Capacity of Wireless Ad-Hoc Networks under Multipacket Transmission and Reception

Shirish Karande†, Zheng Wang†, Hamid R. Sadjadpour†, J.J. Garcia-Luna-Aceves‡
Department of Electrical Engineering† and Computer Engineering‡
University of California, Santa Cruz, 1156 High Street, Santa Cruz, CA 95064, USA
‡ Palo Alto Research Center (PARC), 3333 Coyote Hill Road, Palo Alto, CA 94304, USA
Email:{karandes,wzgold, hamid, jj}@soe.ucsc.edu

***Abstract*—In this paper we deduce the capacity of wireless ad-hoc networks under the assumptions of physical model for cooperative multiple input multiple output (MIMO) communication. We show that the capacity of a random wireless ad-hoc network can be improved significantly by adopting MIMO techniques. In particular, when the nodes are endowed with multi-packet transmission and reception capabilities, the per session capacity increases at least as $n^{1-2/\alpha}R(n)^{3-4/\alpha}$, where $\alpha > 2$ is the path loss parameter and $R(n)$ is the range of cooperation. The proof for the above results utilizes an edge coloring of an appropriately chosen random geometric graph. This approach, as a by product, provides alternative deductions for some previously established results under the physical model. Consequently, we provide an alternative deduction of the classical result for a point to point communication by Gupta and Kumar. Furthurmore, we also deduce a bound that matches a recent result by Wang et. al., for capacity of ad-hoc networks under multipacket reception, within a poly-log factor.**

I. Introduction

The past decade has seen major advances in multiuser detection, multiple input multiple output (MIMO) communication and interference cancelation techniques. Currently, there is an active research interest in modeling and evaluating the impact of these paradigms on the performance of large scale networks. The classical result by Gupta and Kumar [1] essentially states that communication protocols based on point to point communication cannot render networks that perform efficiently with increase in size. It is widely anticipated that advanced cooperative communication can overturn this rather negative conclusion.

Recent efforts to realize (static) scalable ad-hoc networks have met with a mixed degree of success. For example, network coding (NC) [2], which is essentially a generalization of routing, has proved to be incapable of improving the throughput order for multipair unicast transmissions [3]. Nevertheless, other contemporary efforts, which model relatively classical physical layer co-operative techniques, have met with a greater degree of success. In particular, [4] and [5] have exhbited the feasibility of achieving constant per-session capacity. We provided a more detailed taxonomy of related works in Section II.

Section III presents the first contribution of this paper. We model a *random network* with n nodes, homogeneous transmission power, and unicast traffic for k source-destination (S-D) pairs. We introduce a new physical model in which nodes have the ability to decode correctly multiple packets transmitted concurrently from different nodes, and transmit concurrently multiple packets to different nodes. We refer to this as multi-packet transmission and reception (MPTR) physical model. In this section we review some of our recently established results on random geometric graphs

Section IV presents our second contribution, where we define a *combinatorial interference model* based on random geometric graphs. We establish some generic results for the combinatorial models. These results prove useful for the subsequent analysis in the paper, but are also interesting in their own right.

Section V presents our third contribution. We show that for each of: (a) the classical physical model by Gupta and Kumar [1], (b) the multipacket reception physical model by Wang et. al. [6](c) and the MPTR physical model proposed in Section III, there exists a corresponding combinatorial model that performs necessarily worse than the physical model. We utilize this relationship along with properties of random geometric graphs proven in [7], as well as Section IV, to deduce the desired lower bounds. We discuss our results in Section VI.

II. Related Work

There have been many contributions on the capacity study of wireless ad hoc networks that span unicast, multicast and broadcast traffic. Due to space limitations, however, we only mention a few of them that focus on unicasting.

A number of papers have extended the results by Gupta and Kumar [1], which showed a gap between the upper and lower bounds on capacity under the physical model. Franceschetti et al. [8] closed this gap using percolation theory.

Several techniques aimed at improving the capacity of wireless ad hoc networks have been analyzed. Grossglauser and Tse [9] demonstrated that a non-vanishing capacity can be attained at the price of long delivery latencies by taking advantage of long-term storage in mobile nodes.

Someworks demonstrated that changing physical layer assumptions such as using multiple channels [10] or MIMO cooperation [4] can change the capacity of wireless networks.

Ozgur et al. [11] proposed a hierarchical cooperation technique based on virtual MIMO to achieve linear capacity. They showed that the optimal per-session capacity of an ad-hoc network is bounded as $O(n \log n)$, and a constant per-session capacity of $\Theta(1)$ is achievable. Our work is significantly different from this work, in terms of the the model and assumptions used to derive the results. Ozgur et. al. consider the information-theoretic model, and does not assume that nodes are capable of multi-packet transmission (MPT). In contrast, our work is based on the fact that nodes are endowed with MPR and MPT capabilities.

Cooperation can be extended to the simultaneous transmission and reception at the various nodes in the network, which can result in significant capacity improvement [12]. As we have stated, Garcia-Luna-Aceves et al. [5] showed that using MPR at the receivers can increase the order capacity of wireless networks subject to unicast traffic.

III. Preliminaries

A. Network Model

For a continuous region R, we use $|R|$ to denote its area. We denote the cardinality of a set S by $|S|$, and by $\|x-y\|$ the distance between nodes x and y. Whenever convenient, we utilize the indicator function $1_{\{P\}}$, which is equal to one if P is true and zero if P is false. $Pr(E)$ represents the probability of event E. We say that an event E occurs with high probability (w.h.p.) as $n \to \infty$ if $Pr(E) > (1-(1/n))$. We employ the standard order notations $O, \Omega,$ and Θ.

We assume that the traffic in the network is generated by unicast communication between k source-destination (S-D) pairs. We associate a rate vector $\lambda = [\lambda_1, \cdots, \lambda_k]$ with these k pairs. We assume the data rate for each S-D pair to be non-zero. Hence, without loss of generality (w.l.g) the rate vector can be written as $\lambda = [fD_i, \cdots, fD_k]$ where $f \in \mathbb{R}_+$ and $D_i \in [1/2, 1]$ for $1 \leq i \leq k$.We refer to the parameter f as the *concurrent flow rate* and to $D = [D_1, \cdots, D_k]$ as the *demand vector*.

Definition 3.1: Feasible Flow Rate: Given k S-D pairs $\{(s(1), d(1)), \ldots, (s(k), d(k))\}$, a rate vector $\lambda = [fD_i, \cdots, fD_k]$ is feasible if there exists a spatial and temporal scheme for scheduling transmissions such that by operating the network in a multi-hop fashion, and buffering at intermediate nodes when awaiting transmission, every source $s(i)$ can send λ_i bits/sec on average to the chosen destination $d(i)$. A flow rate f is feasible for a demand vector $D = [D_1, \cdots, D_k]$ iff $\lambda = [fD_i, \cdots, fD_k]$ is a feasible rate vector.

Definition 3.2: Capacity of Random Networks: The capacity per commodity of a network is $\Theta(f(n))$ if under a random placement of n nodes,a random choice of k S-D pairs and for an arbitrary demand vector we have:

$$\lim_{n\to\infty} \Pr(cf(n) \text{is feasible flow rate}) = 1 \quad (1)$$

$$\liminf_{n\to\infty} \Pr(c'f(n) \text{is infeasible flow rate}) < 1 \quad (2)$$

for some $c > 0$ and $c < c' < +\infty$.

B. Channel Models

Definition 3.3: Physical Model with Single Packet Reception (SPR) The physical model for plain routing (i.e., single packet reception), was introduced by Gupta and Kumar [1]. In this model a successful communication occurs if signal to interference and noise ratio (*SINR*) of the pair of transmitter i and receiver j satisfies

$$\textit{SINR}_{i\to j} = \frac{Ph_{ij}}{BN_0 + \sum_{k\neq i,k=1}^{n} Ph_{kj}} \geq \beta, \quad (3)$$

where P is the transmit power of a node, h_{ij} is the channel attenuation factor between nodes i and j, and BN_0 is the total noise power. The channel attenuation factors h_{ij} and h_{kj} are completely determined by the path loss model. Hence, $h_{ij} = |X_i - X_j|^{-\alpha}$ in which $\alpha > 2$ is the path-loss parameter and X_i represents the location of node i.

Definition 3.4: Physical Model with Multi-packet Reception (MPR) In a recent work Wang et. al. [6] extended the conventional physical model to incorporate multi-packet reception based on successive interference cancelation. In the MPR model, we continue to assume that nodes operate in a half-duplex mode and each transmitter can transmit at most one packet. Under the MPR condition, if we allow nodes to be decoded jointly or separately, then it can be proven [6] that the node at the circumference of the decoding circle of radius $R(n)$ has the lowest SINR. Therefore, it is sufficient to assure that node satisfies the physical model condition. Let's assume that receiver j receives packets from the farthest transmitter i that satisfies the following *SINR* constraint

$$\textit{SINR}_{i\to j} = \frac{Ph_{ij}}{BN_0 + \sum_{k\notin A_{(j,R(n))}} Pg_{kj}} \geq \beta, \quad (4)$$

where $A_{(j,R(n))}$ is a circle of radius $R(n)$ around node j and $R(n)$ is a range of cooperation.

Definition 3.5: Physical Model with Multi-packet Transmission and Reception (MPTR): In this work, we further extend the physical model to represent the ability to conduct multi-packet transmission along with multi-packet reception. We assume that MPT is achieved by deploying multiple antennas at each node and MPR is again achieved on the basis of successive decoding. So a transmitter i can simultaneously send packets to any receiver j as long as the *SINR* constraint in (4) is satisfied.

C. Graph Theory Results

Definition 3.6: Random Geometric Graph G_R: Consider a *directed* graph $G_R(V_R, E_R)$ formed by randomly distributing n nodes uniformly in a unit square, s.t. if the locations of these nodes are represented by $\{X_1, \cdots, X_n\}$, then edge-set is given by $E = \{(i,j) \mid \|X_i - X_j\| \leq R(n)\}$.

Now consider a sub-graph $H_R \subseteq G_R$ obtained by employing location based constraints on the edge-set. In order to describe these constraints, divide the network area into l^2 squarelets of side-length $a = R(n)/3$ as shown in Figure 1. We obtain H_R by removing all edges, except those connecting two nodes in vertically or horizontally adjacent squarelets.

The following results have been previously previously established.

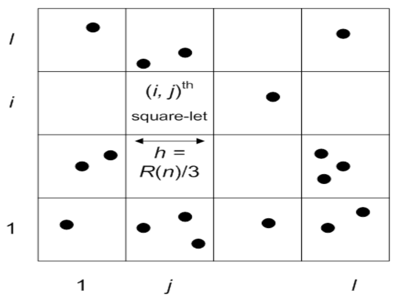

Fig. 1. Decomposition of network area into l^2 squarelets

Lemma 3.7: [13] If $R(n) \geq R_c(n)$, where $R_c(n) = \Theta(\sqrt{\log n/n})$ then w.h.p. the total number of nodes in any squarelet are $\Theta(nR^2(n))$.

Lemma 3.8: [1] The graph G_R and H_R are connected with high probability as $n \to \infty$, iff. $R(n) \geq R_c(n) = \Theta(\sqrt{\log n/n})$.

Lemma 3.9: [7] If $R(n) \geq R_c(n)$, then w.h.p. the degree of each node in G_R and H_R is $\Theta(nR^2(n))$.

Lemma 3.10: [7] The per-session interference free throughput capacity on graph G_R is greater than that on H_R. Moreover, if $R(n) \geq R_c(n)$ then the per-session interference free throughput order on both these graphs is given by $\Theta(n^2R^3(n)/k)$.

IV. Combinatorial Interference Model

We describe the interference of a network by the following generic model.

Definition 4.1: Combinatorial Interference Model: The interference model for the graph[1] $G(V,E)$ is determined by a function $I : E \to P(E)$, where $P(E)$ is the power set of E, i.e., the set of all possible subsets of E. For every $e \in E$, $I(e)$ represents an *interference set* such that, a transmission on edge e is successful if and only if (iff) there are no concurrent transmissions on any $\hat{e} \in I(e)$. An interference model can be restricted to a sub-graph $H(V_H, E_H)$ by defining a function $I_H : E_H \to P(E_H)$ such that $I_H(e) = I(e) \bigcap E_H$.

In the following section, we define specific combinatorial interference models which can be used to indirectly provide lower bounds on the physical model considered in this work. Prior to that, we establish some additional terminology and review few generic results for interference models.

Definition 4.2: Dual-Interference-Set: Consider an edge set E and an interference set $I(e)$ for an edge $e \in E$, as defined in Definition 4.1. The dual interference-set for e is defined by $F(e) = \{\hat{e} \in E \mid e \in I(\hat{e})\}$, which is the set of edges that experience a collision on account of a transmission on edge e.

Definition 4.3: Dual Conflict Graph: Given a graph $G(V,E)$ and an interference function I , we define the *dual conflict graph* as $G_D(E, E_D)$, where $E_D = \{(e, \hat{e}) \mid \hat{e} \in I(e))\}$.

[1]Note that G_r denotes random geometric graph while G represents general graph.

Definition 4.4: Total Degree in Dual Conflict Graph: The total degree of each node in a dual conflict graph is equal to $|M(e)|$ where $M(e) = I(e) \bigcup F(e)$.

Definition 4.5: Interference Clone: Two edges e_1, e_2 are said to be interference-clones under function I if they satisfy the conditions that $M(e_1) = M(e_2)$.

Lemma 4.6: [7] Consider a graph $G(V,E)$ and interference I. Let $\kappa = \max_{e \in E} |M(e)|$. If f is a feasible flow rate in the absence of interference, then flow rate $f_l = f/(1+\kappa)$ is feasible in presence of interference I.

Lemma 4.7: Clone Piggy-backing Lemma: [7] Consider a graph $G(V,E)$ along with interference functions I_A and I_B, then I_A and I_B are such that:

1) $\kappa = \max_{e \in E} |M_A(e)|$
2) $\forall e \in E$ there exists a set $M_{A,\bar{B}(e)} \subseteq M_A(e)$ such that every edge belonging $M_{A,\bar{B}(e)}$ is an interference-clone of e under I_B . Further, let $\mu = \min_{e \in E} |M_{A\bar{B}}(e)|$.

If f is a feasible flow rate in G without any interference, $f_{I_A} = f/(1+\kappa)$ is a feasible flow rate in G under the I_A interference function and κ as its corresponding parameter, then $f_{I_B} = f(1+\mu)/(1+\kappa)$ is feasible in presence of interference defined by I_B.

V. Lower Bound for Physical Model

We deduce lower bounds for the physical model by showing that there exists an appropriate combinatorial model that is necessarily more restrictive. Thus, even though in the physical model the transmission range can be made arbitrarily large by choosing a suitable power level and scheduling, we shall limit the hop-length or decoding range to a maximum value $R(n)$. Furthermore, we shall deduce a guard range $D(n)$ as a function of $R(n)$, such that the receiver can decode a direct transmission of an arbitrary hop-length less than $R(n)$ if a receiver has no interfering transmitter within a range $D(n)$. The precise relationship between $D(n)$ and $R(n)$ depends on the chosen physical modality: SPR, MPR or MPTR.

A. Single Packet Reception

In order to obtain lower bound on the physical model, consider the following combinatorial model

Definition 5.1: Restricted Single Packet Reception Model (RSPR) Consider the graph $H_R \subseteq G_R$ with parameter $R(n)$. Let

$$I_{\text{RSPR}}(e) = W(e) - e \quad \text{such that}$$
$$W(e) = \left\{\hat{e} \in E_R \mid \|X_{\hat{e}^+} - X_{\eta(e^-)}\| \leq D(n)\right\}.$$

where $\eta(e^-)$ could be any node that belongs to the same squarelet as e^- and $R(n) \leq D(n)$.

Lemma 5.2: If $R(n) \geq R_c(n)$ and $k \geq \Theta(n)$, then the per-session throughput order under the RSPR model on graph H_R is at least $\Theta(R(n)/(kD(n)^2))$.

Proof: Lemma 3.9 implies that any random geometric graph with parameter $D(n)$ has the property that the maximum node degree is given by $\Theta(nD^2(n))$. Note this fact is equivalent to saying that a circular region of radius $D(n)$ around any node has a maximum of $\Theta(nD^2(n))$ nodes. Now consider the graph H_R with parameter R(n). In such a graph the maximum node degree, again due to Lemma 3.9 is given by $\Theta(nR^2(n))$. Therefore, there exist constants c_6 and c_7 such that

$$\begin{aligned}\gamma_{max} &= max_{e\in E_{R,H}}|W(e)| \\ &\leq \text{(max. nodes in disk of radius D(n))} \\ &\quad \times\text{(max. degree of a vertex in } H_r) \\ &\leq c_6 nD^2(n)\times c_7 nR^2(n) = O(n^2R^2(n)D^2(n))\end{aligned} \quad (5)$$

Theorem 3.10 tell us that the interference free throughput order in H_R is at least $\Theta(n^2R^3(n)/k)$. Thus, invoking Lemma 4.6, we arrive at

$$\begin{aligned}f_{RSPR} &\leq \text{(max. flow rate with no interference)} \\ &\quad \times(1/(1+\gamma_{max})) \\ &= c_8(n^2R^3(n)/k)\times c_9(1/(n^2R^2(n)D^2(n))) \\ &= c_8c_9(R(n)/(kD^2(n)))\end{aligned} \quad (6)$$

■

Now, if we can show that for an appropriate choice of $D(n)$, a feasible transmission under RSPR is necessarily feasible under SPR, then the above lemma can be used to provide a lower bound on the throughput order of the SPR physical model. In order to choose an appropriate $D(n)$, consider a transmission from node i to node j. Now, consider concentric circles of radius $lD(n)$ around receiver j, with $l\in[1,1/D(n)]$. These concentric circles decompose the network into disjoint annular rings of area $lD^2(n)-(l-1)D^2(n)=(2l-1)D^2(n)$. Since in the RSPR model, each node silences a region of $\Theta(D^2(n))$, the maximum number of transmitters in each annular ring are $c_{10}(2l-1)$. Thus, we require that

$$\begin{aligned}\beta &\leq SINR_{i\to j} \\ &= \frac{PR^{-\alpha}(n)}{BN_0+\sum_{l=1}^{1/D(n)}c_{10}(2l-1)l^{-\alpha}D^{-\alpha}(n)}\end{aligned} \quad (7)$$

Since, as $1/D(n)$ approaches infinity, for $\alpha>2$ the summation $\sum_{l=1}^{1/D(n)}c_{10}(2l-1)l^{-\alpha}$ approaches a constant, the required condition is given by $D(n)/R(n)\geq(c_{11}\beta/P)^{1/\alpha}$ which implies

$$D(n)\geq c_{12}R(n)(\beta/P)^{1/\alpha}=\Theta(R(n)) \quad (8)$$

Therefore we have the following corollary

Lemma 5.3: If $R(n)\geq R_c(n)$, then per-session throughput order under the SPR physical model is at least $\Theta(1/(kR(n)))$.

B. Multiple Packet Reception

To deduce the bounds for the MPR physical model, lets consider the following combinatorial model.

Definition 5.4: Restricted MPR (RMPR) Model:

$$\begin{aligned}I_{\text{RMPR}}(e) = W(e)-U(e)\quad \forall e\in E_{R,H}\ \textit{where} \\ U(e)=\left\{\hat{e}\in E_{R,H}\mid \hat{e}^-=e^-\right\}\end{aligned} \quad (9)$$

Lemma 5.5: If $R(n)\geq R_c(n)$ and $k\geq\Theta(n)$, then per-session throughput order under the RMPR model on graph H_R is at least $\Theta(nR^3(n)/(kD^2(n)))$.

Proof: According to Definitions 5.1 and 5.4, $U(e)$ represents a set of edges which are interference clones under I_{RMPR} such that these clones interfere with each other and e, under the interference I_{RSPR}. Since each node of H_R has degree $\Theta(nR^2(n))$, we have $\sigma_{RMPR}=min_{e\in E_{R,H}}|U(e)|=\Theta(nR^2(n))$. Thus, invoking the Clone Piggybacking Lemma 4.7, we have

$$\begin{aligned}f_{RMPR} &\leq \text{(max. flow rate with no interference)} \\ &\quad \times(1/(1+\gamma_{max}))\times\sigma_{RMPR} \\ &= c_8(n^2R^3(n)/k)\times(1/(c_6c_7n^2R^2(n)D^2(n))) \\ &\quad \times c_7(nR^2(n)) = (c_8/c_6)(nR^3(n)/(kD^2(n)))\end{aligned} \quad (10)$$

■

As we did the previous sub-section, we need to identify an appropriate value of $D(n)$ such that any feasible transmission under the $RMPR$ model is necessarily feasible under the MPR physical model.

Let us consider a transmission from node i to node j and, as in the last sub-section, let us decompose the network into annular rings determined by concentric circles of radius $lD(n)$ around node j. In order to evaluate the required $D(n)$, we need to identify the total transmitters in each annular region. For the RSPR model we have shown that the l^{th} annular ring contains $c_{10}(2l-1)$ transmitters. Focus on one such transmitter k in the l^{th} ring and let m be the corresponding receiver. Now if we change the model from RSPR to RMPR, the node m can receive packets from an additional $\theta(nR(n)^2)$ nodes. These nodes could lie in the same ring, in the $(l-1)^{st}$ ring or the $(l+1)^{st}$ ring. Thus the total number of transmitters in each annular ring are at most

$$\begin{aligned}c_7c_{10}((2(l-1)-1)+(2l-1)+(2(l+1)-1))nR^2(n) \\ \leq c_{13}nlR^2(n).\end{aligned} \quad (11)$$

Thus, we require that

$$\begin{aligned}\beta &\leq SINR_{i\to j} \\ &= \frac{PR(n)^{-\alpha}}{BN_0+\sum_{l=1}^{1/D(n)}c_{13}nR^2(n)l^{-\alpha+1}D(n)^{-\alpha}}\end{aligned} \quad (12)$$

Since $\sum_{l=1}^{1/D(n)}c_{13}l^{-\alpha+1}$ approaches a constant, we require $(D(n)^{\alpha}/nR(n)^{\alpha+2})\geq c_{14}(\beta/P)$. Hence,

$$\begin{aligned}D(n) &\geq cn^{1/\alpha}R(n)^{(\alpha+2)/\alpha}(\beta/P)^{1/\alpha} \\ &= \Omega(n^{1/\alpha}R(n)^{(\alpha+2)/\alpha})\end{aligned} \quad (13)$$

Consequently we have the following corollary.

Lemma 5.6: If $R(n)\geq R_c(n)$ and $k\geq\Theta(n)$, then per-session throughput order under the MPR physical model is at least $\Theta((n^{1-2/\alpha}R(n)^{1-4/\alpha})/k)$.

C. Multiple Packet Transmission and Reception

To deduce the lower bounds for the MPTR physical model, we define the following combinatorial model.

Definition 5.7: Restricted MPTR (RMPTR) Model:

$$\begin{aligned}I_{\text{RMPTR}}(e) = W(e)-V(e)\quad \forall e\in E_{R,H}\ \textit{where} \\ V(e)=\bigcup_{\hat{e}:\hat{e}^-\text{ same squarelet as }e^-}U(\hat{e})\end{aligned} \quad (14)$$

Lemma 5.8: If $R(n) \geq R_c(n)$ and $k \geq \Theta(n)$, then per-session throughput order under the RMPTR model on graph H_R is at least $\Theta(n^2 R^5(n)/(kD(n)^2))$.

Proof: Set $V(e)$ represents a set of edges which are interference clones under I_{RMPTR} such that these clones interfere with each other and e, under the interference I_{RSPR}. Therefore

$$\sigma_{RMPTR} = min_{e \in E_{R,H}} |V(e)| \geq [min_{e \in E_{R,H}} |U(e)|] \times \textit{min. nodes per squarelet} = \Theta(n^2 R(n)^4) \quad (15)$$

Hence,

$$\begin{aligned} f_{RMPTR} &\leq \text{(max. flow rate with no interference)} \\ &\quad \times (1/(1+\gamma_{max})) \times \sigma_{RMPTR} \\ &= c_{14} \times (n^2 R^3(n)/k) \times (1/(n^2 R^2(n) D^2(n))) \\ &\quad \times (n^2 R^4(n)) = c_{14}(n^2 R^5(n)/(kD^2(n))) \quad (16) \end{aligned}$$

■

In the previous sub-section, we observed that every transmission under the RMPR combinatorial model is necessarily feasible under the MPR physical model if $D(n) = \Omega(n^{1/\alpha} R(n)^{(\alpha+2)/\alpha})$. This value of $D(n)$ is also sufficient to ensure that every transmission under RMPTR combinatorial model is feasible under the MPTR physical model. The reason for this similarity in behavior can be understood by observing that changing the model from RMPR to RMPTR does not add any additional interference, even though the same node can simultaneously transmit to multiple receivers, as long as the total received power from interference is the same. Hence,

Lemma 5.9: If $R(n) \geq R_c(n)$ and $k \geq \Theta(n)$, then per-session throughput order under the MPTR physical model is at least $\Theta((n^{2-2/\alpha} R(n)^{3-4/\alpha})/k)$.

VI. Discussion

Let us review our results for the special case of $k = \Theta(n)$ and $R(n) = \Theta(R_c(n))$. For the SPR physical model the per node capacity is given by

$$f_{SPR} = \Omega\left(\frac{1}{\sqrt{nlog(n)}}\right) \quad (17)$$

This lower bound exactly matches the bound obtained in the original work by [1]. Meanwhile for the MPR physical model the per node capacity satisfies

$$f_{MPR} = \Omega\left(\frac{log(n)^{(1/2)-(2/\alpha)}}{\sqrt{(n)}}\right) \quad (18)$$

Thus according to the deductions in this paper, the MPR scheme guarantees a capacity gain by factor of $log(n)^{1-(2/\alpha)}$.Thus our bound is weaker than that reported by Wang et. al. [6] by a factor of $log(n)^{1/\alpha}$. Finally, note that the per-node capacity under the MPTR model is given by

$$f_{MPTR} = \Omega\left(\frac{log(n)^{(3/2)-(2/\alpha)}}{\sqrt{n}}\right) \quad (19)$$

Thus multipacket transmission and reception improves upon the result of using MPR only [6] by a factor of at least $log(n)^{1-(1/\alpha)}$.

VII. Acknowledgments

This work was partially sponsored by the U.S. Army Research Office under grants W911NF-04-1-0224 and W911NF-05-1-0246, by the National Science Foundation under grant CCF-0729230, by the Defense Advanced Research Projects Agency through Air Force Research Laboratory Contract FA8750-07-C-0169, and by the Baskin Chair of Computer Engineering. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the U.S. Government.

References

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, 2000.

[2] R. Ahlswede, C. Ning, S.-Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1204–1216, 2000.

[3] J. Liu, D. Goeckel, and D. Towsley, "Bounds on the gain of network coding and broadcasting in wireless networks," in *Proc. of IEEE INFOCOM 2007*, Anchorage, Alaska, USA., May 6-12 2007.

[4] A. Ozgur, O. Leveque, and D. Tse, "Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks," *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 2549–3572, 2007.

[5] J. J. Garcia-Luna-Aceves, H. R. Sadjadpour, and Z. Wang, "Challenges: Towards truly scalable ad hoc networks," in *Proc. of ACM MobiCom 2007*, Montreal, Quebec, Canada, September 9-14 2007.

[6] Z. Wang, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "The capacity and energy efficiency of wireless ad hoc networks with multi-packet reception," in *Proc. of ACM MobiHoc 2008*, Hong Kong SAR, China, May 26-30 2008.

[7] S. Karande, Z. Wang, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "Optimal scaling of multicommodity flows in wireless ad hoc networks: Beyond the gupta-kumar barrier," in *Proc. of IEEE MASS 2008*, Atlanta, USA, September 26-30 2008.

[8] M. Franceschetti, O. Dousse, D. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Transactions on Information Theory*, vol. 53, no. 3, pp. 1009–1018, 2007.

[9] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Transactions on Networking*, vol. 10, no. 4, pp. 477–486, 2002.

[10] P. Kyasanur and N. Vaidya, "Capacity of multi-channel wireless networks: Impact of number of channels and interfaces," in *Proc. of ACM MobiCom 2005*, Cologne, Germany, August 28-September 2 2005.

[11] A. Ozgur, O. Leveque, and D. Tse, "Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks," *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 2549–3572, 2007.

[12] R. M. de Moraes, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "Many-to-many communication: A new approach for collaboration in manets," in *Proc. of IEEE INFOCOM 2007*, Anchorage, Alaska, USA., May 6-12 2007.

[13] S. Kulkarni and P. Viswanath, "A deterministic approach to throughput scaling wireless networks," *IEEE Transactions on Information Theory*, vol. 50, no. 6, pp. 1041–1049, 2004.