UNIVERSITY OF CALIFORNIA
SANTA CRUZ

THE CAPACITY OF WIRELESS AD HOC NETWORKS

A dissertation submitted in partial satisfaction of the
requirements for the degree of

DOCTOR OF PHILOSOPHY

in

ELECTRICAL ENGINEERING

by

Zheng Wang

June 2010

The Dissertation of Zheng Wang
is approved:

Professor Hamid R. Sadjadpour, Chair

Professor J.J. Garcia-Luna-Aceves

Professor Claire X.-G. Gu

Tyrus Miller
Vice Provost and Dean of Graduate Studies

Table of Contents

List of Figures

Abstract

The Capacity of Wireless Ad Hoc Networks

by

Zheng Wang

This thesis studies the fundamental limits on the capacity of wireless ad hoc networks. First, Multi-Packet Reception (MPR) is proposed to increase the capacity under both protocol and physical models. By defining *power efficiency*, it is also shown that, in order to achieve higher capacity, there is a cost to pay in terms of the network power consumption efficiency.

Second, unicast traffic patten is extended into a unified framework in which information is disseminated by means of unicast, multicast, broadcasting, or different forms of anycast with Single-Packet Reception (SPR) and MPR.

Third, the contribution of Network Coding (NC) is investigated and it is proved that NC does not contribute to the order capacity of multicast traffic when nodes are endowed with MPR and Multi-Packet Transmission (MPT) capabilities in the network.

Finally, Opportunistic Interference Management (OIM) scheme is introduced both in cellular and ad hoc networks. The approach is based on a new multiuser diversity concept that achieves the capacity of Dirty Paper Coding (DPC) asymptotically in cellular networks and significantly improve the scalability performance in ad hoc networks.

To my beloved wife, Chao Shi

To my parents Yuying Zhang and Lufang Wang whom I owe my all

Acknowledgments

I am extremely lucky to have support, encouragement and inspiration from many people, without them this work would not have been possible.

My greatest gratitude goes to my advisor Professor Hamid R. Sadjadpour for his guidance and consistent support. His knowledgeable, wise and inspiring discussions have guided me throughout my whole Ph.D. career. It was such a pleasure to work with him for all these years. Facing so many obstacles, I am lucky that he has always been there to show me the right direction and influenced me as an active thinker.

I also owe special gratitude to Professor Jose Joaquin Garcia-Luna-Aceves, for guiding me millions of times when I was stuck, pointing out the right directions, serving on my Ph.D. committee and reading my thesis.

I am also grateful to Professor Claire Gu for serving on my Ph.D. committee and spending time reviewing this work and giving valuable suggestions and comments on my work.

My fellow students made my life at University of California, Santa Cruz cheerful and memorable. I wish to thank Dr. Xianren Wu, Mingyue Ji, Xin Wang, Hui Xu, Christopher Rouchy, Roger Tilly, Bahador Amiri, Lemonia Dritsoula, and Bita Azimdoost for all the enjoyable discussions. Your friendship is my best fortune.

Chapter 1

Introduction

1.1 Overview of Wireless Ad Hoc Networks

Wireless ad hoc networks have matured as a viable means to provide ubiquitous untethered communication. In order to enhance network connectivity, a source communicates with far destinations by using intermediate nodes as relays.

There has been a growing interest to understand the fundamental capacity limits of wireless ad hoc networks. Results on network capacity are not only important from a theoretical point of view, but also provide guidelines for protocol design in wireless networks.

In the seminal work of Gupta and Kumar [1], the per node throughput capacity of random wireless ad hoc network with multi-pair unicast traffic in protocol model scales as $\Theta\left(1/\sqrt{n \log n}\right)$ [1] with plain multi-hop routing, where n is the number of nodes in

[1] Given two functions $f(n)$ and $g(n)$. This thesis defines that $f = O(g(n))$ if $\sup_n (f(n)/g(n)) < \infty$ and $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$. If both $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$, then $f(n) = \Theta(g(n))$.

the network. That means wireless ad hoc networks can not scale which leads to more research and motivate the most of this thesis.

1.2 Research Motivation and Contributions

This thesis is well motivated to study the scalability of wireless ad hoc networks. The main contributions of this thesis are the followings:

- Multi-Packet Reception (MPR) is proposed in wireless ad hoc networks, which allows multiple concurrent transmissions. It is shown that $\Theta\left(R(n)\right)$ and $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$ bits per second constitute tight bounds for the throughput capacity per node in random wireless ad hoc networks for protocol and physical models respectively, where $R(n)$ is the MPR communication range and α is the channel path loss parameter. MPR achieves higher throughput capacity under physical model than techniques proposed in [1, 2]. When $R(n) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$, the throughput capacity is tight bounded by $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$ and $\Theta\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$ for protocol and physical models respectively. This is a gain of $\Theta\left(\log n\right)$ and $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$ compared to the bound in [1]. A new parameter is introduced to quantify how many bits/sec of information are transferred across the network per each unit of power. The power efficiency of some existing techniques [1, 2] are computed and compared with the power efficiency of MPR. It is shown that MPR provides a tradeoff between throughput capacity, node decoding complexity, and power efficiency in random wireless ad hoc networks. It is also shown that achieving higher throughput ca-

pacity leads to a lower power efficiency.

- The first unified modeling framework is presented for the computation of the capacity-delay tradeoff of random wireless ad hoc networks in which receivers perform Single-Packet Reception (SPR) and Multi-Packet Reception (MPR). This framework considers information dissemination by means of unicast routing, multicast routing, broadcasting, or different forms of anycasting. (n, m, k)-casting is defined as a generalization of all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks. In the context of (n, m, k)-casting, n, m, and k denote the number of nodes in the network, the number of destinations for each communication group, and the actual number of communication-group members that receive information optimally[2], respectively. More importantly, capacity-delay tradeoff studies are presented for all kinds of information dissemination as a general function of the transmission range $r(n)$ of SPR and receiver range $R(n)$ of MPR respectively.

- The real contribution of network coding is addressed in terms of increasing order capacity in multicast application in wireless ad hoc networks. First, when each multicast group consists of a constant number of sinks, the combination of NC, MPT and MPR provides a per session throughput capacity of $\Theta(nT^3(n))$, where $T(n)$ is the communication range. Second, this scaling law represents an order gain of $\Theta(n^2T^4(n))$ over a combination of SPR. The combination of only

[2]Optimality is defined as the k closest (in terms of Euclidean distance of the tree) destinations to the source in an (n, m, k)-cast group.

MPT and MPR is sufficient to achieve a per-session multicast throughput order of $\Theta(nT^3(n))$. Consequently, it is proved that NC does not contribute to the multicast capacity when MPR and MPT are used in the network.

- An Opportunistic Interference Management (OIM) technique is presented for the downlink of a wireless cellular network with which independent data streams can be broadcasted to their corresponding mobile stations with single antenna such that these data streams do not interfere with each other. Unlike all prior techniques that attempt to fight individually fading and interference as impairments in wireless channels, OIM takes advantage of one of them (fading channel) to reduce the negative effect of the other one (interference). The result is very effective, and constitutes a powerful technique that achieves high throughput capacity and yet requires minimum feedback and simple point-to-point encoding and decoding complexity for each node. Furthermore, it is extended into wireless ad hoc networks because of no base station challenge. It is shown that the throughput capacity with OIM in wireless ad hoc networks is $\Theta\left(\frac{\log(T(n))}{\sqrt{n}T(n)}\right)$ when $T(n) = \Omega\left(\sqrt{\log n}\right)$ is the transmission range. The approach provides a gain of $\Theta\left(\log(T(n))\right)$ compared to the simple multi-hop point-to-point communications under similar network assumptions. The gain ranges from $\Theta\left(\log\log n\right)$ to $\Theta\left(\log n\right)$, depending on the value of the transmission range, while the encoding and decoding complexity of the new scheme is similar to that of point-to-point communications. The increase of the capacity is essentially because of the powerful nature of fading in wireless

environment.

1.3 Outline of Thesis

The outline of the rest of the thesis is as follows. In Chapter 2, a comprehensive literature survey is first provided to summarize all the important previous research works, then this chapter gives the general and basic network model used to derive capacity in wireless ad hoc networks.

Chapter 3 proposes Multi-Packet Reception (MPR) technique in wireless ad hoc networks which increases the order capacity of random wireless ad hoc networks under both protocol and physical models compared to the capacity of point-to-point communication reported by Gupta and Kumar [1]. The *power efficiency* $\eta(n)$ is also defined as the bits of information transferred per unit time (second) in the network for each unit power, and show that a lower power efficiency is attained in order to achieve higher throughput capacity.

Chapter 4 extends the unicast traffic model to (n, m, k)-cast. A unified modeling framework is first proposed for the computation of the capacity-delay tradeoff of random wireless ad hoc networks. This framework considers information dissemination by means of unicast routing, multicast routing, broadcasting, or different forms of anycasting. (n, m, k)-casting is defined as a generalization of all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks. The capacity-delay tradeoff is described for (n, m, k)-casting in wireless ad hoc networks in

which receivers perform Single-Packet Reception (SPR) and Multi-Packet Reception (MPR).

Chapter 5 studies the contribution of Network Coding (NC) in improving the multicast capacity of random wireless ad hoc networks when nodes are endowed with Multi-Packet Transmission (MPT) and Multi-Packet Reception (MPR) capabilities. Surprisingly, an identical order capacity can be achieved when nodes have only MPR and MPT capabilities. This result proves that NC does not contribute to the order capacity of multicast traffic in wireless ad hoc networks when MPR and MPT are used in the network. The result is in sharp contrast to the general belief (conjecture) that NC improves the order capacity of multicast.

Chapter 6 introduces a new multiuser diversity scheme both in wireless cellular networks and ad hoc networks. With the new technique, multiple antennas of base stations and mobile users (cellular case) or transmitter-receiver pairs (ad hoc case) can communicate without causing significant interference to each other. The new scheme called Opportunistic Interference Management (OIM) significantly reduces the feedback required in distributed MIMO systems, and requires an encoding and decoding complexity that is similar to that of point-to-point communications. Hence, OIM provides an alternative approach to distributed MIMO systems with significantly less feedback requirements among nodes, which makes this approach far more practical than distributed MIMO systems.

Chapter 7 concludes the thesis, and give future research directions.

Chapter 2

Related Works and Network Models

This chapter presents a survey of important literature works in Section 2.1 and gives an overview of network models with preliminaries of wireless ad hoc network used throughout the thesis in Section 2.2 respectively.

2.1 Related Works and Literature Reviews

Gupta and Kumar [1] shows that the per-node throughput capacity of random wireless ad hoc network with multi-pair unicast traffic in protocol model scales as $\Theta\left(1/\sqrt{n \log n}\right)$ in protocol model. Under the physical model assumption, [1] showes that the throughput capacity has lower and upper bounds of $\Theta(\sqrt{1/n \log n})$ and $\Theta(\sqrt{1/n})$, respectively. Franceschetti et al. [2] closed the gap between these two bounds and obtained a tight bound of $\Theta(\sqrt{1/n})$ using percolation theory. In this approach, all communications are simple point-to-point without any cooperation between senders and receivers. Since the landmark work by Gupta and Kumar [1] on the scalability of wire-

less networks, considerable attention has been devoted to improving or analyzing their results.

2.1.1 Multi-Packet Reception

One line of research has been the development of techniques aimed at improving the capacity of wireless networks. Grossglauser and Tse [3] demonstrated that a non-vanishing capacity can be attained at the price of long delivery latencies by taking advantage of long-term storage in mobile nodes. El Gamal et al [4] characterized the fundamental throughput-delay tradeoff for both static and mobile networks. It has also been shown that, if bandwidth is allowed to increase proportionally to the number of nodes in the network [5, 6], higher throughput capacity can be attained for static wireless networks. Other work demonstrated that changing physical layer assumptions such as using multiple channels [7] or MIMO cooperation [8] can change the capacity of wireless networks. Recently, Ozgur et al. [8] proposed a hierarchical cooperation technique based on virtual MIMO to achieve linear per source-destination capacity. Unfortunately, distributed MIMO techniques require significant cooperation and feedback information among nodes to achieve capacity gains using multiple antenna systems. These challenges include synchronization during transmission and cooperation for decoding which makes distributed MIMO systems less practical. Cooperation can be extended to the simultaneous transmission and reception at the various nodes in the network, which can result in significant improvement in capacity [9].

The work by Gupta and Kumar [1] demonstrated that wireless ad hoc networks

do not scale well for the case of multi-pair unicasts when nodes are able to encode and decode at most one packet at a time. This has motivated the study of different approaches to "embrace interference" in order to increase the capacity of wireless ad hoc networks. Embracing interference consists of increasing the concurrency with which the channel is accessed.

One approach to embracing interference consists of allowing a receiver node to decode correctly multiple packets transmitted concurrently from different nodes, which it is called multi-packet reception (MPR) [10, 11]. In practice, MPR can be achieved with a variety of techniques, including multiuser detection (MUD) [12], directional antennas [13, 14] or multiple input multiple output (MIMO) techniques.

The analysis related to MPR will be given in Chapter 3.

2.1.2 Unifying Traffic Patten

The other area of research on the capacity of wireless networks has focused on broadcast and multicast. Tavli [15] was first to show that $\Theta\left(n^{-1}\right)$ is a bound on the per-node broadcast capacity of arbitrary networks. Zheng [16] derived the broadcast capacity of power-constrained networks, together with another quantity called "information diffusion rate." The work by Keshavarz et al. [17] is perhaps the most general case of computing broadcast capacity for any number of sources in the network.

There are prior contributions on the multicast capacity of wireless networks [18, 19]. Jacquet and Rodolakis [18] proved that the scaling of multicast capacity is decreased by a factor of $O(\sqrt{m})$ compared to the unicast capacity result by Gupta

and Kumar [1] where m is the number of destinations for each source. Li et al. [19] compute the capacity of wireless ad hoc networks for unicast, multicast, and broadcast applications.

The analysis related to unifying traffic patten will be addressed in Chapter 4.

2.1.3 Network Coding

A complementary approach to embracing interference consists of increasing the amount of information sent per channel usage. Network coding (NC), which was originally proposed by Ahlswede *et al.* in [20], is one such technique. Unlike traditional store-and-forward routing, network coding scheme encodes the messages received at intermediate nodes, prior to forwarding them to subsequent next-hop neighbors. Network coding (NC) [20] was introduced and shown to achieve the optimal capacity for single-source multicast in directed graphs corresponding to wired networks in which nodes are connected by point-to-point links. Ahlswede *et al.* [20] showed that network coding can achieve a multicast flow equal to the min-cut for a single source and under the assumptions of a directed graph. Since then, many attempts have been made to apply NC to wireless ad hoc networks, and Liu et al. [21] have shown that NC cannot increase the order capacity of wireless ad hoc networks for multi-pair unicast traffic. However, recent work [22, 23, 24, 25] has shown promising results on the advantage of NC in wireless ad hoc networks subject to multicast traffic. An interesting aspect of these works is that nodes are also assumed to have MPT and MPR capabilities in addition to using NC for multicasting. Recently, Katti et al. [22] and Zhang et al. [23] proposed

analog network coding (ANC) and physical-layer network coding (PNC) respectively, as ways to embrace interference. Interestingly, a careful review of ANC and PNC reveals that they consist of the integration of NC with a form of MPR, in that receivers must be allowed to decode successfully concurrent transmissions from multiple senders by taking advantage of the modulation scheme used at the physical layer (e.g., MSK modulation in PNC [22]). This and other works in network coding (NC) [26, 27] has motivated a large number of researchers to investigate the impact of NC in increasing the throughput capacity of wireless ad hoc networks. However, Liu *et al.* [21] recently showed that NC does not increase the order of the throughput capacity for multi-pair unicast traffic. Nevertheless, a number of efforts (analog network coding [22], physical network coding [23]) have continued the quest for improving the multicast capacity of ad-hoc networks by using NC. Despite the claims of throughput improvement by such studies, the impact of NC on the multicast scaling law has remained uncharacterized.

Li and Li [28] were the first to study the benefits of network coding in undirected networks, where each communication link is bidirectional. Their result [28] shows that, for a single unicast or broadcast session, there is no improvement with respect to throughput due to network coding. In the case of a single multicast session, such an improvement is bounded by a factor of two. Meanwhile, the authors of [24, 25] studied the throughput capacity of NC in wireless ad hoc networks. However, the authors of [24, 25] employ network models that are fundamentally inconsistent with the more commonly accepted assumptions of ad-hoc networks [1]. Specifically, the model constraints of [28, 29, 24, 25] differ as follows: All the prior works assume a single source for unicast,

multicast or even broadcast. Aly *et al.* [25] differentiate the total nodes into source set, relay set and destination set. They do not allow all of the nodes to concurrently serve as sources, relays or destinations, as allowed in the work by Gupta and Kumar [1]. Furthermore, these results do not consider the impact of interference in wireless ad hoc networks.

The analysis related to network coding will be addressed in Chapter 5.

2.1.4 Opportunistic Interference Management

Multiuser diversity scheme [30] was introduced as an alternative to more traditional techniques like time division multiple access (TDMA) to increase the capacity of wireless cellular networks. The main idea behind this approach is that the base station selects a mobile station (MS) that has the best channel condition by taking advantage of the time varying nature of fading channels, thus maximizing the signal-to-noise ratio (SNR). This idea was later extended to mobile wireless ad hoc networks [3] and opportunistic beamforming [31] networks.

Knopp and Humblet [30] derived the optimum capacity for the uplink of a wireless cellular network taking advantage of multi-user diversity. They proved that if the "best" channel (i.e., the channel with the highest SNR in the network) is selected, then all of the power should be allocated to the specific user with the "best channel" instead of using a water-filling power control technique. Tse extended this result into the broadcast case of a wireless cellular network [32]. Furthermore, Viswanath et al. [31] used a similar idea for the downlink channel and employed the so called "dumb antennas"

by taking advantage of opportunistic beamforming. Grossglauser et al. [3] extended this multi-user diversity concept into mobile ad hoc networks and took advantage of the mobility of nodes to scale the network capacity.

Interference alignment [33] is another technique to manage interference. The main idea in this approach is to use part of the degrees of freedom available at a node to transmit the information signal and the remaining part to transmit the interference. The drawback of interference alignment is that the system requires full knowledge of the channel state information (CSI). This condition is very difficult to implement in practice, and feedback of CSI is MK complex numbers in a $K \times M$ interference channel. The advantage of interference alignment is that there is no minimum number of users required to implement this technique.

Sharif and Hassibi introduced a new approach [34, 35] to search for the best SINR in the network. Their approach requires M complex numbers for feedback instead of complete CSI information, and achieves the same capacity of $K \log \log M$ similar to DPC. There are major differences between the approach in this thesis and the design in [34, 35]. First, the approach in this thesis does not require beamforming, while the techniques proposed in [34, 35] take advantage of random beamforming. Second, the feedback requirement in the scheme of this thesis is proportional to the maximum of K integers while this value is proportional to M complex numbers in [34, 35]. When M grows, the feedback information in [34, 35] grows linearly, while this complexity is constant with the number of antennas at the base station in the scheme of this thesis. The approach of this thesis achieves DPC capacity of $K \log \log M$ asymptotically in the

presence of reduced feedback requirement.

The analysis related to opportunistic interference management will be addressed in Chapter 6.

2.2 Network Models and Preliminaries

There are two types of networks, namely, dense and extended networks. Both dense and extended wireless ad hoc network are considered throughout the thesis. In Chapter 3, 4, 5, dense network is considered, where n nodes distributed uniformly in a square of unit area while in Chapter 6, extended network is considered because fading needs to be taken account. The area of a dense network is constant independent of the number of nodes while the area of extended network increases with n. The network is assumed as static which means that the nodes are not mobile. This assumption is followed throughout the thesis. The capacity analysis is based on the protocol, physical models or generalized physical model which is introduced by Gupta and Kumar [1]. Throughout this thesis, the distribution of nodes in random networks is uniform, and non-uniform distribution is the topic of future work. All nodes use a common transmission range $r(n)$ for all their communication.

Definition 2.1 The Protocol Model:

Node i at location X_i can successfully transmit to node j at location X_j if, for any node $X_k, k \neq i$ that transmits at the same time as X_i, it is satisfied that $|X_i - X_j| \leq r(n)$ and $|X_k - X_j| \geq (1 + \Delta)r(n)$.

It has been proved [36] that the minimum communication range $r(n)$ in a random geometric graph to assure connectivity in the network, is given in the following lemma.

Lemma 2.2 Connectivity criterion for protocol model in dense networks*:*

For any $\epsilon > 0$ and $n \to \infty$,

$$\begin{aligned} \textit{Prob(existence of an isolated node)} &= 1 \textit{ when } r(n) = (1-\epsilon)\sqrt{\frac{\log n}{n\pi}} \\ \textit{Prob(existence of an isolated node)} &= 0 \textit{ when } r(n) = (1+\epsilon)\sqrt{\frac{\log n}{n\pi}} \end{aligned} \tag{2.1}$$

Thus, to ensure that there is no isolated node in the network, the transmission range $r(n)$ in random dense networks satisfies

$$r(n) = \Omega\left(\sqrt{\log n/n}\right). \tag{2.2}$$

Definition 2.3 *The Physical Model:*

In the physical model [1] of random wireless ad hoc networks, a successful communication occurs if signal to interference and noise ratio (SINR) of the pair of transmitter i and receiver j satisfies

$$SINR_{i\to j} = \frac{Pg_{ij}}{BN_0 + \sum_{k\neq i, k=1}^{n} Pg_{kj}} \geq \beta, \tag{2.3}$$

where P is the transmit power of a node, g_{ij} is the channel between nodes i and j, and BN_0 is the total noise power. The channel attenuation factors g_{ij} and g_{kj} are only functions of the distance under the simple path loss propagation model, i.e., $g_{ij} = |X_i - X_j|^{-\alpha}$ in which $\alpha > 2$ is the path loss parameter which is the same as [1]. β is the threshold of successful transmission which is a constant number.

Definition 2.4 Feasible throughput capacity:

In a wireless ad hoc network with n nodes where each source transmits its packets to its destinations, a throughput of $\lambda(n)$ bits per second for each node is feasible if there is a spatial and temporal scheme for scheduling transmissions, such that, by operating the network in a multi-hop fashion and buffering at intermediate nodes when awaiting transmission, every node can send $\lambda(n)$ bits per second on average to its destination nodes. That is, there is a $T < \infty$ such that in every time interval $[(i-1)T, iT]$ every node can send $T\lambda(n)$ bits to its corresponding destination nodes.

Definition 2.5 Order of throughput capacity:

$\lambda(n)$ is said to be of order $\Theta(f(n))$ bits per second if there exist deterministic positive constants c and c' such that

$$\begin{cases} \lim\limits_{n\to\infty} \textit{Prob } (\lambda(n) = cf(n) \textit{ is feasible}) = 1 \\ \liminf_{n\to\infty} \textit{Prob } (\lambda(n) = c'f(n) \textit{ is feasible}) < 1. \end{cases} \tag{2.4}$$

Definition 2.6 Euclidean Minimum Spanning Tree (EMST):

Consider a connected undirected graph $G = (V, E)$, where V and E are sets of vertices and edges in the graph G, respectively. The EMST of G is a spanning tree of G with the total minimum Euclidean distance between connected vertices of this tree.

In the rest of this thesis, $\|T\|$ denotes the total Euclidean distance of a tree T; $\#T$ is used for the total number of vertices (nodes) in a tree T; and $\overline{\|T\|}$ denotes the $\overline{\#T}$ statistical average of that value.

Steele [37] determined a tight bound for $\overline{\|\text{EMST}\|}$ for large values of n, which

is restated in the following lemma.

Lemma 2.7 *Let $f(x)$ denote the node probability distribution function in the network area. Then, for large values of n and $d > 1$, the $\overline{\|EMST\|}$ is tight bounded as*

$$\overline{\|EMST\|} = \Theta\left(c(d)n^{\frac{d-1}{d}} \int_{R^d} f(x)^{\frac{d-1}{d}} dx\right), \tag{2.5}$$

where d is the dimension of the network. Note that both $c(d)$ and the integral are constants and not functions of n. When $d = 2$, then $\overline{\|EMST\|} = \Theta(\sqrt{n})$.

The distribution of nodes in random networks is uniform. Therefore, if there are n nodes in a unit square, then the density of nodes equals n. Hence, if $|S|$ denotes the area of space region S, the expected number of the nodes, $E(N_S)$, in this area is given by $E(N_S) = n|S|$. Let N_j be a random variable defining the number of nodes in S_j . Then, for the family of variables N_j, the following standard results are known as the Chernoff bound [38]:

Lemma 2.8 *Chernoff bound*

- *For any $\delta > 0$, $P[N_j > (1+\delta)n|S_j|] < \left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^{n|S_j|}$*
- *For any $0 < \delta < 1$, $P[N_j < (1-\delta)n|S_j|] < e^{-\frac{1}{2}n|S_j|\delta^2}$*

Combining these two inequalities then, for any $0 < \delta < 1$:

$$P[|N_j - n|S_j|| > \delta n|S_j|] < e^{-\theta n|S_j|}, \tag{2.6}$$

where $\theta = (1+\delta)\ln(1+\delta) - \delta$ in the case of the first bound, and $\theta = \frac{1}{2}\delta^2$ in the case of the second bound.

Therefore, for any $\theta > 0$, there exist constants such that deviations from the mean by more than these constants occur with probability approaching zero as $n \to \infty$. It follows that, w.h.p. [1], a very sharp concentration on the number of nodes in an area can be gotten, so the achievable lower bound can be found w.h.p., provided that the upper bound (mean) is given. In the followings of the thesis, the upper bound is first derived, and then the Chernoff bound is used to prove the achievable lower bound w.h.p. with multiple times.

In extended networks, to simplify the analysis, it is assumed that the node density is equal to unity. Hence, if $|S|$ denotes the area of space region S, the expected number of the nodes, $E(N_S)$, in this area is given by $E(N_S) = |S|$. Let N_j be a random variable defining the number of nodes in S_j . Then, for the family of variables N_j, the following standard results is known as the Chernoff bound [38].

$$P[|N_j - |S_j|| > \delta|S_j|] < e^{-\theta|S_j|}, \tag{2.7}$$

where θ is some constant value depending δ and δ is a positive arbitrarily small value close to zero.

Table 2.1 summarizes all the abbreviations that are used in this thesis.

[1] An event happens with high probability if the probability of this event is greater than $1 - \frac{1}{n}$ when n goes to infinity.

Table 2.1: Abbreviation Table

EMST	Euclidean Minimum Spanning Tree
MEMT	Minimum Euclidean multicast Tree
MEMTC	Minimum Euclidean multicast Tree Cells
MEMKT	Minimum Euclidean (n, m, k)-cast Tree
MEMKTC	Minimum Euclidean (n, m, k)-cast Tree Cells
MAMKT	Minimum Area (n, m, k)-cast Tree
MAMT	Minimum Area multicast Tree
TAA	Total Active Area
$r(n)$	Transmission Range in SPR
$R(n)$	Receiver Range in MPR
$T(n)$	Transceiver Range in MPT, MPR or OIM

Chapter 3

Multi-Packet Reception Increases Throughput Capacity

In this chapter, the throughput capacity of random dense wireless ad hoc networks is computed for multi-pair unicast traffic in which nodes are endowed with multi-packet reception (MPR) capabilities. This chapter is constructed as follows. Section 3.1 describes the network model used to obtain upper and lower bounds on the throughput capacity of wireless networks with MPR. Section 3.2 presents the derivation of these bounds. In Section 3.3, a new parameter is introduced to quantify how many bits per second of information are transferred across the network per each unit of power. This metric is called as *power efficiency*, computed by normalizing the throughput capacity by the total transmitted power. After the discussion of several possible implications of this study in Section 3.4, this chapter is concluded in Section 3.5.

3.1 Network Model

According to the Gupta-Kumar protocol model in Definition 2.1 for point-to-point communications i.e. Single-Packet Reception (SPR), next the protocol model for MPR is defined.

In wireless ad hoc networks with MPR capability, the protocol model assumption allows multi-packet reception of nodes as long as they are within a radius of $R(n)$ from the receiver and all other transmitting nodes have a distance larger than $(1+\Delta)R(n)$. The difference is that it is allowed that the receiver node to receive multiple packets from different nodes within its disk of radius $R(n)$ simultaneously.

Definition 3.1 *The Protocol Model with Multi-Packet Reception:*
In wireless ad hoc networks with MPR, the protocol model assumption allows MPR capability at nodes as long as they are within a radius of $R(n)$ from the receiver and all other transmitting nodes are at a distance larger than $(1+\Delta)R(n)$. The difference is that it is allowed that the receiver node to receive multiple packets from different nodes within its disk of radius $R(n)$ simultaneously in MPR scheme.

Note that $r(n)$ in point-to-point communication is a random variable while $R(n)$ in MPR is a predefined value which depends on the complexity of receivers. The protocol model of MPR is equivalent of many-to-one communication. It is assumed that nodes cannot transmit and receive at the same time which is equivalent to half duplex communications [1]. The data rate for each transmitter-receiver pair is a constant value of W bits/second and does not depend on n. Given that W does not change the order

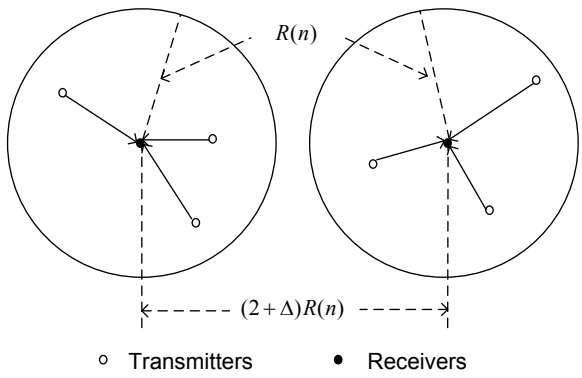


Figure 3.1: MPR protocol model

capacity of the network, its value is normalized to one. The relationship between receiver range of MPR throughout this proposal and transmission range in [1] is defined as

$$R(n) = r(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right). \tag{3.1}$$

$R(n)$ denotes the communication range for MPR model which is a function of decoding complexity of nodes and node density. $r(n)$ denotes the communication range for point-to-point communication, and it is a function of nodes density in the network. Because the distribution of nodes is uniform, these parameters are not a function of node distribution. However when the node distribution in the network is not uniform, these parameters will be a function of node distribution. The MPR protocol model is shown in Fig. 3.1.

Note that this result is independent of the physical layer model used for the network and it is a characteristic of random geometric graphs [36]. Similar to the results in [1], the same minimum communication range $R(n)$ have been adopted to assure connectivity in the network for the protocol model. Note that the successful commu-

nication in the physical model is based on signal to interference and noise ratio and not the distance between nodes, therefore the condition of Definition 3.1 for successful communication in the physical model no longer can be used.

However, in the physical model of MPR, each receiving node has a communication range such that all the nodes transmitting within this range will be decoded by the receiver. Consequently, the definition of physical model should incorporate this fact in order to better represent this new many-to-one communication scheme. The following statement describes the decoding procedure for MPR. Note that, with MPR, the received signal for multiple transmitters can be either decoded jointly using maximum likelihood (ML) decoding or be decoded sequentially utilizing successive interference cancelation (SIC). ML decoding is computationally more complex than SIC but it provides optimal performance. The SIC decoding requires all nodes inside transmission range to be grouped into several smaller sets with each set satisfying the SINR condition in Eq. (2.3). Because the channel model is based on path loss propagation model, the SIC decoding starts from a set of nodes that has the closest distance to the receiver node. Each set may consist of either a single node or multiple nodes. If a set consists more than one node, then the decoding of these nodes are performed jointly. Definition 3.2 below describes the successful transmission for MPR under physical model.

Definition 3.2 *Physical Model with Multi-Packet Reception:*

In the physical model of dense random wireless ad hoc networks [1], the active transmissions from all of the transmitters centered around the corresponding receiver j with

a distance smaller or equal to $R(n)$ occur successfully if the SINR of the transmitter $Z(R(n))$ near to the edge of the circle of the receiver satisfies

$$\mathrm{SINR}_{Z(R(n))\to j} = \frac{P g_{Z(R(n))j}}{BN_0 + \sum_{k,\forall X_k \notin A_{Z(R(n))}} P g_{kj}} \geq \beta, \tag{3.2}$$

where $g_{Z(R(n))j}$ is the channel attenuation factor between nodes $Z(R(n))$ and j and BN_0 is the total noise power. $A_{Z(R(n))} = \pi R^2(n)$ is the area of the circle centered around the receiver j, whose radius is $R(n)$.

Any transmission outside the communication range is considered interference while all the transmissions inside communication range will be decoded jointly or separately depending on the location of nodes inside the transmission circle. The decoding is carried by dividing all the transmitters inside the communication range (circle) into many subsets. The first set of nodes have the closest distance to the receiver. The total number of nodes in each set is selected such that if they are decoded jointly by the receiver, they will satisfy the SINR condition while the remaining nodes inside the transmission circle are considered as interference. Once this set of nodes are decoded jointly, they are subtracted from the received signal and then the next set of nodes are decoded. The selection of nodes for each set depends on the relative locations of nodes with respect to the receiver node. Note that this approach is suboptimal as compared to joint decoding of the entire transmitting nodes inside the communication range which is equivalent to maximum likelihood (ML) decoding. For this reason, the interference inside area $A_{Z(R(n))}$ is denoted as constructive interference, because it consists of transmissions that will be eventually decoded, while all the transmissions from nodes outside

of area A are called destructive interference and are not decoded. Note that in the physical model for the MPR scheme, the communication range $R(n)$ defines the area where the receiver is capable of decoding, which contrasts with point-to-point communication [1], for which the transmission range $r(n)$ defines the possible area where the receiver can decode, given that only one transmission is successful at a receiver.

3.2 Throughput Capacity with Multi-Packet Reception

The capacity of wireless ad hoc networks are computed for both protocol and physical models.

A cut Γ is a partition of the vertices (i.e. nodes in the wireless networks) of a graph into two sets. The cut capacity is defined to be the sum of the capacity of all the active edges crossing the cut that transmit simultaneously and successfully. In this section, random geometric graph (RGG) is used. An edge is active (communication link) in RGG if the protocol or physical model is satisfied for successful communications between the two nodes which is directly a function of distance between nodes. However, an edge in a general graph is not necessarily an active edge for an RGG. Min-cut is a cut whose capacity is the minimum value among the capacity of all cuts. For the wireless networks, the concept of *sparsity cut* is used, which is defined by Liu et al. [21], instead of min-cut, to take into account the differences between wired and wireless links. l_Γ is defined as the length of the cut. For the square region illustrated in Fig. 3.2, the middle line induces a sparsity cut Γ. Because nodes are uniformly deployed in a random

network, such a sparsity cut captures the traffic bottleneck of these random networks on average [21]. The sparsity-cut capacity is upper bounded by the maximum number of simultaneous transmissions across the cut.

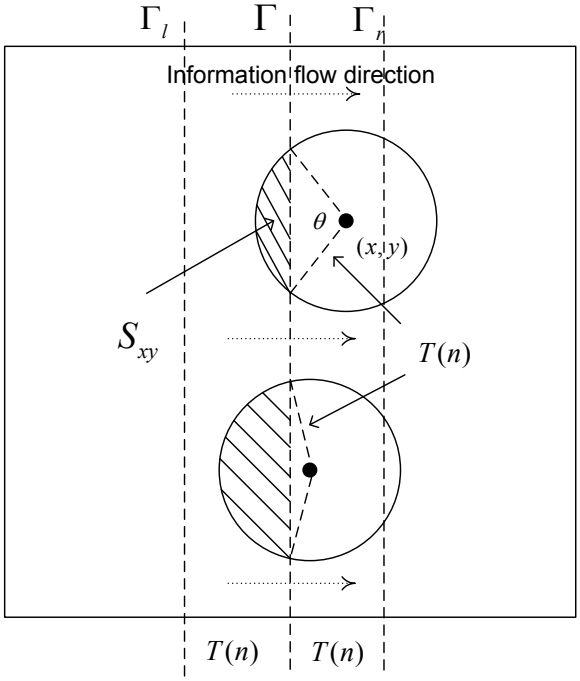


Figure 3.2: For a receiver at location (x, y), all the nodes in the shaded region S_{xy} can send a message successfully and simultaneously.

Definition 3.3 Sparsity Cut: *A sparsity cut for a random network is defined as a cut induced by the line segment with the minimum length that separates the region into two equal area subregions. Note that the definition of sparsity cut does not depend on a specific realization of a random network, it rather focuses on the asymptotic order of some spatial-statistical property of the collection of random networks as a whole. The cut capacity is defined as the transmission bandwidth W multiplied by the maximum possible number of simultaneous transmissions across the cut. This cut capacity constrains the*

information rate that the nodes from one side of the cut as a whole can deliver to the nodes at the other side. The cut length l_Γ is defined as the length of the cut line segment in two dimensional space. Similarly, in 3-D volume, the sparsity cut is a plane, and the cut plane has an area. In another word, sparsity cut can be seen for random geometric graph (RGG) similar to min-cut concept in graph theory.

Let $R(n)$ be the radius of the receiver area A, i.e., $A = \pi R^2(n)$. Given that omnidirectional antennas are assumed for all nodes, the information from any node inside this area is decode-able while the information from all transmitting nodes outside of this region are considered as interference.

It is assumed that each disk with radius $R(n)$ centered at any receiver is disjoint from the other disks centered at the other receivers. It will be shown later that this assumption is necessary in order to guarantee that the physical model condition, SINR $\geq \beta$, is satisfied.

3.2.1 Upper Bound for Protocol Model

The sparsity cut is first derived for a random wireless ad hoc network under the protocol model.

Lemma 3.4 *The asymptotic throughput capacity of a sparsity cut Γ for a unit square region has an upper bound of $c_1 l_\Gamma n R(n)$, where, $c_1 = \pi/2(2+\Delta)$.*

Proof: The cut capacity is the maximum number of simultaneous transmissions across the cut. S_{xy} is defined as the area in the left side of the cut Γ that contains

nodes sending packets to the receiver node located at (x, y) as shown in Fig. 3.2. These nodes lie in the left side of the cut Γ within an area called S_{xy}. The assumption is that all these nodes are sending packets to the right side of the cut Γ.

From the definition of the MPR, for a node at location (x, y), any node in the disk of radius $R(n)$ can transmit information to this receiver simultaneously and the node can successfully decode those packets. In order to obtain an upper bound, edges that cross the cut is only needed to be considered. Let us first consider all possible nodes in the S_{xy} region that can transmit to the receiver node. Because nodes are uniformly distributed, the average number of transmitters located in S_{xy} is $n \times S_{xy}$. The number of nodes that are able to transmit at the same time from left to right is upper bounded as a function of S_{xy}. The area of S_{xy} is $S_{xy} = \frac{1}{2}R^2(n)(\theta - \sin\theta)$ whose area is maximized when $\theta = \pi$, i.e. $\max_{0 \le \theta \le \pi}[S_{xy}] = \frac{1}{2}\pi R^2(n)$. The total number of nodes that can send packets across the cut is upper bounded as

$$\frac{l_\Gamma}{(2+\Delta)R(n)}\frac{1}{2}\pi R^2(n)n = c_1 l_\Gamma n R(n), \tag{3.3}$$

where $c_1 = \pi/2(2+\Delta)$.

Corollary 3.5 *For any arbitrary shape unit area random network, if the minimum cut length l_Γ is not a function of n, then the sparsity cut capacity has an upper bound of $\Theta(nR(n))$.*

Proof: Regardless of the shape of the unit area region, it is clear that the length of l_Γ is $\Theta(1)$. because the network area is unity. If l_Γ is not a function of n, then the capacity is always upper bounded as $\Theta(nR(n))$.

Theorem 3.6 *The per-node throughput of* MPR *scheme in a dense random network is upper bounded by* $\Theta(R(n))$.

Proof: For a sparsity cut Γ in the middle of the unit plain, on average, there are $\Theta(n)$ pairs of source-destination nodes that need to cross Γ in one direction, i.e., $n_{\Gamma_{l,r}} = n_{\Gamma_{r,l}} = \Theta(n)$. Combining this result with Corollary 3.5, this theorem can be easily proved. Note that $n_{\Gamma_{l,r}}$ and $n_{\Gamma_{r,l}}$ are the transmissions from left to right and from right to left respectively.

3.2.2 Lower Bound for Protocol Model

It will be proved that, when n nodes are distributed uniformly over a unit square area, there have simultaneously at least $\frac{l_\Gamma}{(2+\Delta)R(n)}$ circular regions in Fig. 3.2, each one contains $\Theta(nR^2(n))$ nodes. The objective is to find the achievable lower bound using Chernoff bound such that the distribution of the number of edges across the cut is sharply concentrated around its mean, and hence in a randomly chosen network, the actual number of edges crossing the sparsity cut is indeed $\Theta(nR(n))$.

Theorem 3.7 *Each area* A_j *with circular shape contains* $\Theta(nR^2(n))$ *nodes uniformly for all values of* $j, 1 \leq j \leq \left\lceil \frac{l_\Gamma}{(2+\Delta)R(n)} \right\rceil$, *w.h.p.. It can be expressed as*

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] = 1, \tag{3.4}$$

where δ *is a positive small value arbitrarily close to zero.*

Proof: Since l_Γ is not a function of n, using Chernoff bound (Lemma 2.8) and Eq. (2.6), for any given $0 < \delta < 1$, $\theta > 0$ can be found such that

$$P\left[|N_j - E(N_j)| > \delta E(N_j)\right] < e^{-\theta E(N_j)} = e^{-\theta n |A_j|}. \tag{3.5}$$

Thus, it can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n \to \infty$. This is a key step in showing that when all the events $\bigcap_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} |N_j - E(N_j)| < \delta E(N_j)$ occur simultaneously, then all N_j's converge uniformly to their expected values. Utilizing the union bound, it arrived at

$$\begin{aligned}
& P\left[\bigcap_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] \\
= \; & 1 - P\left[\bigcup_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} |N_j - E(N_j)| \geq \delta E(N_j)\right] \\
\geq \; & 1 - \sum_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} P\left[|N_j - E(N_j)| \geq \delta E(N_j)\right] \\
> \; & 1 - \left\lceil \frac{l_\Gamma}{(2+\Delta)R(n)} \right\rceil e^{-\theta E(N_j)} \\
= \; & 1 - \left\lceil \frac{l_\Gamma}{(2+\Delta)R(n)} \right\rceil e^{-\frac{\theta \pi n R^2(n)}{2}}.
\end{aligned} \tag{3.6}$$

The last term is derived from the fact that $E(N_j) = \frac{\pi}{2} n R^2(n)$. In order to guarantee connectivity, $R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$ is needed. Thus the following equations can be gotten as

$$\frac{e^{-\frac{\theta \pi n R^2(n)}{2}}}{R(n)} = O\left(\frac{1}{n^{\frac{\theta\pi}{2} - \frac{1}{2}} \log n}\right) = O\left(\frac{1}{n}\right), \tag{3.7}$$

provided that $\theta > 3/\pi$. Then

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\lceil l_\Gamma/(2+\Delta)R(n)\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] > 1 - \frac{1}{n}, \tag{3.8}$$

which proves this theorem.

The next theorem demonstrates that this capacity is an achievable lower bound.

Corollary 3.8 *The per-node throughput of* MPR *scheme for a dense random network has a lower bound of* $\Theta(R(n))$.

Proof: It is proved in Theorem 3.7, there are $\left\lceil \frac{l_\Gamma}{(2+\Delta)R(n)} \right\rceil$ different circles of radius $R(n)$ each of them having $\Theta(nR^2(n))$ nodes. Therefore, per-node is the multiplications of these two values which is divided by the total number of nodes.

$$\left\lceil \frac{l_\Gamma}{(2+\Delta)R(n)} \right\rceil \times \frac{nR^2(n)}{n} = R(n) \tag{3.9}$$

3.2.3 Upper Bound for Physical Model

The division range $D(n)$ is defined as the minimum distance required between receiving nodes such that each node can decode all transmitters within the communication range $R(n)$ successfully. Equivalently, $D(n)$ is the minimum distance that separates simultaneous active receivers far from each other such that receiver nodes can have successful communications. Based on the above, $D(n)$ is a function that depends on n which is willing to be minimized between two concurrent receivers as shown in Fig. 3.3 such that the physical model constraint is satisfied. It will be proven that $D(n)$ is a function of $R(n)$.

Lemma 3.9 *The asymptotic throughput capacity of a sparsity cut* Γ *for a unit square region has an upper bound of* $\pi l_\Gamma n \frac{R^2(n)}{D(n)}$ *, where,* $R(n)$ *and* $D(n)$ *are communication range and division range of MPR respectively as illustrated in Fig. 3.3.*

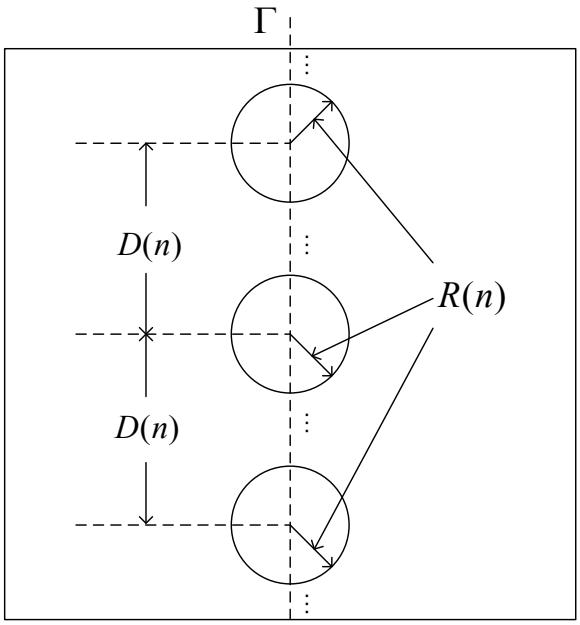


Figure 3.3: Upper bound design of the network

Proof: The cut capacity is upper bounded by the maximum number of simultaneous transmissions across the cut. Based on the results from section 3.2.1 and the total number of nodes in each area S_{xy}, the total information capacity (i.e. the total capacity) C_j can be computed for one receiver j at the right side of the cut as

$$C_j = \frac{1}{2}\pi n R^2(n). \tag{3.10}$$

The constraint to guarantee that Eq. (3.10) is true for all of the nodes inside the circle of radius $R(n)$, is to satisfy $\mathrm{SINR}_{i \in \mathrm{S}_{xy}} \geq \beta$. For this reason, the circles in which nodes are transmitting concurrently must be away from each other far enough to satisfy SINR criterion.

Therefore, the total throughput capacity $C(n)$ across the sparsity cut is

$$C(n) \leq \left(\left\lfloor \frac{l_\Gamma}{D(n)} \right\rfloor + 1 \right) C_j < \frac{\pi n R^2(n)(l_\Gamma + D(n))}{2D(n)}. \tag{3.11}$$

Since $l_\Gamma \geq D(n)$, then $l_\Gamma + D(n) \leq 2l_\Gamma$ and the proof follows.

Lemma 3.10 *The per-node throughput of MPR scheme in a dense random network is upper bounded by* $O\left(\frac{R^2(n)}{D(n)}\right)$.

Proof: From lemma 3.9, there are $\lceil l_\Gamma / D(n) \rceil$ different circles of radius $R(n)$ each of them having $\Theta(nR^2(n))$ nodes on average. Therefore, the average per node throughput capacity can be derived as

$$\lambda(n) = \frac{C(n)}{n} = O\left(\frac{R^2(n)}{D(n)}\right). \tag{3.12}$$

To derive an upper bound for the throughput capacity, a minimum $D(n)$ need to be obtained, such that it guarantees $\text{SINR}_{Z(R(n))} \geq \beta$. The decoding is conducted from the nearest nodes to the farthest nodes by decoding the strongest signals first and then subtract them from the received signal. So if the SINR of the outmost node can be decoded, then all of the nodes inside that circle can be decoded successfully because the nodes closer to the receiver provide higher SINR if they are decoded either jointly or separately depending on the location of nodes in the network. Based on this assumption, the SINR of the farthest nodes $Z(R(n))$ (i.e., at the conjunction edge of the communication circle) is only needed to be computed to make sure that $\text{SINR}_{Z(R(n))} \geq \beta$. Hence, to obtain the upper bound of the capacity is equivalent to maximize the

following function.

$$\max_{\mathrm{SINR}_{Z(R(n))}\geq\beta} \lambda(n) = \max_{\mathrm{SINR}_{Z(R(n))}\geq\beta} O\left(\frac{R^2(n)}{D(n)}\right) \tag{3.13}$$

Note that the throughput capacity is maximized by minimizing $D(n)$ since $R(n)$ is a network parameter that is determined in advance. If the value of $D(n)$ is too small, then Eq. (3.2) will not be satisfied. The aim is to find the optimum value for $D(n)$ such that both conditions are satisfied. The following theorem and its applications establish the optimum value that will satisfy Eq. (3.2).

Theorem 3.11 *The per-node throughput of MPR scheme in a dense random network is upper bounded by* $O\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$.

Proof: In order to compute the upper bound, the SINR for the node that is in a circle close to the edge of the network is derived.

For this receiver node, the Euclidean distances of interfering nodes are at $(iD(n)+R(n))$ assuming that all interfering nodes are at the farthest distance from the receiver node. Then the SINR of the transmitter node that is located at the circumference of the communication circle is given by

$$\begin{aligned}\mathrm{SINR}_{Z(R(n))} &\leq \frac{P/R^{\alpha}(n)}{\frac{\pi}{2}nR^2(n)\sum_{i=1}^{\lceil l_\Gamma/D(n)\rceil}\frac{P}{(iD(n)+R(n))^{\alpha}}} \\ &\leq \left(\frac{D(n)}{R(n)}\right)^{\alpha}\frac{1}{\frac{\pi}{2}nR^2(n)\sum_{i=1}^{\lceil l_\Gamma/D(n)\rceil}\frac{1}{(i+\frac{1}{2})^{\alpha}}}.\end{aligned} \tag{3.14}$$

The second inequality above stems from the fact that $\frac{R(n)}{D(n)} \leq \frac{1}{2}$. Note that $\lceil l_\Gamma/D(n)\rceil$ approaches infinity when $n \to \infty$; therefore, the summation $\sum_{i=1}^{\lceil l_\Gamma/D(n)\rceil}\frac{1}{(i+\frac{1}{2})^{\alpha}}$ converges

to a bounded value when $\alpha > 2$. This means that there are constant values c_3 and c_4 such that

$$c_3 \leq \sum_{i=1}^{\lceil l_\Gamma / D(n) \rceil} \frac{1}{(i+\frac{1}{2})^\alpha} \leq \sum_{i=1}^{\lceil l_\Gamma / D(n) \rceil} \frac{1}{(i)^\alpha} \leq c_4. \tag{3.15}$$

Combining (3.14) and (3.15), the SINR constraint can be revised as

$$\beta \leq \text{ SINR}_{Z(R(n))} \leq \left(\frac{D(n)}{R(n)}\right)^\alpha \frac{2}{\pi c_3 n R^2(n)}. \tag{3.16}$$

Then the relationship between $R(n)$ and $D(n)$ can be expressed as

$$D(n) \geq \left(\frac{c_3 \beta \pi}{2}\right)^{\frac{1}{\alpha}} n^{\frac{1}{\alpha}} (R(n))^{(1+2/\alpha)}. \tag{3.17}$$

From Eqs. (3.12) and (3.17), the upper bound of the throughput capacity is computed as

$$\lambda(n) = O\left(\frac{R^2(n)}{D(n)}\right) = O\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right). \tag{3.18}$$

The above upper bound is derived based on the assumption that the SINR for the nodes that are located on the circumference of communication circle A of radius $R(n)$ satisfy the physical model, i.e., $\text{SINR}_{Z(R(n))} \geq \beta$. It will be shown that this upper bound in Theorem 3.11 is also an achievable capacity.

3.2.4 Lower Bound for Physical Model

Given the upper bound derived in the previous section, the Chernoff Bound is used to prove the achievable lower bound. It is proven that, when n nodes are distributed uniformly over a square area, there are simultaneously $\left\lceil \frac{l_\Gamma}{D(n)} \right\rceil$ circular regions (see fig. 3.2), each one containing $\Theta(nR^2(n))$ nodes. The objective is to find the achievable lower

bound using the Chernoff bound, such that the distribution of the number of edges across the cut is sharply concentrated around its mean, and hence in a randomly chosen network, the actual number of edges crossing the sparsity cut is indeed $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{(1/\alpha)}}\right)$.

Theorem 3.12 *Each area A_j with circular shape of radius $R(n)$ contains $\Theta(nR^2(n))$ nodes uniformly and w.h.p. for all values of $j, 1 \leq j \leq \left\lceil \frac{l_\Gamma}{D(n)} \right\rceil$ under the condition that $R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$. Equivalently, this can be expressed as*

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\lceil l_\Gamma/D(n)\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] = 1, \tag{3.19}$$

where δ is a positive arbitrarily small value close to zero.

Proof: From Eq. (2.6), for any given $0 < \delta < 1$, there exists a $\theta > 0$ such that

$$P\left[|N_j - E(N_j)| > \delta E(N_j)\right] < e^{-\theta E(N_j)} = e^{-\theta n |A_j|}. \tag{3.20}$$

Thus, it can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n \to \infty$. This is a key step in showing that when all the events $\bigcap_{j=1}^{\lceil l_\Gamma/D(n)\rceil} |N_j - E(N_j)| < \delta E(N_j)$ occur simultaneously, then all N_js converge uniformly to their expected values. Utilizing the same technique as in 3.2.2, it can be obtained that

$$\begin{aligned} & P\left[\bigcap_{j=1}^{\lceil l_\Gamma/D(n)\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] \\ \geq\ & 1 - \sum_{j=1}^{\lceil l_\Gamma/D(n)\rceil} P\left[|N_j - E(N_j)| \geq \delta E(N_j)\right] \\ >\ & 1 - \left\lceil \frac{l_\Gamma}{D(n)} \right\rceil e^{-\theta E(N_j)}. \end{aligned} \tag{3.21}$$

Because $E(N_j) = \frac{\pi}{2} n R^2(n)$, the final result is

$$\begin{aligned}
& \lim_{n\to\infty} P\left[\bigcap_{j=1}^{\lceil l_\Gamma / D(n)\rceil} |N_j - E(N_j)| > \delta E(N_j)\right] \\
\geq\ & 1 - \left\lceil \frac{l_\Gamma}{D(n)} \right\rceil e^{-\frac{\theta\pi n R^2(n)}{2}} \\
\geq\ & 1 - \left\lceil \frac{l_\Gamma}{2R(n)} \right\rceil e^{-\frac{\theta\pi n R^2(n)}{2}}. \qquad (3.22)
\end{aligned}$$

If $R(n) \geq \sqrt{\frac{c_5 \log n}{n}}$ and as $n \to \infty$, then $\frac{e^{-\frac{\theta\pi n R^2(n)}{2}}}{R(n)} \to 0$, when $\theta > 1/\pi c_5$. Here, the key constraint of $R(n)$ is given as

$$R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right). \qquad (3.23)$$

Eq. (3.23) is equivalent to the connectivity condition in the protocol model [1]. It is interesting to note that connectivity criterion is not really used in the physical model, however, it turns out that the minimum distance for the communication range in MPR model is equivalent to the connectivity constraint in protocol model for random networks.

The above theorem demonstrates that there are indeed $\Theta(nR^2(n))$ nodes in each communication region with the constraint in Eq. (3.23). The achievable capacity is only feasible when the communication range of each node in MPR scheme is at least equal to the connectivity criterion of transmission range in point-to-point communication [1]. Combining the result of Eq. (3.18) in Theorem 3.11 and Eq. (3.23) in Theorem 3.12, the following theorem can be stated for the lower bound of throughput capacity. It implies that the lower bound order capacity achieves the upper bound in physical model.

Theorem 3.13 *The per-node throughput capacity of MPR scheme in a dense wireless ad hoc network is bounded by* $\Omega\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$*, provided that* $R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$*. The achievable lower bound is* $\Omega\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$ *for* $\alpha > 2$*.*

Proof: It is first proved that Eq. (3.18) is an achievable bound and then by applying the minimum communication range constraint in Eq. (3.23), the lower bound for this theorem is derived.

To derive the achievable lower bound, a scheme is designed for separating decode-able transmitter nodes inside the communication circle and interference, such that $\text{SINR}_{Z(R(n))} \geq \beta_1$. Similar to the derivations in Eq. (3.14) and using Fig. 3.3, it is clear that the SINR is minimized when the largest value for interference is considered. This value is achieved when the interference is computed for a receiver node in the middle of the network and use the closest possible distance to the receiver node[1]. This lower bound can be written as

$$\text{SINR}_{Z(R(n))} \geq \frac{\frac{P}{R^{\alpha}(n)}}{BN_0 + \frac{\pi}{2} n R^2(n) \sum_{i=1}^{\lceil l_\Gamma / 2D(n) \rceil} \frac{2P}{(iD(n) - R(n))^{\alpha}}}. \tag{3.24}$$

Assume that $D(n)$ satisfies the condition in Eq. (3.17). If the constraint for $R(n)$ is used in Eq. (3.23), it arrives at

$$\frac{D(n)}{R(n)} \geq \left(\frac{c_3 \beta \pi}{2}\right)^{\frac{1}{\alpha}} n^{\frac{1}{\alpha}} (R(n))^{2/\alpha} \geq \Theta\left((\log n)^{\frac{1}{\alpha}}\right), \tag{3.25}$$

which illustrates that $R(n)$ can be ignored compared with $D(n)$ for large values of n, i.e., $n \to \infty$. Now the asymptotic behavior of Eq. (3.24) is evaluated when $n \to \infty$.

[1]Note that the difference between maximum and minimum value of interference is a constant value.

Combining Eqs. (3.24) and (3.25), $\mathrm{SINR}_{Z(R(n))}$ can be lower bounded by

$$\begin{aligned}
\lim_{n\to\infty} \mathrm{SINR}_{Z(R(n))} &\geq \left(\frac{D(n)}{R(n)}\right)^{\alpha} \frac{1}{\pi n R^2(n) \sum_{i=1}^{\lceil l_\Gamma / D(n) \rceil} \frac{1}{i^\alpha}} \\
&\geq \left(\frac{D(n)}{R(n)}\right)^{\alpha} \frac{1}{\pi c_4 n R^2(n)} \\
&\geq \frac{c_3}{2c_4}\beta = \beta_1.
\end{aligned}$$

This inequality is derived using Eqs. (3.15) and (3.17), together with the fact that the second term in the denominator of SINR goes to infinity when $n \to \infty$ and, therefore, the first term related to the noise can be dropped. Using the same arguments introduced for the computation of the upper bound, it can be shown that a non-zero value for $\mathrm{SINR}_{Z(R(n))}$ can be achieved which implies that the throughput capacity can be achieved asymptotically.

The above theorem demonstrates that a gain of at least $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$ can be achieved compared with the results by Gupta and Kumar [1] and Franceschetti et al. [2]. Combining Theorems 3.11 and 3.13, it arrives at the next major contribution of this chapter.

Theorem 3.14 *The per-node throughput capacity of MPR scheme in a dense wireless ad hoc network is tight bounded as* $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$. *The communication range is lower bounded as* $R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$, *which implies a bound of* $\Theta\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$.

Note that this result shows that the gap can be closed in the physical model similar to the results derived by Franceschetti et al. [2] but achieving higher throughput capacity with MPR.

3.3 Power Efficiency

Many wireless sensor and ad hoc networks are energy and power limited systems and it is natural to ask what the price of achieving higher capacities in wireless ad hoc networks is.

The capacity was originally defined in [1] based on bits per second for random networks. The definition of bits-per Joule was defined in [39]. To incorporate the effect of energy consumption for communication in wireless networks, bits per second per Watts for random networks as *power efficiency* is defined in the followings. This new metric is a measure for evaluating the power efficiency of the capacity in wireless sensor and ad hoc networks. The formal definition is as follows.

Definition 3.15 *Power Efficiency: In wireless ad hoc networks with limited energy, the power efficiency is defined as*

$$\eta(n) = \frac{\lambda(n)}{P(n)}, \tag{3.26}$$

where $\lambda(n)$ is the capacity of the network and $P(n)$ is the total minimum average power required to achieve $\lambda(n)$ for each source-destination pair in the network. The metric is "bits per Joule" or "bits per second per Watts" in random wireless ad hoc networks."

With this definition of efficiency, the relationship between the capacity and the power efficiency is computed for the various approaches defined to increase the throughput capacity of wireless ad hoc networks, including MPR.

3.3.1 Power efficiency with Single-Packet Reception

In the paper [1] by Gupta and Kumar, it is easy to show that the minimum transmit power P for each hop to guarantee SINR $\geq \beta$ is

$$\min(P) = \Theta(s_n^{\alpha}) = \Theta\left(\left(\frac{\log n}{n}\right)^{\frac{\alpha}{2}}\right), \tag{3.27}$$

Where, $s_n = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$. The total average power to transmit this information is

$$P(n) = \min(P) \times \text{total number of hops} = \Theta\left(\left(\frac{\log n}{n}\right)^{\frac{\alpha}{2}-\frac{1}{2}}\right). \tag{3.28}$$

The power efficiency for this scheme can be computed by dividing the throughput capacity by the total average power required to achieve this capacity. This renders

$$\eta(n) = \Theta\left(\frac{n^{\frac{\alpha}{2}-1}}{(\log n)^{\frac{\alpha}{2}}}\right). \tag{3.29}$$

3.3.2 Power efficiency with Percolation Theory

The communication in the approach by Franceschetti et al. [2] is based on dividing the transfer of packets into four phases. In the first phase, the source transmits a packet to a relay inside a path that is called "highway path." The distance between the source and highway path is considered a long range communication and is proportional to $\Theta(\frac{\log n}{\sqrt{n}})$. Inside the highway path in phases two and three, multiple hop communication occurs horizontally and vertically respectively. The communication range is of short range and proportional to $\Theta(\frac{1}{\sqrt{n}})$. Communication in phase four is similar to phase one and it is between relay and destination.

Assume that $P_h(n)$ is the transmit power at the highway path in phases two and three. Following the definition in [2], the interference from the other cells can be expressed as

$$I(d,n) \leq P_h(n)\left(s_n(d+1)\right)^{-\alpha} c_6. \tag{3.30}$$

where c_6 is a constant value. The signal power at the receiver is lower bounded as

$$S(d,n) \geq P_h(n)\left(s_n\sqrt{2}(d+1)\right)^{-\alpha}. \tag{3.31}$$

Using the above results, the SINR is derived as

$$\text{SINR} = \frac{S(d,n)}{BN_0 + I(d,n)} \geq \frac{P_h(n)\left(\sqrt{2}\right)^{-\alpha}}{BN_0(s_n(d+1))^{\alpha} + P_h(n)c_6}. \tag{3.32}$$

In the limit, the minimum required power to guarantee that the SINR satisfies the physical model when $n \to \infty$ is $\min(P_h(n)) = \Theta((s_n(d+1))^{\alpha}) = \Theta\left((n)^{-\alpha/2}\right)$.

For the long-range communications in the first and fourth phase, there is no interference. Therefore, the SINR can be expressed as

$$\text{SINR} = \frac{P_u(n)\left(\frac{\log n}{\sqrt{n}}\right)^{-\alpha}}{BN_0}. \tag{3.33}$$

The minimum required power for this case to guarantee the physical model condition is given by

$$\min(P_u(n)) = \Theta\left(\left(\frac{(\log n)^2}{n}\right)^{\frac{\alpha}{2}}\right). \tag{3.34}$$

Using the definition of power efficiency, its value for this case can be computed

as

$$\begin{aligned}\eta(n) &= \lambda(n)/P(n) \\ &= \frac{\lambda(n)}{2\min(P_u(n)) + \sqrt{n}\min(P_h(n))} \\ &= \Theta\left(n^{\frac{\alpha}{2}-1}\right). \end{aligned} \tag{3.35}$$

3.3.3 Power Efficiency with Multi-Packet Reception

It is demonstrated that MPR closes the gap between the upper and lower bounds of the capacity of wireless ad hoc networks by achieving higher throughput capacity. However, it is important to find out the power efficiency of this approach. From the derivation of throughput capacity for MPR in Eq. (3.24), the SINR is given by

$$\text{SINR} \geq \frac{P(R(n))^{-\alpha}}{BN_0 + \frac{\pi}{2}nR^2(n)\sum_{i=1}^{\lceil l_\Gamma/2D(n)\rceil} 2P(iD(n) - R(n))^{-\alpha}} \tag{3.36}$$

The physical model constraint is guaranteed for SINR asymptotically when the minimum transmit power $P_{\text{MPR}}(n)$ is

$$\min(P_{\text{MPR}}(n)) = \Theta\left(R^\alpha(n)\right) = \left(\frac{\log n}{n}\right)^{\frac{\alpha}{2}}. \tag{3.37}$$

Eq. (3.37) is derived using Eqs. (3.23) and (3.25) when $n \to \infty$.

The relationship between $\lambda(n)$ and $P_{\text{MPR}}(n)$ can be computed from Theorem 3.14 as

$$\lambda(n) = n^{-1/\alpha}\left(P_{\text{MPR}}(n)\right)^{\frac{\alpha-2}{\alpha^2}}. \tag{3.38}$$

Because the communication range in MPR is equal to $R(n)$, the total minimum transmit power from source to destination is equal to $\frac{P_{\text{MPR}}(n)}{R(n)}$.

The power efficiency of MPR scheme is given by

$$\begin{aligned}\eta(n) &= \frac{\lambda(n)R(n)}{P_{\mathrm{MPR}}(n)} \\ &= \lambda(n)(R(n))^{1-\alpha} \\ &= n^{-\frac{\alpha-1}{\alpha-2}}\lambda(n)^{\frac{-(\alpha-1)^2-1}{\alpha-2}}. \end{aligned} \tag{3.39}$$

3.4 Discussion

The reason for the significant increase in capacity with MPR is that, unlike point-to-point communication in which nodes compete to access the channel, MPR embraces (strong) interference by utilizing higher decoding complexity for all nodes. As it has been pointed out, recent work on network coding [22, 23] implicitly assumes some form of MPR. These results clearly demonstrate that embracing interference is crucial to improve the performance of wireless ad hoc networks, and that MPR constitutes an important component of that.

Another interesting observation is the fact that increasing the communication range $R(n)$ increases the throughput capacity. This is in sharp contrast with point-to-point communication in which increasing the communication range actually decreases the throughput capacity and it is again due to the fact that MPR embraces the interference.

Fig. 3.4 shows the tradeoff between the total minimum transmit power and the throughput capacity. From this figure, it is clear that the total transmit power for the network must be increased in order to increase the per-node throughput capacity

in random wireless ad hoc networks.

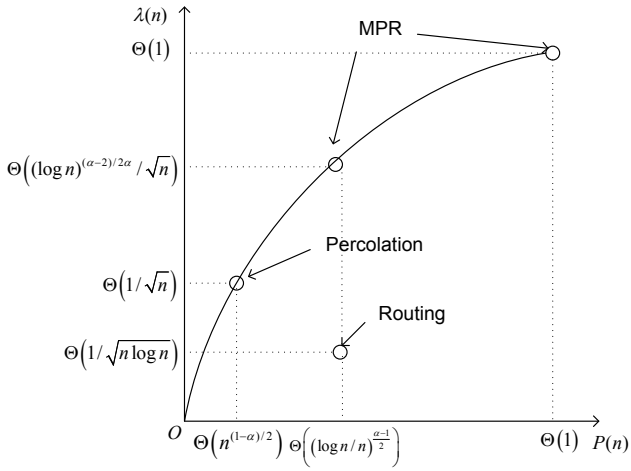


Figure 3.4: Power and capacity relationship

Fig. 3.5 shows that, by increasing the throughput capacity in wireless ad hoc networks, the power efficiency of all the schemes analyzed decreases. Many wireless ad hoc networks are limited in total available energy or power for each node. Therefore, increasing the throughput capacity may not be feasible if the required power to do so is not available. This result also shows that the throughput capacity should not be the only metric used in evaluating and comparing the merits of different schemes. The power efficiency of these schemes is also very important. Based on different values for $R(n)$, different throughput capacities can be attained. In general, MPR allows to have tradeoff between receiver complexity and throughput capacity.

There are certain issues that do not been discussed in this thesis. The analysis does not include the energy required for increased decoding complexity, which is

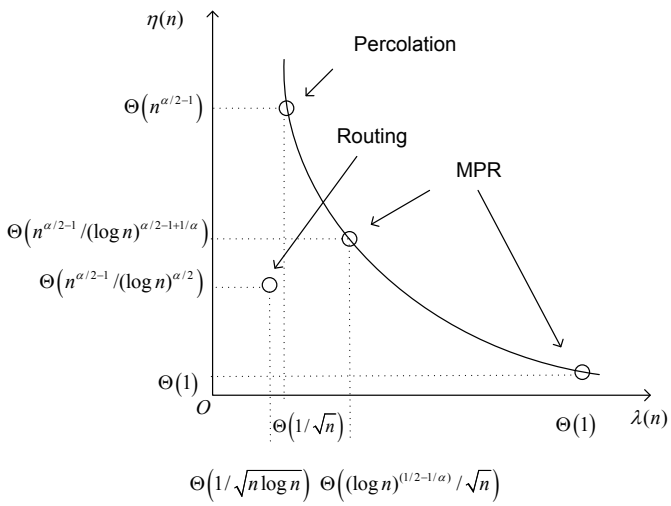


Figure 3.5: Capacity and power efficiency tradeoff

necessary for MPR. The analysis also does not include the additional required overhead related to cooperation among nodes. Such topics are the subject of future studies.

3.5 Conclusion

This chapter shows that the use of MPR can close the gap for the throughput capacity in random wireless ad hoc networks under the physical model, while achieving much higher capacity gain than that of [2]. The tight bounds are $\Theta(R(n))$ and $\Theta\left(\frac{(R(n))^{1-2/\alpha}}{(n^{(1/\alpha)})}\right)$ where $R(n)$ is the communication range in MPR model for protocol and physical models respectively.

A new definition related to power efficiency is introduced. The results show that increasing the throughput capacity by means of MPR or any of the other techniques proposed to date [1, 2] results in a reduction of power efficiency in the network.

Accordingly, there is a tradeoff to be made between increasing capacity and decreasing power efficiency. Determining what is the optimum tradeoff between capacity and power efficiency is an open problem.

Chapter 4

A Unifying Perspective of Throughput Capacity

In this chapter, the unified modeling framework is presented for the computation of the capacity-delay tradeoff of random wireless ad hoc networks. This framework considers information dissemination by means of unicast routing, multicast routing, broadcasting, or different forms of anycasting. (n, m, k)-casting is defined as a generalization of all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks. In the context of (n, m, k)-casting, n, m, and k denote the number of nodes in the network, the number of destinations for each communication group, and the actual number of communication-group members that receive information optimally[1], respectively. Section 4.1 describes the network model and necessary concepts for the development of the framework. The capacity-delay tradeoff is

[1]Optimality is defined as the k closest (in terms of Euclidean distance of the tree) destinations to the source in an (n, m, k)-cast group.

presented for (n, m, k)-casting in wireless ad hoc networks in which receivers perform Single-Packet Reception (SPR) in Section 4.2 and Multi-Packet Reception (MPR) in Section 4.3 respectively. The results are consistent with prior results in wireless networks and extend them to the general (n, m, k)-cast case. This chapter is concluded in Section 4.4.

4.1 Network Model

This chapter studied the case in which all n nodes in the network act as sources that communicates with a group of m receivers (with $m \leq n$) and that k of those receivers obtain the information reliably. This characterization of information dissemination from sources to receivers is called as (n, m, k)-casting. This characterization is useful because it can model all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks.

Definition 4.1 Feasible throughput capacity of (n, m, k)-cast:

A throughput of $\lambda(n)$ bits per second for each node is feasible if a scheduling transmission scheme allows each node in the network to transmit $\lambda(n)$ bits per second on average to its k out of m destinations.

Definition 4.2 (n, m, k)-cast tree:

An (n, m, k)-cast tree is a set of nodes that connect a source node of an (n, m, k)-cast with all its intended k receivers out of m, in order for the source to send information to k of those receivers. It can be shown as unicast ($k = m = 1$), broadcast ($k = m = n$),

multicast ($k = m < n$), anycast ($n = m, k = 1$) and all forms of manycast ($k \leq m \leq n$).

The total Euclidean length of an (n, m, k)-cast tree is a function of transmission range $r(n)$ or receiver range of MPR $R(n)$. Therefore, the optimum (n, m, k)-cast tree that has the minimum Euclidean distance is a function of $r(n)$ or $R(n)$. For this reason, changing the transmission or receiver range will change the optimum (n, m, k) tree. Specifically, a multicast tree is also denoted by an (n, m, m)-cast tree (i.e., when $m = k$) which can include the unicast, multicast and broadcast cases.

The construction of (n, m, k)-cast tree starts with connecting the source to m destinations using minimum number of relays or hops. After constructing this tree, k out of m nodes is picked in this tree that have minimum total Euclidean distance to the source. This selection of k nodes is referred as "optimum" because it results in maximum throughput capacity for the network. Note that there are $\binom{m}{k}$ choices for selecting k nodes and in this chapter, the above criterion have been selected for this selection.

When communicating over a broadcast channel, a transmission from a source or relay in an (n, m, k)-cast may interfere with other transmissions in the same or different (n, m, k)-casts. For a given (n, m, k)-cast to succeed, the packet from the source must reach k of the m receivers in the group reliably at least once. Furthermore, any given relay forwards a packet only once. Accordingly, one or multiple (n, m, k)-cast trees can be defined by the set of transmissions that reach each relay and destination of a given (n, m, k)-cast for the first time. When $m = k$, the resulting (n, m, m)-cast tree is also

called a multicast tree. For the case in which $k \leq m$, the selection of the subset of k receivers that correctly receive the packet from the source is such that each of them is reached through a branch of the (n, m, k)-cast tree.

Given the distribution of nodes in the plane and the protocol model it is assumed that, the possible (n, m, k)-cast trees needed to consider include only those that render the minimum number of transmissions for a packet from the source to reach all the intended receivers (k or m) at least once. Because transmissions occur over a common broadcast channel, this implies that the (n, m, k)-cast trees in which this work are interested are those that involve the minimum number of relay nodes needed to connect the source and intended receivers of an (n, m, k)-cast. That is, it is focused on (n, m, k)-cast trees built by the aggregation of shortest paths (minimum-hop paths) between a source and all of its intended destinations. Accordingly, the following definition for (n, m, k)-cast trees is adopted in the rest of this chapter.

An (n, m, k)-cast tree is a function of the transmission range $r(n)$. Therefore, the optimum tree that has the minimum Euclidean distance is a function of $r(n)$. For this reason, changing the transmission range will change the optimum (n, m, k)-cast tree.

Definition 4.3 Minimum Euclidean (n, m, k)-cast Tree (MEMKT $(r(n))$):
The MEMKT$(r(n))$ of an (n, m, k)-cast is an (n, m, k)-cast tree, in which the k destinations receive information from the source among the m receivers of the (n, m, k)-cast, and have the minimum total Euclidean distance. When $k = m$ for instance, minimum

Euclidean multicast tree ($MEMT(r(n))$) is denoted by an (n, m, m)-cast tree with a total minimum Euclidean distance. $MEMKT(R(n))$ can be defined similarly when the nodes in the networks have MPR capabilities.

Definition 4.4 Minimum Area (n, m, k)-cast Tree (MAMKT($r(n)$)):

The MAMKT($r(n)$) in a (n, m, k)-cast tree with k out of m destinations for each source is a (n, m, k)-cast tree that has minimum total area. The area of a (n, m, k)-cast tree is defined as the total area covered by the circles centered around each source or relay with radius $r(n)$.

Note that EMST is spanning tree that consider only the source and destinations, while MEMKT and MAMKT are related to a real routing tree that includes the relays needed to connect the source with the destinations.

In the delay analysis, it is assumed that the delay associated with packet transmission is negligible and the delay is essentially proportional to the number of hops from source to destination. When the packet size is large, then the transmission delay is considerable and this delay no longer can be ignored. The analysis does not consider this case and this is the subject of future study.

Definition 4.5 Delay of an (n, m, k)-Cast:

In an (n, m, k)-cast, the delay of a packet in a network is the time it takes the packet to reach all *k destinations after it leaves the source.*

The queuing delay at the source is not taken into account, because the interest is in the network delay. The average packet delay for a network with n nodes, $D_{m,k}(n)$,

is obtained by averaging over all packets, all source-destination pairs, and all random network configurations.

Definition 4.6 Total Active Area (TAA $(\Delta, R(n))$):

The TAA$(\Delta, R(n))$ is the total area of the network multiplied by the average maximum number of simultaneous transmissions and receptions inside a communication region of $\Theta(R^2(n))$.

It can be shown that this value has an upper bound of $O(1)$ and $O(nR^2(n))$ for SPR and MPR respectively.

4.2 Capacity of (n, m, k)-cast with Single-Packet Reception

4.2.1 Upper Bound

Note that MEMKT includes intermediate relays while EMST(m) only includes m destinations. Lemma 2.7 computes the average total Euclidean distance for EMST(m). To compute the upper bound for (n, m, k)-cast, the relationship between $\overline{S(\text{MAMKT})}$ and $\overline{\|\text{EMST}\|}(m)$ will be first demonstrated.

Lemma 4.7 *The average area for MEMKT$(r(n))$ has the following lower bound.*

$$\overline{S\left(MAMKT(r(n))\right)} = \begin{cases} \Omega\left(kr(n)/\sqrt{m}\right), m = O\left(r^{-2}(n)\right) \\ \Omega\left(kr^2(n)\right), \Omega(k) = r^{-2}(n) = O(m) \\ \Omega\left(1\right), \quad k = \Omega\left(r^{-2}(n)\right) \end{cases} \tag{4.1}$$

Proof: From Lemma 2.7, if only m destinations is selected ($m+1$ nodes including source and m destinations) out of n nodes to construct an EMST(m), then the total average Euclidean distance of the EMST(m) is at least $\Theta(\sqrt{m})$. Given that there are m destinations for the tree, then the average Euclidean distance between any two nodes for this tree is $\Theta\left(\sqrt{m}/m\right)$, so the k closest destinations and the source construct a tree with average length of $\Theta\left(\sqrt{m}k/m\right)$. If the k destinations is just selected randomly, then the problem is an (n,k,k)-cast in the formulation and then the distance of that tree is $\Theta(\sqrt{k})$ based on Lemma 2.7. Here, it is assumed to construct m multicast tree first, and then choose the optimal (smallest length of the tree) ones as the real destinations.

It has been proved [19] that the average area of a tree T with transmission range $r(n)$ is lower bounded by the multiplication between the length of the tree and transmission range $r(n)$ when the number of the actual destinations satisfies $m = O\left(r^{-2}(n)\right)$. Thus, when the transmission range is not a large value, then the total area in such a tree is lower bounded by $\Omega\left(kr(n)/\sqrt{m}\right)$. This is the top lower bound in Eq. (4.1). When the transmission range is larger, given that only the closest k nodes are needed in the set, then the area of that tree is lower bounded by $\Omega\left(kr^2(n)\right)$ ($\pi r^2(n)$ is the area covered by one node). This is the second lower bound in Eq. (4.1). Once $k = \Omega\left(r^{-2}(n)\right)$, then the MAMKT($r(n)$) covers the entire network and $\Omega(1)$ can be used as the lower bound, which is the last value in Eq. (4.1). The threshold for $r(n)$ is derived when the first two lower bounds are equal, i.e., $\Theta\left(kr(n)/\sqrt{m}\right) = \Theta\left(kr^2(n)\right)$. The solution to the value of m_b is $m_b = \Theta\left(r^{-2}(n)\right)$. This result means that, when $m = O(m_b)$ or $m = \Omega(m_b)$, the lower bound of $\overline{S\,(\text{MAMKT})}$ is $\Omega\left(kr(n)/\sqrt{m}\right)$ or $\Theta\left(kr^2(n)\right)$, respectively.

Theorem 4.8 *The upper bound of the per-node (n, m, k)-cast throughput capacity in dense wireless ad hoc networks is*

$$C_{m,k}(n) = \begin{cases} O\left(\sqrt{m}(nkr(n))^{-1}\right), \ m = O\left(r^{-2}(n)\right) \\ O\left(\left(nkr^2(n)\right)^{-1}\right), \ \Omega(k) = r^{-2}(n) = O(m) \\ O\left(n^{-1}\right), \ k = \Omega\left(r^{-2}(n)\right) \end{cases} \tag{4.2}$$

Proof: The proof is immediate by combining Lemma 4.7 with the fact that total area for a unit network was "1". Using the same argument in [1], the total throughput capacity is the total area divided by the consumed area for one (n, m, k)-cast tree. The result is immediate by normalizing the result by n.

Note that $\overline{S\,(\text{MAMKT})}$ can have some overlap for different (n, m, k)-cast sessions. The exclusive area for each multicast session is in the same order as the $\overline{S\,(\text{MAMKT})}$.

In [1], disks of radius $\Delta r(n)/2$ centered at each receiver are disjoint in order to guarantee the protocol model. Therefore, the actual minimum exclusive area for each (n, m, k)-cast session is at least

$$\overline{S\,(\text{MAMKT})} \times \pi \left(\frac{\Delta r(n)}{2}\right)^2 \times \frac{1}{\pi r^2(n)} = \frac{\Delta^2}{4}\overline{S\,(\text{MAMKT})}. \tag{4.3}$$

The difference is at most $\Delta^2/4$ which does not change the order. Hence, the capacity is the network area divided by the total occupied area of one (n, m, k)-cast tree normalized by n, which leads to the per-node capacity.

4.2.2 Lower Bound

To derive the achievable lower bound, a TDMA scheme is used for random dense networks similar to the approach used in [40]. The network area is divided into

square cells. Each square cell has an area of $r^2(n)/2$, which makes the diagonal length of square equal to $r(n)$. Under this condition, connectivity inside all cells is guaranteed and all nodes inside a cell are within transmission range of each other. A cell graph is builded over the cells that are occupied with at least one vertex (node). Two cells are connected if there exist a pair of nodes, one in each cell, which are less than or equal to $r(n)$ distance apart. Because the whole network is connected when Eq. (2.2) is satisfied, it follows that the cell graph is connected.

To satisfy the protocol model, cells in groups should be designed such that simultaneous transmissions within each group do not violate the condition for successful communication in the protocol model. Let L represent the minimum number of cell separations in each group of cells that communicate simultaneously. Utilizing the protocol model, L is given as $L = \left\lfloor 1 + \frac{r(n)+(1+\Delta)r(n)}{r(n)/\sqrt{2}} \right\rfloor = \lfloor 1 + \sqrt{2}(2+\Delta) \rfloor$.

If time is divided into L^2 time slots and assign each time slot to a single group of cells, interference is avoided and the protocol model is satisfied. Given that the parameter L is not a function of n, the TDMA scheme does not change the order capacity of the network.

Definition 4.9 Minimum Euclidean (n, m, k)-Cast Tree Cells(MEMKTC($r(n)$)):
The MEMKTC($r(n)$) of an (n, m, k)-cast tree is the total cells containing all the nodes in the (n, m, k)-cast tree.

The following lemma establishes the achievable lower bound for the (n, m, k)-cast capacity as a function of $\overline{\#\text{MEMKTC}(r(n))}$, the total number of cells that contain

all the nodes in an (n, m, k)-cast group.

Lemma 4.10 *The achievable lower bound of the per-node (n, m, k)-cast throughput capacity in dense wireless ad hoc networks is given by*

$$C_{m,k}(n) = \Omega\left(1/\overline{\#MEMKTC(r(n))} \times 1/nr^2(n)\right), \tag{4.4}$$

Proof: There are $\frac{1}{(r(n)/\sqrt{2})^2}$ cells in the unit square network area. From the definition of $\overline{\#\text{MEMTC}(r(n))}$ and the fact that our TDMA scheme does not change the order capacity, it is clear that there are at most $\overline{\#\text{MEMTC}(r(n))}$ interfering cells for any (n, m, m)-cast communications. Therefore, at any given time there can be at least $\Omega(\frac{1}{\overline{\#\text{MEMTC}(r(n))}}\frac{1}{r^2(n)})$ simultaneous communications in the network. Accordingly, the per-node lower bound capacity is given by $\Omega(\frac{1}{\overline{\#\text{MEMTC}(r(n))}}\frac{1}{nr^2(n)})$, which proves the lemma.

Lemma 4.11 *The average number of cells in MEMKT$(r(n))$ tree is tight bounded as*

$$\overline{\#MEMKTC(r(n))} = \begin{cases} \Theta\left(k(\sqrt{m}r(n))^{-1}\right), \ m = O\left(r^{-2}(n)\right), \\ \Theta\left(k\right), \ \Omega(k) = r^{-2}(n) = O(m), \\ \Theta\left(r^{-2}(n)\right), \ k = \Omega\left(r^{-2}(n)\right). \end{cases} \tag{4.5}$$

Proof: Because the maximum number of cells in this network is equal to $\Theta\left(\frac{1}{r^2(n)}\right)$, it is clear that one upper bound for $\overline{\#\text{MEMTC}(r(n))}$ is this value. That is, $\overline{\#\text{MEMTC}(r(n))}$ cannot exceed the total number of cells in the network. On the other hand, the total Euclidean distance of the (n, m, m)-cast tree was shown earlier to be $\Theta(\sqrt{m})$. Because $r(n)$ is the transmission range of the network, the maximum number

of cells for this (n, m, m)-cast tree must be at most $\Theta\left(\frac{\sqrt{m}}{r(n)}\right)$, i.e., $\overline{\#\text{MEMTC}(r(n))} \leq \Theta\left(\frac{\sqrt{m}}{r(n)}\right)$. This upper bound can be achieved only if every two adjacent nodes in the (n, m, m)-cast tree belong to two different cells in the network. However, in practice, it is possible that some adjacent nodes in the (n, m, m)-cast tree are located in a single cell. Consequently, this value is the upper bound. The actual upper bound clearly is the minimum of these two extreme values in the network, which is a function of the topology of the network and this proves the lemma.

Theorem 4.12 *The achievable lower bound of the (n, m, k)-cast throughput capacity in dense wireless ad hoc networks is*

$$C_{m,k}(n) = \begin{cases} \Omega\left(\sqrt{m}(nkr(n))^{-1}\right), \ m = O\left(r^{-2}(n)\right) \\ \Omega\left(\left(nkr^2(n)\right)^{-1}\right), \ \Omega(k) = r^{-2}(n) = O(m) \\ \Omega\left(n^{-1}\right), \ k = \Omega\left(r^{-2}(n)\right) \end{cases} \tag{4.6}$$

Proof: The proof is immediate by combining Lemmas 4.10 and 4.11.

It is clear that by combining Theorems 4.8 and 4.12, a tight bound for the capacity of (n, m, k)-cast can be derived.

It is noticed from the above results that there are three distinct capacity regions for (n, m, k)-casting. These three different regions are achieved based on different values of transmission range $r(n)$, m, and k. In the first region, the order capacity of wireless ad hoc networks is similar to that of unicast communication. Therefore, this first capacity region is referred as unicast region. This unicast capacity region also includes the capacity for multicasting or any type of anycast communication. Once the number of

receiver nodes is smaller than $\Theta\left(r^{-2}(n)\right)$, then it enters into a second capacity region, which is called the multicast capacity region. The last region is defined for the case when both m and k are larger than $\Theta\left(r^{-2}(n)\right)$. The network capacity in this region is equivalent to the broadcast capacity of the network, and hence this region is called the broadcast capacity region. The (n, m, k)-cast tree associated to this region spans all the elements of the graph and it is equivalent to a connected dominating set for the entire network. Therefore, regardless of having multicast, broadcast, or any type of anycast communications, the capacity reaches its minimum possible value for a given transmission range which is the same as broadcast capacity.

4.2.3 Delay Analysis of (n, m, k)-Cast

In this section, the delay of (n, m, k)-casting and its tradeoff with capacity are presented. As Definition 4.5 states, the packet delay is proportional to the total number of hops required from each source to its destinations. In order to compute this delay, the following lemma is proved.

Lemma 4.13 *The delay of (n, m, k)-cast in a random dense wireless ad hoc network with SPR is*

$$D_{m,k}(n) = \Theta\left(\overline{\#MEMKTC(r(n))}\right). \tag{4.7}$$

Proof: From the definition of $\overline{\#\text{MEMKTC}(r(n))}$ and Lemma 4.11, it concludes that $\overline{\#\text{MEMKTC}(r(n))}$ is proportional to the minimum number of hops in which the information is routed from source to all its destinations. Because a TDMA scheme

is assumed to achieve the lower bound for the capacity, it is clear that, to transport the information from one cell to the next adjacent cell, one to two hops are required. Therefore, $\overline{\#\text{MEMKTC}(r(n))}$ is also in the same order as the total number of hops needed. Based on the definition of delay, it is clear that $\overline{\#\text{MEMKTC}(r(n))}$ is also the same order bound as the total delay, which proves the Lemma.

It is clear that a tight bound for delay can be computed in (n, m, k)-cast as a function of $r(n)$ by combining lemmas 4.11 and 4.13.

Theorem 4.14 *The relationship between capacity and delay for (n, m, k)-cast is given by*

$$C_{m,k}(n)D_{m,k}(n) = \Theta\left(\left(nr^2(n)\right)^{-1}\right). \tag{4.8}$$

Proof: The results can be easily derived by comparing Theorems 4.12, 4.8 with lemmas 4.11 and 4.13.

4.2.4 Discussion of Results

There is much valuable insight to be gained from modeling the capacity of unicasting, multicasting, broadcasting and anycasting using the same framework. The (n, m, k)-cast framework allows us to analyze the throughput capacity of wireless networks as a function of the number of receivers of a communication group, which can range from 1 up to the number of nodes in the network, as well as a function of the transmission range. Accordingly, the results obtained in all prior work can be derived from the model by selecting the appropriate values for $r(n)$ and m in the capacity results

obtained in Sections 4.1. In addition, the framework also provides new insight on the capacity of information dissemination techniques that are becoming more prevalent with the availability of in-network storage, namely anycasting, and allows us to reason about the nature that route signaling should be rendered more scalable wireless networks.

4.2.4.1 $C_{m,k}(n)$ as Function of Transmission Range $r(n)$ and Group Size m

The relationship between $C_{m,k}(n)$ and the transmission range $r(n)$ can be seen in Fig. 4.1. From this figure, it can be shown that maximum capacity can be attained when the transmission range has its minimum value, i.e., $r(n) = \Omega\left(\sqrt{\log n/n}\right)$. It can conclude that the throughput capacity of dense wireless ad hoc networks is proportional with the $\Theta\left(\sqrt{m}/k\right)$ and inversely proportional with the transmission range $r(n)$. Besides, the broadcast threshold m_b will be decreased when $r(n)$ increases.

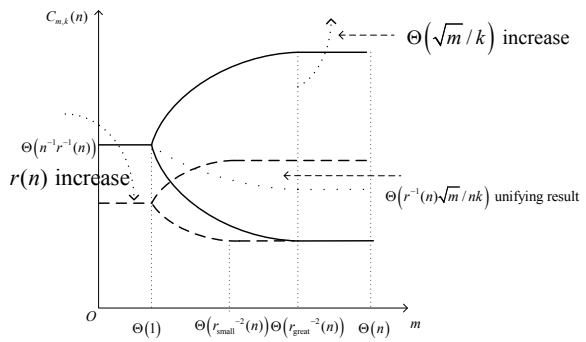


Figure 4.1: $C_{m,k}(n)$ as a function of transmission range $r(n)$, real number of destinations k, and the number of destination group choices m.

Fig. 4.2 shows $C_{m,k}(n)$ as a function of m. As it was the case for $C_{m,m}(n)$, if m varies from 1 to $m_u = \Theta(1)$, the capacity of the network does not change and equals $\Theta\left(\frac{1}{\sqrt{n\log n}}\right)$. For values of m larger than m_u, the (n, m, k)-cast order capacity

can increase or decrease depending on the value of k. The smallest order capacity corresponds to the case when $k = m$, i.e., multicasting ($m < n$) or broadcasting ($m = n$), and the largest order capacity is attained for anycasting ($k = 1$). The shaded area in the figure shows the achievable capacity for manycasting ($1 < k < m$) for different values of m and k.

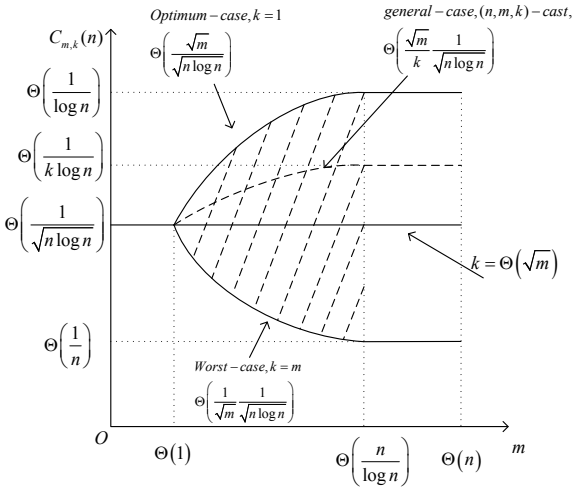


Figure 4.2: Unifying view of throughput capacity

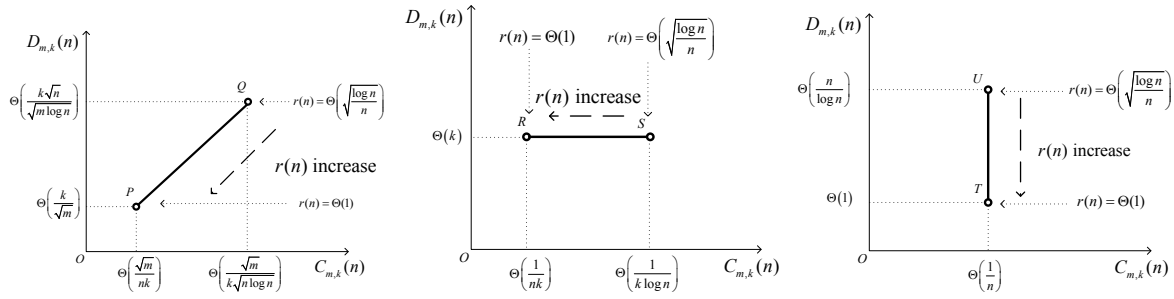
It can be observed that, regardless of the value of k, the capacity of wireless ad hoc networks becomes constant when $m = \Omega\left(n/\log n\right)$ and an increase in the value of m does not change the throughput capacity. This result can be understood by the fact that, when the number of destinations reaches $\Theta\left(n/\log n\right)$, this set becomes the connected dominating set (CDS($r(n)$)) of the entire network as long as the transmission range $r(n)$ is chosen such that the network is a connected network. Equivalently, if a broadcast is made to the entire network, the capacity does not change because all the

nodes in the network are either inside this set or within one hop from an element in this set.

Note that the capacity of anycast or manycast is greater than the capacity of unicast if $k = O(\sqrt{m})$, even if each node requires to transmit its packets to more than one destination. This result shows that as long as $k = O(\sqrt{m})$, the total number of hops required to transmit packet to k destinations is always, on average, less than sending the packet from the same source to a single randomly selected destination in unicast communications. Equivalently, the total Euclidean distance for a manycast tree is on average less than the Euclidean distance between any randomly selected source and destination in unicast communication. However, this Euclidean distances become the same, on average, when $k = \Theta(\sqrt{m})$. As it can be predicted from this figure, the total Euclidean distance in a manycast tree increases as k increases and for $k = \Omega(\sqrt{m})$, the capacity of manycast becomes less than unicast because of the total Euclidean distance in the manycast tree.

4.2.4.2 $D_{m,k}(n)$ as a Function of Transmission Range ($r(n)$) and Tradeoff between $D_{m,k}(n)$ and $C_{m,k}(n)$

Figs. 4.3(a), 4.3(b), and 4.3(c) depict the relationship between $D_{m,k}(n)$ and $C_{m,k}(n)$ when SPR is used in a wireless ad hoc network. With the model, the unifying relationship between capacity and delay can be generalized into multicast and broadcast, as shown in Figs. 4.3(b) and 4.3(c). In the unicast capacity region, the transmission range $r(n)$ should be made as small as possible to increase the capacity of the network



(a) $m = O\left(r^{-2}(n)\right)$ (b) $\Omega(k) = r^{-2}(n) = O(m)$ (c) $k = \Omega\left(r^{-2}(n)\right)$

Figure 4.3: The relationship between delay and capacity

and to avoid interference, with the corresponding cost of increasing delay. To decrease the delay, the transmission range $r(n)$ should be increased, so that the number of hops required to disseminate information is reduced; however, doing so decreases the capacity of the network by increasing multiple access interference (MAI). In the multicast capacity region (see Fig. 4.3(b)), it is observed that the transmission range should be made as small as possible to increase the capacity with no penalty of delay increases. However for the broadcast capacity region (see Fig. 4.3(c)), increasing the transmission range decreases the delay in the network with no penalty for capacity. In this region, maximizing the transmission range should be the strategy.

The above results indicate that there are different tradeoffs between the capacity $C_{m,k}(n)$ and the delay $D_{m,k}(n)$ in terms of transmission range $r(n)$ for the three capacity regions of wireless ad hoc networks.

4.3 Capacity of (n, m, k)-Cast with Multi-Packet Reception

This section presents capacity and delay scaling laws for random wireless ad hoc networks under all information dissemination modalities (unicast, multicast, broadcast, anycast) when nodes are endowed with multi-packet reception (MPR) capabilities.

4.3.1 Upper Bound

The following Lemma provides an upper bound for the per-session capacity as a function of $\overline{\mathrm{TAA}(\Delta, R(n))}$ and $\overline{S\,(\mathrm{MAMKT}(R(n)))}$. Essentially, $\overline{S\,(\mathrm{MAMKT}(R(n)))}$ equals the minimum area consumed to (n, m, k)-cast a packet to k destinations out of m choices (see Fig. 4.4), and $\overline{\mathrm{TAA}(\Delta, R(n))}$ represents the maximum area which can be supported when MPR are used.

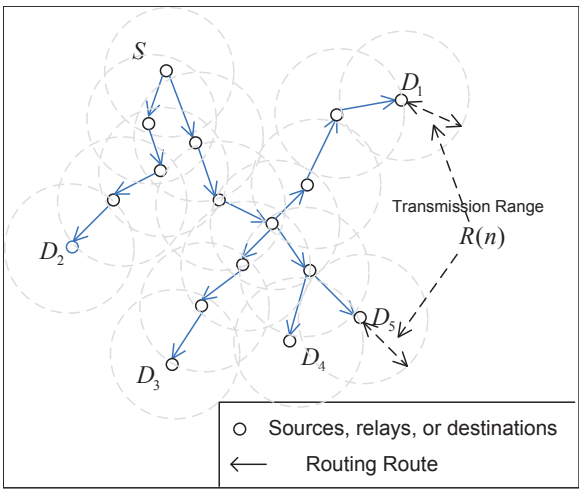


Figure 4.4: Area coverage by one multicast Tree

Lemma 4.15 *In random dense wireless ad hoc networks, the per-node throughput ca-*

pacity of (n, m, k)-cast with MPR is given by $O\left(\frac{1}{n} \times \frac{\overline{TAA(\Delta,R(n))}}{\overline{S(MAMKT(R(n)))}}\right)$.

Proof: With MPR, it is observed that $\overline{S\left(\mathrm{MAMKT}(R(n))\right)}$ represents the total area required to transmit information from a multicast source to all its m destinations. The ratio between average total active area, $\overline{\mathrm{TAA}(\Delta, R(n))}$, and $\overline{S\left(\mathrm{MAMT}(R(n))\right)}$ represents the average number of simultaneous (n, m, k)-cast communications that can occur in the network. Normalizing this ratio by n provides per-node capacity with the definition of $\overline{\mathrm{TAA}(\Delta, R(n))}$ which leads to the Lemma.

Lemma 4.15 provides the upper bound for the (n, m, k)-cast throughput capacity with MPR as a function of $\overline{S\left(\mathrm{MAMKT}(R(n))\right)}$ and $\overline{\mathrm{TAA}(\Delta, R(n))}$. In order to compute the upper bound, the upper bound of $\overline{\mathrm{TAA}(\Delta, R(n))}$ and the lower bound of $\overline{S\left(\mathrm{MAMKT}(R(n))\right)}$ are derived. Combining these results provides an upper bound for the (n, m, k)-cast throughput capacity with MPR. To compute the lower bound for $\overline{S\left(\mathrm{MAMKT}(R(n))\right)}$, the relationship between $\overline{S\left(\mathrm{MAMKT}(R(n))\right)}$ and the total length of Euclidean Minimum Spanning Tree (EMST), $\overline{\|\mathrm{EMST}\|}$ is found.

Lemma 4.16 *In (n, m, k)-cast applications, the average area of a (n, m, k)-cast tree with transmission range $R(n)$, $\overline{S\left(MAMKT(R(n))\right)}$ has the following lower bound as*

$$\overline{S\left(MAMKT(R(n))\right)} = \begin{cases} \Omega\left(kR(n)/\sqrt{m}\right) & \text{for } m = O\left(R^{-2}(n)\right) \\ \Omega\left(kR^2(n)\right) & \text{for } \Omega(k) = R^{-2}(n) = O(m) \\ \Omega\left(1\right) & \text{for } k = \Omega\left(R^{-2}(n)\right) \end{cases} \cdot \tag{4.9}$$

Proof: Note that $\overline{S\left(\mathrm{MAMKT}(R(n))\right)}$ is the same value for MPR and SPR and they only depend on the communication range in the network. After substitute the

$r(n)$ of Lemma 4.7 to $R(n)$, then it completes the proof.

The next lemma states the upper bound for $\overline{\text{TAA}(\Delta, R(n))}$ for a network using MPR.

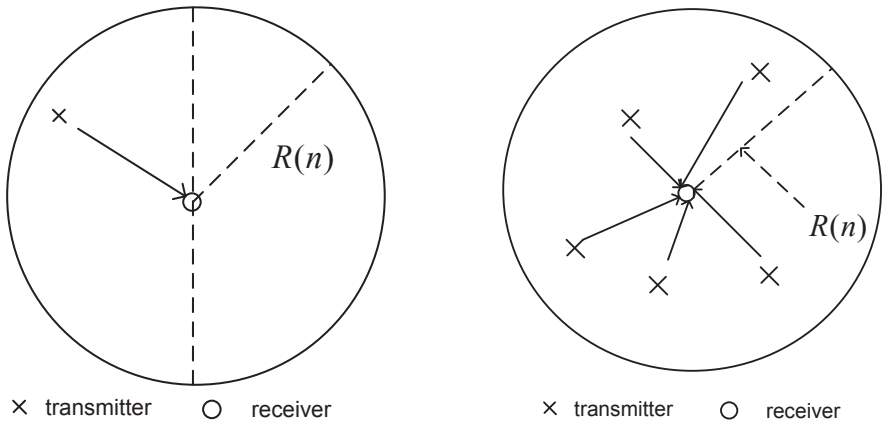
Lemma 4.17 *The average total active area, $\overline{TAA(\Delta, R(n))}$, has the following upper bound in networks with MPR.*

$$\overline{TAA(\Delta, R(n))} = O\left(nR^2(n)\right) \tag{4.10}$$

Proof: As discussed earlier, the $\overline{\text{TAA}(\Delta, R(n))}$ for SPR is equal to 1 since for each circle of radius $R(n)$, there is only a single pair of transmitter-receiver nodes (see Fig. 4.5(a)). For the case of MPR, the number of transmitters in a circle of radius $R(n)$ is upper bounded as $O(nR^2(n))$. The upper bound for $\overline{\text{TAA}(\Delta, R(n))}$ is achieved when the maximum number of transmitters are employed in this circle. Fig. 4.5(b) demonstrates an example that can achieve this upper bound simultaneously for transmitters. Given the fact that this value also is the maximum possible number of transmitter and receiver nodes, the result follows immediately.

Lemma 4.17 implies that the total active area with MPR is upper bounded by $\Theta\left(nR^2(n)\right)$. By contrast, for the case of SPR, it is only $\Theta(1)$. Combining Lemmas 4.15, 4.16, and 4.17, the upper bound of (n, m, k)-cast capacity for MPR can be computed in the following theorem.

Theorem 4.18 *In wireless ad hoc networks with MPR, the upper bound on the per-node*



(a) SPR

(b) MPR

Figure 4.5: Upper bound of total available area based on protocol model with MPR

throughput capacity of (n, m, k)*-cast is*

$$C_{m,k}(n) = \begin{cases} O\left(k^{-1}\sqrt{m}R(n)\right) & \textit{for } m = O\left(R^{-2}(n)\right) \\ O\left(k^{-1}\right) & \textit{for } \Omega(k) = R^{-2}(n) = O(m) \\ O\left(R^{2}(n)\right) & \textit{for } k = \Omega\left(R^{-2}(n)\right) \end{cases}, \tag{4.11}$$

4.3.2 Lower Bound

To derive an achievable lower bound, a TDMA scheme is used for random dense wireless ad hoc networks similar to the approach used in Section 4.2. The difference is that the transmission range $r(n)$ is changed to receiver range $R(n)$.

To satisfy the MPR protocol model, similarly, let $L = \left\lceil 1 + \frac{R(n)+(1+\Delta)R(n)}{R(n)/\sqrt{2}} \right\rceil = \lceil 1+\sqrt{2}(2+\Delta) \rceil$ represent the minimum number of cell separations in each group of cells that communicate simultaneously. If time is divided into L^2 time slots and assign each

time slot to a single group of cells, interference is avoided and the protocol model is satisfied. The separation example can be shown for the upper two receiver circles in Fig. 4.6. For the MPR protocol model, the distance between two adjacent receiving nodes is $(2+\Delta)R(n)$. Because this distance is smaller than $(L-1)R(n)$, this organization of cells guarantees that the MPR protocol model is satisfied. Fig. 4.6 represents one of these groups with a cross sign inside those cells for $L = 4$. The capacity reduction caused by the TDMA scheme is a constant factor and does not change the order capacity of the network.

Next it is proved that, when n nodes are distributed uniformly over a unit square area, with MPR scheme, there are simultaneously at least $\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil$ circular regions (see Fig. 4.6), each one containing $\Theta(nR^2(n))$ nodes w.h.p.. The objective is to find an achievable lower bound using the Chernoff bound, such that the distribution of the number of edges in this unit space is sharply concentrated around its mean, and hence the actual number of simultaneous transmissions occurring in the unit space in a randomly chosen network is indeed $\Theta(n)$ w.h.p..

Lemma 4.19 *The circular area of radius $R(n)$ corresponding to the receiver range of a receiver j contains $\Theta(nR^2(n))$ nodes w.h.p. for all values of j, $1 \leq j \leq \left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil$.*

Proof: The statement of this lemma can be expressed as

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] = 1, \tag{4.12}$$

where N_j and $E(N_j)$ are the random variables that represent the number of nodes in

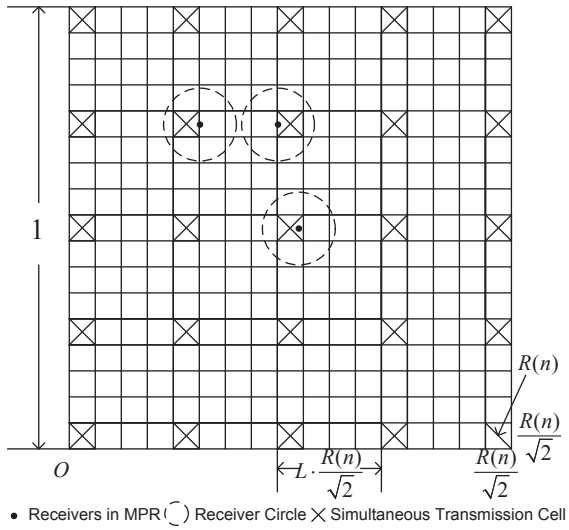


Figure 4.6: Cell construction used to derive a lower bound on capacity

the receiver circle of radius $R(n)$ centered around node j and the expected value of this random variable respectively, and δ is a positive arbitrarily small value close to zero.

From the Chernoff bound in Eq. (2.6), for any given $0 < \delta < 1$, $\theta > 0$ can be found such that $P\left[|N_j - E(N_j)| > \delta E(N_j)\right] < e^{-\theta E(N_j)}$. Thus, it can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n \to \infty$. This is a key step in showing that when all the events $\bigcap_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil} |N_j - E(N_j)| < \delta E(N_j)$ occur simultaneously, then all N_j's converge uniformly to their expected values. Utilizing the union bound, it

arrives at

$$
\begin{aligned}
& P\left[\bigcap_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2}\right\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] \\
= \; & 1 - P\left[\bigcup_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2}\right\rceil} |N_j - E(N_j)| > \delta E(N_j)\right] \\
\geq \; & 1 - \sum_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2}\right\rceil} P\left[|N_j - E(N_j)| > \delta E(N_j)\right] \\
> \; & 1 - \left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil e^{-\theta E(N_j)}. \qquad (4.13)
\end{aligned}
$$

Given that $E(N_j) = \pi n R^2(n)$, then

$$
\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2}\right\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] \geq 1 - \lim_{n\to\infty} \left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil e^{-\theta\pi n R^2(n)} \qquad (4.14)
$$

Utilizing the connectivity criterion in Eq. (3.1), $\lim_{n\to\infty} \frac{e^{-\theta\pi n R^2(n)}}{R^2(n)} \to 0$, which completes the proof.

The previous lemma proves that, w.h.p., there are indeed $\Theta(n)$ simultaneous transmitters which are in $\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil$ circles of radius $R(n)$ around the receivers, who can transmit simultaneously, as shown in Fig. 4.6. With Lemmas 4.19, the preparation for the following achievable lower bound has been done.

Let us define $\overline{\#\text{MEMKTC}(R(n))}$ as the total number of cells that contain all the nodes in an (n, m, k)-cast group. Also, $\overline{\#\text{MEMTC}(R(n))}$ is defined as the total number of cells that contain all the nodes in an (n, m, m)-cast group. The following lemma establishes the achievable lower bound for the (n, m, k)-cast throughput capacity

of MPR or MPR as a function of $\overline{\#\text{MEMKTC}(R(n))}$. Note that $\overline{\#\text{MEMKTC}(R(n))}$ only depends on the (n, m, k)-cast network parameters regardless of using MPR techniques. Next Lemma provides a tight bound for $\overline{\#\text{MEMKTC}(R(n))}$.

Lemma 4.20 *The average number of cells covered by the nodes in MEMKTC$(R(n))$, is tight bounded w.h.p. as follows:*

$$\overline{\#MEMKTC(R(n))} = \begin{cases} \Theta\left(k\left(\sqrt{m}R(n)\right)^{-1}\right) & \text{for } m = O\left(R^{-2}(n)\right) \\ \Theta\left(k\right) & \text{for } \Omega(k) = R^{-2}(n) = O(m) \\ \Theta\left(R^{-2}(n)\right) & \text{for } k = \Omega\left(R^{-2}(n)\right) \end{cases} \tag{4.15}$$

Proof: The proof is similar to the Lemma 4.11. The difference is that transmission range $r(n)$ is substituted with receiver range $R(n)$.

Next the routing scheme is discussed to achieve the lower bound capacity which is similar to the scheme used in [41]. According to the model, each (n, m, k)-cast session creates a (n, m, k)-cast tree $\overline{\#\text{MEMKT}(R(n))}$ to connect the source and destinations. The trees are denoted as T_is, where $i = 1, 2, \cdots, n$. The multi-hop routing between source and destination transfers the packets by using cells that are only intersected by T_i. There is a bound on the number of trees each cell needs to serve, which means the probability that the trees will intersect a particular cell is bounded.

Lemma 4.21 *For any* $R(n) = \Omega\left(\sqrt{\log n/n}\right)$,

$$\lim_{n\to\infty} Prob\left(\sup_{(k,j)}\{Number\ of\ trees\ T_i s\ intersecting\ S_{k,j}\} = O\left(nR^2(n)\overline{\#MEMKTC(R(n))}\right)\right) = 1 \tag{4.16}$$

Proof: For every tree T_i and cell S_{k_0,j_0},

$$p = \text{Prob}\{\text{Tree } T_i \text{ intersects } S_{k_0,j_0}\} = \Theta\left(R^2(n)\overline{\#\text{MEMKTC}(R(n))}\right) \tag{4.17}$$

First the number of trees served by one particular cell S_{k_0,j_0} is bounded. Define i.i.d. random variable $I_i, i \leq i \leq n$, as follows:

$$I_i = \begin{cases} 1, \text{ if } T_i \text{ intersects } S_{k_0,j_0} \\ \\ 0, \text{ if not} \end{cases} \tag{4.18}$$

Then $\text{Prob}(I_i = 1) = p, \forall i$, where p is defined in Eq. (4.17). Denote by Z_n the total number of trees served by S_{k_0,j_0}. Then $Z_n := I_1 + I_2 + \cdots + I_n$. Thus by the Chernoff Bounds [38], for all positive b and a, $\text{Prob}(Z_n > b) \leq \frac{E[e^{aZ_n}]}{e^{ab}}$.

$$E\left[e^{aZ_n}\right] = (1+(e^a-1)p)^n \leq \exp(n(e^a-1)p) = \Theta\left(\exp\left((e^a-1)nR^2(n)\overline{\#\text{MEMKTC}(R(n))}\right)\right) \tag{4.19}$$

Now choosing $b = \Theta\left(nR^2(n)\overline{\#\text{MEMKTC}(R(n))}\right)$, if a is selected small enough,

$$\text{Prob}\left(Z_n = \Omega\left(nR^{-2}(n)\overline{\#\text{MEMKTC}(R(n))}\right)\right) = O\left(\exp\left(-nR^2(n)\overline{\#\text{MEMKTC}(R(n))}\right)\right). \tag{4.20}$$

Thus by the union bound,

$$\begin{aligned} & \text{Prob}\left(\text{Some cell intersects } \Omega(nR^2(n)\overline{\#\text{MEMKTC}(R(n))}) \text{ trees }\right) \\ \leq & \sum_{k,j} \text{Prob}\left(\text{Cell } S_{jk} \text{ intersects} \Omega(nR(n)) \text{ trees }\right) \\ = & O\left(\frac{1}{R^2(n)} \exp\left(-nR^2(n)\overline{\#\text{MEMKTC}(R(n))}\right)\right) \end{aligned} \tag{4.21}$$

The right hand side tends to zero for $R(n) = \Omega\left(\sqrt{\log n/n}\right)$ as n goes to infinity for all three different regions of $\overline{\#\text{MEMKTC}(R(n))}$ from Eq. (4.15).

There exists a transmitting schedule such that in every L^2 (L is constant) slots, each cell transmits at rate W bits/second with maximum transmission distance $R(n)$. Therefore, the rate for each cell is $\Theta\left(nR^2(n)\right)W/L^2$. From Lemma 4.21, each cell needs to transmit at rate $O\left(C_{m,k}(n)nR^2(n)\overline{\#\text{MEMKTC}(R(n))}\right)$, with probability approaching one. In order to accommodate this requirement by all cells, it needs

$$C_{m,k}(n)nR^2(n)\overline{\#\text{MEMKTC}(R(n))} = \Omega\left(\left(nR^2(n)\right)W/L^2\right) \tag{4.22}$$

Thus the achievable throughput have been proven for Lemma 4.22 in order to guarantee each cell can support this capacity.

Lemma 4.22 *The achievable lower bound for the (n, m, k)-cast capacity is given by*

$$C_{m,k}(n) = \Omega\left(\left(\overline{\#MEMKTC(R(n))}\right)^{-1}\right). \tag{4.23}$$

Proof: There are $(R(n)/\sqrt{2})^{-2}$ cells in the unit square network area. With the Lemma 4.21 and the fact that the TDMA scheme does not change the order capacity, it is clear that there are at most in the order of $\overline{\#\text{MEMKTC}(R(n))}$ interfering cells for any (n, m, k)-cast communication. For each cell, the order of nodes in each cell is $\Theta\left(\pi R^2(n)n\right)$. Accordingly, the total lower bound capacity is given by $\Omega\left((R(n)/\sqrt{2})^{-2} \times \left(\pi R^2(n)n\right) \times \left(\overline{\#\text{MEMKTC}(R(n))}\right)^{-1}\right)$. Normalizing this value by total number of nodes in the network, n, proves the lemma.

Combining Lemmas 4.20 and 4.22, it arrives at the achievable lower bound of the (n, m, k)-cast throughput capacity in dense random wireless ad hoc networks with MPR.

Theorem 4.23 *The achievable lower bound of the (n, m, k)-cast throughput capacity with MPR is*

$$C_{m,k}(n) = \begin{cases} \Omega\left(k^{-1}\sqrt{m}R(n)\right) & \textit{for} \quad m = O\left(R^{-2}(n)\right), \\ \Omega\left(k^{-1}\right) & \textit{for} \quad \Omega(k) = R^{-2}(n) = O(m), \\ \Omega\left(R^{2}(n)\right) & \textit{for} \quad k = \Omega\left(R^{-2}(n)\right) \end{cases} \tag{4.24}$$

Proof: There are $(R(n)/\sqrt{2})^{-2}$ cells in the unit square network area and only $(LR(n)/\sqrt{2})^{-2}$ of these cells can communicate simultaneously because of the TDMA scheme which is described earlier. From the definition of $\overline{\#\text{MEMKTC}(R(n))}$, it is clear that there are in the order of $\overline{\#\text{MEMKTC}(R(n))}$ transmissions required in order to transfer a packet from source to all its destinations in any (n, m, k)-cast communication scheme. It is clear from Lemma 4.19 that for each of $(LR(n)/\sqrt{2})^{-2}$ simultaneous transmitting cells, there are $\Theta\left(\pi R^2(n)n\right)$ nodes transmitting packets to their respected receiver nodes using MPR. Since each one of (n, m, k)-cast group requires $\overline{\#\text{MEMKTC}(R(n))}$ transmissions, the total throughput capacity lower bound for the network is equal to $\Omega\left(\frac{(R(n)/\sqrt{2})^{-2}\times(\pi R^2(n)n)}{\overline{\#\text{MEMKTC}(R(n))}}\right)$. If this value is divided by the total number of nodes in the network, n, and substitute $\overline{\#\text{MEMKTC}(R(n))}$ with the results from Lemma 4.20, then the theorem will be proved.

It has been proved there is no congestion in relay nodes. Furthermore, it will be proved there is not any congestion in destination. Suppose each source selects a destination randomly and in dependently. Then it will be proved with high probability, a node can be destination for at most $\frac{3\log n}{\log\log n}$ sources. This problem is similar to the " bins and balls problems" in [42].

Lemma 4.24 *The probability of a particular destination having k sources selected is*

$$\lim_{n\to\infty} Prob[\textit{destination } i \textit{ has at least } k \textit{ sources}] \leq \left(\frac{e}{k}\right)^k \tag{4.25}$$

Proof: Look at any subset of sources of size k, then the probability that the subset of sources select destination i is $(\frac{1}{n})^k$. Then a union bound of these probabilities over all $\binom{n}{k}$ subsets of size k is taken. The events are being summed over, though, are not disjoint. Therefore, it can be only shown that the probability of a destination having at least k balls is at most $\binom{n}{k}\binom{n}{1}^k$. Using Stirling's approximation $\lim_{n\to\infty} \frac{n!}{\sqrt{2\pi n}(\frac{n}{e})^n} = 1$,

$$\lim_{n\to\infty} \frac{\binom{n}{k}\binom{n}{1}^k}{\left(\frac{e}{k}\right)^k} = 1, \tag{4.26}$$

which proved the lemma.

Lemma 4.25 *With high probability, i.e. with probability greater than $1-\frac{1}{n}$, there exist at most $\frac{3\log n}{\log\log n}$ sources for each destination.*

Proof: Let $k = \frac{3\log n}{\log\log n}$. From Lemma 4.24,

$$\begin{aligned}
&\lim_{n\to\infty} \text{Prob[destination } i \text{ has at least } k \text{ sources]} \\
\leq\ & \left(\frac{e}{k}\right)^k = \left(\frac{e\log\log n}{3\log n}\right)^{\frac{3\log n}{\log\log n}} \\
\leq\ & \exp\left(\frac{3\log n}{\log\log n}(\log\log\log n - \log\log n)\right) \\
=\ & \exp\left(-3\log n + \frac{3\log n\log\log\log n}{\log\log n}\right) \\
\leq\ & \exp(-2\log n) = \frac{1}{n^2}
\end{aligned} \tag{4.27}$$

Using Union Bound, $\lim_{n\to\infty}$ Prob[any destination has at least k sources] $\leq n\frac{1}{n^2} = \frac{1}{n}$, which implies that $\lim_{n\to\infty}$ Prob[all destinations have at most k sources] $\geq 1-\frac{1}{n}$ It proved the lemma.

For MPR (n, m, k)-cast, it is required that for all destinations, there does not exist any one whose traffic load congestion is larger than the total throughput it can support. It means that the maximum throughput for each destination should always be greater than the total traffic load. In MPR case, the total throughput of each destination is $nR^2(n)$. The traffic load congestion for each destination is the multiplication of throughput per node of $C_{m,k}(n)$ and the maximum possible sources that select a node, i.e. $\frac{3\log n}{\log\log n}$. Hence,

$$nR^2(n) \geq C_{m,k}(n)\frac{3\log n}{\log\log n}, \tag{4.28}$$

As long as $R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$, it can easily verified for three different regions, $C_{m,k}(n)$ can achieve the lower bound of Theorem 4.23.

4.3.3 Capacity-Delay Tradeoff with Multi-Packet Reception

4.3.3.1 Capacity of (n, m, k)-Cast with MPR

From Theorems 4.18 and 4.23, the tight bound throughput capacity is provided for the (n, m, k)-cast when the nodes have MPR capability in dense random wireless ad hoc networks as follows.

Theorem 4.26 *The throughput capacity of (n, m, k)-cast in a random dense wireless ad hoc network with MPR is*

$$C_{m,k}(n) = \begin{cases} \Theta\left(k^{-1}\sqrt{m}R(n)\right) & \textit{for } m = O\left(R^{-2}(n)\right) \\ \Theta\left(k^{-1}\right) & \textit{for } \Omega(k) = R^{-2}(n) = O(m) \\ \Theta\left(R^2(n)\right) & \textit{for } k = \Omega\left(R^{-2}(n)\right) \end{cases} \cdot \tag{4.29}$$

The receiver range of MPR should satisfy $R(n) = \Omega\left(\sqrt{\log n/n}\right)$. *Note that the thresholds for different values for* m *and* k *provide various capacities for* (n, m, k)*-cast in MPR.*

The throughput capacity of (n, m, k)-cast in a random dense wireless ad hoc network with SPR is derived and there are two major different between SPR and MPR. First, for SPR, the receiver range $R(n)$ must be changed into the transmission range $r(n)$. Second, in SPR, there can be at most a single successful transmission inside a circle of radius of $r(n)$ centered around each receiver node.

4.3.3.2 Delay of (n, m, k)-Cast with MPR and its Relationship with Capacity

In this section, the result is presented regarding the tradeoff between delay and capacity. As defined delay earlier, packet delay is proportional to the total number of hops required from each source to its destinations. In order to compute this delay, the following lemma is first proved.

Lemma 4.27 *The delay of* (n, m, k)*-cast in a random dense wireless ad hoc network with MPR is*

$$D_{m,k}(n) = \Theta\left(\overline{\#MEMKTC(R(n))}\right) \tag{4.30}$$

Proof: From the definition of $\overline{\#\text{MEMKTC}(R(n))}$ and Lemma 4.20, it concludes that $\overline{\#\text{MEMKTC}(R(n))}$ is proportional to the minimum number of hops in which the information is routed from source to all its destinations. Since TDMA scheme is used to achieve the lower bound for the capacity, it is clear that in order to transport

the information from one cell to the next adjacent cell, one to two hops is needed (see Fig. 4.6). Therefore, $\overline{\#\text{MEMKTC}(R(n))}$ is also in the same order as the total number of hops. Based on the definition of delay, it is clear that $\overline{\#\text{MEMKTC}(R(n))}$ is also the same order bound as the total delay which proves the Lemma.

Theorem 4.28 *The relationship between capacity and delay for (n, m, k)-cast with MPR is given below*

$$C_{m,k}(n)D_{m,k}(n) = \Theta(1) \tag{4.31}$$

Proof: The results can be easily derived by comparing Theorem 4.26 with Lemmas 4.27.

The relationship between capacity and delay in (n, m, k)-cast with SPR is given below. The capacity-delay tradeoff in [4] is a special case of the results for $m = k = 1$ which can be shown as $D_{1,1}(n) = \Theta(nC_{1,1}(n))$.

4.3.4 Discussion of Results

Theorems 4.26 provides capacity information for MPR whose fundamental difference from the SPR is due to the fact that the MPR scheme embraces interference, while SPR is based on avoiding interference by limiting transmission range. The details are given as follow aspects.

4.3.4.1 $C_{m,k}(n)$ as a Function of Group Size (m)

Comparing the capacities attained with MPR and SPR for unicast traffic (see Fig. 4.7), the ratio is equal to $\Theta\left(R(n)\sqrt{n\log n}\right)$. The same ratio is equal to $\Theta\left(R^2(n)n\right)$

for the case of broadcasting. If a larger value is chosen for the communication range for MPR, i.e., $R(n) = \Omega\left(\sqrt{\log n/n}\right)$, then it is easy to show that the capacity gain for MPR compared to SPR is larger in broadcast communication than for unicasting. The larger gains attained with MPR for broadcast communication are a consequence of the fact that, as the number of broadcast destinations increases, more copies of the same packets must be sent to a larger number of nodes. In a network using MPR, concurrent broadcast transmissions can be decoded by the receivers while at most one broadcast transmission can succeed at a time when SPR is assumed.

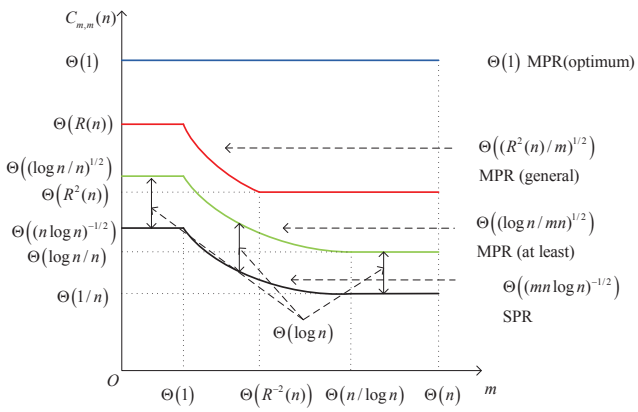


Figure 4.7: Order throughput capacity of $(n,\ m,\ m)$-cast with SPR and MPR as a function of number of destinations m and receiver range $R(n)$

Fig. 4.8 compares the throughput capacity of MPR to that of SPR. Comparing the results for both cases when the number of destinations for each session is smaller than $\Theta\left(R^{-2}(n)\right)$, it appears that they both have the same term as $\sqrt{m}/k$. However, for MPR this term is multiplied by $R(n)$, while for SPR this term is divided by $r(n)$. If $R(n) = r(n)$ is assumed, it appears that increasing the receiver range increases the capacity for the MPR scheme, while it decreases the capacity for SPR. This fundamental

difference is due to the fact that the MPR scheme embraces interference, while SPR is based on avoiding it by limiting transmissions around receivers.

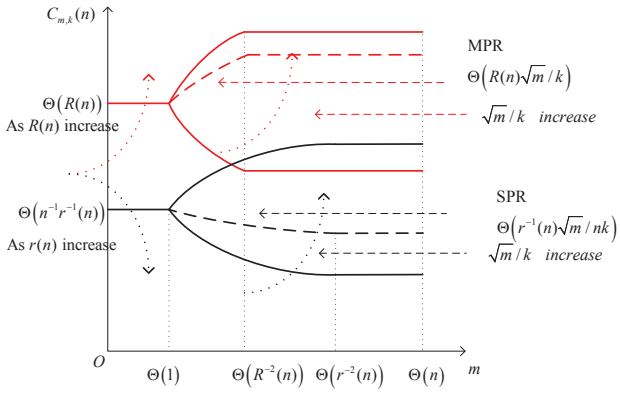


Figure 4.8: Order throughput capacity of $(n,\ m,\ k)$-cast with SPR and MPR

Note that the capacity of anycast or manycast is greater than the capacity of unicast if $k = O(\sqrt{m})$, even if each node requires to transmit its packets to more than one destination. This result shows that, as long as $k = O(\sqrt{m})$, the total number of hops required to transmit packet to k destinations is always, on average, less than sending the packet from the same source to a single randomly selected destination in unicast communications. Equivalently, the total Euclidean distance for a manycast tree is on average less than the Euclidean distance between any randomly selected source and destination in unicast communication. However, these Euclidean distances become the same, on average, when $k = \Theta(\sqrt{m})$. As it can be predicted from this figure, the total Euclidean distance in a manycast tree increases as k increase and for $k = \Omega(\sqrt{m})$, the capacity of manycast becomes less than unicast because of the total Euclidean distance in the manycast tree.

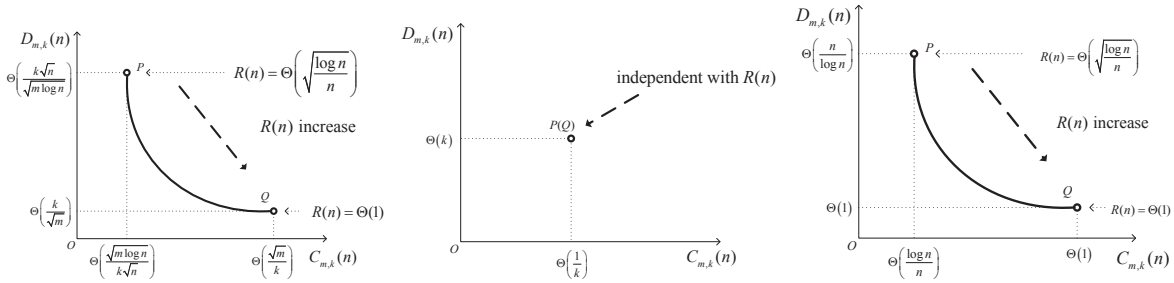
4.3.4.2 $C_{m,k}(n)$ as a Function of Receiver Range ($R(n)$)

Eq. (4.29) show that the throughput capacity of wireless ad hoc networks do increase with the increase in the receiver range $R(n)$ when the receivers decode more than one packet at a time. This result is in sharp contrast to results attained with SPR, with which increasing the communication range decreases the capacity. In networks with MPR, by increasing the receiver range in the network, the total number of simultaneous transmissions is actually increased at a given time! In contrast, for networks with SPR, a larger transmission range leads to increased interference at larger number of nodes, which forces these nodes to be silent during a communication session.

Clearly, the capacity of the network is maximized if the number of simultaneous transmissions is maximized in the network. Ideally, if the receiver range can be made $\Theta(1)$, then a network using MPR can scale linearly with n. Obviously, the receiver range is restricted in practice by the complexity of the nodes. However, even the receiver range is assumed to have the minimum value, which is the connectivity criterion in Eq. (3.1), MPR still renders a capacity gain compared to SPR. Furthermore, this gain is still an order gain equal to $\Theta(\log n)$ compared to the capacity attained with SPR for (n, m, k)-casting.

4.3.4.3 Capacity Delay Tradeoff

Theorems 4.26 provide capacity information for MPR. There are three different capacity regions depending on the values of k and m in (n, m, k)-cast. Figs. 4.9(a), 4.9(b) and 4.9(c) compare the tradeoff between throughput capacity and delay for MPR



(a) $m = O\left(R^{-2}(n)\right)$ (b) $\Omega(k) = R^{-2}(n) = O(m)$ (c) $k = \Omega\left(R^{-2}(n)\right)$

Figure 4.9: The tradeoff between capacity and delay with MPR

and SPR for all these three regions of capacity. By observing the capacity for MPR and SPR, it is noticed that the receiver range $R(n)$ is multiplied for capacity computation in MPR in two regions in Eq. (4.29) and in one region is independent of $R(n)$ while the transmission range $r(n)$ is divided for capacity computation in SPR in the first two regions of capacity. These behaviors are shown in Figs. 4.9(a), 4.9(b) and 4.9(c). This fundamental difference is due to the fact that the MPR scheme embraces interference, while SPR is based on avoiding interference by limiting transmission range.

The above result indicates that large capacity increases can be attained by embracing interference with MPR and embracing opportunism by appropriate use of in-network storage and information dissemination from the nearest site(s) of a communication group, rather than from pre-defined origins hosting the content. If the communication group is the entire network ($m = n$), information flows from the closest neighbor(s) to each node and the maximum capacity gain is attained. If the group size

is independent of the size of the network ($m = \Theta(1)$), the order capacity is the same as for unicast.

Fig. 4.9(a) is the first region in capacity for MPR. Interesting observation is the fact that unlike SPR that increasing capacity results in increasing delay, capacity can be increased and delay can be decreased simultaneously with MPR. This is a significant advantage of using MPR and stems from the fact that MPR embraces interference and consequently, it does not need to sacrifice capacity or delay to improve the other parameter.

Fig. 4.9(b) shows the capacity-delay tradeoff in the second capacity region. For the case of MPR, the capacity or delay is not a function of $R(n)$ and therefore, there is no tradeoff. For this case in SPR, increasing $r(n)$ decreases capacity but has no effect on the delay.

Fig. 4.9(c) is the third region of capacity for MPR. This is the broadcasting region of capacity and it is clear that SPR does not provide any tradeoff. In general, by increasing the transmission range delay can be decreased while the capacity remains constant. The reason for this behavior is the fact that all nodes in broadcasting region are receiving the packet and increasing transmission range does not create any interference. On the other hand, when MPR is used and the receiver range is increased, again both capacity and delay are improved similar to the first case. Clearly, the capacity of the network with MPR is maximized if the number of simultaneous transmissions is maximized in the network. Ideally, if the receiver range can be made $\Theta(1)$, then a network using MPR can scale linearly with n. Obviously, the receiver range is restricted

in practice by the complexity of the receivers. However, even with the minimum value for the receiver range, which is the connectivity criterion, MPR still renders a capacity gain compared to SPR. Furthermore, this gain is still an order gain equal to $\Theta(\log n)$ compared to the capacity attained with SPR for (n, m, k)-casting.

In summary, the tradeoff between capacity $C_{m,k}(n)$ and delay $D_{m,k}(n)$ with MPR is in sharp contrast to SPR. The results in this chapter provide new directions and opportunities for future research activities in wireless ad hoc networks. Another important aspect that it is not discussed in this chapter is related to practical limitations and decoding complexity that can have with MPR scheme. This aspect is important and its investigation is the subject of future studies.

4.4 Conclusion

A unifying framework is introduced for the modeling of the order capacity of wireless networks subject to different types of information dissemination. To do so, (n, m, k)-casting is defined as a generalization of all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks. The modeling framework provides a unique perspective to the understanding of the capacity of wireless ad hoc networks. The approach unifies existing results on the order capacity of wireless networks subject to unicasting , multicasting, or broadcasting and provides new capacity and delay results for anycasting and manycasting.

Chapter 5

Network Coding Does Not Increase the Order Capacity

This chapter studies the contribution of NC in improving the multicast capacity of random wireless ad hoc networks when nodes are endowed with MPT and MPR capabilities. This chapter is constructed as follows. Section 5.1 describes the network model used. Sections 5.2 and 5.3 present the capacity of MPT plus MPR and NC plus MPT plus MPR respectively. Section 5.4 concludes this chapter.

5.1 Network Model

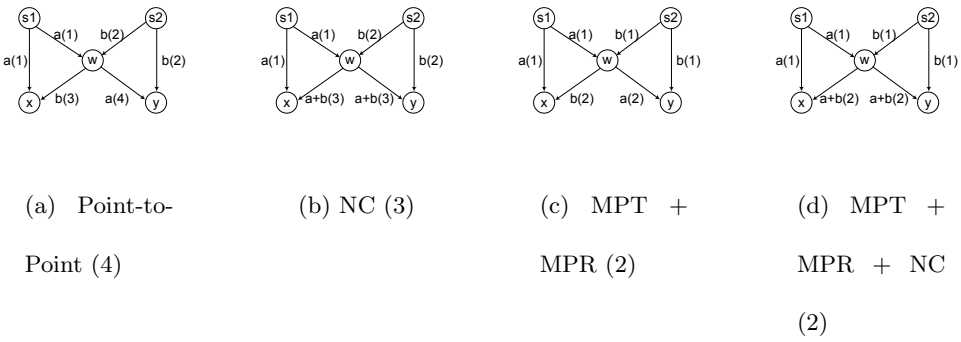
The following extensions is made to account for MPT and MPR capabilities at the transmitters and receivers, respectively. In wireless ad hoc networks with MPT (MPR) capability, any transmitter (receiver) node can transmit (receive) different information simultaneously to (from) multiple nodes within the circle whose radius is $T(n)$.

For the rest of this chapter, it is assumed that $T(n) = r(n)$ for simplicity. The possibility to achieve ideal MPT and MPR is assumed. In an non-ideal scenario, realization of MPT or MPR leads to a loss in the rate provided to an individual receiver. Certain special cases in the rate region can also be expressed as reduction in transmission range, i.e. $T(n) \leq r(n)$. In idealized conditions $r(n) = T(n)$. It is further assumed that nodes cannot transmit and receive at the same time, which is equivalent to half-duplex communications [1]. From system point of view, MPT and MPR are dual if the source and destination duality is considered.

In [20] it was proved that the max-flow min-cut is equal to multicast capacity of a directed graph with single source. The directed graph model is more applicable for wired networks. However, this chapter wishes to study the utility of NC in a wireless environment where links are bidirectional [24, 25].

In a single-source network, the cut capacity is equal to the maximum flow. Thus [25] provides an upper bound on the multicast capacity of a network with single source and NC+MPT+MPR capability. However, in [24, 25], the source, relays and destinations are strictly different and information can not be transmitted directly towards the destinations. These two assumptions will be eventually relaxed in this chapter.

Before analyzing the scaling law of throughput capacity with NC, MPT and MPR in wireless ad hoc networks, one example is illustrated to claim the consequences of MPT and MPR as Fig. 5.1. From this figure, it is observed that combination of MPT, MPR and NC can increase the information flow and the gain is equivalent to only combining MPT and MPR. Intuitively, it can be conjectured that the gain of NC



(a) Point-to-Point (4)

(b) NC (3)

(c) MPT + MPR (2)

(d) MPT + MPR + NC (2)

Figure 5.1: One example for SPR, MPT, MPR and NC

is given actually by MPT and/or MPR which will be proved in the following sections. In Section 5.2 and 5.3, it will be proved that the scaling law of MPT and MPR abilities without and with NC respectively, and then show they are the same order.

5.2 The Throughput Capacity with MPT and MPR

In this section, the scaling laws in random geometric graphs is computed when nodes are endowed with MPR and MPT capabilities. The approach is based on the results for SPR and extending it to MPR and MPT cases.

5.2.1 Upper Bound

The following Lemma provides an upper bound for the per-session capacity as a function of $\overline{\mathrm{TAA}(\Delta, T(n))}$ and $\overline{S\,(\mathrm{MAMT}(T(n)))}$. Essentially, $\overline{S\,(\mathrm{MAMT}(T(n)))}$ equals the minimum area consumed to multicast a packet to m destinations (see Fig. 4.4), and $\overline{\mathrm{TAA}(\Delta, T(n))}$ represents the maximum area which can be supported when

MPT and MPR are used.

Lemma 5.1 *In random dense wireless ad hoc networks, the per-node throughput capacity of multicast with MPT and MPR is given by* $O\left(\frac{1}{n} \times \frac{\overline{TAA(\Delta,T(n))}}{\overline{S(MAMT(T(n)))}}\right)$.

Proof: With MPT and MPR, it is observed that $\overline{S\left(\mathrm{MAMT}(T(n))\right)}$ represents the total area required to transmit information from a multicast source to all its m destinations. The ratio between average total active area, $\overline{\mathrm{TAA}(\Delta, T(n))}$, and $\overline{S\left(\mathrm{MAMT}(T(n))\right)}$ represents the average number of simultaneous multicast communications that can occur in the network. Normalizing this ratio by n provides per-node capacity.

Lemma 5.1 provides the upper bound for the multicast throughput capacity with MPT and MPR as a function of $\overline{S\left(\mathrm{MAMT}(T(n))\right)}$ and $\overline{\mathrm{TAA}(\Delta, T(n))}$. In order to compute the upper bound, the upper bound of $\overline{\mathrm{TAA}(\Delta, T(n))}$ and the lower bound of $\overline{S\left(\mathrm{MAMT}(T(n))\right)}$ are derived. Combining these results provides an upper bound for the multicast throughput capacity with MPT and MPR.

Lemma 5.2 *The average area of a multicast tree with transmission range* $T(n)$, $\overline{S\left(MAMT(T(n))\right)}$ *is lower bounded by* $\Omega\left(T(n)\right)$, *when* m *is a constant value.*

Proof: From [19], it can be deduced that $\overline{S\left(\mathrm{MAMT}(T(n))\right)}$ is lower bounded as $\Omega\left(\overline{\|\mathrm{EMST}\|} \times T(n)\right)$. Even for the case of the minimum value for $T(n)$ to assure connectivity, this upper bound is guaranteed for constant values of m. Lemma 2.7 states that $\overline{\|\mathrm{EMST}\|} = \Theta\left(\sqrt{m}\right) = \Theta(1)^*$. The proof follows immediately.

$^*m$ is a constant value.

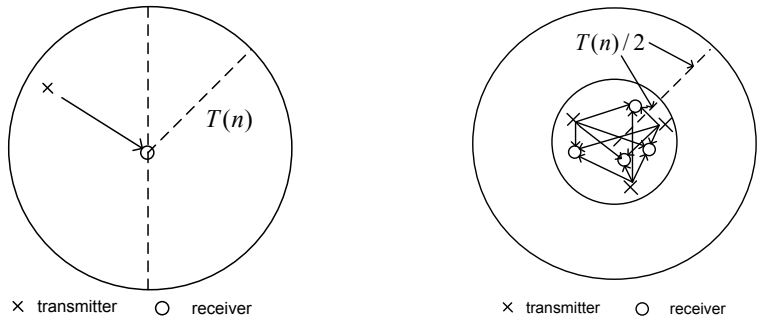
Lemma 5.3 *The average total active area, $\overline{TAA(\Delta, T(n))}$, has the following upper bound in networks with MPT and MPR.*

$$\overline{TAA(\Delta, T(n))} = O\left(n^2 T^4(n)\right) \tag{5.1}$$

Proof: As discussed earlier, the $\overline{\mathrm{TAA}(\Delta, T(n))}$ for SPR is equal to 1 since for each circle of radius $T(n)$, there is only a single pair of transmitter-receiver nodes (see Fig. 5.2(a)). For the case of MPR and MPT, the number of nodes in a circle of radius $T(n)$ is upper bounded as $O(nT^2(n))$. This is also upper bound for the number of transmitters or receivers in this region. The upper bound for $\overline{\mathrm{TAA}(\Delta, T(n))}$ is achieved when the maximum number of transmitters and receivers are employed in this circle. Fig. 5.2(b) demonstrates an example that can achieve this upper bound simultaneously for transmitters and receivers. Let a circle of radius $\frac{T(n)}{2}$ located at the center of another circle of radius $T(n)$. Note that with this construction, any two nodes inside the small circle are connected. If half of the nodes is randomly assigned inside the small circle as transmitters and the other half as receiver nodes, then the average number of transmitters and receivers in this circle are proportional to $\Theta(nT^2(n))$. Given the fact that this value also is the maximum possible number of transmitter and receiver nodes, the result follows immediately.

Combining Lemmas 5.1, 5.2 and 5.3, the upper bound for multicast capacity of MPT and MPR can be computed.

Theorem 5.4 *In wireless ad hoc networks with MPT and MPR, the upper bound on*



(a) SPR

(b) MPT and MPR

Figure 5.2: Upper bound of total available area based on protocol model with SPR and MPT plus MPR

the per-node throughput capacity of multicast with constant number of destinations is

$$C_m(n) = O\left(nT^3(n)\right) \tag{5.2}$$

5.2.2 Lower Bound

To derive an achievable lower bound, a TDMA scheme is used for random dense wireless ad hoc networks similar to the approach used in [40, 43].

The network area is divided into square cells. Each square cell has an area of $T^2(n)/2$, which makes the diagonal length of square equal to $T(n)$, as shown in Fig. 5.3. Under this condition, connectivity inside all cells is guaranteed and all nodes inside a cell are within communication range of each other. A cell graph is builded over the cells that are occupied with at least one vertex (node). Two cells are connected if there exist a pair of nodes, one in each cell, that are less than or equal to $T(n)$ distance apart. Because the whole network is connected when $T(n) = r(n) = \Omega\left(\sqrt{\log n/n}\right)$, it follows

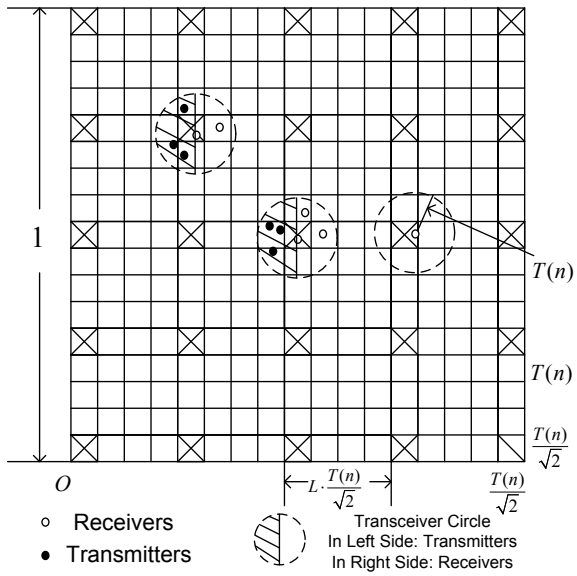


Figure 5.3: Cell construction used to derive a lower bound on capacity

that the cell graph is connected [40, 43].

To satisfy the MPT and MPR protocol model, cells are organized in groups so that simultaneous transmissions within each group does not violate the conditions for successful communication in the MPT and MPR protocol model. Let L represent the minimum number of cell separations in each group of cells that communicate simultaneously. Utilizing the protocol model, L satisfies the following condition:

$$L = \left\lceil 1 + \frac{T(n) + (1+\Delta)T(n)}{T(n)/\sqrt{2}} \right\rceil = \lceil 1 + \sqrt{2}(2+\Delta) \rceil \tag{5.3}$$

If time is divided into L^2 time slots and assign each time slot to a single group of cells, interference is avoided and the protocol model is satisfied. The separation example can be shown for the upper two receiver circles in Fig. 5.3. For the MPT and

MPR protocol model, the distance between two adjacent receiving nodes is $(2+\Delta)T(n)$. Because this distance is smaller than $(L-1)T(n)$, this organization of cells guarantees that the MPT and MPR protocol model is satisfied. Fig. 5.3 represents one of these groups with a cross sign inside those cells for $L = 4$. An achievable multicast capacity for MPT and MPR can be derived by taking advantage of this cell arrangement and TDMA scheme. The capacity reduction caused by the TDMA scheme is a constant factor and does not change the order capacity of the network.

Next the objective is to find an achievable lower bound using the Chernoff bound, such that the distribution of the number of edges in this unit space is sharply concentrated around its mean, and hence the actual number of simultaneous transmissions occurring in the unit space in a randomly chosen network is indeed $\Theta(n^2T^2(n))$ w.h.p..

Lemma 5.5 *The circular area of radius $T(n)$ corresponding to the transceiver range of any node j in the cross area in Fig. 5.3 contains $\Theta(nT^2(n))$ nodes w.h.p., and is uniformly distributed for all values of j, $1 \leq j \leq \frac{1}{(LT(n)/\sqrt{2})^2}$.*

Proof: The statement of this lemma can be expressed as

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\frac{1}{(LT(n)/\sqrt{2})^2}} |N_j - E(N_j)| < \delta E(N_j)\right] = 1, \tag{5.4}$$

where N_j and $E\left(N_j\right)$ are the random variables that represent the number of transmitters in the receiver circle of radius $T(n)$ centered by the receiver j and the expected value

of this random variable respectively, and δ is a positive arbitrarily small value close to zero.

From the Chernoff bound in Eq. (2.6), for any given $0 < \delta < 1$, $\theta > 0$ can be found such that $P\left[|N_j - E(N_j)| > \delta E(N_j)\right] < e^{-\theta E(N_j)}$. Thus, it can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n \to \infty$. This is a key step in showing that when all the events $\bigcap_{j=1}^{\frac{1}{(LT(n)/\sqrt{2})^2}} |N_j - E(N_j)| < \delta E(N_j)$ occur simultaneously, then all N_j's converge uniformly to their expected values. Utilizing the union bound, it arrives at

$$\begin{aligned} P\left[\bigcap_{j=1}^{\frac{1}{(LT(n)/\sqrt{2})^2}} |N_j - E(N_j)| < \delta E(N_j)\right] &\geq 1 - \sum_{j=1}^{\frac{1}{(LT(n)/\sqrt{2})^2}} P\left[|N_j - E(N_j)| > \delta E(N_j)\right] \\ &> 1 - \frac{1}{(LT(n)/\sqrt{2})^2} e^{-\theta E(N_j)}. \end{aligned} \tag{5.5}$$

Given that $E(N_j) = \pi n T^2(n)$, then

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\frac{1}{(LT(n)/\sqrt{2})^2}} |N_j - E(N_j)| < \delta E(N_j)\right] \geq 1 - \lim_{n\to\infty} \frac{1}{(LT(n)/\sqrt{2})^2} e^{-\theta \pi n T^2(n)} \tag{5.6}$$

Utilizing the connectivity criterion, $\lim_{n\to\infty} \frac{e^{-\theta\pi n T^2(n)}}{T^2(n)} \to 0$, which finishes the proof.

Furthermore, all of the nodes can be arranged in the left side of the corresponding transceiver circle be the transmitters, and all of the nodes in the right side of the corresponding transceiver circle be the receivers. Thus, the following lemma arrives at

Lemma 5.6 *In the unit square area for a wireless ad hoc network shown in Fig. 5.3, the total number of transmitter-receiver links (simultaneous transmissions) is* $\Omega\left(n^2T^2(n)\right)$.

Proof: From Lemma 5.5, for any node in the cross cell in the whole network shown in Fig. 5.3, there are $\Theta(nT^2(n))$ nodes in the transceiver circle. the total nodes are divided into two categories, transmitters in the left of the transceiver circles and receivers in the right of the transceiver circles. To guarantee all of the transmitters and receivers are in the transceiver range, only the nodes in the circle with radius $T(n)/2$ are considered. Because of the MPT and MPR capabilities, so that every transmitter in the left of the transceiver circle with $T(n)/2$ radius can transmit successfully to every receiver in the right, then the total number of successful transmissions is $\pi^2n^2T^4(n)/16$ which is the achievable lower bound. The actual number of the transmissions can be much larger than this because only $T(n)/2$ is considered instead of $T(n)$. Using the Chernoff Bound in Eq. 2.6 and Lemma 5.5, it can be shown that the total number of successful transmissions is

$$\Omega\left(\frac{1}{\left(LT(n)/\sqrt{2}\right)^2} \times \frac{\pi^2n^2T^4(n)}{16}\right) = \Omega\left(n^2T^2(n)\right). \tag{5.7}$$

The above results enables us to obtain the following achievable lower bound.

Let us define $\overline{\#\text{MEMTC}(T(n))}$ as the total number of cells that contain all the nodes in a multicast group. Note that $\overline{\#\text{MEMTC}(T(n))}$ also represents the average number of channel uses required to transport a packet from a source to its m destinations in a multicast tree. The following lemma establishes the achievable lower bound for the multicast throughput capacity of MPT and MPR as a function of $\overline{\#\text{MEMTC}(T(n))}$.

Lemma 5.7 *The achievable lower bound of the multicast capacity is given by*

$$C_m(n) = \Omega\left(\frac{nT^2(n)}{\overline{\#MEMTC(T(n))}}\right). \tag{5.8}$$

Proof: There are $(T(n)/\sqrt{2})^{-2}$ cells in the unit square network area. From the definition of $\overline{\#\text{MEMTC}(T(n))}$ and the fact that the TDMA scheme does not change the order capacity, it is clear that there are at most in the order of $\overline{\#\text{MEMTC}(T(n))}$ interfering cells for multicast communication. Hence, from Lemma 5.6, there are a total of $\Theta\left(n^2T^2(n)\right)$ nodes transmitting simultaneously, which are distributed over all the $(T(n)/\sqrt{2})^{-2}$ cells. Accordingly, the total lower bound capacity is given by $\Omega\left(\left(n^2T^2(n)\right) \times \left(\overline{\#\text{MEMTC}(T(n))}\right)^{-1}\right)$ which is the ratio between the total number of active links at any time divided by the number of channel uses required to complete a multicast communication group. Normalizing this value by total number of nodes in the network, n, proves the lemma.

Given the above lemma, to express the lower bound of $C_m(n)$ as a function of network parameters, the upper bound of $\overline{\#\text{MEMTC}(T(n))}$ need to be computed.

Lemma 5.8 *The average number of cells covered by a multicast tree, $\overline{MEMTC(T(n))}$, is upper bounded as*

$$\overline{\#MEMTC(T(n))} = O\left(\frac{\sqrt{m}}{T(n)}\right) = O\left(\frac{1}{T(n)}\right). \tag{5.9}$$

Proof: Because $T(n)$ is the transceiver range of the network, the maximum number of cells for this multicast tree must be $O\left(\sqrt{m}T^{-1}(n)\right)$, i.e., $\overline{\#\text{MEMTC}\left(T(n)\right)} = O\left(\sqrt{m}T^{-1}(n)\right)$. This upper bound can be achieved only if every two adjacent nodes in

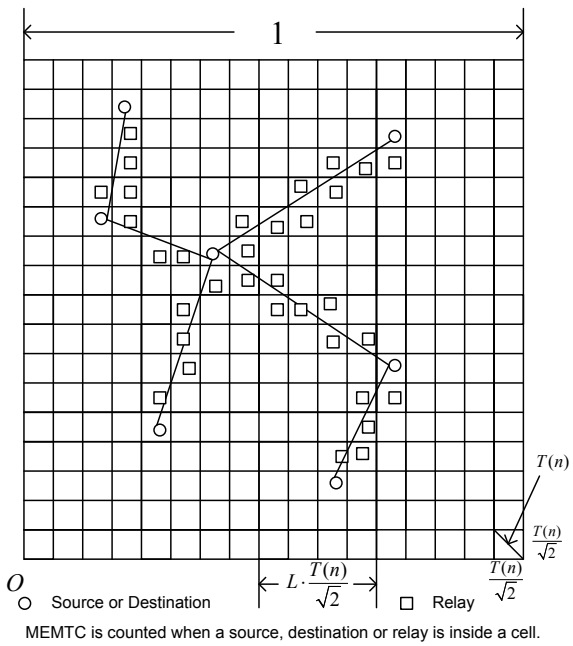


Figure 5.4: Illustration of $\overline{\#\text{MEMTC}(T(n))}$, the Euclidean distance of neighbor relay is smaller than $T(n)$

the multicast tree belong to two different cells in the network. However, in practice, it is possible that some adjacent nodes in multicast tree locate in a single cell. Consequently, this value is upper bound as described in (5.9). Note that the optimum multicast tree in wireless ad hoc network, may not necessarily cover the same route. However, since the intention is to derive the achievable lower bound, a scheme that follows the MEMT routing can be designed, so that each relay in that real routing tree is in the cells which is crossed by MEMT or the neighbor cell of MEMT. Therefore, all of those cells including those real relays as MEMTC are counted (see Fig. 5.4). Since as described later, this technique will provide the same order bound for the capacity as the upper bound, clearly the optimum multicast tree cannot achieve lower order bound.

Combining Lemmas 5.7 and 5.8, it arrives at the achievable lower bound of the multicast throughput capacity in dense random wireless ad hoc networks with MPT and MPR.

Theorem 5.9 *When the number of the destinations m is a constant, the achievable lower bound of the multicast throughput capacity with MPT and MPR is*

$$C_m(n) = \Omega\left(nT^3(n)\right). \tag{5.10}$$

Next the routing scheme is discussed to achieve the lower bound capacity which is similar to the scheme used in [41]. According to our model, each multicast session creates a multicast tree $\overline{\#\mathrm{MEMT}(T(n))}$ to connect the source and destinations. The trees are denoted as T_is, where $i = 1, 2, \cdots, n$. The routing scheme between source and destination is such that packets are forwarded by using cells that are intersected only by T_i. There is a bound on the number of trees that each cell needs to serve, which means that the probability that the trees intersects a particular cell can be bounded.

The following lemma with MPT and MPR case for multicast communications will be proved in the follows.

Lemma 5.10 *There has, for any $T(n) = \Omega\left(\sqrt{\log n/n}\right)$,*

$$\begin{aligned}&\lim_{n\to\infty} \mathit{Prob}\left(\sup_{(k,j)}\{\mathit{Number\ of\ trees\ T_i s\ intersecting\ S_{k,j}}\}\right.\\&\left.= O\left(nT(n)\right)\right) = 1\end{aligned} \tag{5.11}$$

Proof: For every tree T_i and cell S_{k_0,j_0}, with Lemma 4.20,

$$\begin{aligned} p &= \text{Prob}\{\text{Tree } T_i \text{ intersects } S_{k_0,j_0}\} \\ &= \Theta\left(T^2(n)\overline{\#\text{MEMTC}(T(n))}\right) \\ &= O(T(n)) \end{aligned} \tag{5.12}$$

First, the number of trees served by one particular cell S_{k_0,j_0} is bounded. Define i.i.d. random variables $I_i, 1 \leq i \leq n$, as follows:

$$I_i = \begin{cases} 1, \text{ if } T_i \text{ intersects } S_{k_0,j_0} \\ \\ 0, \text{ if not} \end{cases} \tag{5.13}$$

Then $\text{Prob}(I_i = 1) = p, \forall i$, where p is defined in Eq. (5.12). Denote by Z_n the total number of trees served by S_{k_0,j_0}. Then $Z_n := I_1 + I_2 + \cdots + I_n$. Thus by the Chernoff Bounds Eq. (2.6), for all positive b and a, $\text{Prob}(Z_n > b) \leq \frac{E[e^{aZ_n}]}{e^{ab}}$. Because of $1 + x \leq e^x$, there has

$$E[e^{aZ_n}] = (1 + (e^a - 1)p)^n \leq \exp(n(e^a - 1)p) \tag{5.14}$$

$$= O(\exp(e^a - 1)nT(n)) \tag{5.15}$$

Now choosing $b = \Theta(nT(n))$, it gets $\text{Prob}(Z_n = \Omega(nT(n))) = O(\exp(nT(n)))$. Thus by the union bound, it has

$$\begin{aligned} &\text{Prob}\left(\text{Some cell intersects } \Omega(nT(n)) \text{ trees }\right) \\ \leq\ & \sum_{k,j} \text{Prob}\left(\text{Cell } S_{jk} \text{ intersects more than } \Omega(nT(n)) \text{ trees }\right) \\ =\ & O\left(\frac{1}{T^2(n)} \exp\left(-nT(n)\right)\right) \end{aligned} \tag{5.16}$$

The right hand side tends to zero for $T(n) = \Omega\left(\sqrt{\log n/n}\right)$ as n goes to infinity.

It is known that there exists a transmitting schedule such that in every L^2 (L is constant) slots, each cell transmits at rate W bits/second with maximum transmission distance $T(n)$. Therefore, the rate for each cell is $\Theta\left(n^2T^4(n)\right)W/L^2$. From Lemma 5.10, each cell needs to transmit at rate $O\left(C_m(n)nT(n)\right)$, with probability approaching one. In order to accommodate this requirement by all cells, it is needed that

$$C_m(n)nT(n) = \Omega\left(\left(n^2T^4(n)\right)W/L^2\right) \tag{5.17}$$

Thus it have been proven that the achievable throughput for Theorem 4.23 in order to guarantee each cell can support this capacity. It can be written as

$$C_m(n) = \Omega\left(nT^3(n)\right) \tag{5.18}$$

It has been proved that there is no congestion in relay nodes. Furthermore, it will be proved that there is not any congestion in destination. Suppose each source selects a destination randomly and in dependently. Then it will be proved with high probability, a node can be destination for at most $\frac{3\log n}{\log\log n}$ sources. This problem is similar to the " bins and balls problems" in [42].

Lemma 5.11 *The probability of a particular destination having k sources selected is*

$$\lim_{n\to\infty} Prob[\textit{destination } i \textit{ has at least } k \textit{ sources}] \leq \left(\frac{e}{k}\right)^k \tag{5.19}$$

Proof: If observing any subset of sources of size k, then the probability that the subset of sources select destination i is $(\frac{1}{k})^k$. Then a union bound of these probabil-

ities is taken over all $\binom{n}{k}$ subsets of size k. The events are summed over, though, are not disjoint. Therefore, we can only show that the probability of a destination having at least k balls is at most $\binom{n}{k}\binom{n}{1}^k$. Using Stirling's approximation $\lim_{n\to\infty} \frac{n!}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n} = 1$, it has

$$\lim_{n\to\infty} \frac{\binom{n}{k}\binom{n}{1}^k}{\left(\frac{e}{k}\right)^k} = 1, \tag{5.20}$$

which proved the lemma.

Lemma 5.12 *With high probability, i.e. with probability greater than* $1 - \frac{1}{n}$*, there exist at most* $\frac{3\log n}{\log\log n}$ *sources for each destination.*

Proof: Let $k = \frac{3\log n}{\log\log n}$. From Lemma 5.11, it has

$$\begin{aligned}
& \lim_{n\to\infty} \text{Prob[destination } i \text{ has at least } k \text{ sources]} \\
\leq\ & \left(\frac{e}{k}\right)^k = \left(\frac{e\log\log n}{3\log n}\right)^{\frac{3\log n}{\log\log n}} \\
\leq\ & \exp\left(\frac{3\log n}{\log\log n}(\log\log\log n - \log\log n)\right) \\
=\ & \exp\left(-3\log n + \frac{3\log n\log\log\log n}{\log\log n}\right) \\
\leq\ & \exp(-2\log n) = \frac{1}{n^2}
\end{aligned} \tag{5.21}$$

Using Union Bound, it has

$$\begin{aligned}
& \lim_{n\to\infty} \text{Prob[any destination has at least } k \text{ sources]} \\
\leq\ & n\frac{1}{n^2} = \frac{1}{n}
\end{aligned} \tag{5.22}$$

which implies that

$$\lim_{n\to\infty} \text{Prob[all destinations have at most } k \text{ sources]} \geq 1 - \frac{1}{n} \tag{5.23}$$

It proved the lemma.

For MPTR unicast, it is required that for all destinations, there does not exist any one whose traffic load congestion is larger than the total throughput it can support. It means that the maximum throughput for each destination should always be greater than the total traffic load. In MPTR case, the total throughput of each destination is $nT^2(n)$. The traffic load congestion for each destination is the multiplication of throughput per node of $nT^3(n)$ and the maximum possible sources that select a node, i.e. $\frac{3\log n}{\log\log n}$. Hence,

$$nT^2(n) > nT^3(n)\frac{3\log n}{\log\log n}, \tag{5.24}$$

which is

$$T(n) = O\left(\frac{\log\log n}{\log n}\right). \tag{5.25}$$

Therefore, $T(n)$ is bounded as

$$\Omega\left(\sqrt{\frac{\log n}{n}}\right) = T(n) = O\left(\frac{\log\log n}{\log n}\right). \tag{5.26}$$

The left side is the connectivity constraint and the right side is the traffic load constraint to guarantee that $C(n) = nT^3(n)$ can be achieved.

5.2.3 Tight Bound and Comparison with SPR

From Theorems 5.4 and 5.9, a tight bound throughput capacity can be provided for multicasting when nodes have MPT and MPR capabilities in dense random wireless ad hoc networks as follows.

Theorem 5.13 *The throughput capacity of multicast with constant number m destinations (i.e., m is not a function of n) in a random dense wireless ad hoc network with MPT and MPR is*

$$C_m^{MPT+MPR}(n) = \Theta\left(nT^3(n)\right). \tag{5.27}$$

The transceiver range of MPT and MPR should satisfy $\Omega\left(\sqrt{\frac{\log n}{n}}\right) = T(n) = O\left(\frac{\log\log n}{\log n}\right)$.

Following similar proof procedure, the tight capacity for MPT or MPR only can be derived in the following theorem.

Theorem 5.14 *The throughput capacity of multicast with constant m number of destinations in a random dense wireless ad hoc network with MPT or MPR is given by*

$$C_m^{MPT}(n) = C_m^{MPR}(n) = \Theta\left(T(n)\right). \tag{5.28}$$

The transceiver range of MPT and MPR should satisfy $\Omega\left(\sqrt{\frac{\log n}{n}}\right) = T(n) = O\left(\frac{\log\log n}{\log n}\right)$.

The multicast throughput capacity with SPR is given by the following lemma.

Lemma 5.15 *In multicast with a constant m number of destinations, without MPR or MPR ability, the capacity is*

$$C_m^{PTP}(n) = \Theta\left(\frac{1}{nr(n)}\right) \tag{5.29}$$

where, $r(n) = \Omega\left(\sqrt{\log n/n}\right)$, *and PTP means SPR. When* $r(n) = \Theta\left(\sqrt{\log n/n}\right)$ *for the minimum transmission range to guarantee the connectivity, then the maximum capacity is obtained as* $C_m^{PTP\text{-}Max}(n) = \Theta\left(\frac{1}{\sqrt{n\log n}}\right)$.

Combining Theorem 5.13 with Lemma 5.15, the gain of throughput capacity with MPT and MPR capability in wireless ad hoc networks can be stated as follows.

Theorem 5.16 *In multicast with a constant m number of destinations, with MPT and MPR ability, the gain of per-node throughput capacity compared with SPR is $\Theta\left(n^2T^4(n)\right)$ (with only MPT or MPR, the gain is $\Theta\left(nT^2(n)\right)$), where, $\Omega\left(\sqrt{\frac{\log n}{n}}\right) = T(n) = r(n) = O\left(\frac{\log\log n}{\log n}\right)$. When $T(n) = \Theta\left(\sqrt{\log n/n}\right)$, the gain of per-node capacity is at least $\Theta\left(\log^2 n\right)$ (with only MPT or MPR, the gain is $\Theta(\log n)$).*

5.3 Capacity with NC, MPT and MPR with Finite m

This section studies the multi-source multicast capacity of a wireless network when nodes use NC, MPT and MPR. The results presented serve as an upper-bound for what can be achieved by combining NC, MPT and MPR in the presence of interference. The arguments are generic and can be used to deduce upper bounds for the multicast capacity of other interesting cases where NC is used along with only one of MPT or MPR, or even the scenario where NC is used with traditional SPR.

In the proof, the characteristic of network coding is used which does not change the capacity information flow across the sparsity cut and has been widely used in [21].

The bounds for the case of multi-source multicasting is deduced by reducing it to a suitable unicast routing problem. Under the reduction, an upper bound for the unicast problem also serves for the original multicast routing problem. Thus consider the following simple yet powerful lemma.

Lemma 5.17 *Consider a network with n nodes $V = \{a_1, \ldots, a_n\}$ and k multicast sessions. Each session consists of one of the n nodes acting as a source with an arbitrary finite subset of V acting as the set of destinations. Let s_i be the source of the ith session and let $D_i = \{d_{i1}, \ldots, d_{im_i}\}$ be the set of m_i destinations. Let $\lambda = [\lambda_1, \ldots \lambda_k]$ be a feasible rate vector, i.e. there exists a joint routing-coding-scheduling scheme that can realize a throughput of λ_i for the ith session. Then λ is also a feasible vector for any unicast routing problem in the same network such that the traffic consists of k unicast sessions with s_i being the source of the ith session and the destination d_i is any arbitrary element of the set D_i.*

If a multicast capacity from a source to multiple destinations is feasible, then clearly it is feasible to achieve the same capacity to any one arbitrarily chosen node from this set of destinations.

Lemma 5.18 *Consider a random geometric network with n nodes distributed uniformly in a unit square. Consider a decomposition of the unit-square into two disjoint regions R and R^c such that the area of each region is of order $\Theta(1)$. Now consider a multicast traffic scenario consisting of n sessions with each node being the source of a session and m randomly chosen nodes being the destination of the session. A source satisfies property P if the source belongs to region R and at least one of its destination belongs to R^c OR if the source belongs to region R^c and at least one of its destination belongs to R. It can be easily shown that the number of sources satisfying property P are $\Theta(n)$.*

Since nodes are uniformly deployed in a random network, a sparsity cut cap-

tures the traffic bottleneck of these random networks. The cut capacity represents the information rate that the nodes from one side of the cut as a whole can deliver to the nodes at the other side. This is the maximum information (bits per second) that can be transmitted across the cut from left to right (or from right to left). The sparsity cut capacity is upper bounded by deriving the maximum number of simultaneous transmissions across the cut.

Lemma 5.19 *The capacity of a sparsity cut Γ for a unit region has an upper bound of $O\left(T^{-1}(n)\right)$, $O\left(nT(n)\right)$, and $O\left(n^2T^3(n)\right)$ for SPR with network coding, MPT or MPR with network coding and MPT+MPR with network coding respectively.*

Proof: The capacity for SPR with network coding has been derived in [21] as $O\left(T^{-1}(n)\right)$. According to the protocol model of [1], the disks of radius $T(n)$ centered at each receiver are disjoint. This fact has been utilized in [21]. However, [1] does not consider many-to-one (or one-to-many) communications, which is the case for MPR (or MPT) scheme. Hence, some additional arguments are needed to prove the remaining claims.

Let us consider the combination of only MPR with NC. The cut capacity is upper bounded by the maximum number of simultaneous transmissions across the cut. It is easy to see in Fig. 3.2 that all the nodes located in the shaded area S_{xy} can send their packets to the receiver node located at (x, y). These nodes lie in the left side of the cut Γ within an area called S_{xy} and the assumption is that all these nodes are sending packets to the right side of the cut Γ. For a node at location (x, y), any node in the disk

of radius $T(n)$ can transmit information to this receiver simultaneously and the node can successfully decode those packets. In order to obtain an upper bound, only edges that cross the cut need to be considered. Let's first consider all possible nodes that can transmit to the receiver node in the S_{xy} region. The average number of transmitters located in S_{xy} is $n \times S_{xy}$. The number of nodes that are able to transmit at the same time from left to right is upper bounded as a function of S_{xy}.

The area of S_{xy} is computed as

$$S_{xy} = \frac{\theta}{2\pi}\pi T^2(n) - T^2(n)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = \frac{1}{2}T^2(n)(\theta - \sin\theta). \tag{5.30}$$

This area is maximized when $\theta = \pi$.

$$\max_{0\leq\theta\leq\pi}[S_{xy}] = \frac{1}{2}\pi T^2(n) \tag{5.31}$$

For the case of MPR (or MPT) with network coding, the disk with radius $T(n)$ centered at any receiver(transmitter) should be disjoint from the other disks centered at the other receivers[21]. Thus, the total number of nodes that can send packets across the cut is upper bounded as

$$\frac{l_\Gamma}{(2+\Delta)T(n)}\frac{1}{2}\pi T^2(n)n = c_1 l_\Gamma n T(n), \tag{5.32}$$

where $c_1 = \pi/2(2+\Delta)$. Similar arguments can be used to bound the cut-capacity when only MPT is combined with NC.

Now, the case of combining MPT+MPR with network coding is considered. Note that each node has a maximum of $\Theta\left(nT^2(n)\right)$ neighbors, which implies that each node can simultaneously receive packets from a maximum of $\Theta\left(nT^2(n)\right)$ transmitters.

Moreover, any node that receives transmission from across the cut Γ has to lie in the region enclosed by the dotted line Γ_r on the right side in Fig. 3.2, where Γ_r is at a $T(n)$ from Γ. Thus, the rectangular region enclosed by Γ and Γ_r has an area of $\Theta(T(n))$. Hence, an average of $\Theta(nT(n))$ nodes can receive packets from across the cut. Therefore the total number of transmissions across the cut are bounded by

$$O\left(nT(n) * nT^2(n)\right) = O\left(n^2T^3(n)\right) \tag{5.33}$$

Theorem 5.20 *In a wireless ad hoc network formed by n nodes distributed randomly in a unit square with traffic formed by each node acting as source for a multicast sessions with $m = \Theta(1)$ randomly chosen nodes as destinations, the per-session multicast capacities are*

$$\begin{aligned} C_m^{NC+PTP} &= O\left(\frac{1}{nT(n)}\right), \\ C_m^{NC+MPT} &= C_m^{NC+MPR} = O\left(T(n)\right), \\ C_m^{NC+MPT+MPR} &= O\left(nT^3(n)\right), \end{aligned} \tag{5.34}$$

where NC + PTP denotes the use of NC with SPR (no MPT or MPR), i.e., a node can only transmit or receive at most one packet at a time.

Proof: For any sparsity cut of the unit area, lemmas 5.18 and 5.17 tell us that a unicast routing problem can be constructed satisfying the property that any rate for the unicast problem is feasible for the original multicast problem and there are $\Theta(n)$ source-destination pairs across the cut. Thus, the capacity of the sparsity cut provides a bound for the unicast problem, which can in turn be used to provide an upper bound for

the multicast problem. From Lemma 5.19, finally, such arguments can be extended to show that the combination of NC+MPT (or NC+MPR) and NC+MPT+MPR allows us to simultaneously transmit a maximum of $O\left(nT(n)\right)$ and $O\left(n^2T^3(n)\right)$ packets across the cut respectively. The result of the theorem then follows from the fact that the cut capacity has to be divided among the $\Theta(n)$ source-destination pairs across the cut. Finally, because multicast capacity must be upper bounded by the unicast capacity which finishes the proof.

5.4 Conclusion

By combining the results from theorems 5.13 and 5.20, the main contribution of this chapter is stated in the following theorem.

Theorem 5.21 *In wireless ad hoc networks with multi-pair multicast sessions and with a finite number of m destinations for each source, the throughput capacity utilizing NC, MPT and MPR capabilities for all nodes is the same order as when the nodes are endowed only with MPT and/or MPR.*

$$\begin{aligned} C_m^{MPT+NC}(n) &= C_m^{MPT}(n) \\ C_m^{MPR+NC}(n) &= C_m^{MPR}(n) \\ C_m^{MPT+MPR+NC}(n) &= C_m^{MPT+MPR}(n) \end{aligned} \tag{5.35}$$

Proof: Because from Theorems 5.13 and 5.4, it is realized that the multicast capacity of NC with MPT and MPR is tightly bounded by the tight bound of multicast

capacity of MPT and MPR without NC, so the multicast capacity of MPT and MPR with or without NC are the same.

It is also important to emphasize that, as Theorem 5.20 shows, NC does not provide any order capacity gain for multi-source multicasting when the size of receiver groups is $m = \Theta(1)$ and nodes use SPR. Hence, the result in Theorem 5.21 implies that NC does not provide an order capacity gain when MPT or MPR or combination of MPR and MPT is used, and that MPT and MPR are the real contributing factors for order capacity increases in wireless ad hoc networks.

Chapter 6

Opportunistic Interference Management

In this chapter, a new multiuser diversity scheme is introduced in cellular networks which allows parallel communication in the network without any cooperation among mobile stations. If the network does not have enough mobile stations, then some of the users need to jointly decode their corresponding data streams. The result suggests the existence of a tradeoff between multiuser diversity and cooperation in the downlink of cellular networks. The opportunistic interference management (OIM) approach is based on a new multiuser diversity concept that achieves the capacity of dirty paper coding (DPC) asymptotically. Surprisingly, this gain is achieved without requiring full channel state information (CSI) and only number of antennas CSI are fed back from mobile stations to the base station. An additional advantage of this scheme is the fact that the encoding and decoding of signals for this distributed MIMO system is based on simple point-to-point communications. Furthermore, OIM has been extended in to wireless ad hoc networks, which increased the capacity significantly.

This chapter is constructed as follows. Sections 6.1 and 6.2 present the capacity of wireless cellular and ad hoc networks respectively. Section 6.3 concludes this chapter.

6.1 OIM in Wireless Cellular Networks

The problem of optimal transmission is investigated in the downlink of a cellular networks when the base station has independent messages for the mobile stations in the network.

6.1.1 Wireless Cellular Network Model

The problem of optimal transmission in the downlink of a cellular network is investigated when the base station has independent messages for the mobile stations in the network. Clearly if the base station has only K antennas, it can transmit at most K independent data streams at any given time. It is assumed that all mobile stations have a single antenna for communication. The channel between the base station and mobile stations $\mathbf{H}$ is a $M \times K$ matrix with elements h_{ji}, where $i \in [1, 2, \ldots, K]$ is the antenna index of the base station and $j \in [1, 2, \ldots, M]$ is the mobile user index. Block fading model is considered where the channel coefficients are constant during coherence interval of T. Then the received signal $\mathbf{Y}^{M\times 1}$ is expressed as

$$\mathbf{Y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{6.1}$$

where $\mathbf{x}$ is the transmit $K \times 1$ signal vector and $\mathbf{n}$ is the $M \times 1$ noise vector. The noise at each of the receive antennas is i.i.d. with $\mathcal{CN}(0, \sigma_n^2)$ distribution.

6.1.2 Scheduling protocol

During the first phase of communication, the base-station antennas sequentially transmit K pilot signals. In this period, all the mobile stations listen to these known messages. After the last pilot signal is transmitted, mobile stations evaluate the SNR for each antenna. If the SNR satisfies certain conditions for a mobile node, that particular mobile station will be selected by the base station. The mobile station is selected when the SNR for one transmit antenna is greater than a pre-determined threshold $\mathrm{SNR}_{\mathrm{tr}}$ and below another pre-determined threshold of $\mathrm{INR}_{\mathrm{tr}}$ for the remaining $K-1$ antennas.

In the second phase of communication, the mobile stations that satisfy SNR criteria will notify the base station that they have the required condition to receive packets during the remaining time period of T. It will not be discussed that the channel access protocol required for these mobile stations to contact the base station or the case when two mobile stations satisfy OIM condition for the same base station antenna. It is assumed that this will be resolved by some handshake between the mobile stations and the base station. Note that, if appropriate values for $\mathrm{SNR}_{\mathrm{tr}}$ and $\mathrm{INR}_{\mathrm{tr}}$ is chosen such that $\mathrm{SNR}_{\mathrm{tr}} \gg \mathrm{INR}_{\mathrm{tr}}$, then the base station can simultaneously transmit different packets from its antennas to different mobile stations. The mobile stations only receive their respective packets with a strong signal and can treat the rest of the packets as noise. The value of $\mathrm{SNR}_{\mathrm{tr}}$ (or $\mathrm{INR}_{\mathrm{tr}}$) can be selected as high (or low) as required for a given system, as long as M is large enough.

In general, there is a relationship between average number of antennas with OIM condition, $D = \mathbf{E}(d)$, and number of mobile stations, M. Clearly, OIM decreases the encoding and decoding complexity of MIMO broadcasting channel significantly[1] at the expense of the presence of large number of mobile stations. Fig. 6.1 demonstrates the system that is used here. Without loss of generality, it is assumed that the user i for $i \in [1, 2, \ldots, d]$ is assigned to antenna i in the base station. In this figure, solid and dotted lines represent strong and weak channels between an antenna at the base station and a mobile station respectively. Note that if there is no line between the base station and mobile stations, then it means the channel is a random parameter based on the channel probability distribution function. For simplicity, Fig. 6.1 only illustrates the strong channel case.

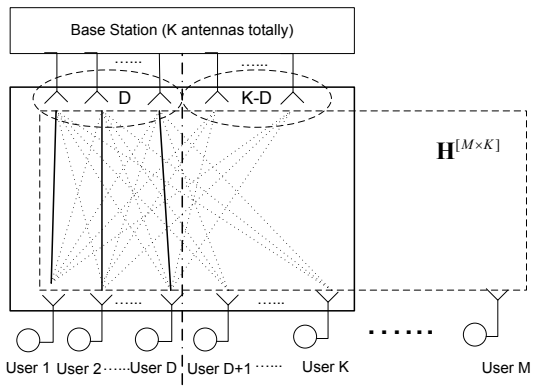


Figure 6.1: Wireless cellular network model

[1]For OIM technique, the encoding and decoding of multiple antennas reduces to simple point-to-point communication because the channels are decoupled from each other and no longer interfere significantly with each other.

6.1.3 Theoretical Analysis and Numerical Results

Let's define SNR_{ji} as the signal-to-noise ratio when antenna j is transmitting packet to mobile station i in the downlink. Further denote INR_{ji} as the interference-to-noise ratio between transmit antenna j and receiver i. The objective of OIM is to identify d mobile stations out of M choices to satisfy the following criteria

$$\begin{aligned} &\mathrm{SNR}_{ii} \geq \mathrm{SNR}_{\mathrm{tr}}, 1 \leq i \leq d, \\ &\mathrm{INR}_{ji} \leq \mathrm{INR}_{\mathrm{tr}}, 1 \leq j \leq K, 1 \leq i \leq d, j \neq i \end{aligned} \tag{6.2}$$

The above condition in (6.2) states that each one of the d mobile stations has a very good channel to a single antenna of the base station and strong fading to the other $K-1$ antennas of base station as shown in Fig. 6.1. After all the mobile users with OIM condition return their feedback to the base station, then the base station will select those mobile stations to participate in the communication phase such that the maximum multiplexing gain is achieved. Note that it is possible that two mobile users satisfy OIM condition for the same base station antenna.

The sum rate in the downlink of wireless cellular channel can be written as

$$\begin{aligned} R_{\text{proposed}} &= \sum_{i=1}^{d} \log\left(1 + \mathrm{SINR}_{ii}\right) \\ &= \sum_{i=1}^{d} \log\left(1 + \frac{\mathrm{SNR}_{ii}}{\sum_{j=1, j\neq i}^{d-1} \mathrm{INR}_{ji} + 1}\right) \\ &\geq d \log\left(1 + \frac{\mathrm{SNR}_{\mathrm{tr}}}{(K-1)\mathrm{INR}_{\mathrm{tr}} + 1}\right) \\ &= d \log(1 + \mathrm{SINR}_{\mathrm{tr}}) \end{aligned} \tag{6.3}$$

where SINR_{ii} and $\mathrm{SINR}_{\mathrm{tr}}$ are defined as

$$\mathrm{SINR}_{ii} = \frac{\mathrm{SNR}_{ii}}{\sum_{j=1,j\neq i}^{d-1} \mathrm{INR}_{ji} + 1}, \forall i = 1, 2, \cdots, d \tag{6.4}$$

and

$$\mathrm{SINR}_{\mathrm{tr}} = \frac{\mathrm{SNR}_{\mathrm{tr}}}{(K-1)\mathrm{INR}_{\mathrm{tr}} + 1}, \tag{6.5}$$

respectively.

First, the mean value of multiplexing gain d is derived. Then, it will be proved that for any value of $\mathrm{SINR}_{\mathrm{tr}}$, there exists a minimum value of M that satisfies Eq. (6.3). Finally, it is proved that our approach achieves the optimum capacity of DPC asymptotically.

For the rest of paper, the channel distribution is considered to be Rayleigh fading but OIM can be implemented for other time-varying channel distributions. Note that for an i.i.d. Rayleigh fading channel $\mathbf{H}$, the probability distribution function (pdf) of SNR (or INR) is given by [44]

$$p(\mathrm{z}) = \begin{cases} \frac{1}{\sigma} \exp\left(-\frac{z}{\sigma}\right), & z > 0 \\ 0, & z \leq 0 \end{cases} \tag{6.6}$$

where z is the SNR (or INR) value and $E_{\mathbf{H}}(z) = \sigma$, $\mathbf{Var}_{\mathbf{H}}(z) = \sigma^2$. Equivalently, $\sqrt{\sigma/2}$ is the parameter for Rayleigh fading distribution which shows the strength of the fading channel.

6.1.3.1 Exact Analysis

Let's define event A for any mobile station that satisfies the condition in Eq. (6.2). Since the channels between the base station and the mobile stations are i.i.d., then the probabilities of these two events can be derived as

$$\Pr(A) = \binom{K}{1} \int_{\mathrm{SNR_{tr}}}^{\infty} p(z)dz \left(\int_{0}^{\mathrm{INR_{tr}}} p(z)dz \right)^{K-1} = \binom{K}{1} e^{-\frac{\mathrm{SNR_{tr}}}{\sigma}} \left(1 - e^{-\frac{\mathrm{INR_{tr}}}{\sigma}}\right)^{K-1} \tag{6.7}$$

Our objective is to maximize this probability based on network parameters. Maximizing $\Pr(A)$ will minimize the number of required mobile stations M as will be proved later. Note that among all network parameters $K, \mathrm{SNR_{tr}}, \mathrm{INR_{tr}}$, and σ, the values of K and σ are really related to the physical properties of the network and are not design parameters. Further, the parameters $\mathrm{SNR_{tr}}$ and $\mathrm{INR_{tr}}$ can be replaced with a single parameter $\mathrm{SINR_{tr}}$ using Eq. (6.5).

Let X be the random variable related to the number of mobile stations satisfying the OIM condition for Eq. (6.2). Note that it is possible that two mobile stations satisfy OIM condition for the same base-station antenna. The probability of $X = x$ is computed as

$$\Pr(X = x) = \binom{M}{x} (\Pr(A))^{x} (1 - \Pr(A))^{M-x} . \tag{6.8}$$

It is planed to solve this problem by formulating it as "bins and balls" problem. Note that there are x balls that satisfy the OIM condition. The probability distribution of x is given in Eq. (6.8). Let's define the conditional probability of choosing y base-station antennas (or bins) when there are x mobile stations (or balls) satisfying the OIM

condition and denote it as $\Pr_B(d = y|X = x)$. Note that this probability includes the possibility that some of y antennas are not associated to any of x mobile stations and some correspond to more than one mobile station, i.e., some bins are empty and some bins have more than one ball in them. This conditional probability is equal to

$$\Pr_B(d = y|X = x) = \left(\frac{y}{K}\right)^x, \qquad y \leq K \tag{6.9}$$

Let's define $\Pr_C(d = y|X = x)$ the probability that all of x mobile stations are associated to y base-station antennas and there is no antenna in this set that is not associated to at least one of the x mobile stations. Then, this conditional probability can be derived as

$$\Pr_C(d = y|X = x) = \begin{cases} \Pr_B(d = 1|X = x), & y = 1 \\ \Pr_B(d = y|X = x) - \sum_{j=1}^{y-1} \binom{y}{j} & \\ \cdot (\Pr_C(d = j|X = x)), 1 < y \leq \min(x, K) & \\ 0. & y > \min(x, K) \end{cases} \tag{6.10}$$

This equation is derived iteratively and in order to initialize it for $y = 1$, $\Pr_B(d = 1|X = x)$ is utilized. Since $\Pr_C(d = y|X = x)$ represents the probability of selecting a specific combination of y antennas, the total possible choices can be derived as

$$\Pr_D(d = y|X = x) = \binom{K}{y} \Pr_C(d = y|X = x). \tag{6.11}$$

Finally, the expected value of d is derived using law of total probability.

$$\begin{aligned} D &= \mathbf{E}(d) = \sum_{y=1}^{K}\sum_{x=1}^{M} y\mathrm{Pr}_D(d=y|X=x)\mathrm{Pr}(X=x) \\ &= \sum_{x=1}^{M} \mathbf{E}(d|X=x)\mathrm{Pr}(X=x) \\ &= \sum_{x=1}^{M} \mathbf{E}(d|X=x)\binom{M}{x}(\mathrm{Pr}(A))^x(1-\mathrm{Pr}(A))^{M-x} \end{aligned} \tag{6.12}$$

where $\mathbf{E}(d|X=x)$ is defined as

$$\mathbf{E}(d|X=x) = \sum_{y=1}^{K} y\mathrm{Pr}_D(d=y|X=x). \tag{6.13}$$

6.1.3.2 Numerical Results

Our simulation results are based on exact analysis of interference management technique. Fig. 6.2 illustrates the minimum required value for M when D varies and for $k = 3$ or 5 and $\sigma = 100$. As it can be seed from this result, when the SINR_{tr} requirement increases, the number of mobile stations required to implement this technique increases significantly. Therefore, using capacity approaching techniques such as Turbo code or LDPC that requires very low SINR_{tr} will help to implement this technique with modest number of MS users. Besides, from this figure it is noticed that there is a tradeoff between the total number of the mobile stations M and the number of the nodes $K-D$ needed to do cooperative communication utilizing technique such as distributed MIMO. For example when $K = 3$, the capacity of the network increases twofold with only 100 mobile stations in the network.

Fig. 6.3 demonstrates the relationship between the minimum number of mobile

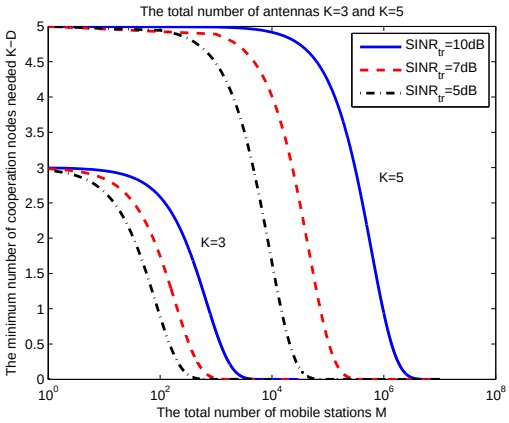


Figure 6.2: Simulation results for different values of SINR

stations required for different channel fading conditions. The result clearly shows that as the fading of the channel increases, the minimum required number for M decreases. As it is mentioned earlier, the new multiuser diversity scheme performs better when the fading strength in channel increases to take advantage of both strong and weak channels. Note that the original multiuser diversity concept performs better by only taking advantage of strong channels.

In order to reduce the minimum required number of mobile users further, each mobile user can be allowed to utilize two antennas and try to select one of the antennas that satisfies OIM condition. However, such increase in the number of antennas does not require space-time encoding or decoding. From base station point of view, the additional antenna for each mobile user is equivalent of increasing the number of mobile users twofold or equivalently, the actual minimum number of mobile users required to achieve a multiplexing gain is reduced by a factor of 2.

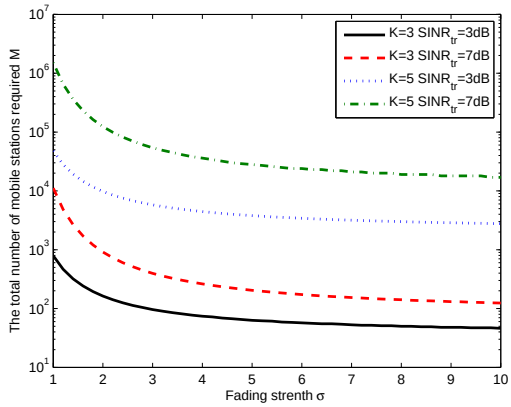


Figure 6.3: Simulation results for different fading channel environments and total number of mobile stations M required

6.1.3.3 Scaling Law Analysis

In this subsection, it will be proved that the sum-rate of the proposed scheme under OIM condition achieves the optimum asymptotic DPC capacity, i.e. $K \log \log M$. Let's define x as the number of mobile stations that satisfy Eq. (6.2). The probability that the first user associated to any of the antennas at the base station is $\Pr(A)$, and this probability for the second user is $\frac{K-1}{K}\Pr(A)$ and this probability can be similarly computed for all other users. The probability for the last (d^{th}) user to satisfy Eq. (6.2) is $\frac{K-d+1}{K}\Pr(A) > \frac{1}{K}\Pr(A)$. From this argument, it is clear that these probabilities are lower bounded as $\frac{1}{K}\Pr(A)$.

The lower bound for the expected value of d is given by

$$D = \mathbf{E}(d) \geq \frac{M}{K}\Pr(A). \tag{6.14}$$

It is noteworthy to mention again that the number of mobile stations that satisfy OIM condition is a random variable and D is simply the average value of this random variable.

Thus,

$$M \leq DK(\Pr(A))^{-1}. \tag{6.15}$$

Note that M is upper bounded by the inverse of $\Pr(A)$. Therefore, in order to minimize M, it is necessary to minimize $(\Pr(A))^{-1}$ such that the $\mathrm{SINR}_{\mathrm{tr}}$ condition in Eq. (6.5) is satisfied.

$$\text{minimize} \qquad (\Pr(A))^{-1} \tag{6.16}$$

$$\text{subject to} \qquad \mathrm{SINR}_{\mathrm{tr}} = \frac{\mathrm{SNR}_{\mathrm{tr}}}{(K-1)\mathrm{INR}_{\mathrm{tr}} + 1} \tag{6.17}$$

This optimization problem can be rewritten as

$$\begin{aligned}
\min_{Eq.(6.17)} \left((\Pr(A))^{-1}\right) &= \tfrac{1}{K} \min_{Eq.(6.17)} \left(\frac{e^{\frac{\mathrm{SNR}_{\mathrm{tr}}}{\sigma}}}{\left(1-e^{-\frac{\mathrm{INR}_{\mathrm{tr}}}{\sigma}}\right)^{K-1}} \right), \\
&\overset{(a)}{=} \tfrac{1}{K} e^{\frac{\mathrm{SINR}_{\mathrm{tr}}}{\sigma}} \min_{\mathrm{INR}_{\mathrm{tr}}} \left(\frac{e^{(K-1)\frac{\mathrm{SINR}_{\mathrm{tr}}\mathrm{INR}_{\mathrm{tr}}}{\sigma}}}{\left(1-e^{-\frac{\mathrm{INR}_{\mathrm{tr}}}{\sigma}}\right)^{K-1}} \right), \\
&\overset{(b)}{=} \tfrac{1}{K} e^{\frac{\mathrm{SINR}_{\mathrm{tr}}}{\sigma}} \sigma^{K-1} \min_{\mathrm{INR}_{\mathrm{tr}}} \left(\frac{e^{(K-1)\frac{\mathrm{SINR}_{\mathrm{tr}}\mathrm{INR}_{\mathrm{tr}}}{\sigma}}}{(\mathrm{INR}_{\mathrm{tr}})^{K-1}} \right).
\end{aligned} \tag{6.18}$$

The equality (a) is derived by replacing $\mathrm{SNR}_{\mathrm{tr}}$ with $\mathrm{INR}_{\mathrm{tr}}$ and $\mathrm{SINR}_{\mathrm{tr}}$ using Eq. (6.5). Since in practice a successful communication occurs when there is a predetermined minimum value for SINR, therefore the value of $\mathrm{SINR}_{\mathrm{tr}}$ is fixed and attempt to optimize the above equation based on $\mathrm{INR}_{\mathrm{tr}}$. The limitation in (b) is derived by assuming $\frac{\mathrm{INR}_{\mathrm{tr}}}{\sigma} \to 0$ and the fact that $\lim_{x\to 0}(1-\exp(-x)) = x$. Note that the unique characteristic of this new scheme is to take advantage of strong fading and clearly, under that circumstance the value of $\frac{\mathrm{INR}_{\mathrm{tr}}}{\sigma}$ is small.

The minimum value of $\left(\frac{e^{\frac{(K-1)\mathrm{SINR}_{\mathrm{tr}}}{\sigma}\mathrm{INR}_{\mathrm{tr}}}}{\mathrm{INR}_{\mathrm{tr}}^{K-1}}\right)$ can be derived by taking its first derivative with respect to INR_{tr} and making it equal to zero.

$$e^{\frac{(K-1)\mathrm{SINR}_{\mathrm{tr}}}{\sigma}\mathrm{INR}_{\mathrm{tr}}} \times \left(\frac{(K-1)\mathrm{SINR}_{\mathrm{tr}}}{\sigma}\mathrm{INR}_{\mathrm{tr}}^{K-1} - (K-1)\mathrm{INR}_{\mathrm{tr}}^{K-2}\right) = 0 \tag{6.19}$$

Note that $\mathrm{SINR}_{\mathrm{tr}}$ is usually a pre-determined variable for most applications and it is needed to optimize this equation with respect to $\mathrm{INR}_{\mathrm{tr}}^{*}$. The solution for $\mathrm{INR}_{\mathrm{tr}}^{*}$ is

$$\mathrm{INR}_{\mathrm{tr}}^{*} = \frac{\sigma}{\mathrm{SINR}_{\mathrm{tr}}}. \tag{6.20}$$

Then the optimum value for $(\Pr(A))^{-1}$ is given by

$$M^{*} \leq DK(P^{*}(A))^{-1} = De^{\frac{\mathrm{SINR}_{\mathrm{tr}}}{\sigma}}\left(\mathrm{SINR}_{\mathrm{tr}}e\right)^{K-1}. \tag{6.21}$$

Now the asymptotic behavior of the network (i.e. $M \to \infty$) is investigated and the maximum achievable capacity and scaling laws for this scheme are tried to be computed. When M tends to infinity, $\mathrm{SINR}_{\mathrm{tr}}$ goes to infinity too. Note that the property that $\lim_{x\to\infty}\frac{x^c}{e^x} = 0$ is used, where c is a constant, then

$$\lim_{\mathrm{SINR}_{\mathrm{tr}}\to\infty} e^{\frac{\mathrm{SINR}_{\mathrm{tr}}}{\sigma}}(\mathrm{SINR}_{\mathrm{tr}})^{K-1} = O\left(e^{2\frac{\mathrm{SINR}_{\mathrm{tr}}}{\sigma}}\right). \tag{6.22}$$

Therefore, the value of $(\Pr(A))^{-1}$ is asymptotically derived as

$$\Omega\left(\frac{M}{DK}\right) = \lim_{M\to\infty}(\Pr(A))^{-1*} = \frac{1}{K}e^{K-1}e^{\frac{\mathrm{SINR}_{\mathrm{tr}}}{\sigma}}(\mathrm{SINR}_{\mathrm{tr}})^{K-1} = O\left(\frac{1}{K}e^{K-1}\cdot e^{2\frac{\mathrm{SINR}_{\mathrm{tr}}}{\sigma}}\right). \tag{6.23}$$

The lower bound of $\mathrm{SINR}_{\mathrm{tr}}$ is asymptotically computed as

$$\lim_{M\to\infty} \mathrm{SINR}_{\mathrm{tr}}^{\max} = \Omega\left(\frac{\sigma}{2}\log\left(\frac{1}{D}\left(\frac{1}{e}\right)^{K-1} M\right)\right) = \Omega(\log M). \tag{6.24}$$

Thus, the $\mathrm{SINR}_{\mathrm{tr}}^{\max}$ scales at least with $\Omega(\log M)$. If it is assumed that $\mathrm{SINR}_{\mathrm{tr}} = \Theta\left(\frac{\sigma}{4}\log M\right) = \Theta\left(\frac{\sigma}{2}\log M^{1/2}\right)$. and with Eq. (6.23), because $D \leq K$, then there is $M^{1/2} = O\left(De^{K}\right) = O\left(Ke^{K}\right) = O\left(e^{2K}\right)$. This result implies that $K = \Omega\left(\log M\right)$ is achievable. Then the scaling laws of OIM scheme is

$$R_{\mathrm{proposed}} = \Omega(K \log\log M). \tag{6.25}$$

It have been proved that OIM achieves DPC asymptotic capacity.

$$\lim_{M\to\infty} R_{\mathrm{OIM}} = R_{\mathrm{DPC}} = \Theta(K \log\log M) \tag{6.26}$$

This result implies that

$$\lim_{M\to\infty} \mathrm{SINR}_{\mathrm{tr}} = \Theta(\log M). \tag{6.27}$$

Our objective is to show, via simulation, that when $\mathrm{SINR}_{\mathrm{tr}}$ grows proportional to $\Theta(\log M)$, the maximum multiplexing gain of k can be achieved when M tends to infinity. Let's define $\mathrm{SINR}_{\mathrm{tr}}$ as

$$\mathrm{SINR}_{\mathrm{tr}} = \frac{\sigma}{c_0}\log\left(\left(\frac{1}{e}\right)^{K-1} M\right). \tag{6.28}$$

where c_0 is a constant value. In practical cellular systems, it is possible that the minimum number of mobile users may not be available in a cell. Note that it is easy to show that for any value of K, M and σ, the designer can select the appropriate value for

$\mathrm{SINR}_{\mathrm{tr}}$ such that the maximum multiplexing gain is achieved at the expense of reduced rate for each individual mobile user, i.e., $D = K$.

Fig. 6.4 confirms that when $\mathrm{SINR}_{\mathrm{tr}}$ grows logarithmically with M, this approach achieves the maximum multiplexing gain for different values of c_o based on Eq. (6.28).

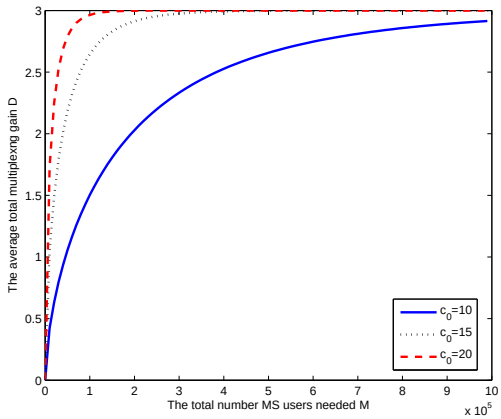


Figure 6.4: Simulation results demonstrate DPC capacity and maximum multiplexing gain are achieved simultaneously.

It is noteworthy to point out that when the value of σ is small or equivalently, if the channel fading is not strong, then OIM cannot converge to the maximum multiplexing gain of K rapidly. In the new multiuser diversity scheme that is introduced in this paper, both strong and weak channels are important. When the fading coefficient σ is stronger, then this technique performs better. Fig. 6.5 illustrates this important point.

When $K = 1$, then our approach is similar to that of [30]. Moreover if $M \to \infty$ and $D = K$, then our scheme has the same asymptotic scaling laws capacity result as

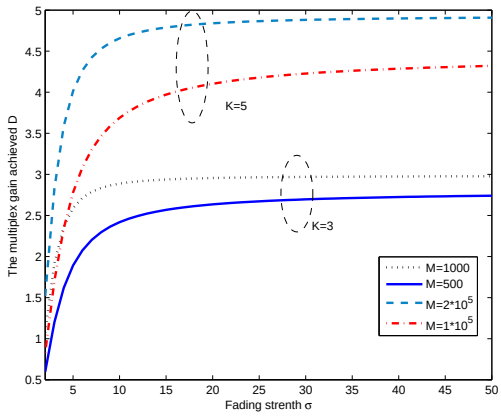


Figure 6.5: Simulation results demonstrate relationship between fading strength and multiplexing gain.

that of [34]. The cost of the proposed scheme is the need for a minimum number of mobile stations, M. In most practical cellular systems, in any given frequency and time inside a cell, there is only one assigned mobile station while this technique suggests that there can be up to the number of base-station antennas utilizing the same spectrum at the same time with no bandwidth expansion. Clearly, this approach can increase the capacity of wireless cellular networks significantly. This gain is achieved with modest feedback requirement which is proportional to the number of antennas at the base station.

6.1.3.4 Feedback requirements

A natural question regarding our OIM scheme is what the number of MS users is that satisfies the interference management criterion. Clearly, this number is a random variable, which is denoted by X. It will be proved that this value is at most

K with probability arbitrarily close to one if the network parameters are appropriately selected. More specifically, the probability that $X \leq K$ MS users satisfy the interference management criteria denoted as η can be arbitrarily close to 1 if proper SINR_{tr} is selected based on network parameters such as fading parameter σ and M.

For any MS, the probability that it satisfies the interference management condition is $K \times \Pr(A)$, i.e., the MS has a very strong channel with a single BS antenna and a very weak channel (deep fade) with all other BS antennas. The number of the MSs satisfying the interference management criteria is a random variable X satisfying binomial distribution whose probability density function (pdf) is given by Eq. (6.8). Therefore, the cumulative distribution function can be expressed as

$$\Pr(X \leq K) \quad = \quad \sum_{i=0}^{K} \binom{M}{i} (\Pr(A))^i (1 - \Pr(A))^{M-i} \geq \eta, \tag{6.29}$$

where $0 < \eta < 1$ can be arbitrarily close to 1 , i.e., $\eta = 99\%$.

It will be shown that the number of mobile users X (which is a random variable) with OIM constraint is always smaller than K with probability arbitrarily close to 1 with the correct choice of network parameters. Note that, for any value of K, M and σ, the designer can select the appropriate value for SINR_{tr} such that with probability close to 1 the value of random variable X is less than K as numerically shown in Fig. 6.6. Given that the number of active MSs in a cell is known to the BS, the BS can adjust the SINR_{tr} value such that the number of MS users qualifying the OIM condition does not increase significantly. This is a significant improvement compared to the dirty paper coding or techniques introduced in [34, 35], which require $K \times M$ and M CSI feedback

information respectively. When M increases, the feedback information also increases accordingly. However, OIM requires $\Theta(K)$ CSI feedback regardless of the number of mobile stations with probability arbitrarily close to 1 as long as the SINR_{tr} is adjusted appropriately. For any values of K, M and σ, the designer can select the appropriate value for SINR_{tr} such that with probability close to 1 the value of random variable X is less than K as shown in Fig. 6.6.

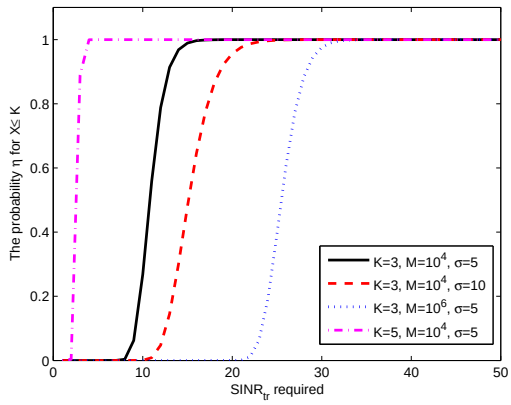


Figure 6.6: The feedback is at most K with almost sure

6.1.4 Practical Related Issues

There are still two important issues with OIM scheme. One is the fact that in current cellular systems, the assignment of users is based on pre-determined schemes such as time-division. The other issue is the fairness problem which is important so that all users have minimum access to the channel. For example, some mobile users may be close to the base station for a long period of time with line of sight. In the

following section, an approach is provided to incorporate OIM scheme into existing TDMA systems to assure fairness in terms of accessing the channel for all users. The extension of this approach to other standards such as CDMA is straightforward.

6.1.4.1 Fairness under TDMA Scheme

In this section, one practical approach is proposed for existing GSM cellular systems to guarantee the fairness and Quality of Service (QoS) for TDMA users while allowing other users to take advantage of OIM scheme without interrupting the main user. For any TDMA user, the received signal vector can be written as

$$\mathbf{R}_{\mathrm{TDMA}}^{\mathrm{T}} = \mathbf{S}_{\mathrm{TDMA}}^{\mathrm{T}} h_{\mathrm{TDMA}} + \sum_{i=1}^{d} S_i h_i \mathbf{V}^{\mathrm{T}} + \mathbf{n}^{\mathrm{T}}, \tag{6.30}$$

where $\mathbf{R}_{\mathrm{TDMA}}$ and $\mathbf{S}_{\mathrm{TDMA}}$ are the TDMA signal vectors received by a mobile user and transmitted by an antenna in the base station respectively, provided that this antenna does not participate in OIM scheme, i.e., $d < K$. The superscript T represents transpose of a vector, S_i and V^T are the signal transmitted by the antenna that is utilizing OIM scheme and a vector with unit weight that will be multiplied by each signal S_i respectively. $\mathbf{n}$ is the additive Gaussian noise vector with zero mean i.i.d. elements and variance of σ_n. h_{TDMA} and h_i are the CSI between base station and mobile users that are participating in TDMA and OIM scheme respectively.

At the receiver, the received vector is multiplied by a vector $\mathbf{U}$. This vector is

orthonormal to $\mathbf{V}$, i.e., $\mathbf{U}\mathbf{V}^T = 0$. Thus, the received signal will be equal to

$$\begin{aligned}\mathbf{U}\mathbf{R}_{\mathrm{TDMA}}^{\mathrm{T}} &= \mathbf{U}\mathbf{S}_{\mathrm{TDMA}}^{\mathrm{T}} h_{\mathrm{TDMA}} + \sum_{i=1}^{d} s_i h_i \mathbf{U}\mathbf{V}^T + \mathbf{U}\mathbf{n}^{\mathrm{T}} \\ &= \mathbf{U}\mathbf{S}_{\mathrm{TDMA}}^{\mathrm{T}} h_{\mathrm{TDMA}} + \mathbf{n}' \end{aligned} \tag{6.31}$$

Note that the signals transmitted utilizing OIM scheme are now multiplied by this new vector $\mathbf{V}$. Even though the TDMA user does not have the OIM capability and therefore other users are interfering with this user, but when the orthogonal vector $\mathbf{U}$ is multiplied by the received vector, these interfering signals can be get rid of. Further, the vector $\mathbf{V}$ does not have any relationship with CSI and any of beamforming scheme is not really used. The criterion for selecting this vector will be later described. For block fading channel, this vector only requires to be of length 2. It is noticed that by the new transmission policy, the actual rate of signals participating in OIM scheme have been reduced by a factor proportional to the length of vector $\mathbf{V}$. However, the rate of TDMA signal is still one symbol per channel use.

If the wireless channel is block fading, then $\mathbf{U} = [u_1, u_2]$ and $\mathbf{V} = [v_1, v_2]$ are enough for implementation. However, for fast fading the implementation of this technique is more complicated and it is omitted here. For the rest of the paper, it is assumed that the QPSK signals are used for transmission. Since the TDMA vector signal is multiplied by $\mathbf{U}$ as shown in Eq. (6.31), then our criterion for designing this signal is based on the condition that the combination of multiple QPSK signals results in optimum separation of points in the two-dimensional space. This condition will help in decoding performance of the received signal. Note that again this vector is not really

a function of channel matrix as it is common in beamforming techniques.

For a combination of two QPSK signals, an appropriate choice would be a 16-QAM signal. It has been shown in [45] that any combination of QPSK signals can be mapped into M-QAM signals. For the specific case of 16-QAM, there is

$$\text{16-QAM} = \sum_{i=0}^{1} 2^i \left(\frac{\sqrt{2}}{2}\right) (j^{x_i}) \exp\left(\frac{\pi j}{4}\right) \tag{6.32}$$

where $x_i \in Z_4 = \{0, 1, 2, 3\}$. The QPSK constellation can be realized as QPSK $= j^{x_i}$. Thus, one can use shift and rotation operation to create M-QAM constellations from QPSK symbols. It is easy from Eq. (6.32) to show that the normalized values of vectors $\mathbf{U}$ and $\mathbf{V}$ are

$$\mathbf{U} = \sqrt{\frac{2}{5}} \exp\left(\frac{\pi j}{4}\right) \left[\frac{\sqrt{2}}{2}, \sqrt{2}\right] \tag{6.33}$$

and

$$\mathbf{V} = \sqrt{\frac{2}{5}} \left[\sqrt{2}, -\frac{\sqrt{2}}{2}\right] \tag{6.34}$$

respectively. Since the vector $\mathbf{U}$ is normalized, then the variance of Gaussian noise remains the same.

Note that with this signalling at the base station, the Quality of Service (QoS) and fairness for all users are guaranteed in a time-division approach while other users can utilize the spectrum taking advantage of OIM scheme.

6.1.4.2 Signaling requirement

One of the main advantages of this technique is the fact that, by taking advantage of multiuser diversity, a distributed MIMO system in the downlink of wireless

cellular networks is reduced into a group of parallel single-input single output (SISO) systems. For this reason, all challenges and complexities related to space-time signal processing design can be replaced by simple point-to-point communications while achieving maximum capacity as long as the number of mobile stations is adequate. This significant simplification of the signalling in the cellular systems is an additional advantage of our OIM scheme.

6.2 OIM in Wireless Ad Hoc Networks

In this section, OIM scheme is extended for cellular networks to distributed version in wireless ad hoc networks. Surprisingly, by fully taking advantage of fading channels in multiuser environments, the feedback requirement is proportional to a small value, while the encoding and decoding scheme is very simple and similar to the point-to-point communications. The original multiuser diversity concept was based on looking for the best channels, while the interference management approach is based on searching simultaneously for the best and worse channels. The increase of the capacity is essentially because of the powerful nature of fading in wireless environment.

6.2.1 Wireless Ad Hoc Network Model

The extend network has been assumed in this section. It is assumed that the movement of nodes causes fading. However, this is a restricted movement such that any node only moves within its transmission range and the network topology and routing does not change with time. If the nodes have unrestricted mobility in the entire network,

it is assumed that the time duration that causes the topology of the network to change is always smaller than the duration for transporting information from each source to its destination. Therefore, at any snapshot during packet transmission from any source to its destination in the network, the topology is static.

Let X_i and $X_{R(i)}$ denote the locations of node i and its receiving node $R(i)$ respectively. Let $P_{iR(i)}$ be the received signal power at node $R(i)$. The wireless channel is subject to fading as described below. P is defined as the transmit power at node i and $|X_i - X_{R(i)}|$ as the Euclidean distance between nodes i and $R(i)$. $P_{iR(i)}$ is modeled as

$$P_{iR(i)} = |H_{iR(i)}|^2 \frac{P}{(|X_i - X_{R(i)}|)^\alpha} \tag{6.35}$$

where $H_{iR(i)}$ is a random variable that incorporates the channel fading and α is the path-loss exponent whose typical values are between 2 and 6. Under Rayleigh fading model, $H_{iR(i)}$ and $|H_{iR(i)}|^2$ have Rayleigh and exponential distributions respectively.

Definition 6.1 Generalized Physical Model

In this analysis, the data rate between the transmitter-receiver pair i and $R(i)$ in bits/second is defined as

$$C_{iR(i)} = W \log\left(1 + SINR_{iR(i)}\right), \tag{6.36}$$

where W is the bandwidth and $SINR_{iR(i)}$ between the transmitting node X_i and the receiving node $X_{R(i)}$ is defined as

$$SINR_{iR(i)} = \frac{\frac{P}{(|X_i - X_{R(i)}|)^\alpha}|H_{iR(i)}|^2}{N + \sum_{k \neq i} \frac{P}{(|X_k - X_{R(i)}|)^\alpha}|H_{kR(i)}|^2}, \tag{6.37}$$

where N is the ambient noise power and X_k's$(k \neq i)$ are the interfering nodes. Note that the channel model consists of large scale fluctuation $|X_i - X_{R(i)}|^{-\alpha}$ and small scale fluctuation $|H_{kR(i)}|^2$.

6.2.2 Scheduling Protocol

Fig. 6.7 illustrates the system involved in OIM. Without loss of generality, it is assumed that the receiver $R(i) = i$ for $i \in [1, 2, \ldots, K]$ in Fig. 6.7. In this figure, solid line and dotted line represent a strong and weak channel between transmitters and receivers respectively.

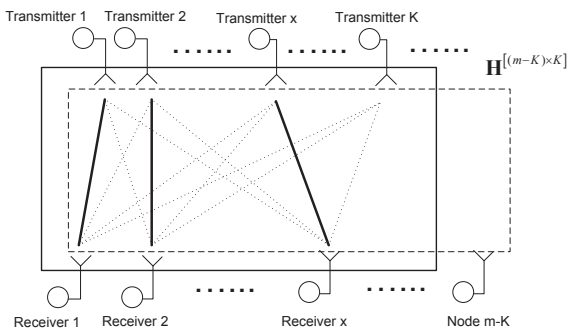


Figure 6.7: Opportunistic interference management system model

It is assumed that, for each node, there is always some traffic demand to any other neighbor node at any time slot. Each packet is either destined for a particular neighbor node or is relayed through a route that need the node to relay.

For any time slot $\mathbf{T}$, there are x potential transmitters out of K that satisfy the OIM condition, where x is a random variable with mean value of $D = \mathbf{E}(x)$. The probability distribution function of x and the relationship between D, K and the rest

of the nodes $m - K$ will be defined subsequently. In practice, K transmitter nodes are selected who are close by in order to make coordination easier. During the first phase of communication, the K transmitters sequentially transmit K pilot signals. In this period, all the other $m - K$ nodes listen to these known messages. After the last pilot signal is transmitted, all of the other nodes evaluate the SNR for each transmitter. If the SNR for only one transmitter is greater than a pre-determined threshold $\mathrm{SNR}_{\mathrm{tr}}$ and below another pre-determined threshold of $\mathrm{INR}_{\mathrm{tr}}$ for the remaining $K - 1$ transmitters, that particular receiver selects that particular transmitter. In the second phase of communication, these receivers notify the transmitters that they have the required criterion to receive packets during the remaining time period of $\mathbf{T}$. If appropriate values for $\mathrm{SNR}_{\mathrm{tr}}$ and $\mathrm{INR}_{\mathrm{tr}}$ are chosen, such that $\mathrm{SNR}_{\mathrm{tr}} \gg \mathrm{INR}_{\mathrm{tr}}$, then the transmitters can transmit different packets to different receivers concurrently. The receivers only receive their perspective packets with strong signal and can treat the rest of packets as noise. The value of $\mathrm{SNR}_{\mathrm{tr}}$ (or $\mathrm{INR}_{\mathrm{tr}}$) can be selected as high (or low) as required for a given system as long as m is large enough. Their relationship will be shown in details later.

6.2.3 Theoretical Analysis and Numerical Results

Let's define $\mathrm{SNR}_{iR(i)}$ and $\mathrm{INR}_{jR(i)}$ as the signal-to-noise ratio and interference-to-noise ratio between transmitter i, other transmitter $j, j \neq i$ and i's corresponding receiver $R(i)$ respectively. Note that only fading (small scale fluctuation of channel) is considered for the analysis of OIM as explained earlier. The objective of OIM is to find x receiver nodes out of $m - K$ choices to satisfy the following criteria. Since x is a

random variable, the average value of x receiver that satisfies OIM requirement is used, i.e., $D = \mathbf{E}(x)$. Then for any associate transmitter $i, i \in 1, 2, \cdots, K$, i's corresponding receiver $R(i)$ and other transmitter $j, j \in 1, 2, \cdots K, j \neq i$, there has

$$\begin{aligned} &\mathrm{SNR}_{iR(i)} \geq \mathrm{SNR}_{\mathrm{tr}}, i \in, 1, 2, \cdots, K, R(i) \in 1, 2, \cdots, x \\ &\mathrm{INR}_{jR(i)} \leq \mathrm{INR}_{\mathrm{tr}}, j \in 1, 2, \cdots, K, j \neq i \end{aligned} \tag{6.38}$$

The above condition states that each one of the x receiver nodes has a very good channel to a single transmitter node and weak channel (strong fading) to the other $K-1$ receiver nodes as shown in Fig. 6.7.

Then, $\mathrm{SINR}_{iR(i)}$ is defined as

$$\mathrm{SINR}_{iR(i)} = \frac{\mathrm{SNR}_{iR(i)}}{\sum_{j=1, j\neq i}^{K-1} \mathrm{INR}_{jR(i)} + 1}, \tag{6.39}$$

and $\mathrm{SINR}_{\mathrm{tr}}$ as

$$\mathrm{SINR}_{\mathrm{tr}} = \frac{\mathrm{SNR}_{\mathrm{tr}}}{(K-1)\mathrm{INR}_{\mathrm{tr}} + 1}. \tag{6.40}$$

respectively.

Hence, supposing $D = \mathbf{E}(x)$ transmitter-receiver pairs satisfying Eq. (6.38), then the sum rate can be written as

$$\begin{aligned} C_{\text{proposed}} &= \sum_{i=1}^{D} \log\left(1 + \mathrm{SINR}_{iR(i)}\right), \\ &= \sum_{i=1}^{D} \log\left(1 + \frac{\mathrm{SNR}_{iR(i)}}{\sum_{j=1, j\neq i}^{K-1} \mathrm{INR}_{jR(i)} + 1}\right), \\ &\geq D \log\left(1 + \frac{\mathrm{SNR}_{\mathrm{tr}}}{(K-1)\mathrm{INR}_{\mathrm{tr}} + 1}\right), \\ &= D \log(1 + \mathrm{SINR}_{\mathrm{tr}}) \end{aligned} \tag{6.41}$$

In the following, it will be proved that for any given value of $\mathrm{SINR}_{\mathrm{tr}}$, there exists a relationship between m and D that will satisfy Eq. (6.41). To prove the existence of this algorithm, it needs to be proved that there are $D = \mathbf{E}(x)$ transmitter-receiver pairs that satisfy Eq. (6.38) on average.

To prove the condition in Eq. (6.41), it is assumed that the channel distribution is Rayleigh fading channel which has been defined as Eq. 6.6.

Assuming the probability distribution function, expected value and variance of x are $\Pr(x)$, $D = \mathbf{E}(x)$ and $\Delta^2 = \mathbf{Var}(x)$ respectively. Note that by selecting the average value of x, in practice the actual number of nodes satisfying OIM is either larger or smaller than this average value. Therefore, a constant value is chosen such that with a probability arbitrarily close to zero, the actual number of nodes satisfying OIM criterion is always smaller than this value. By utilizing Chebyshev's inequality,

$$\Pr\left(|x - D| \geq c_0\Delta\right) \leq \frac{1}{c_0^2}. \tag{6.42}$$

This equation implies that for any given c_0, the value of x is smaller than $D + c_0\Delta$ with probability greater than $1 - \frac{1}{c_0^2}$. Clearly this probability can be selected arbitrarily close to one. The practical price is increase in transmission of pilot signals during the first phase of communications. In the followings, $D = \Theta(K)$ will be proved.

6.2.4 Throughput Capacity Analysis

The achievable bound for the capacity analysis is based on the TDMA scheme that was originally introduced in [43]. In this approach, the network is divided into

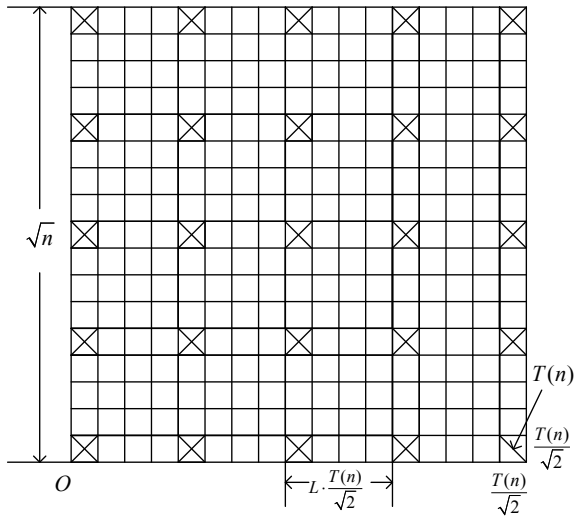


Figure 6.8: The cell construction in extended wireless ad hoc network

smaller square cells each one with an area such that all the nodes inside each cell are connected. Therefore, each square cell has an area of $T^2(n)/2$ which makes the diagonal length of square equal to $T(n)$ as shown in Fig. 6.8. Under this condition, if the transmission range is at most $T(n)$ for each hop, then all nodes inside a cell are within cooperation range of each other. A cell graph is built over the network that are occupied with at least one vertex (node) [43]. Cells are organized into groups such that simultaneous transmissions within each group does not violate the OIM condition for successful communication. Let L represent the minimum number of cell separations in each group of cells that communicate simultaneously. In every $1/L^2$ time slots, each cell receives one time slot to communicate. In an active cell, each transmitter node either sends a packet to one of the nodes inside the cell or a node in adjacent cells. Fig. 4.6 shows a group of active cells with cross symbol inside the cells. Note that the distance

between interfering cells is at least $qT(n)L/\sqrt{2} - T(n)/\sqrt{2}$ for $q = 1, 2, \cdots$, centered around any active cell.

The analysis is based on computing SINR for two cases of interference within a cell and interference from outside the cell. The former one is denoted as $\text{SINR}_{\text{inner}}$ and the latter as $\text{SINR}_{\text{outer}}$. Note that in general, the SNR can be computed as

$$\text{SNR}_{iR(i)} = \frac{\frac{P}{|X_i - X_{R(i)}|^\alpha}|H_{iR(i)}|^2}{N} \geq \text{SNR}_{\text{tr}}. \tag{6.43}$$

The lower bound is derived based on the OIM condition. If $|H_{iR(i)}|^2 \geq c_3$ is assumed, then c_3 can be selected as $c_3 = \text{SNR}_{\text{tr}} N \frac{|X_i - X_{R(i)}|^\alpha}{P}$. To compute the lower bound for $\text{SINR}_{\text{outer}}$, note that $\mathbf{E}[|H_{kR(i)}|^2] = \sigma$ and $\mathbf{Var}[|H_{kR(i)}|^2] = \sigma^2$ because of the characteristic of exponential distribution $|H_{kR(i)}|^2$ for any k in Eq. (6.6). Due to $|X_i - X_{R(i)}| \leq T(n)$ for neighbor cell, then

$$\begin{aligned}
\text{SINR}_{\text{outer}} &= \frac{\frac{P}{|X_i - X_{R(i)}|^\alpha}|H_{iR(i)}|^2}{N + \sum_{k \neq i} \frac{P}{|X_k - X_{R(i)}|^\alpha}|H_{kR(i)}|^2} \\
&\geq \frac{\frac{c_3 P}{(T(n))^\alpha}}{N + \sum_{q=1}^{\infty} 8q \frac{P}{(qT(n)L/\sqrt{2} - T(n)/\sqrt{2})^\alpha}|H_{qR(i)}|^2} \\
&= \frac{\frac{c_3 P}{(\sqrt{2})^\alpha}}{N\left(\frac{T(n)}{\sqrt{2}}\right)^\alpha + \frac{8P}{L^\alpha}\sum_{q=1}^{\infty} \frac{q}{(q - \frac{1}{L})^\alpha}|H_{qR(i)}|^2}
\end{aligned} \tag{6.44}$$

where, the second term of denominator needs to be proved bounded provided that P increases with $T(n)$ in extended networks. $S_q = \frac{q}{(q - \frac{1}{L})^\alpha}$ is defined. The sum $S =$

$\sum_{q=1}^{\infty} S_q$ is bounded by a constant c_4 as follows

$$\begin{aligned}
S &= \sum_{q=1}^{\infty} \frac{1}{(q-\frac{1}{L})^{\alpha-1}} + \frac{1}{L}\sum_{q=1}^{\infty} \frac{1}{(q-\frac{1}{L})^{\alpha}} \\
&\leq \frac{1}{(1-\frac{1}{L})^{\alpha-1}} + \int_{1-\frac{1}{L}}^{\infty} \frac{1}{x^{\alpha-1}} dx \\
&\quad + \frac{1}{L}\left(\frac{1}{\left(1-\frac{1}{L}\right)^{\alpha}} + \int_{1-\frac{1}{L}}^{\infty} \frac{1}{x^{\alpha}} dx \right) \\
&= \frac{1}{\left(1-\frac{1}{L}\right)^{\alpha-1}} + \frac{\left(1-\frac{1}{L}\right)^{-(\alpha-2)}}{\alpha-2} \\
&\quad + \frac{1}{L}\frac{1}{\left(1-\frac{1}{L}\right)^{\alpha}} + \frac{1}{L}\frac{\left(1-\frac{1}{L}\right)^{-(\alpha-1)}}{\alpha-1} \\
&= c_4
\end{aligned}$$

as [41] when $\alpha > 2$. When L is selected sufficiently large, then the effect of interference from outside cells can be reduced to any desired value based on Eq. (6.44). Next it needs to proved that $S = \sum_{q=1}^{\infty} S_q |H_{qR(i)}|^2$ is bounded. Because $|H_{qR(i)}|^2$ is a random variable, then $\Pr\left(\sum_{q=1}^{\infty} S_q |H_{qR(i)}|^2 \to \infty\right) = 0$ should be proved.

Let's define $\mathbf{E}\left[|H_{qR(i)}|^2\right] = \sigma$, then

$$\mu = \mathbf{E}\left[\sum_{q=1}^{\infty} S_q |H_{qR(i)}|^2\right] = \sum_{q=1}^{\infty} S_q \mathbf{E}[|H_{qR(i)}|^2] \leq c_4\sigma. \tag{6.45}$$

From Chebyshev's inequality,

$$\Pr\left(\left|\sum_{q=1}^{\infty} S_q |H_{qR(i)}|^2 - \mu\right| \geq \alpha\right) \leq \frac{V^2}{\alpha^2} \tag{6.46}$$

where, $V = \mathbf{Var}\left[\sum_{q=1}^{\infty} S_q |H_{qR(i)}|^2\right]$. Because of $S_q^2 \leq S_q, \forall q$, then $V = \sum_{q=1}^{\infty} S_q^2 \mathbf{Var}\left[|H_{qR(i)}|^2\right] \leq c_4\sigma^2$ provided that $|H_{qR(i)}|$ are i.i.d for different values

of q. Clearly, if $\alpha \to \infty$, then

$$\Pr\left(\sum_{q=1}^{\infty} S_q |H_{kR(i)}|^2 \to \infty\right) = 0 \tag{6.47}$$

Thus, the Eq. (6.44) is bounded as

$$\mathrm{SINR}_{\mathrm{outer}} \geq \mathrm{SINR}_{\mathrm{tr}}(\mathrm{outer}) \tag{6.48}$$

where, $\mathrm{SINR}_{\mathrm{tr}}(\mathrm{outer})$ is a constant term derived from Eq. (6.44) that is defined based on the communication requirements for each node. From Eq. (6.40), the lower bound for $\mathrm{SINR}_{\mathrm{inner}}$ is given by

$$\mathrm{SINR}_{\mathrm{inner}} \geq \mathrm{SINR}_{\mathrm{tr}} = \mathrm{SINR}_{\mathrm{tr}}(\mathrm{inner}). \tag{6.49}$$

Combining Eq. (6.48) and Eq. (6.49), SINR is given by

$$\begin{aligned}
\mathrm{SINR}_{iR(i)} &= \frac{\mathrm{Signal}}{N + \mathrm{Interference}_{\mathrm{outer}} + \mathrm{Interference}_{\mathrm{inner}}}, \\
&\geq \frac{\mathrm{Signal}}{N + \frac{\mathrm{Signal}}{\mathrm{SINR}_{\mathrm{tr}}(\mathrm{outer})} - N + \frac{\mathrm{Signal}}{\mathrm{SINR}_{\mathrm{tr}}(\mathrm{inner})} - N}, \\
&\geq \frac{1}{\frac{1}{\mathrm{SINR}_{\mathrm{tr}}(\mathrm{outer})} + \frac{1}{\mathrm{SINR}_{\mathrm{tr}}(\mathrm{inner})}}, \\
&= \frac{\mathrm{SINR}_{\mathrm{tr}}(\mathrm{inner})\mathrm{SINR}_{\mathrm{tr}}(\mathrm{outer})}{\mathrm{SINR}_{\mathrm{tr}}(\mathrm{inner}) + \mathrm{SINR}_{\mathrm{tr}}(\mathrm{outer})}, \\
&= \mathrm{SINR}_{\mathrm{tr}}(\mathrm{total})
\end{aligned} \tag{6.50}$$

Next, the relationship between $D = \mathbf{E}(x)$, K and $m = \Theta(T^2(n))$ is derived in order to compute the throughput capacity for each cell. Based on Eq. (6.41), the order capacity for each cell can be computed.

Let's define event A is for a receiver node that satisfies the condition in Eq. (6.38), and that the channels between the transmitter and receiver nodes are i.i.d., then

this probability can be derived as Eq. (6.7). The following derivation is similar with the one in Section 6.1. The event that $y = d$ receiver nodes satisfy the OIM constraint satisfies binomial distribution as follows:

$$\Pr(y = d) = \binom{m-K}{d} (\Pr(A))^d (1 - \Pr(A))^{m-K-d}. \tag{6.51}$$

The lower bound for the expected value of x is given by

$$D = \mathbf{E}(x) \geq \frac{m-K}{K} \Pr(A). \tag{6.52}$$

It is noteworthy to mention again that the number of receivers that satisfy OIM condition x is a random variable and D is simply the average value of this random variable. Thus,

$$m \leq K(D\,(\Pr(A))^{-1} + 1). \tag{6.53}$$

Follow the same derivation in as Section 6.1, with optimum value for $(\Pr(A))^{-1}$ using Eq. (6.18), the optimum m is derived from Eq. (6.53) as

$$m \leq K + De^{\frac{\mathrm{SINR}_{\mathrm{tr}}}{\sigma}} (\mathrm{SINR}_{\mathrm{tr}} e)^{K-1}. \tag{6.54}$$

This value is derived by replacing the optimum value of $\mathrm{INR}^*_{\mathrm{tr}}$ into Eq. (6.18) and using the limitation (b) in this equation.

Now the asymptotic behavior of the network (i.e. $m \to \infty$) is investigated and the maximum achievable capacity and scaling laws is computed for this scheme. Note that σ represents the strength of fading channel and as this parameter increases or equivalently the channel experience more severe fade, then the value of D increases. The main reason is the fact that fading environment helps to combat interference.

From Eq. (6.54), if $D = \Theta(K)$ is selected, then

$$m = O\left(De^K\right) = O\left(Ke^K\right) = O\left(e^{2K}\right) \tag{6.55}$$

Thus, when $m = \Theta\left(T^2(n)\right)$,

$$D = \Theta(K) = \Theta(\log m) = \Theta(\log T(n)) \tag{6.56}$$

Then by utilizing Eq. (6.41), the scaling laws of OIM scheme for each cell is

$$\begin{aligned} \mathbf{E}\left(C_{\text{proposed}}\right) &= \Theta\left(D \log\left(1 + \text{SINR}_{\text{tr}}\right)\right) \\ &= \Theta\left(\log T(n)\right) \end{aligned} \tag{6.57}$$

It is worthy to point out that when σ tends to zero, this technique cannot achieve the optimum value of K. Equivalently, this condition occur when the channel fading is not strong. This is contrary to the current belief for point-to-point communications that fading reduces the network capacity. In a multi-user environment, fading actually is very helpful. The proposed multi-user diversity scheme also is different from the original scheme that requires the transmitter to search for the node with the best channel condition. As it has been shown, fading is very important and when the channel fading strength increases, it can achieve better capacity performance in the network.

Next it is proved that when n nodes are distributed uniformly over a square area, each cell contains $\Theta(T^2(n))$ nodes w.h.p.. The objective is to find an achievable bound using the Chernoff bound, such that the distribution of the number of nodes in each cell space is sharply concentrated around its mean.

Lemma 6.2 *The square cells of side length $T(n)/\sqrt{2}$ for concurrent transmission contains $\Theta(T^2(n))$ nodes w.h.p., and is uniformly distributed for all j cells, $1 \leq j \leq \left\lceil \frac{n}{(LT(n)/\sqrt{2})^2} \right\rceil$, when $T(n) = \Omega\left(\sqrt{\log n}\right)$.*

Proof The statement of this lemma can be expressed as

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\left\lceil \frac{n}{(LT(n)/\sqrt{2})^2} \right\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] = 1, \tag{6.58}$$

where N_j and $E\left(N_j\right)$ are the random variables that represent the number of nodes in the square cell with diagonal distance of $T(n)$ centered around cell j and the expected value of this random variable respectively, and δ is a positive arbitrarily small value close to zero.

From the Chernoff bound in Eq. (2.7), for any given $0 < \delta < 1$, $\theta > 0$ can be found depending δ such that $P\left[|N_j - E(N_j)| > \delta E(N_j)\right] < e^{-\theta E(N_j)}$. Thus, it can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n \to \infty$. This is a key step in showing that when all the events $\bigcap_{j=1}^{\left\lceil \frac{n}{(LT(n)/\sqrt{2})^2} \right\rceil} |N_j - E(N_j)| < \delta E(N_j)$ occur simultaneously, then all N_j's converge uniformly to their expected values. Utilizing the

union bound, it arrives at

$$
\begin{aligned}
& P\left[\bigcap_{j=1}^{\left\lceil \frac{n}{(LT(n)/\sqrt{2})^2}\right\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] \\
= \; & 1 - P\left[\bigcup_{j=1}^{\left\lceil \frac{n}{(LT(n)/\sqrt{2})^2}\right\rceil} |N_j - E(N_j)| > \delta E(N_j)\right] \\
\geq \; & 1 - \sum_{j=1}^{\left\lceil \frac{n}{(LT(n)/\sqrt{2})^2}\right\rceil} P\left[|N_j - E(N_j)| > \delta E(N_j)\right] \\
> \; & 1 - \left\lceil \frac{n}{(LT(n)/\sqrt{2})^2} \right\rceil e^{-\theta E(N_j)}. \qquad (6.59)
\end{aligned}
$$

Given that $E(N_j) = \frac{T^2(n)}{2}$, then

$$
\begin{aligned}
& \lim_{n\to\infty} P\left[\bigcap_{j=1}^{\left\lceil \frac{n}{(LT(n)/\sqrt{2})^2}\right\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] \\
& \geq 1 - \lim_{n\to\infty} \left\lceil \frac{n}{(LT(n)/\sqrt{2})^2} \right\rceil e^{-\theta T^2(n)/2} \qquad (6.60)
\end{aligned}
$$

If $T(n) \geq \sqrt{2\log n/\theta}$, $\lim_{n\to\infty} \frac{ne^{-\theta T^2(n)/2}}{T^2(n)} \to 0$, which completes the proof.

Next the routing scheme is discussed to achieve the achievable lower bound capacity which is similar with the routing scheme in [41]. This routing scheme is extended from the dense-network model into the extended-network model to accommodate fading. According to the model, each node $i, 1 \leq i \leq n$, generates data packets at a rate $C(n)$ with each destination chosen as the node nearest to a randomly chosen location Y_i. Denote by $X_{dest(i)}$ the node nearest to Y_i, and by L_i the straight-line segment connecting X_i and Y_i (see Fig. 6.9). The packets generated by X_i are forwarded toward $X_{dest(i)}$ in a multi-hop fashion, from cell to cell in the order that they are intersected

by L_i. In each hop, the packet is transmitted from one cell to the next cell intersecting L_i. Any node in the cell can be chosen as a receiver. Finally, after reaching the cell containing Y_i, the packet will be forwarded to $X_{dest(i)}$ in the next active slot for that cell. This can be done because $X_{dest(i)}$ is within a range of $T(n)$ to any node in that cell. There is a bound on the number of routes each cell needs to serve, which means the probability that a line will intersect a particular cell is bounded.

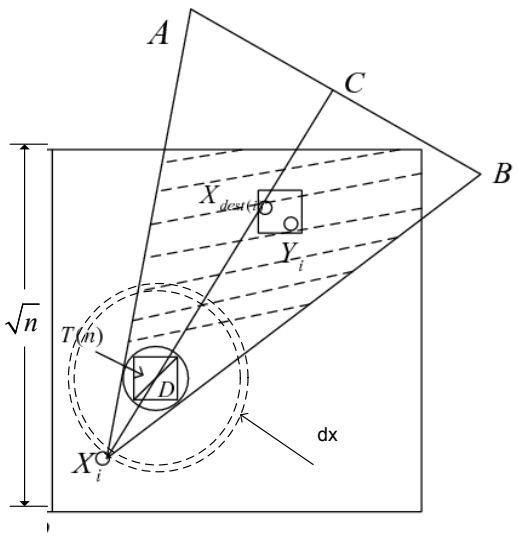


Figure 6.9: Routing scheme proof

For completeness, the following two lemmas are presented for the extended network.

Lemma 6.3 *For every line L_i and cell S_{k_0,j_0},*

$$Pr\{Line\ L_i\ intersects\ S_{k_0,j_0}\} = p = O\left(\frac{T(n)}{\sqrt{n}}\right) \tag{6.61}$$

Proof S_{k_0,j_0} is defined as the cell which is contained in a disk of radius $T(n)/2$

centered at D as shown in Fig. 6.9. Suppose X_i is at distance x from the disk. The two tangent lines originating from X_i equally are extended such that $|X_iA| = |X_iB|$ and $|X_iC| = \sqrt{2n}$, where C is the mid-point of AB.

Then L_i intersects S_{k_0,j_0} only if Y_i is in the shaded area. Its area is less than the minimum of n and the area of the triangle, which is $\sqrt{2n} \times \frac{\sqrt{2n}\frac{T(n)}{2}}{\sqrt{(x+T(n)/2)^2-(T(n)/2)^2}} < nT(n)/x$.

The location of X_i is uniformly distributed, therefore, the probability density function that X_i is at distance x from the disk is a ring that is bounded by $O\left(\frac{x+T(n)/2}{n}dx\right)$. Hence,

$$\begin{aligned} & \Pr\{\text{Line } L_i \text{ intersects } S_{k_0,j_0}\}, \\ = \; & O\left(\frac{1}{n}\int_{T(n)/2}^{\sqrt{2n}} (\min(nT(n)/x, n)) \left(\frac{x+T(n)/2}{n}\right) dx\right), \\ = \; & O\left(\frac{T(n)}{\sqrt{n}}\right). \end{aligned} \tag{6.62}$$

Based on the above lemma, it can arrive the following uniform bound on the number of routes served by each cell.

Lemma 6.4 *It can be proved that*

$$\begin{aligned} & \lim_{n\to\infty} Pr\left(\sup_{(k,j)}\{\textit{Number of lines } L_i \textit{ intersecting } S_{k,j}\}\right. \\ & \left. = O\left(\sqrt{n}T(n)\right)\right) = 1. \end{aligned} \tag{6.63}$$

Proof First the bound for the number of routes served by one particular cell

S_{k_0,j_0} is derived. Define i.i.d. random variable $I_i, i \leq i \leq n$, as follows.

$$I_i = \begin{cases} 1, \text{ if } L_i \text{ intersects } S_{k_0,j_0} \\ \\ 0, \text{ if not} \end{cases} \tag{6.64}$$

Let $\Pr(I_i = 1) = p \ \forall i$, where p is defined in Lemma 6.3. Denote Z_n the total number of routes served by S_{k_0,j_0}. Then $Z_n := I_1 + I_2 + \cdots + I_n$. Using Chernoff bound, for all positive values of b and a, $\Pr(Z_n > b) \leq \frac{E[e^{aZ_n}]}{e^{ab}}$. Since $1 + x \leq e^x$,

$$\begin{aligned} E[e^{aZ_n}] &= (1 + (e^a - 1)p)^n \leq \exp(n(e^a - 1)p), \\ &= O(\exp((e^a - 1)\sqrt{n}T(n))). \end{aligned} \tag{6.65}$$

Now by choosing $b = c\sqrt{n}T(n))$ for any constant $c > 1$, $\Pr(Z_n = \Omega(\sqrt{n}T(n))) = O(\exp(-\sqrt{n}T(n)))$ if a is small enough.

Thus by the union bound,

$$\begin{aligned} & \Pr\left(\text{Some cell intersects } \Omega(\sqrt{n}T(n)) \text{ lines }\right) \\ \leq & \sum_{k,j} \Pr\left(\text{Cell } S_{jk} \text{ intersects } \Omega(nT(n)) \text{ lines }\right) \\ = & O\left(\frac{n}{T^2(n)} \exp\left(-\sqrt{n}T(n)\right)\right) \end{aligned} \tag{6.66}$$

The right hand side tends to zero for any value of $T(n)$.

From earlier discussion, it is known that there exists a transmission schedule such that in every L^2 (L is a constant) time slots, each cell receives one time slot to transmit at rate $C_{\text{proposed}}W$ bits/second as shown in Eq. (6.57) with maximum transmission distance $T(n)$. So the rate at which each cell can transmit is $\log(T(n))\, W/L^2$.

From Lemma 6.4, each cell needs to transmit at rate $O\left(C(n)\sqrt{n}T(n)\right)$ where $C(n)$ is the throughput capacity of the network. This can be accommodated by all cells if

$$C(n)\sqrt{n}T(n) = \Theta\left(\log\left(T(n)\right)W/L^2\right) \tag{6.67}$$

Note that in each cell, the traffic passing through that cell can be handled by any designated node in that cell. The following theorem describes the main result of this chapter.

Theorem 6.5 *In extended wireless ad hoc networks, the unicast throughput capacity in multipath fading environment with multi-hop communication when nodes utilize OIM is*

$$C(n) = \Theta\left(\frac{\log\left(T(n)\right)}{\sqrt{n}T(n)}\right), \tag{6.68}$$

where $T(n) = \Omega\left(\sqrt{\log n}\right)$.

Next theorem presents the throughput capacity of this network in the absence of OIM.

Theorem 6.6 *In extended wireless ad hoc networks, the unicast throughput capacity with multi-hop point-to-point communication is*

$$C(n) = \Theta\left(\frac{1}{\sqrt{n}T(n)}\right), \tag{6.69}$$

where $T(n) = \Omega\left(\sqrt{\log n}\right)$.

The proof procedure for this theorem is very similar to that of Theorem 6.5 except that the OIM effect is not considered in Eq. (6.38). Also note that because there is no OIM, there is only a single transmission in each cell.

When $T(n) = \Omega\left(\sqrt{\log n}\right)$, $C(n) = \Theta\left(\frac{\log\log n}{\sqrt{n\log n}}\right)$ for fading channel utilizing OIM with $\Theta(\log\log n)$ gain compared to point-to-point communications and when $T(n) = \Theta\left(\sqrt{n}\right)$, $C(n) = \Omega\left(\frac{\log n}{n}\right)$ for fading channel utilizing OIM with $\Theta(\log n)$ gain compared to point-to-point communications. The capacity of these two schemes are illustrated in Fig. 6.10.

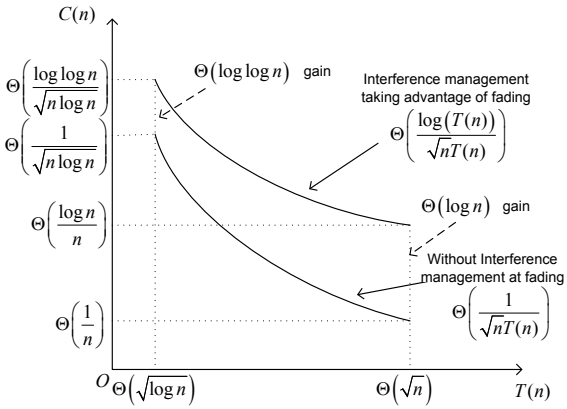


Figure 6.10: The throughput capacity with and without OIM in extended wireless ad hoc network with fading channel

Next figure demonstrates theoretical and simulation results for the capacity of wireless ad hoc networks with and without OIM. The results clearly show that the theoretical results matches simulation results. The simulation has been done with 10^4 nodes in the network. Note that by increasing σ, or by decreasing $\mathrm{SINR}_{\mathrm{tr}}$ or transmission range $T(n)$, the throughput capacity increases as predicted by the analysis.

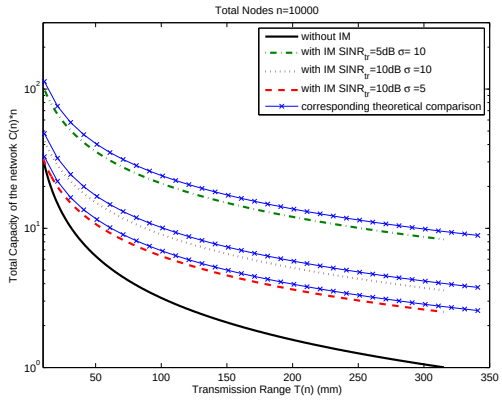


Figure 6.11: The throughput capacity simulation with and without OIM as a function of σ, $\mathrm{SINR_{tr}}$, and $T(n)$.

6.3 Conclusion

This chapter proposes an opportunistic interference management technique that takes advantage of the fading in the channel to minimize the negative effect of interference both in wireless cellular and ad hoc networks. This technique reduces the encoding and decoding complexity for the downlink of wireless cellular networks to that of point-to-point communications, which is much simpler than proposed MIMO systems in literature. Finally, it is proved that it is not necessary to perform cooperative communication in a multiuser environment, which requires significant feedback between cooperating nodes. It is also proved that increasing fading actually enhances the performance of the OIM scheme and increases the capacity of wireless ad hoc networks significantly compared to simple point-to-point communications.

Chapter 7

Conclusion and Future Research

7.1 Conclusion

In this thesis, the capacity of wireless ad hoc networks has been studied. The work related to this research has been studied and new designs and analysis are proposed for wireless networks to improve its overall behavior.

First, Multi-Packet Reception (MPR) technique is proposed in wireless ad hoc networks which increases the order capacity of random wireless ad hoc networks under both protocol and physical models. The *power efficiency* $\eta(n)$ is also defined as the bits of information transferred per unit time (second) in the network for each unit power, and show that a lower power efficiency is attained in order to achieve higher throughput capacity.

Then a unifying framework is introduced for the modeling of the order capacity of wireless networks subject to different types of information dissemination with

SPR and MPR respetively. To do so, (n, m, k)-casting is defined as a generalization of all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks. The modeling framework provides a unique perspective to the understanding of the capacity of wireless ad hoc networks. The approach unifies existing results on the order capacity of wireless networks subject to unicasting , multicasting, or broadcasting and provides new capacity and delay results for anycasting and manycasting.

Another important contribution of this thesis is to prove the following statement. In wireless ad hoc networks with multi-pair multicast sessions and with a finite number of destinations for each source, the throughput capacity utilizing NC, MPT and MPR capabilities for all nodes is the same order as when the nodes are endowed only with MPT and MPR.

Finally, this thesis proposes an opportunistic interference management technique that takes advantage of the fading in the channel to minimize the negative effect of interference in both wireless cellular and ad hoc networks. This technique reduces the encoding and decoding complexity to that of point-to-point communications, which is much simpler than proposed MIMO systems in literature. Finally, it is proved that it is not necessary to perform cooperative communication in a multiuser environment, which requires significant feedback between cooperating nodes.

7.2 Future Research

First, this thesis discusses homogeneous networks where the distribution of nodes is uniform and all nodes have the same communication range. However in many practical applications, the distribution of nodes is not uniform and nodes may have different communication range. The impact of non-uniform node distribution and asymmetric transmission ranges on the throughput capacity, delay and power efficiency is the subject of future study.

Second, the multicast throughput and delay has been investigated recently for mobile environments [46]. Future studies should investigate the (n, m, k)-cast for mobile ad hoc networks.

Third, it has been known that there is still no order gain in multicast communications when only NC is used in SPR scenario and the number of destinations in a multicast group is a function of n [47], but it is still not clear what the constant gain is. This problem is important and will be the subject of future investigation in identifying the actual capacity contribution of NC in wireless ad hoc networks. It is also important to note that NC provides many other advantages in random wireless ad hoc networks for different applications such as secrecy that are not investigated in this thesis.

Finally, the real implementation of opportunistic interference management technique in wireless ad hoc networks and its application is still worth being studied.

Bibliography

[1] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, March 2000.

[2] M. Franceschetti, O. Dousse, D. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Transactions on Information Theory*, vol. 53, no. 3, pp. 1009–1018, March 2007.

[3] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Transactions on Networking*, vol. 10, no. 4, pp. 477–486, August 2002.

[4] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Optimal throughput-delay scaling in wireless networks part I: The fluid model," *IEEE Transactions on Information Theory*, vol. 52, no. 6, pp. 2568–2592, June 2006.

[5] H. Zhang and J. Hou, "Capacity of wireless ad-hoc networks under ultra wide band with power constraint," in *Proc. of IEEE INFOCOM 2005*, Miami, Florida, USA, March 13-17 2005.

[6] R. Negi and A. Rajeswaran, "Capacity of power constrained ad-hoc networks," in *Proc. of IEEE INFOCOM 2004*, Hong Kong, March 7-11 2004.

[7] P. Kyasanur and N. Vaidya, "Capacity of multi-channel wireless networks: Impact of number of channels and interfaces," in *Proc. of ACM MobiCom 2005*, Cologne, Germany, August 28-September 2 2005.

[8] A. Ozgur, O. Leveque, and D. Tse, "Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks," *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 2549–3572, October 2007.

[9] R. de Moraes, H. Sadjadpour, and J. J. Garcia-Luna-Aceves, "Many-to-many communication for mobile ad hoc networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 5, pp. 2388–2399, May 2008.

[10] S. Ghez, S. Verdu, and S. Schwartz, "Stability properties of slotted aloha with multipacket reception capability," *IEEE Transactions on Automatic Control*, vol. 33, no. 7, pp. 640–649, July 1988.

[11] ——, "Optimal decentralized control in the random access multipacket channel," *IEEE Transactions on Automatic Control*, vol. 34, no. 11, pp. 1153–1163, November 1989.

[12] S. Verdu, *Multiuser Detection.* Cambridge University Press, 1998.

[13] C. Peraki and S. Servetto, "On the maximum stable throughput problem in random networks with directional antennas," in *Proc. of ACM MobiHoc 2003*, Annapolis, Maryland, USA, June 1-3 2003.

[14] S. Yi, Y. Pei, and S. Kalyanaraman, "On the capacity improvement of ad hoc wireless networks using directional antennas," in *Proc. of ACM MobiHoc 2003*, Annapolis, Maryland, USA, June 1-3 2003.

[15] B. Tavli, "Broadcast capacity of wireless networks," *IEEE Communications Letters*, vol. 10, no. 2, pp. 68–69, February 2006.

[16] R. Zheng, "Information dissemination in power-constrained wireless networks," in *Proc. of IEEE INFOCOM 2006*, Barcelona, Catalunya, Spain, April 23-29 2006.

[17] A. Keshavarz, V. Ribeiro, and R. Riedi, "Broadcast capacity in multihop wireless networks," in *Proc. of ACM MobiCom 2006*, Los Angeles, California, USA, September 23-29 2006.

[18] P. Jacquet and G. Rodolakis, "Multicast scaling properties in massively dense ad hoc networks," in *Proc. of IEEE ICPADS 2005*, Fukuoka, Japan, July 20-22 2005.

[19] X.-Y. Li, S.-J. Tang, and O. Frieder, "Multicast capacity for large scale wireless ad hoc networks," in *Proc. of ACM MobiCom 2007*, Montreal, Canada, September 9-14 2007.

[20] R. Ahlswede, C. Ning, S.-Y. Li, and R. Yeung, "Network information flow," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1204–1216, July 2000.

[21] J. Liu, D. Goeckel, and D. Towsley, "Bounds on the gain of network coding and broadcasting in wireless networks," in *Proc. of IEEE INFOCOM 2007*, Anchorage, Alaska, USA, May 6-12 2007.

[22] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," in *Proc. of ACM SIGCOMM 2007*, Kyoto, Japan, August 27-31 2007.

[23] S. Zhang, S. Liew, and P. Lam, "Hot topic: Physical-layer network coding," in *Proc. of ACM MobiCom 2006*, Los Angeles, California, USA, September 23-29 2006.

[24] A. Ramamoorthy, J. Shi, and R. Wesel, "On the capacity of network coding for random networks," *IEEE Transactions on Information Theory*, vol. 51, no. 8, pp. 2878–2885, August 2005.

[25] S. Aly, V. Kapoor, J. Meng, and A. Klappenecker, "Bounds on the network coding capacity for wireless random networks," in *Proc. of NetCod 2007*, San Diego, California, USA, January 29 2007.

[26] S.-Y. Li, R. Yeung, and N. Cai, "Linear network coding," *IEEE Transactions on Information Theory*, vol. 49, no. 2, pp. 371–381, February 2003.

[27] R. Koetter and M. Medard, "An algebraic approach to network coding," *IEEE/ACM Transactions on Networking*, vol. 11, no. 5, pp. 782–795, October 2003.

[28] Z. Li and B. Li, "Network coding in undirected networks," in *Proc. of CISS 2004*, Princeton, NJ, USA, March 17-19 2004.

[29] Z. Li, B. Li, and L. Lau, "On achieving maximum multicast throughput in undirected networks," *IEEE/ACM Transactions on Special Issue on Networking and Information Theory*, vol. 52, pp. 2467–2485, June 2006.

[30] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. of IEEE ICC 1995*, Seattle, Washington, USA, June 18-22 1995.

[31] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1277–1294, June 2002.

[32] D. Tse, "Optimal power allocation over parallel gaussian broadcast channels," in *Proc. of IEEE ISIT 1997*, Ulm, Germany, June 29 - July 4 1997.

[33] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3425–3439, August 2008.

[34] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channels with partial side information," *IEEE Transactions on Information Theory*, vol. 51, no. 2, pp. 506–522, February 2005.

[35] ——, "A comparison of time-sharing, DPC, and beamforming for MIMO broadcast channels with many users," *IEEE Transactions on Communications*, vol. 55, no. 1, pp. 11–15, January 2007.

[36] M. Penrose, "The longest edge of the random minimal spanning tree," *The Annals of Applied Probability*, vol. 7, no. 2, pp. 340–361, 1997.

[37] M. Steele, "Growth rates of euclidean minimal spanning trees with power weighted edges," *The Annals of Probability*, vol. 16, no. 4, pp. 1767–1787, 1988.

[38] R. Motwani and P. Raghavan, *Randomized Algorithms.* Cambridge University Press, 1995.

[39] S. Verdu, "On channel capacity per unit cost," *IEEE Transactions on Information Theory*, vol. 36, no. 5, pp. 1019–1030, September 1990.

[40] A. Giridhar and P. Kumar, "Computing and communicating functions over sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 4, pp. 755–764, April 2005.

[41] F. Xue and P. Kumar, *Scaling Laws for Ad-Hoc Wireless Networks: An Information Theoretic Approach.* Now Publishers Inc., 2006.

[42] J. Kleinberg and E. Tardos, *Algorithm Design.* Addison Wesley, 2005.

[43] S. Kulkarni and P. Viswanath, "A deterministic approach to throughput scaling wireless networks," *IEEE Transactions on Information Theory*, vol. 50, no. 6, pp. 1041–1049, June 2004.

[44] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication.* Cambridge University Press, 2005.

[45] B. Tarokh and H. R. Sadjadpour, "Construction of OFDM M-QAM sequences with low peak-to-average power ratio," *IEEE Transactions on Communications*, vol. 51, no. 1, pp. 25– 28, January 2003.

[46] C. Hu, X. Wang, and F. Wu, "Motioncast: On the capacity and delay tradeoffs," in *Proc. of ACM MobiHoc 2009*, New Orleans, Louisiana, May 18-21 2009.

[47] S. Karande, Z. Wang, H. Sadjadpour, and J. J. Garcia-Luna-Aceves, "Multicast throughput order of network coding in wireless ad-hoc networks," *Submitted to IEEE Transactions on Communications*, 2009.