been proven to exhibit the small-world phenomenon [3], and to model this phenomenon, Watts and Strogatz divided the edges of the network into local and long-range contacts [7]. The famous work by Kleinberg [3] studied a two-dimensional grid network with the small-world property. The source node s in his work selects any other node v as its long-range contact with a probability proportional to $d^{-\alpha}(s, v)$, where $d(s, v)$ is the lattice distance between s and v. Li et al. studied an extended network's capacity considering almost the same assumptions [2].

In this paper, we study the dense composite networks that consist of social and communication networks in which each node has local and one long-range contacts. To simplify the analysis and routing protocol, for each node only four local contacts are selected in random from four possible directions that are located within one hop distance. Probability that node t is the destination for source node s follows the power-law distribution, $Pr(t \text{ is long-range contact of } s) = \frac{d(s,t)^{-\alpha}}{\sum d(s,v)^{-\alpha}}$, where $d(s, v)$ is the Euclidean distance between s and any other node v. This power-law distribution for the probability of selecting a node as long-range contact is one of the main characteristics of social networks. The destination for each node in the network is its long-range contact. All source nodes know the locations of their social contacts.

The main contribution of this paper is to compute the throughput capacity of composite networks. This is the first paper in literature to analytically compute the scaling laws for such networks. This analysis is important since in many practical wireless communication networks, nodes communicate only with their social contacts. This interaction between social and communication networks has remained an open problem in literature. This paper is the first attempt to study this interaction for a simple model for social networks that was first introduced by Kleinberg in [3].

The rest of the paper is organized as follows. In section II, we introduce the model which is used for the network, and the

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main results of our work on the lower bound for throughput capacity are derived in section III. Section IV computes the upper bound of throughput capacity for composite networks and section IV discusses the results. The paper is concluded in section V.

II. Preliminaries

In this work, a dense network with a unit square area containing n uniformly and randomly distributed nodes is considered. To guarantee connectivity in this network [6], the transmission range ($r(n)$) is assumed to be $r(n) = K_0\sqrt{logn/n}$.

The protocol model defined in [4] considers a common transmission range $r(n)$ for all nodes in the network. Node i at position X_i can successfully transmit to node j at position X_j if for any node k at position X_k, $k \neq i$, that transmits at the same time as i, then $|X_i - X_j| \leq r(n)$ and $|X_k - X_j| \geq (1+\Delta)r(n)$, where X_i, X_j and X_k are the cartesian positions in the unit square network for these nodes, and $\Delta > 0$ is the guard zone factor.

The TDMA medium access control scheme is shown in figure 1. The network area is divided into square-lets with side-length $C_1 r(n), (C_1 < \frac{1}{4})$, and at any given time only cells separated by M square-lets distance are allowed to transmit as shown in grey color in figure 1 where $M \geq (2+\Delta)/C_1$.

The decentralized routing protocol used in this work is very simple. Each node knows the location of its long-range contact. Therefore, the node selects one of its four local contacts which is the closest one to the destination and transmits the packets to this local contact. This multi-hop transmission of packets continues until the packets reach the destination. We assume that there is one local contact in each of the 4 adjacent cells of the source which gaurantees that this simple routing protocol converges. It is important to note that if each node has more than four local contacts, i.e., all nodes within transmission range are local contacts, then the order throughput capacity computation will not change and the same results can be derived. The four local contacts and decentralized routing protocol assumptions were first considered in [3].

III. Throughput Capacity of Composite Networks - Lower bound

In this network model, each node is a source transmitting data at a rate λ. Let X be the number of hops traveled by each bit from source to destination. Thus, the total number of concurrent transmissions in this network would be $n\lambda E[X]$, where $E[X]$ is the average number of hops for any source-destination pair. This value is upper bounded by the total bandwidth W available divided by the number of non-interfered groups in TDMA scheme $((MC_1r(n))^2 = M^2C_1^2r^2(n))$ as shown in figure 1. Therefore, the maximum rate of transferring data in this network is

$$\lambda \leq \lambda_{max} = \frac{\frac{W}{M^2C_1^2r^2(n)}}{nE[X]}. \qquad (1)$$

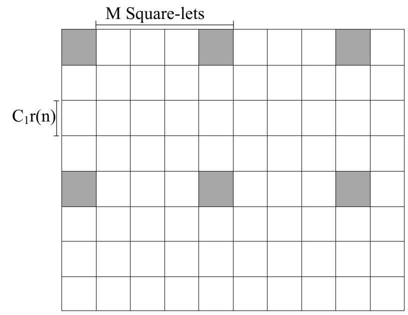

Fig. 1. TDMA Scheme, grey square-lets can transmit at the same time.

The average number of hops can be computed through $E[X] = \sum xP(X=x)$. $P(X=x)$ is the probability that the packets travel x hops from s to reach destination (long-range contact) t. Let's assume the range of Euclidean distance for longe-range contact is $\frac{1}{n^{1+\delta}} = d_{min} \leq d \leq d_{max} = K_1$ for any $\delta > 0$ [5]. Then we have

$$\begin{aligned} P(X=1) &\leq \int_{d_{min}}^{r(n)} \frac{2\pi nx.x^{-\alpha}dx}{\sum_v d^{-\alpha}(s,v)} \\ &= \frac{2\pi n}{\sum_v d^{-\alpha}(s,v)} \int_{d_{min}}^{r(n)} x^{1-\alpha}dx. \qquad (2) \end{aligned}$$

To compute $P(X=x)$ for $x > 1$, the long-range contact outside the circle with radius $r(n)$ centered at the source node should be considered. Thus, it is easy to show from figure 2 that $P(X=x) = 0,\ for\ 1 < x < \lceil \frac{1}{C_1} + 1 \rceil$. The maximum number of hops is $\frac{2}{C_1 r(n)}$. Thus, $P(X=x)$ should be calculated for $x = \lceil \frac{1}{C_1} + 1 \rceil, ..., \lceil \frac{2}{C_1 r(n)} \rceil$.

To compute $P(X=x)$, we need to compute the number of nodes in a distance of x hops from the source and their corresponding Euclidean distances from the source. The geometric place of such nodes is a rhombus around the source node as shown in figure 2. The probability that the number of hops between source and destination is x hops is equal to the probability that the destination is located in one of the cells on the boundaries of this rhombus.

$$\begin{aligned} P(X=x) &= \sum_{i=1}^{4x} P(destination\ is\ located\ inside\ s_i) \\ &= \sum_{i=1}^{4x} \sum_{t\ in\ s_i} \frac{d^{-\alpha}(s,t)}{\sum_v d^{-\alpha}(s,v)} \\ &\leq 8 \sum_{i=1}^{1+\lfloor \frac{x}{2} \rfloor} \sum_{t\ in\ s_i} \frac{d^{-\alpha}(s,t)}{\sum_v d^{-\alpha}(s,v)} \end{aligned}$$

The probability that node t is the destination is inversely proportional to its Euclidean distance from s. In our calculation, we compute the distance for square-lets h_i (in light grey

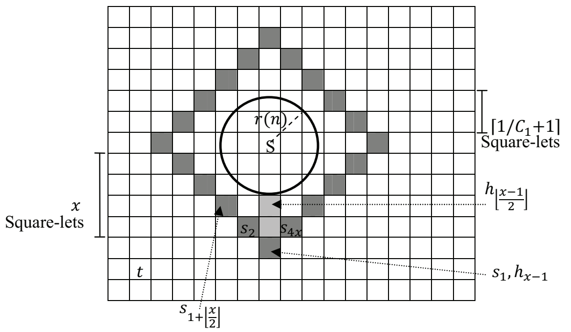

Fig. 2. Dark grey square-lets (s_i's) show the region containing the nodes with $P(X=x)$. Light grey square-lets (h_i's) are used in calculations instead of the actual geometric place.

in the figure) instead of s_i square-lets which provides an upper bound for the $P(X=x)$ probability.

$$
\begin{aligned}
P(X=x) &\leq 8\sum_{i=\lfloor\frac{x-1}{2}\rfloor}^{x-1}\sum_{t\ in\ h_i}\frac{d^{-\alpha}(s,t)}{\sum_v d^{-\alpha}(s,v)}\\
&\leq 8[nC_1^2r^2(n)\frac{((x-1)C_1r(n))^{-\alpha}}{\sum_v d^{-\alpha}(s,v)}\\
&+ nC_1^2r^2(n)\frac{((x-2)C_1r(n))^{-\alpha}}{\sum_v d^{-\alpha}(s,v)}+...\\
&+ nC_1^2r^2(n)\frac{(\lfloor\frac{x-1}{2}\rfloor C_1r(n))^{-\alpha}}{\sum_v d^{-\alpha}(s,v)}]\\
&= \frac{8C_1^{2-\alpha}n}{\sum_v d^{-\alpha}(s,v)}r^{2-\alpha}(n)\sum_{j=\lfloor\frac{x-1}{2}\rfloor}^{x-1}j^{-\alpha}\\
&\leq \frac{K_2nr^{2-\alpha}(n)}{\sum_v d^{-\alpha}(s,v)}\int_{\lfloor\frac{x-1}{2}\rfloor}^{x}(u-1)^{-\alpha}du
\end{aligned}
$$

Note that $K_2=8C_1^{2-\alpha}$. Thus,

$$
\begin{aligned}
E[X] &\leq P(X=1)+\sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1r(n)}}xP(X=x)\\
&\leq \frac{2\pi n}{\sum_v d^{-\alpha}(s,v)}\int_{d_{min}}^{r(n)}u^{1-\alpha}du\\
&+ \sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1r(n)}}\frac{K_2nr^{2-\alpha}(n)}{\sum_v d^{-\alpha}(s,v)}x\int_{\lfloor\frac{x-1}{2}\rfloor}^{x}(u-1)^{-\alpha}du.
\end{aligned}
\tag{3}
$$

A. Lower bound throughput for $\alpha=0$

When $\alpha=0$, this is equivalent of assuming that the long-range contact is uniformly and randomly selected among all the nodes in the network. This is almost the same scenario as Gupta and Kumar and therefore, we expect to arrive at the same throughput capacity. Average number of hops traveled by the data between source and destination is obtained by replacing α with 0 in eq. (3) and is given by

$$
\begin{aligned}
E[X] &\leq 2\pi\int_{d_{min}}^{r(n)}udu+\sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1r(n)}}K_2r^2(n)x\int_{\lfloor\frac{x-1}{2}\rfloor}^{x}du,\\
&= \pi(r^2(n)-d_{min}^2)\\
&+ \sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1r(n)}}K_2r^2(n)x(x-\lfloor\frac{x-1}{2}\rfloor),\\
&\leq \pi r^2(n)+\sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1r(n)}}K_2r^2(n)(\frac{x^2}{2}+x),\\
&\leq \pi r^2(n)+K_2r^2(n)\int_{\frac{1}{C_1}+1}^{\frac{2}{C_1r(n)}+1}(\frac{x^2}{2}+x)dx,\\
&= r^2(n)(\frac{32}{3C_1r^3(n)}+\frac{32}{r^2(n)}+\frac{24C_1}{r(n)}+K_3),\\
&\overset{a}{\leq} \frac{64}{3C_1r(n)}=K_4\sqrt{\frac{n}{\log n}},
\end{aligned}
$$

where (a) holds for large n.

The maximum rate of transferring information in this network is

$$
\lambda_{max} \geq \frac{K_{\alpha_0}^L\frac{n}{\log n}}{n\sqrt{\frac{n}{\log n}}}=\frac{K_{\alpha_0}^L}{\sqrt{n\log n}},
$$

where $K_{\alpha_0}^L=\frac{3W}{64M^2C_1K_0}$.

B. Lower bound throughput for $\alpha>3$

The probability that the destination is within one hop from source when $\alpha>3$ is derived from eq. (2).

$$
P(X=1) = \frac{2\pi n}{\sum_v d^{-\alpha}(s,v)}\frac{(-r(n)^{2-\alpha}+d_{min}^{2-\alpha})}{\alpha-2}\leq 1
$$

The average number of hops between source and destination for $x>1$ is derived using the second term of eq. (3).

$$
\begin{aligned}
&\sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1r(n)}}xP(X=x)\\
&\leq \frac{K_2nr^{2-\alpha}(n)}{(1-\alpha)\sum_v d^{-\alpha}(s,v)}\times\\
&\sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1r(n)}}x((x-1)^{1-\alpha}-(\lfloor\frac{x-1}{2}\rfloor-1)^{1-\alpha})\\
&\leq \frac{K_2nr^{2-\alpha}(n)}{\sum_v d^{-\alpha}(s,v)}\frac{1}{\alpha-1}\sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1r(n)}}\frac{2^{\alpha-1}x}{(x-4)^{\alpha-1}}
\end{aligned}
$$

$$\leq \frac{2^{\alpha-1}K_2 n}{(\alpha-1)r^{\alpha-2}(n)\sum_v d^{-\alpha}(s,v)}\int_{\frac{1}{C_1}+1}^{\frac{2}{C_1 r(n)}+1}\frac{(x-1)dx}{(x-5)^{\alpha-1}}$$
$$\leq \frac{K_2 n(K_3+K_4 r^{-1}(n)+...+K_{\alpha+1}r^{2-\alpha})}{\sum_v d^{-\alpha}(s,v)}$$
$$\overset{b}{\leq} \frac{K_2 n 2K_{\alpha+1}r^{2-\alpha}(n)}{\sum_v d^{-\alpha}(s,v)}$$

Inequality (b) is valid for sufficiently large values of n.

Moreover, the lower bound for $\sum d(s,v)^{-\alpha}$ for large values of n can be obtained as follows.

$$\begin{aligned}\sum d(s,v)^{-\alpha} &\geq \int_{C_1 r(n)}^{K_1/2} 2\pi n.xdx.x^{-\alpha} \quad (4)\\ &= \frac{2\pi n}{\alpha-2}((C_1 r(n))^{2-\alpha}-(K_1/2)^{2-\alpha})\\ &\geq \frac{\pi n C_1^{2-\alpha}r^{2-\alpha}(n)}{\alpha-2}\end{aligned}$$

Thus $E[X]$ is upper bounded by:

$$E[X] \leq \frac{K_2 n 2K_{\alpha+1}r^{2-\alpha}(n)}{\pi n C_1^{2-\alpha}r^{2-\alpha}(n)}+1=K_5$$

Replacing $E[X]$ in eq. (1) leads to the following maximum rate of transferring information.

$$\lambda_{max} \geq \frac{K_{\alpha_4}^L\frac{n}{\log n}}{n}=\frac{K_{\alpha_4}^L}{\log n}$$

where

$$K_{\alpha_4}^L=\frac{\pi W}{M^2C_1^2K_0^2(16K_{\alpha+1}+\pi)},$$

and

$$K_{\alpha+1}=\frac{2^{\alpha-1}(\alpha-2)}{\alpha-1}(\frac{1}{\alpha-3}(\frac{C_1}{1-4C_1})^{\alpha-3}+\frac{4}{\alpha-2}(\frac{C_1}{1-4C_1})^{\alpha-2}).$$

IV. Throughput Capacity of Composite Network : Upper bound

In order to calculate the upper bound throughput capacity of eq. (1), the lower bound of $E[X]$ is needed.

$$\begin{aligned}E[X] &= P(X=1)+\sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1 r(n)}} xP(X=x)\\ &\geq \sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1 r(n)}} xP(X=x)\\ &\geq \sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1 r(n)}} x\frac{1}{4}4x(C_1r(n))^2 n\frac{((x+1)C_1 r(n))^{-\alpha}}{\sum_v d^{-\alpha}(s,v)}\\ &= \frac{C_1^{2-\alpha}nr^{2-\alpha}(n)}{\sum_v d^{-\alpha}(s,v)}\sum_{x=\frac{1}{C_1}+1}^{\frac{2}{C_1 r(n)}} x^2(1+x)^{-\alpha} \quad (5)\end{aligned}$$

and the upper bound on the normalization factor is obtained as

$$\sum d(s,v)^{-\alpha} \leq \int_{r(n)}^{K_1} 2\pi n.xdx.x^{-\alpha}. \quad (6)$$

Combining eq.s (5), (6), and (1) and following the same derivations, we arrive at the same upper bound throughput capacity as lower bound with different constant factors which are given below.

$$K_{\alpha_0}^U=\frac{3W}{4M^2C_1K_0},K_{\alpha_1}^U=\frac{\pi WK_1}{M^2C_1K_0},K_{\alpha_2}^U=\frac{4\pi(1+\delta)W}{M^2C_1K_0}$$
$$K_{\alpha_3}^U=\frac{16\pi W}{M^2C_1^2K_0^2},K_{\alpha_4}^U=\frac{4\pi(1+2C_1)^{\alpha-3}W(\alpha-3)}{M^2C_1^{\alpha-1}K_0^2(\alpha-3)}$$

The details derivation of throughput capacity for different values of α is omitted here.

V. Discussion

Table 1 summarizes the tight bound on the average number of hops to be traversed from source to reach destination, $E[X]$, and the tight bound for the maximum throughput capacity, λ_{max}, that can be obtained as a function of α. Note that parameter α determines the power law distribution in social networks.

If there is no constraint or preference for each source node to choose its destination, the source-destination pair is selected uniformly in the network area, i.e., $\alpha = 0$. The analysis of composite networks in this case shows that the average number of hops between source and destination is proportional to the inverse of the transmission range, and the throughput capacity is similar to the results derived by Gupta and Kumar [1].

For large values of α ($\alpha > 3$), the destination is within the transmission range with probability close to 1. Thus, the information needs to pass just one hop to reach the destination. Consequently, the maximum throughput capacity is achieved. The results can be justified by noticing that when a node is activated, based on protocol model all nodes within transmission range ($r(n)=\sqrt{\frac{\log n}{n}}$) must be silenced. The number of these nodes is equal to $\pi r^2(n)n=\Theta(\log n)$. Therefore, by using a TDMA approach with parameter proportional to $\frac{1}{\log n}$, all nodes can transmit their packets to their destinations which is the maximum capacity with a point-to-point communication scheme. Therefore, the results implies that when the members of a social group are physically located close to each other, the maximum throughput capacity in wireless ad hoc networks can be obtained.

Increasing the value of α will decrease the average distance between source and long-range contact (or destination in this paper) which results in increase in capacity.

The constant factor K_{α_i} for $i \geq 0$ plays an important role in the analysis of the network. However, in order to better understand the asymptotic behavior of the network, we define *degradation factor* ($DF(n)$) as the rate of throughput capacity decrease as a function of the number of nodes when both values are computed in logarithmic scale. This factor is independent of constant factors of capacity bounds and demonstrates the asymptotic behavior of the composite networks more accurately. $DF(n)$ is defined as

$$DF(n) = -\frac{\partial(\log \lambda_{max})}{\partial(\log n)}.$$

Figure 3 illustrates degradation factor for different values of α. As can be seen, the degradation factor reaches a constant value when the number of nodes increases. For example for $\alpha = 0, 1, 2$ the degradation factor goes to the constant value of 0.5. This result implies that for large number of nodes, the value of the throughput capacity bound is degraded by a factor of $10^{-0.5}$ if the number of nodes increased tenfold.

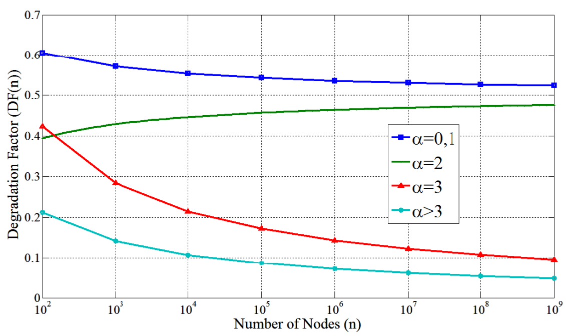

Fig. 3. λ_{max} Degradation Factor.

Figure 4 demonstrates the behavior of throughput capacity for composite networks when the number of nodes increases. The figure shows that when α increases, the rate of throughput degradation as a function of n decreases. It appears that when $\alpha = 1$, the long-range social contacts behave as if they are distributed uniformly and randomly. Therefore, the throughput capacity is the same for α equal to 0 (no social network) and 1.

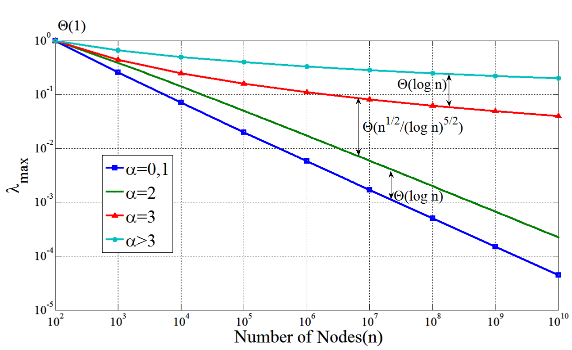

Fig. 4. Maximum throughput capacity vs. number of nodes.

	$E[X]$	λ_{max}
$\alpha = 0$	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(\frac{1}{\sqrt{n \log n}})$
$\alpha = 1$	$\Theta(\sqrt{\frac{n}{\log n}})$	$\Theta(\frac{1}{\sqrt{n \log n}})$
$\alpha = 2$	$\Theta(\frac{1}{\log n}\sqrt{\frac{n}{\log n}})$	$\Theta(\sqrt{\frac{\log n}{n}})$
$\alpha = 3$	$\Theta(\log n)$	$\Theta(\frac{1}{(\log n)^2})$
$\alpha > 3$	$\Theta(1)$	$\Theta(\frac{1}{\log n})$

TABLE I
λ_{max} AND $E[X]$ FOR DIFFERENT VALUES OF α

VI. Conclusion and Future Work

This paper defines composite networks as a combination of social and communication networks. Each node is associated with four local contacts inside its transmission range and a single long-range contact with its distance from source following power law distribution. There is no direct link from source to destination and source-destination pairs communicate using multi-hop communications. The interaction between social and wireless communication networks are studied. A tight bound of the throughput capacity for different values of α is derived. The results demonstrate that when the distance between source and destination decreases, the throughput capacity increases. For $\alpha > 3$, maximum throughput capacity of $\Theta(1/\log n)$ is achieved by utilizing simple point-to-point communications. This is the maximum feasible throughput that can be achieved in point-to-point communications.

As the future work, we can generalize this work by allowing each source to have more than one long-range contact, one of which is the destination.

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