Multicast Throughput Order of Network Coding in Wireless Ad-hoc Networks

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***Abstract*—We consider a network with n nodes distributed uniformly in a unit square. We show that, under the protocol model, when $n_s = \Omega\left(log(n)^{1+\alpha}\right)$ out of the n nodes, each act as source of independent information for a multicast group consisting of m randomly chosen destinations, the per-session capacity in the presence of network coding (NC) has a tight bound of $\Theta\left(\frac{\sqrt{n}}{n_s\sqrt{m log(n)}}\right)$ when $m = O(\frac{n}{log(n)})$ and $\Theta(\frac{1}{n_s})$ when $m = \Omega(\frac{n}{log(n)})$. In the case of the physical model, we consider $n_s = n$ and show that the per-session capacity under the physical model has a tight bound of $\Theta\left(\frac{1}{\sqrt{mn}}\right)$ when $m = O\left(\frac{n}{(log(n))^3}\right)$, and $\Theta\left(\frac{1}{n}\right)$ when $m = \Omega\left(\frac{n}{log(n)}\right)$. Prior work has shown that these same order bounds are achievable utilizing only traditional store-and-forward methods. Consequently, our work implies that the network coding gain is bounded by a constant for all values of m. For the physical model we have an exception to the above conclusion when m is bounded by $O\left(\frac{n}{(log(n))^3}\right)$ and $\Omega\left(\frac{n}{log(n)}\right)$. In this range, the network coding gain is bounded by $O\left((log(n))^{\frac{1}{2}}\right)$.**

***Index Terms*—Capacity, multicast, network coding.**

I. Introduction

The concept of network coding was first introduced by Yeung and Zhang [1] and subsequently generalized by Ahlswede et al. [2] for a single source multicast in arbitrary directed graphs. Since then, many studies have investigated the benefits of using network coding (NC) in wireless networks.

Recent work [3], [4] has shown that the throughput gain due to the use of NC in a wireless network is bounded by a constant when the traffic in the network consists of multiple unicast sessions. However, the original motivation for the work by Ahlswede et. al [2] was the improvement in network performance for multicasting, not unicasting. Furthermore, under conventional routing, several works [5]–[12] have demonstrated that broadcasting and multicasting can significantly alter the throughput order of wireless networks. Hence, conclusions about the order gain for the unicast capacity cannot be used to determin whether NC can provide any order increase in the multicast capacity of wireless networks.

Recently, widely cited experiments [13], [14] have been reported in which NC has been used successfully in combination with other mechanisms to attain large throughput gains compared to approaches based on conventional protocol stacks. These empirical results have led many to believe that the combination of NC with wireless broadcasting can lead to significant improvements in the multicast throughput order of wireless networks. However, the exact characterization of the multicast order capacity of NC in wireless networks has remained unresolved, with only limited results having been reported to date on the subject.

In this work, under the same standard assumptions in the literature, we undertake the characterization of the multicast and broadcast throughput order of static wireless ad-hoc networks in presence of network coding. Namely, we consider a network consisting of n nodes distributed randomly in the network space, with n_s of the n nodes acting as a multicast source each of a group of m randomly chosen nodes in the network.

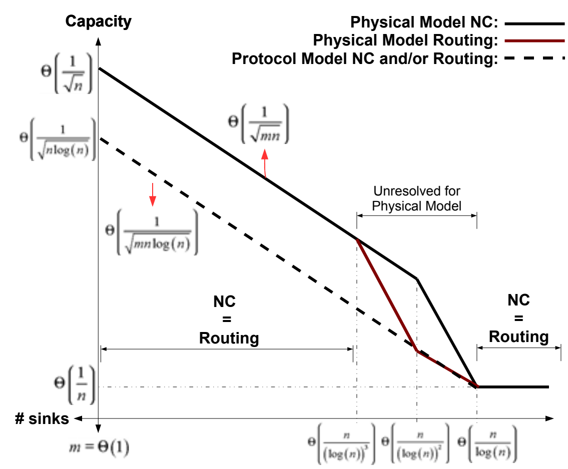

Fig. 1. Throughput Order of Network Coding with $n_s = n$

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The first contribution of this paper is to show that, under the protocol model and with $n_s = \Omega\left(log(n)^{1+\alpha}\right)$ s.t. $\alpha > 0$, the per-session multicast capacity of random wireless ad hoc network in the presence of arbitrary NC [1] has a tight bound of $\Theta\left(\frac{\sqrt{n}}{n_s\sqrt{mlog(n)}}\right)$ when $m = O(\frac{n}{log(n)})$, and of $\Theta(\frac{1}{n_s})$ when $m = \Omega(\frac{n}{log(n)})$. The second contribution of this paper is to show that, under the physical model, the per-session multicast capacity of random wireless ad hoc network with $n_s = n$ and arbitrary NC has a tight bound of $\Theta\left(\frac{1}{\sqrt{mn}}\right)$ when $m = O\left(\frac{n}{log(n)^3}\right)$, and $\Theta\left(\frac{1}{n}\right)$ when $m = \Omega\left(\frac{n}{log(n)}\right)$.

It has already been established in the literature that the above bounds are achievable using traditional store-and-forward routing methods. Consequently, as described by Fig. 1, our analysis demonstrates conclusively that the throughput gain due to NC for multicasting and broadcasting is bounded by a constant factor! We have an exception to the above conclusion for the physical model when m is bounded by $O\left(\frac{n}{(log(n))^3}\right)$ and $\Omega\left(\frac{n}{log(n)}\right)$. It is the subject of future work to investigate whether this gap can be closed.

The remainder of this paper is organized as follows. Section II surveys relevant prior work. Section III describes the network models and other concepts used in proofs. Section IV deduces the capacity results under the protocol model, and Section V addresses the physical model. Section VI summarizes our conclusions.

II. Related Work

Our literature review focuses on prior work addressing the capacity of multicasting and broadcasting in wireless networks, and the capacity of NC in wireless networks.

A. Prior Results Assuming Traditional Store-and-Forward

We first summarize prior results on the order capacity of broadcasting and multicasting under conventional store-and-forward routing.

Tavli [5] showed that the per-node broadcast capacity of arbitrary networks is bounded by $\Theta\left(n^{-1}\right)$, where n is the number of network nodes. Zheng [6] derived the broadcast capacity of power-constrained networks, together with another quantity called "information diffusion rate." Lastly, Keshavarz et al. [7] computed the broadcast capacity for any number of sources in the network.

Several efforts have addressed the multicast capacity of wireless networks, primarily under the protocol model. Jacquet and Rodolakis [8] proved that the scaling of the multicast capacity is decreased by a factor of $O(\sqrt{m})$ compared to the unicast capacity result by Gupta and Kumar [15]. This result implies that the gain attained with multicasting compared to transmitting the same information to each of the m multicast receivers as unicasts is at least $\Theta(\sqrt{m})$.

The work by Shakkottai et al [9] assumes there are n^{ϵ} multicast sources and $n^{1-\epsilon}$ destinations per flow for some $\epsilon > 0$. The results from this work are limited in scope, because of its constraints on the number of sources and destinations.

[1] Arbitrary NC implies that a transmitted symbol can be an arbitrary function of all the symbols received and generated at a node.

Li et al. [10] compute the capacity of wireless ad hoc networks for unicast, multicast, and broadcast applications. Wang et al. [11] independently generalized this work and introduced (n, m, k)-casting as a framework for the characterization of all types of information dissemination in wireless networks.

Keshavarz et al. [12] studied the multicast and broadcast capacity of wireless networks, considered the physical model, and generalized the work in [16] to the multicast regime. Recently, Li et al. reported results on the multicast capacity of wireless networks under a Gaussian Channel model [17]. For n sources, the throughput order reported for the Gaussian Channel model [17] is identical to that attained under the Physical model [10], [11].

B. Prior Results on Network Coding

Ahlswede et al. [2] showed that NC can achieve the min-cut bound for a single source multicast on a directed graph. Since then, a number of theoretical results have been reported for NC. We mention a select few, which provide bounds on the NC gain over routing or provide max-flow min-cut type inequalities that can be used to provide outer-bounds on the rate region under NC.

Li et. al [18], [19] have studied the benefits of NC in undirected networks. The result shows that, for a single unicast or broadcast session, there is no throughput improvement due to NC. In the case of a single multicast session, such an improvement is bounded by a factor of two.

Borade [20] used the classical multi-terminal cut-set bounds [21] to derive edge-cut outer bounds on the rate region under NC for multi-source unicast and multicast. Subsequent studies [22], [23] have shown that the (vertex) cut-set bounds are not tight and improved bounds can be obtained by employing more sophisticated edge-cuts.

Studies such as those summarized above [20], [22], [23] do not readily capture the geometric constraints of multi-hop communication in wireless ad-hoc networks. Nevertheless, there has been prior work to determine the unicast throughput order in wireless networks under NC. Liu et al. [3] have shown that the gain of NC for unicast traffic in a random network (i.e., a network in which the nodes are distributed randomly in an Euclidean space and the sources and destinations are also placed randomly) is bounded by a constant factor. Keshavarz et al. [4] extended these conclusions to arbitrary networks and an arbitrary unicast traffic pattern.

Physical network coding (PNC) [24] and analog network coding (ANC) [25] have been proposed recently, which combine NC with advanced processing at the physical layer that allow receivers to decode multiple concurrent transmissions. ANC was shown [25] to provide throughput gains when compared with digital network coding (i.e., receivers decode at most one packet at a time) and traditional routing (i.e., no NC and receivers decode at most one packet at a time) operating in simple network topologies in which ideal scheduling (i.e., no MAI) is assumed for channel access. Throughput gains have also been reported for PNC in simple topologies [24].

Recently, we have shown that the order throughput of a wireless network can be increased by embracing interference at the physical layer through multi-packet transmission (MPT) or reception (MPR), without the use of NC [26], [27]. Furthermore, we have also shown [28] that using NC together with MPT and MPR does not increase the order throughput of a wireless network for multicasting compared to what MPR and MPT can provide by themselves! What these results imply is that similar throughput gains to those observed with ANC could be attained in practice by embracing concurrency at the physical layer without the need for NC.

Hence, the question remains as to whether NC by itself can provide any gains on the multicast throughput order in wireless networks.

The work presented in the rest of this paper differs from our own recent results [28] in three important ways. In our previous work [28], the sinks associated with each multicast source are bounded by a constant, whereas in this paper the number of sinks is a function of the network size n. Our previous work [28] assumes that a node is capable of MPT and MPR (i.e., receiving or transmitting distinct information from multiple transmitters to multiple receivers at the same time), whereas this paper assumes single-packet transmission and reception. Lastly, our previous work [28] does not present any results for an SINR model, while this paper addresses the physical model.

III. PRELIMINARIES

For a continuous region A, we use $|A|$ to denote its area. We denote the cardinality of a set $\mathbb{S}$ by $|\mathbb{S}|$, and by $\|X_i - X_j\|$ the distance between nodes i and j. Whenever convenient, we utilize the indicator function $1_{\{P\}}$, which is equal to one if P is true and zero if P is false. $Pr(E)$ represents the probability of event E. We say that an event E occurs with high probability (w.h.p.) if $Pr(E) > (1 - (1/n))$ as $n \to \infty$. We employ the standard order notations O, Ω, and Θ.

We assume that the topology of a network is described by a uniformly random distribution of n nodes in a unit square. Let $V = 1, \ldots, n$ represent the node-set and let X_i be the location of node $i \in V$. To avoid boundary effects, it is typical to assume that the network surface is placed upon a toroid or sphere. However, for mathematical convenience, in this work we ignore edge effects and thus assume that the network is placed in a 2-D plane. Further, in our model, as n goes to infinity, the density of the network also goes to infinity. Therefore, our analysis is applicable only to dense networks. We do not consider mobility of nodes and assume a static stationary distribution of nodes. Our capacity analysis is based on both the protocol model and the physical model introduced by Gupta and Kumar [15].

Definition 3.1: The Protocol Model
We assume that all nodes use an identical transmission range $r(n)$ for all their communication. Node i can successfully transmit to node j if for any node $k \neq i$, that transmits at the same time as i it is true that $|X_i - X_j| \leq r(n)$ and $|X_k - X_j| \geq (1 + \Delta)r(n)$.

We shall utilize the following well known property [29] in our analysis

Lemma 3.2: Connectivity Criteria
For a random distribution of n nodes in a unit-square, the network connectivity under the protocol model can be guaranteed w.h.p if and only if (iff)

$$r(n) \geq r_c(n) = \sqrt{\frac{15log(n)}{n}}. \tag{1}$$

Definition 3.3: The Physical Model
All transmissions at all nodes utilize an identical transmission power P. Node i can successfully transmit to node j iff the signal-to-interference/noise ratio (*SINR*) satisfies

$$\mathit{SINR}_{i \to j} = \frac{Ph_{ij}}{BN_0 + \sum_{k \neq i, k=1}^{n} Ph_{kj}} \geq \beta, \tag{2}$$

where h_{ij} is the channel attenuation factor between nodes i and j, and BN_0 is the total ambient noise power. We assume that the channel attenuation factors are completely determined by the path loss model and hence $h_{ij} = \|X_i - X_j\|^{-\alpha}$. We assume that $\beta \geq 1$ in all our analysis.

We assume that the data rate for each successful transmission is W bits/second, which is a constant value and does not depend on n. Given that W does not change the order capacity of the network, we normalize its value to one.

We should emphasize that the above model allows the broadcast of common information from a transmitter to all neighboring receivers that satisfy the interference and attenuation conditions for successful reception. However, we do not consider the case of MPT (or MPR), which allows transmission (or reception) of unique information to (from) multiple nodes at the same time. Thus, our model is similar to those used by Li et al. [3] and Gupta and Kumar [15].

To appropriately model NC, we assume that the information transmitted by a node can be an arbitrary function of the information previously received by the node. Hence, our results apply to any type of NC.

We focus on the traffic scenario in which each of n_s nodes of the wireless network acts as a multicast source for a randomly chosen set of m destinations. [2] We assume that $n_s = \Omega\left((log(n))^{1+\alpha}\right)$ for the protocol model, while we restrict our attention to $n_s = n$ for the physical model.

Definition 3.4: Feasible rate
In a wireless ad hoc network with n nodes in which each source transmits its packets to m destinations, a throughput of $\lambda_m(n)$ bits per second for each multicast session is feasible if there is a spatial and temporal scheme for scheduling network-coded transmissions, such that every source node can send $\lambda_m(n)$ bits per second on average to its m chosen destination nodes, by operating the network in a multi-hop fashion, and using coding and buffering at intermediate nodes when awaiting transmission. That is, there is a $T < \infty$ such that every node can send $T\lambda_m(n)$ bits to its corresponding destination nodes in every time interval $[(i-1)T, iT]$. We denote by $C_m(n)$ the maximum feasible rate.

Definition 3.5: Throughput Order
$C_m(n)$ is said to be of order $\Theta(f(n))$ bits/second if there exist deterministic positive constants c and c' such that

$$\begin{cases} \lim_{n \to \infty} \text{Prob } (C_m(n) = cf(n) \text{ is feasible}) = 1 \\ \lim_{n \to \infty} \text{Prob } (C_m(n) = c'f(n) \text{ is feasible}) < 1. \end{cases} \tag{3}$$

[2]There exist $\binom{n}{m}$ distinct choices for node-sets of size m. Each of these node-sets are chosen with equal probablity.

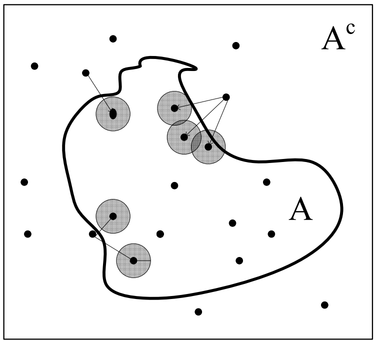


Fig. 2. Generalized Sparsity Cut

Definition 3.6: Vertex Cut
Given a node set V, a cut is the separation of the vertex set V into two disjoint and exhaustive subsets (S, S^C). Here, a vertex partition can be completely described by partitioning the network-area into two region (A, A^c) as shown in Fig. 2. Thus, we also refer to a closed region A as a cut. The cut-capacity $C(A)$ is defined as the maximum number of packets that can be transmitted from A^C to A in a time unit.

Definition 3.7: Multicast Cut-Demand
Given a cut A, a source node in A^c is said to have demand across the cut iff at least one of its destination lies in A. The multicast demand $D(A)$ across the cut is defined as the total number of sources in A^c such that there is at least one destination in the multicast group across the cut.

Definition 3.8: Sparsest Cut
We define the sparsity Γ_A of cut A as the ratio

$$\Gamma_A = \frac{C(A)}{D(A)} \tag{4}$$

Hence, the sparsest cut is given by

$$A^* = arg \min_A \Gamma_A \tag{5}$$

where A^* has the least possible sparsity, denoted as Γ_{A^*}.

The notion of sparsity cut has been utilized in a number of studies related to NC. The definition of Sparsity cut used by Leighton and Rao [30] is applicable only to unicast traffic [3]. We employ a more generalized definition.

Studies such as that by Harvey et al. [23] define sparsity-cuts in terms of edge-cuts, i.e., a cut does not lead to a graph (vertex) separation [23]. We shall use the sparsity of a cut to provide an upper bound on the rate achievable under NC. It is important to understand that we are employing a definition that is distinct from prior studies [23], because they [23] show that, under an alternate definition, NC can exceed the bound provided by a sparsity cut.

Finally we state the well-known Chernoff Bounds [31], which shall be used repeatedly in the rest of this paper.

Lemma 3.9: Chernoff Bounds: Consider n i.i.d random variables $Y_i \in \{0, 1\}$ with $p = \Pr(Y_i = 1)$. Let $Y = \sum_{i=1}^{n} Y_i$. Then, for any $1 \geq \delta \geq 0$ and $\delta_2 \geq 0$, we have

$$\Pr\left(Y \leq (1-\delta_1)np\right) \leq 2e^{\frac{-\delta_1^2 np}{3}} \tag{6}$$

$$\Pr\left(Y \geq (1+\delta_2)np\right) \leq 2e^{\frac{-\delta_2^2 np}{3}} \tag{7}$$

IV. Bounds for The Protocol Model

It is well-known that under its conventional definition, the sparsity cut can be used to obtain an upper bound on the unicast traffic flow in a wireless network [3], [30]. In a similar way, our generalized definition provides an upper bound for multicast flows. The following lemma is applicable to both the protocol as well as the physical model.

Lemma 4.1: Let $C_m(n)$ be the maximum multicast flow-rate in a network and let A^* be the sparsest cut with sparsity Γ_{A^*}, then we have

$$C_m(n) \leq \Gamma_{A^*}. \tag{8}$$

Proof: Let f be the total maximum feasible average rate at which bits can be transmitted from A^c to A, where A is any arbitrary cut. Then by Def. 3.6 we have

$$f \leq C(A) \tag{9}$$

The total information flow across a cut has to be greater than or equal to the sum of the data rates associated with individual multicast sessions that communicate across the cut. Hence,

$$\begin{aligned} f &\geq \sum_{i=1}^{n} C_m(n) 1_{\{\text{source } i \text{ has demand across cut } A\}} \\ &= C_m(n) \sum_{i=1}^{n} 1_{\{\text{source } i \text{ has demand across cut } A\}} \\ &= C_m(n) D(A). \end{aligned} \tag{10}$$

Inserting the above equation in Eq. 9 we have

$$C_m(n) \leq \frac{C(A)}{D(A)} = \Gamma_A \leq \Gamma_{A^*} \tag{11}$$

■

The above deductions imply that the maximum multicast flow-rate is less than the sparsity of any arbitrary cut. Thus, to obtain an upper bound on the network capacity, we are free to choose a region A of any arbitrary shape and size. In this section, we utilize cuts of square shape with length $L_A = 4l_A$, i.e., each side of the square A has length l_A. This is illustrated in Fig. 3. The parameter l_A plays a crucial role in deducing the required upper bounds. In particular, we choose l_A so as to guarantee that the demand $D(A) = \Theta(n)$.

Lemma 4.2: In a random network with $n_s = \Omega\left((log(n))^{1+\alpha}\right)$ of the n nodes act as source for groups of m randomly chosen destination nodes, for every $\alpha > 0$, $\epsilon \geq 0$ and $1 \geq \delta_1 \geq 0$ if

$$l_A = \frac{1}{\sqrt{(1+\epsilon)m}} \qquad \text{for } m \leq \frac{1}{4(1+\epsilon)r(n)^2} \tag{12}$$

$$l_A = 2r(n) \qquad \text{for } m \geq \frac{1}{4(1+\epsilon)r(n)^2} \tag{13}$$

then as $n \to \infty$, w.h.p we have

$$D(A) \geq (1-\delta_1)n_s c_1 \tag{14}$$

where $c_1 = \left(1 - \frac{1}{1+\epsilon}\right)\left(1 - \frac{1}{e^{\frac{1}{1+\epsilon}}}\right)$.

Proof: Let q be the probability that a randomly chosen node i has demand across cut A. Thus,

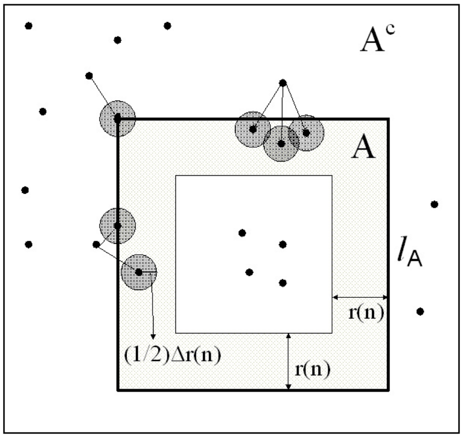


Fig. 3. Cut Capacity under Protocol Model

$$\begin{aligned} q &= Pr(i \in A^c) \times Pr(\text{at least one destination of } i \in A) \\ &\geq (1 - \|A\|)(1 - (1 - \|A\|)^m) \\ &= \left(1 - l_A^2\right)\left(1 - \left(1 - l_A^2\right)^m\right) \end{aligned} \quad (15)$$

Now, note that $\frac{1}{\sqrt{1+\epsilon}} \geq l_A \geq \frac{1}{\sqrt{(1+\epsilon)m}}$ for all m. Hence, we have

$$\begin{aligned} q \geq & \left(1 - \frac{1}{1+\epsilon}\right)\left(1 - \left(1 - \frac{1}{(1+\epsilon)m}\right)^m\right) \\ \geq & \left(1 - \frac{1}{1+\epsilon}\right)\left(1 - \frac{1}{e^{\frac{1}{1+\epsilon}}}\right) = c_1 \end{aligned} \quad (16)$$

where the second inequality follows from the well-known fact that $e^{-x} \geq (1-x)$ for any $0 \leq x \leq 1$.

Let Y_i be an indicator variable that is equal to one if the node i has demand across cut A. Thus, $Pr(Y_i = 1) = q$ and $D(A) = \sum_{i=1:n} Y_i$, and the Chernoff bound of Eq. (6) from Lemma 3.9 further implies that

$$Pr(D(A) \leq (1-\delta_1)n_s q) \leq 2e^{\frac{-\delta_1^2 n_s q}{3}} \quad (17)$$

Now $\frac{(log(n))^{1+\alpha}}{log(2n)} \geq \frac{3}{\delta_1^2 c_1}$ guarantees that $Pr(D(A) \leq (1-\delta_1)n_s q) \leq \frac{1}{n}$ ∎

A choice of $l_A = \frac{1}{\sqrt{(1+\epsilon_1)m}}$ can be used in the above lemma for all m, and such a condition would be sufficient to prove the required result that demand $D(A) \geq (1-\delta_1)n_s c_1$ w.h.p. However, in the following analysis we require that $l_A \geq 2r(n)$. Therefore, we introduce the condition that $l_A = 2r(n)$ for $m \geq \frac{1}{4(1+\epsilon_1)r(n)^2}$. Note that if $m \geq \frac{1}{4(1+\epsilon_1)r(n)^2}$, then $2r(n) \geq \frac{1}{\sqrt{(1+\epsilon_1)m}}$.

We invoke the following important observation to obtain an upper bound on the cut-capacity.

Remark 4.3: Gupta and Kumar [15] observed that, in any time slot, a disk of radius $\frac{\Delta r(n)}{2}$ centered at each receiver in that slot should be disjoint. However, this fact does not apply to the case in which nodes exploit broadcast transmissions, as is done when nodes are capable of employing NC. Indeed, as Fig. 3 illustrates, the disks can overlap if the associated nodes are receiving identical information from a common transmitter. Nevertheless, as highlighted by Li et al. [3], even under the NC assumption, the union of the disks centered at the receivers of one transmission should be disjoint from the union of the disks centered at the receivers of another transmission, given that no MPR is assumed.

Lemma 4.4: Under the protocol model, if a square-shaped cut A has side length $l_A \geq 2r(n)$, then the cut capacity satisfies

$$C(A) \leq \frac{16L_A}{\pi\Delta^2 r(n)} \quad (18)$$

Proof: In the protocol model, the distance between a transmitter and a receiver is bounded by $r(n)$. Hence, any node in A that receives a transmission from A^c should lie within a distance $r(n)$ from the boundary of the cut, i.e., all the receivers must be placed within an annular region of area

$$\begin{aligned} l_A^2 - (l_A - 2r(n))^2 &= 4l_A r(n) - 4r(n)^2 \\ &\leq 4l_A r(n) = L_A r(n) \end{aligned} \quad (19)$$

where the length L_A of the cut is the perimeter of the region A.

We observe that each transmission across the cut does not allow any more transmissions within an area of at least $\frac{\pi\Delta^2 r(n)^2}{4}$. Additionally, at least $\frac{1}{4}$ of this area has to fall within the annular region near the cut boundary. Therefore,

$$\begin{aligned} C(A) = & \text{ max. no. of transmissions from } A^c \text{ to } A \\ \leq & \frac{\text{Area of annular region}}{\frac{\pi\Delta^2 r(n)^2}{4\times 4}} = \frac{16L_A}{\pi\Delta^2 r(n)} \end{aligned} \quad (20)$$

∎

Theorem 4.5: Under the protocol model, as $n \to \infty$, the multicast capacity of a random geometric network with NC has the following upper bound w.h.p

$$C_m(n) = \frac{c_2\sqrt{n}}{n_s\sqrt{15(1+\epsilon_1)mlog(n)}} \quad \text{if m} \leq \frac{n(1+\epsilon_1)^{-1}}{60log(n)} \quad (21)$$

$$C_m(n) = \frac{2c_2}{n_s} \quad \text{if m} \geq \frac{n(1+\epsilon_1)^{-1}}{60log(n)} \quad (22)$$

where $n_s = \Omega\left((log(n))^{1+\alpha}\right)$ s.t. $\alpha > 0$, $c_2 = \frac{64(1+\epsilon_1)e^{\frac{1}{1+\epsilon_1}}}{\pi\Delta^2\epsilon_1(1-\delta_1)(e^{\frac{1}{1+\epsilon_1}}-1)}$ and $\delta_1, \epsilon_1 \geq 0$

Proof: On account of Lemma 4.1, we can obtain an upper bound on the network capacity by just providing a bound for the sparsity Γ_A. Furthermore, note that $L_A = 4l_A$; hence, due to Lemma 4.4 we can say that for all $l_a \geq 2r(n)$ we have

$$C_m(n) \leq \frac{64l_A}{\pi\Delta^2 r(n)D(A)}. \quad (23)$$

Consider $m \geq \frac{1}{4(1+\epsilon_1)r(n)^2}$. If we choose $l_A = 2r(n)$, then from Lemma 4.2 w.h.p we have $D(A) \geq (1-\delta_1)n_s c_1$. Therefore,

$$C_m(n) \leq \frac{128}{\pi\Delta^2(1-\delta_1)n_s c_1} \tag{24}$$

Similarly, if we choose $l_A = \frac{1}{\sqrt{(1+\epsilon)m}}$ for all $m \leq \frac{1}{4(1+\epsilon_1)r(n)^2}$, we have

$$C_m(n) \leq \frac{64\frac{1}{\sqrt{(1+\epsilon_1)m}}}{\pi\Delta^2(1-\delta_1)r(n)n_s c_1}. \tag{25}$$

Note that $C_m(n)$ is maximized for all $m \leq \frac{1}{4(1+\epsilon_1)r(n)^2}$ by choosing the smallest possible value of $r(n)$. Nevertheless the Connectivity Criteria (Lemma 3.2) requires that $r(n) \geq \sqrt{\frac{15log(n)}{n}}$. The final result is obtained by substituting the value of c_1 and $r(n) = \sqrt{\frac{15log(n)}{n}}$ in Eqs. 24-25. ■

The multicast capacity under pure store-and-forward routing has been characterized by Li et al. [10] and Wang et al. [11] and it is stated in the following theorem for the sake of completeness.

Theorem 4.6: [10], [11] Under the protocol model, the multicast capacity of a random geometric network with store-and-forward routing has a tight bound of

$$C_m(n) = \Theta\left(\frac{1}{\sqrt{mnlog(n)}}\right) \quad \text{if } m = O\left(\frac{n}{log(n)}\right) \tag{26}$$

$$C_m(n) = \Theta\left(\frac{1}{n}\right) \quad \text{if } m = \Omega\left(\frac{n}{log(n)}\right) \tag{27}$$

Network coding (NC) is a generalization of store-and-forward routing and thus any capacity achieved by routing is necessarily achieved by NC. Hence,

Theorem 4.7: Under the protocol model, the multicast capacity of a random geometric network with NC has a tight bound equal to

$$C_m(n) = \Theta\left(\frac{1}{\sqrt{mnlog(n)}}\right) \quad \text{if } m = O\left(\frac{n}{log(n)}\right) \tag{28}$$

$$C_m(n) = \Theta\left(\frac{1}{n}\right) \quad \text{if } m = \Omega\left(\frac{n}{log(n)}\right) \tag{29}$$

Finally, we can arrive at the following conclusion.

Corollary 4.8: The multicast throughput order gain provided by NC over store-and-forward routing in a random geometric network is $O(1)$ under the protocol model.

V. Bounds for The Physical Model

To prove the upper bound under the physical model we utilize a circular cut, instead of square shaped cut, with radius r_A as shown in Fig. 4. Additionally, we utilize the following property of the physical model. A similar property of "*straight-lined cuts*" has also been utilized by Liu et al. [3].

Lemma 5.1: Consider a circular cut A of radius r_A with its center at point O. Let S_1 and S_2 be two nodes outside A transmitting across the cut in the same slot. We claim that the arc subtended by angle $\angle S_1OS_2$ on cut A has a length of at least

$$\frac{\Delta_1 r_A max\{L_1, L_2\}}{r_A + max\{L_1, L_2\}} \tag{30}$$

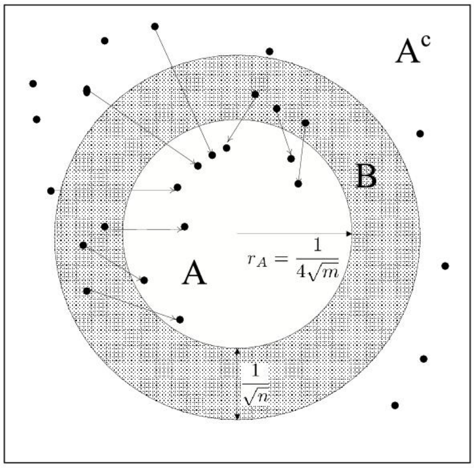


Fig. 4. Cut Capacity under Physical Model

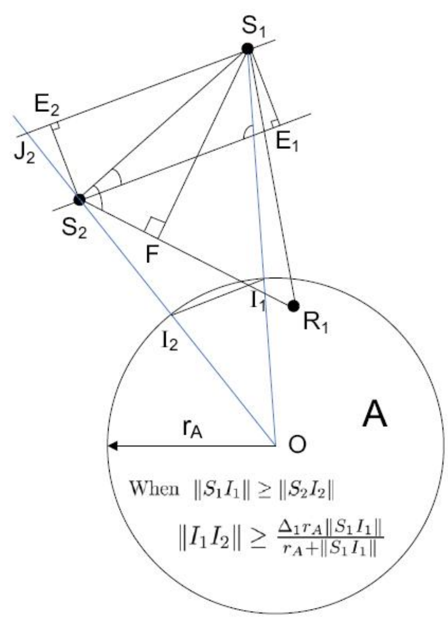


Fig. 5. Geometric property of transmissions across the cut

where $\Delta_1 = \left(\beta^{\frac{1}{\alpha}} - 1\right)$ and L_i represents the (minimum) distance of transmitter S_i from cut A.

Proof: Without loss of generality we can assume that S_1, S_2 are placed as shown in Fig. 5 and $L_1 \geq L_2$. In Fig. 5 the rays OS_1 and OS_2 intersect the cut A at I_1 and I_2 respectively. Therefore, $L_1 = \|S_1I_1\|$ and $L_2 = \|S_2I_2\|$. Furthermore, the length of segment I_1I_2 is smaller than the length of the arc subtended by $\angle S_1OS_2$. Hence, in order to prove the claim, it is sufficient to show that

$$\|I_1I_2\| \geq \frac{\Delta_1 r_A\|S_1I_1\|}{r_A + \|S_1I_1\|} \tag{31}$$

Consider a receiver R_1 that lies inside A and can successfully decode a transmission from S_1. It follows from Eq. 2 in Definition 3.3 that

$$\frac{P\|S_1R_1\|^{-\alpha}}{BN_o + P\|S_2R_1\|^{-\alpha}} \geq \beta$$

$$\implies \quad \|S_2R_1\| \geq \beta^{\frac{1}{\alpha}}\|S_1R_1\| = (1+\Delta_1)\|S_1R_1\| \tag{32}$$

Consider the triangle formed by S_1, S_2 and R_1, as shown in Fig. 5. Now draw a perpendicular from S_1 to F, which is a point on segment S_2R_1. Note that $\|FR_1\| \leq \|S_1R_1\|$ and hence it is easy to show that $\|S_2F\| \geq \Delta_1|S_1R_1|$. Now draw a line through S_2 parallel to segment I_1I_2 and drop a perpendicular S_1E_1 on this line. Since $\angle S_1S_2E_1 \leq \angle S_1S_2R_1$, we have $\cos(\angle S_1S_2E_1) \geq \cos(\angle S_1S_2R_1)$, which implies that $|S_2E_1| \geq |S_2F|$. Similarly, draw a line through S_1 parallel to I_1I_2. Let this line intersect the ray OS_2 at J_2. Drop a perpendicular S_2E_2 on line S_1J_2. Because the triangle S_1OJ_2 is isosceles, $\angle S_1J_2S_2$ is acute and hence E_2 should lie within the segment S_1J_2. Hence, $\|S_1J_2\| \geq \|S_1E_2\|$. Because $S_2E_1S_1E_2$ forms a rectangle we get $\|S_1J_2\| \geq \Delta_1|S_1R_1|$. Finally, we note that $\|S_1R_1\| \geq \|S_1I_1\|$ because S_1I_1 is the shortest distance between S_1 and circle A. Hence,

$$\|S_1J_2\| \geq \Delta_1\|S_1I_1\| \tag{33}$$

Consider the triangle OS_1J_2. The Basic Proportionality Theorem implies that

$$\|I_1I_2\| = \frac{\|S_1J_2\|\|OI_1\|}{\|OS_1\|} \tag{34}$$

Substituting Eq. 33 in Eq. 34 proves the claim in Eq. 31 ■

Theorem 5.2: Under the physical model, the multicast capacity of a random geometric network with NC has the following upper bound w.h.p $C_m(n) = O\left(\frac{1}{\sqrt{mn}}\right)$, when $m = O\left(\frac{n}{log(n)^2}\right)$ and $n \to \infty$.

Proof: Consider a circular cut A with radius $r_A = \frac{1}{4\sqrt{m}}$. Divide the region A^c, as shown in Fig. 4, into sub-region B and $A^c - B$, where the B is an annular region of width $\frac{1}{\sqrt{n}}$. Let n_B and n_{A^C-B} be the maximum number of nodes, from region B and region $A^c - B$ respectively,that can transmit to region A in a single time slot. Hence,

$$C(A) \leq n_B + n_{A^c-B} \tag{35}$$

A transmission from any node in region $A^c - B$ to any node in region A has a minimum hop length of $\frac{1}{\sqrt{n}}$. Consequently, Lemma 5.1 implies that any two transmitters in $A^c - B$ that transmit in the same slot have to be separated such that they subtend an arc on A of length at least $\frac{\Delta_1 r_A \frac{1}{\sqrt{n}}}{r_A + \frac{1}{\sqrt{n}}}$. Given that the circumference of A is $2\pi r_A$, we have

$$\begin{aligned} n_{A^c-B} &\leq 2\pi r_A \times \frac{r_A + \frac{1}{\sqrt{n}}}{\Delta_1 r_A \frac{1}{\sqrt{n}}} \\ &= \frac{2\pi}{\Delta_1}\left(\frac{\sqrt{n}}{4\sqrt{m}} + 1\right) \leq \frac{5\pi\sqrt{n}}{2\Delta_1\sqrt{m}} \end{aligned} \tag{36}$$

To obtain a bound on n_B, observe that the area of region B is given by

$$\begin{aligned} |B| &= \pi\left(r_A + \frac{1}{\sqrt{n}}\right)^2 - \pi r_A^2 \\ &= \frac{2\pi r_A}{\sqrt{n}} + \frac{\pi}{n} \leq \frac{\pi}{2\sqrt{mn}} + \frac{\pi}{\sqrt{mn}} \leq \frac{3\pi}{2\sqrt{mn}} \end{aligned} \tag{37}$$

If $m = O\left(\frac{n}{(log(n))^2}\right)$, there exists a constant $c_3 \geq 0$ such that

$$|B| \leq \frac{c_3 log(n)}{n} \tag{38}$$

The total number of nodes in B is necessarily greater than n_B. Therefore, the Chernoff bound of Eq. 6 implies that, for any $\delta_2 \geq 0$, we have

$$\begin{aligned} \Pr\left(n_B \leq \frac{3\pi(1+\delta_2)\sqrt{n}}{2\sqrt{m}}\right) &\leq 2e^{\frac{-\delta_2^2 n|B|}{3}} \\ &\leq 2e^{\frac{-\delta_2^2 log(n)}{3c_3}} = \frac{2}{n^{\frac{\delta_2^2}{3c_3}}} \end{aligned}$$

Consequently, if we choose $\delta_2 \geq 3c_3$, then as $n \to \infty$ w.h.p we have

$$\begin{aligned} C(A) &\leq \frac{3\pi(1+\delta_2)\sqrt{n}}{2\sqrt{m}} + \frac{5\pi\sqrt{n}}{2\Delta_1\sqrt{m}} \\ &= \frac{\pi(3(1+\delta_2)\Delta_1 + 5)\sqrt{n}}{2\Delta_1\sqrt{m}} \end{aligned} \tag{39}$$

In the previous section, we have already shown that w.h.p the demand across a square shaped cut with area $O(\frac{1}{m})$ is of order $\Theta(n)$. Such a property is valid for circular cuts also. Let q_1 be probability that a source node in A^c has at least one of its m destinations in the circle A. We can show that

$$\begin{aligned} q_1 &\geq \left(1 - \frac{1}{16}\right)\left(1 - \left(1 - \frac{1}{16m}\right)^m\right) \\ &= \frac{15\left(1 - e^{\frac{1}{16}}\right)}{16} = c_4 \end{aligned} \tag{40}$$

The Chernoff Bound of Eq. 7 implies that there exists a $1 \geq \delta_1 \geq 0$ such that as $n \to \infty$ w.h.p. $D(A) \geq (1-\delta_1)c_4 n$. Therefore, the Sparsity bound from Lemma 4.1, along with Eqs. 39 and 40 implies that w.h.p.

$$C_m(n) \leq \left(\frac{\pi(3(1+\delta_2)\Delta_1 + 5)}{2\Delta_1(1-\delta_1)c_4}\right)\frac{1}{\sqrt{mn}} \tag{41}$$

■

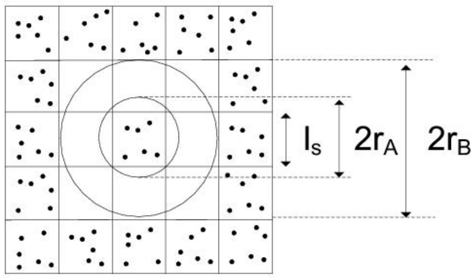
The mathematical techniques used in the above proof cannot be used to obtain an upper bound on multicast capacity of NC for all values of m. In particular, note that Eq. 38 and hence the convergence condition in Eq. 39 requires that $m = O\left(\frac{n}{(log(n))^2}\right)$. Therefore, we consider an alternative approach to obtain upper bounds. This approach shall give us a tighter upper bound for $m = \Omega\left(\frac{n}{log(n)}\right)$.

Theorem 5.3: Under the physical model, the multicast capacity in a random geometric network with NC has the following upper bound w.h.p.

$$C_m(n) = O\left(\frac{1}{mlog(n)}\right) \qquad \text{if } m \leq \frac{n}{log(n)} \tag{42}$$

$$C_m(n) = O\left(\frac{1}{n}\right) \qquad \text{if } m \geq \frac{n}{log(n)} \tag{43}$$

Proof: Decompose the network into square-lets of side-length $\sqrt{\frac{log(n)}{9n}}$. Let J be the event that there exists a square-let containing *at least* $\frac{(1-\delta_3)log(n)}{9n}$ nodes, where $1 \geq \delta_3 \geq 0$, with all its eight adjoining square-lets empty. The event J is



$$l_s = \frac{1}{3}\sqrt{\frac{log(n)}{n}}, r_A = \frac{l_s}{\sqrt{2}}, r_B = \frac{3l_s}{2}$$

Fig. 6. Clustering of nodes

illustrated in Fig. 6. We are interested in showing that the event J occurs w.h.p. Let η represent the total number of nodes in a square-let, $p_1 = Pr(\eta = 0)$ and $p_2 = Pr\left(\eta \leq \frac{(1-\delta_3)log(n)}{9n}\right)$, where $1 \geq \delta_3 \geq 0$. Using the fact that $lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n = e^{-a}$, p_1 can be computed as

$$p_1 = \left(1 - \frac{log(n)}{9n}\right)^n = e^{\frac{-log(n)}{9}} = n^{\frac{-1}{9}}. \tag{44}$$

In addition, Eq. 6 implies that

$$\begin{aligned} p_2 \quad &= Pr\left(\eta \leq \frac{(1-\delta_3)log(n)}{9n}\right) \\ &\leq 2e^{\frac{-\delta_3^2 log(n)}{27}} = 2n^{\frac{-\delta_3^2}{27}}. \end{aligned} \tag{45}$$

Therefore, as $n \to \infty$, in the limit we have

$$\begin{aligned} Pr(J) \quad &\geq \quad 1 - (1-(1-p_2)p_1^8)^{\frac{9n}{log(n)}} \\ &\geq \quad 1 - (1-(1-2n^{\frac{-\delta_3^2}{27}})n^{\frac{-8}{9}})^{\frac{9n}{log(n)}} \\ &\geq \quad 1 - \left(\left(1 - \frac{n^{\frac{1}{9}}(1-2n^{\frac{-1}{27}})}{n}\right)^n\right)^{\frac{9}{log(n)}} \\ &= \quad 1 - e^{-9\frac{n^{\frac{1}{9}}(1-2n^{\frac{-1}{27}})}{log(n)}} = 1. \end{aligned} \tag{46}$$

because $e^{-9\frac{n^{\frac{1}{9}}(1-2n^{\frac{-1}{27}})}{log(n)}}$ approaches zero faster than $\frac{1}{n}$ when $n \to \infty$.

Let us choose a circular cut A of radius $r_A = \frac{l_s}{\sqrt{2}}$ such that A circumscribes a square-let satisfying event J. Observe that we can draw another circle B of radius $r_B = \frac{3l_s}{2}$ concentric to A, such that all nodes that transmit across the cut A are placed outside B. Therefore, the minimum hop-length of any transmission across the cut A is at least $r_B - r_A$. Accordingly, Lemma 5.1 implies that

$$\begin{aligned} C(A) \quad &\leq \quad 2\pi r_A \times \frac{r_A + (r_B - r_A)}{\Delta_1 r_A (r_B - r_A)} = \frac{2\pi r_A}{\Delta_1 (r_B - r_A)} \\ &= \quad \frac{2\pi\frac{l_s}{\sqrt{2}}}{\Delta_1\left(\frac{3l_s}{2} - \frac{l_s}{\sqrt{2}}\right)} = \frac{\pi 2\sqrt{2}}{\Delta_1(3-\sqrt{2})} = c_5 \end{aligned} \tag{47}$$

Now let p_3 be the probability that a source has demand across cut A. Observe that all the nodes inside the circle A are within the middle square-let. Hence, the Chernoff Bound can be used to show that, as $n \to \infty$, w.h.p the total number of nodes outside the circle A are at least $n - \frac{(1+\delta_4)log(n)}{9}$, where $\delta_4 \geq 0$. Therefore, as $n \to \infty$ w.h.p.,

$$\begin{aligned} p_3 &= \\ &\left(1 - \frac{(1+\delta_4)log(9n)}{n}\right)\left(1 - \left(1 - \frac{(1-\delta_3)log(n)}{9n}\right)^m\right) \\ &= \left(1 - e^{\frac{-m(1-\delta_3)log(n)}{9n}}\right) \end{aligned} \tag{48}$$

In the above equation we have $p_3 = \Theta(1)$ when $m = \Omega\left(\frac{n}{log(n)}\right)$, while when $m = O\left(\frac{n}{log(n)}\right)$ we have that

$$p_3 \geq \frac{m(1-\delta_3)log(n)}{9n} \tag{49}$$

Therefore, an application of Eq. 7 allows us to show that $D(A) = \Omega(mlog(n))$ when $m = O\left(\frac{n}{log(n)}\right)$, while $D(A) = \Omega(n)$ when $m = \Omega\left(\frac{n}{log(n)}\right)$. We get the final result by calculating the sparsity $\Gamma_A = \frac{C(A)}{D(A)}$ which, as established by Lemma 4.1 provides an upper bound for the capacity $C_m(n)$. ■

The upper bounds stated in the above theorem are identical to those of Theorem 2 in the work by Keshavarz-Haddad and Riedi [12] and the initial steps in our proof are similar to those they use [12]. However, we highlight that our eventual argument utilizes the geometric properties of the cut and hence is distinct from their work. In particular, the claims and the proof by Keshavarz-Haddad and Riedi [12] are applicable only to store-and-forward routing, while our bounds apply to NC.

Keshavarz-Haddad and Riedi [12] have established the following lower bound on the multicast capacity under store-and-forward routing.

Theorem 5.4: Under the physical model, the multicast capacity of a random geometric network with store-and-forward routing has the following lower bound w.h.p.

$$C_m(n) = \Omega\left(\frac{1}{\sqrt{mn}}\right) \qquad \text{if } m \leq \frac{n}{log(n)^3} \tag{50}$$

$$C_m(n) = \Omega\left(\frac{1}{m\sqrt{log(n)^3}}\right) \qquad \text{if } \frac{n}{log(n)^3} \leq m \leq \frac{n}{log(n)^2} \tag{51}$$

$$C_m(n) = \Omega\left(\frac{1}{\sqrt{mnlog(n)}}\right) \qquad \text{if } \frac{n}{log(n)^2} \leq m \leq \frac{n}{log(n)} \tag{52}$$

$$C_m(n) = \Omega\left(\frac{1}{n}\right) \qquad \text{if } \frac{n}{log(n)} \leq m \tag{53}$$

Given that any capacity achieved with store-and-forward routing is necessarily achievable with NC, putting together the results we have presented up to this point, we arrive at the following result.

Theorem 5.5: Under the physical model, the multicast capacity in a random geometric network with NC has a tight bound w.h.p. of

$$C_m(n) = \Theta\left(\frac{1}{\sqrt{mn}}\right) \qquad \text{if } m \leq \frac{n}{log(n)^3} \tag{54}$$

$$C_m(n) = \Theta\left(\frac{1}{n}\right) \qquad \text{if } \frac{n}{log(n)} \leq m \tag{55}$$

Accordingly, we have the following result.

Corollary 5.6: In a random geometric network with n nodes and for values of $m \leq \frac{n}{log(n)^3}$ and $\frac{n}{log(n)} \leq m$, the multicast throughput order gain provided by NC over store-and-forward routing is $O(1)$ under the physical model.

VI. CONCLUSION

Network coding (NC) has received considerable attention, and recent results for specific instantiations of NC have led many to infer that NC could lead to order throughput gains for multicasting in wireless networks. In this work, under standard assumptions made in prior work regularly such as uniform power, random traffic, and random node deployment, we used the protocol and physical models to show that the order throughput gain derived from NC for multicasting and broadcasting in wireless networks is bounded by a constant. That is, as the network size increases, NC renders the same order throughput as traditional store-and-forward routing.

Despite our negative result on the multicast order throughput for NC, the constant-factor gains that can be attained with NC over store-and-forward routing should not be ignored, and they may be of importance in practical settings. Hence, the exact characterization of the constant remains an open problem that merits further investigation. In addition, we highlight that, heterogeneity in node deployment and traffic patterns, power control and mobility, and signaling overhead can all significantly impact the scaling law of the ad-hoc network.

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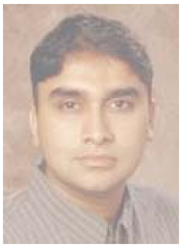

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