# Construction of non-square $M$-QAM sequences with low PMEPR for OFDM systems

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**Abstract:** The author considers the use of coding to reduce the peak-to-mean envelope power ratio (PMEPR) for orthogonal frequency division multiplexing (OFDM) systems. Most of the existing schemes that use coding for PMEPR reduction assume a PSK constellation. The author presents the construction of non-square $M$-QAM symbols from a combination of QPSK and BPSK signals when $M = 2^n$ and $n$ is an odd number. By using QPSK and BPSK Golay (or Golay-like) sequences, $M$-QAM sequences with low PMEPR are generated. An upper bound for the instantaneous envelope power of Golay-like $M$-QAM sequences was derived. After the mean envelope power for these sequences was computed, the general upper bound for the PMEPR of $M$-QAM sequences with non-square constellations was derived.

## 1 Introduction

Orthogonal frequency division multiplexing (OFDM) [1] is a multicarrier modulation technique that has been adopted for many types of applications in wireless systems such as the physical layer of IEEE802.11a [2] and digital audio broadcasting [3]. In wireless applications, severe multipath propagation makes the recovery of the transmitted signal very challenging in a mobile communication channel. In particular, many mobile channels are subject to frequency-selective multipath fading. An OFDM system divides the channel spectrum into several sub-channels (also called bins) and modulates the transmitted data using these sub-channels, and this approach is therefore suitable for this kind of mobile channel.

In asymmetric digital subscriber line (ADSL) modems [4], a similar approach, called discrete multitone (DMT) technology is used. For twisted pair wire applications, the available spectrum is limited. The channel characteristics in a twisted pair wire are a function of many factors such as frequency, loop length, gauge etc. In this application, it is desirable to use transmission techniques that are bandwidth-efficient. The DMT technology can provide an efficient way to transmit the data using an optimum bit-loading algorithm [5].

As mentioned earlier, an OFDM system divides the available spectrum into $N$ sub-channels. In applications such as ADSL modems, the value of $N$ is as large as 256. Such a high peak-to-mean envelope power ratio (PMEPR) requires a large dynamic range for the power amplifier within its linear amplification region which is not available for many applications. This is the motivation behind a lot of the work to reduce the PMEPR of the signal in OFDM systems. In [6], the use of block coding with a small PMEPR was recommended. A simplified version of this approach is also proposed in the literature [7, 8]. There are many other approaches (utilising coding) and the interested reader should refer to [9–15]. There are also many other PMEPR reduction techniques in the literature that do not use coding, i.e. [16–19].

For coding-based PMEPR reduction approaches, the phase shift keying (PSK) signal constellation is usually considered. In order to utilise the available bandwidth better, many applications use high level quadrature amplitude modulation (QAM). Recently, there was an attempt to generalise these codes to a 16-QAM constellation [20]. We presented the general solution for an $M$-QAM ($M = 2^n$) signal constellation for even values of $n$ in [21]. In this paper, the solution for odd values of n together with its PMEPR bound is derived.

## 2 Problem statements

Suppose the $i$th carrier of an OFDM symbol is defined as $a_i$, $0 \le i \le N-1$, then the transmitted signal is represented as

$$S_a(t) = \sum_{i=0}^{N-1} a_i \exp(2\pi \mathrm{j}(f_o + i f_s)t) \tag{1}$$

where $f_o$ is the carrier frequency and $f_s$ is the bandwidth of each sub-channel. In some applications, such as an ADSL modem, the complex conjugates of $N$ $M$-QAM symbols are produced and a total of 2N complex values are at the input of the inverse fast Fourier transform (IFFT) block. In this case, $S_a(t)$ is a real signal. In other applications, an $N$-point IFFT is used and $S_a(t)$ is a complex signal. Throughout the paper, we assume that when the signal $S_a(t)$ is mentioned, it can be either a complex or a real signal. The details of the operation of the transmitter and the receiver of an OFDM system are beyond the scope of this paper and can be found in [1, 22].

The vector $\boldsymbol{a}$ represents the codeword of $N$ symbols, i.e. $\boldsymbol{a} = (a_0, a_1, \ldots, a_{N-1})$, $C$ the ensemble of all possible codewords ($\boldsymbol{a} \in C$) and $||\boldsymbol{a}||^2$ the power associated with each codeword $\boldsymbol{a}$. Let $p(\boldsymbol{a})$ denote the probability of codeword $\boldsymbol{a}$ being transmitted, then the mean envelope power of the transmitted signal is defined as

$$P_{av} = \sum_{\boldsymbol{a} \in C} ||\boldsymbol{a}||^2 p(\boldsymbol{a}) \tag{2}$$



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If the instantaneous envelope power is $P(t) = |S_a(t)|^2$, then the PMEPR of the codeword $\boldsymbol{a}$ is defined as

$$PMEPR(\boldsymbol{a}) = \frac{\max |S_a(t)|^2}{P_{av}} \tag{3}$$

The maximisation is during one OFDM symbol period. Our objective is to design codes $C$ with a small PMEPR.

Suppose each element $a_i$ at the input of the IFFT block is a complex number representing a point in the quadrature PSK (QPSK) or binary PSK (BPSK) constellation symbol. Later we will show that a non-square M-QAM symbol can be represented as a sum of QPSK and BPSK signals. Therefore equivalently $a_i$ can be shown as $a_i = \exp(a_i')$. Then (1) can be written as

$$S_a(t) = \sum_{i=0}^{N-1} \exp(a_i' + 2\pi \mathrm{j}(f_o + if_s)t) \tag{4}$$

Therefore the instantaneous envelope power can be given as

$$\begin{aligned} P(t) &= |S_a(t)|^2 \\ &= \sum_{i=0}^{N-1}\sum_{j_1=0}^{N-1} \exp(a_i' - a_{j_1}' + 2\pi \mathrm{j}(i-j_1)f_s t) \end{aligned} \tag{5}$$

By substituting $j_1 = i+u$, $P(t)$ can be written as

$$P(t) = \sum_{i=0}^{N-1}\sum_{u=-i}^{N-1-i} \exp(a_i' - a_{i+u}' - 2\pi \mathrm{j}uf_s t) \tag{6}$$

For sequence $\boldsymbol{a}' = (a_0', a_1', \cdots, a_{N-1}')$, the aperiodic autocorrelation of $\boldsymbol{a}'$ at displacement $u$ is defined as

$$C_{\boldsymbol{a}'}(u) = \sum_{i=0}^{N-1} \exp(a_i' - a_{i+u}') \tag{7}$$

Replacing (7) into (6) gives the following equation:

$$P(t) = N + \sum_{u \neq 0} C_{\boldsymbol{a}'}(u)\exp(-2\pi \mathrm{j}uf_s t) \tag{8}$$

The above derivation has been described earlier in many references, e.g. [10] and [23]. It can be seen [10] that the peak envelope power is upper bounded as $N^2$. The mean envelope power over one symbol period is $N$. Therefore the PMEPR is equal to $N$ for a sequence without any autocorrelation properties.

## 3 Construction of non-square $M$-QAM signals from a combination of QPSK and BPSK signals

The QPSK constellation can be realised as QPSK $= \mathrm{j}^{x_i}$, where $x_i \in Z_4 = \{0, 1, 2, 3\}$. Thus any QPSK sequence $\boldsymbol{a} = (a_0 a_1 \cdots a_{N-1})$ can be associated with another (unique) sequence $\boldsymbol{x}_i = (x_i^0 x_i^1 \cdots x_i^{N-1})$ where the elements of $\boldsymbol{x}_i$ are in $Z_4$. The BPSK constellation can also be realised as the set $BPSK = \{(-1)^{y_i} | y_i = 0, 1\}$. Thus one can associate with any BPSK sequence $\boldsymbol{a} = a_0 a_1 \cdots a_{N-1}$ a unique sequence $\boldsymbol{y} = y_0 y_1 \cdots y_{N-1}$, where $y_i \in Z_2$.

By direct computation, it can be shown that the non-square $M$-QAM symbol ($M = 2^n$, $n$ is an odd number) is a set sum of QPSK and BPSK signals:

$$\begin{aligned} M\text{-}QAM = &\left(\frac{\sqrt{2}}{2}\right)(BPSK)\exp\left(\frac{\pi \mathrm{j}}{4}\right) \\ &+ \sum_{i_1=0}^{\frac{n-3}{2}} (2^{i_1})(\sqrt{2})(\mathrm{j}^{x_{i_1}})\exp\left(\frac{\pi \mathrm{j}}{4}\right) \end{aligned} \tag{9}$$

for $M = 2^n$, $n = 3, 5, 7, \cdots$. Thus any point of our general $M$-QAM constellation at time $k$ can be written as:

$$\exp\left(\frac{\pi \mathrm{j}}{4}\right)\left[\left(\frac{\sqrt{2}}{2}\right)(-1)^{y^k} + (\sqrt{2})(\mathrm{j}^{x_0^k}) + \cdots + 2^{\frac{n-3}{2}}(\sqrt{2})\left(\mathrm{j}^{x^k_{\frac{n-3}{2}}}\right)\right]$$

for $x_0^k, x_1^k, x_2^k, \cdots \in Z_4$ and $y^k \in Z_2$. In this way one can associate with any $M$-QAM sequence (with an odd value of $n$) ($\boldsymbol{a} = a_0 a_1 \cdots a_{N-1}$) a unique sequence:

$$(y^0 x_0^0 x_1^0 \cdots x^0_{\frac{n-3}{2}}, y^1 x_0^1 x_1^1 \cdots x^1_{\frac{n-3}{2}}, \cdots, y^{N-1} x_0^{N-1} x_1^{N-1} \cdots x^{N-1}_{\frac{n-3}{2}}) \\ \in Z_2^N \times \underbrace{Z_4^N \times Z_4^N \times \cdots \times Z_4^N}_{\frac{n-1}{2}}$$

In particular, we can write

$$\begin{aligned} &S_{y,x_0,x_1,\cdots,x_{\frac{n-3}{2}}}(t) = S_a(t) \\ &= \sum_{k=0}^{N-1}\left\{\begin{array}{l} \frac{\sqrt{2}}{2}(-1)^{y^k}\exp\left(2\pi \mathrm{j}(f_0 + kf_s)t + \frac{\pi}{4}\right) + \\ \sum_{i_1=0}^{\frac{n-3}{2}} 2^{i_1}\sqrt{2}(\mathrm{j}^{x^k_{i_1}})\exp\left(2\pi \mathrm{j}(f_0 + kf_s)t + \frac{\pi}{4}\right) \end{array}\right\} \end{aligned} \tag{10}$$

The instantaneous power for $M$-QAM and odd values of $n$ is given by

$$\begin{aligned} P_{y,x_0,x_1,\cdots,x_{\frac{n-3}{2}}}(t) = &\left|\sum_{k=0}^{N-1}\left(\frac{\sqrt{2}}{2}(-1)^{y^k}\exp(2\pi \mathrm{j}kf_s t)\right)\right. \\ &\left. + \sum_{i_1=0}^{\frac{n-3}{2}} 2^{i_1}\sqrt{2}(\mathrm{j}^{x^k_{i_1}})\exp(2\pi \mathrm{j}kf_s t))\right|^2 \end{aligned} \tag{11}$$

By defining

$$S_y(t) = \sum_{k=0}^{N-1}(-1)^{y^k}\exp\left(2\pi \mathrm{j}(f_0 + kf_s)t + \frac{\pi}{4}\right) \text{ and}$$

$$S_{x_i}(t) = \sum_{k=0}^{N-1} \mathrm{j}^{x_i^k}\exp\left(2\pi \mathrm{j}(f_o + kf_s)t + \frac{\mathrm{j}\pi}{4}\right)$$

the instantaneous power can be written as

$$P_{y,x_0,x_1,\cdots,x_{\frac{n-3}{2}}}(t) = \left|\frac{\sqrt{2}}{2}S_y(t) + \sum_{i_1=0}^{\frac{n-3}{2}} 2^{i_1}\sqrt{2}S_{x_{i_1}}(t)\right|^2 \tag{12}$$

The above results can be used to compute PMEPR bounds when the $M$-QAM sequence is constructed based on a combination of QPSK and BPSK Golay (or Golay-like) sequences.

## 4 Golay sequences and Reed–Muller codes

Golay sequences were first introduced in [24]. Any two sequences $\boldsymbol{a}$ and $\boldsymbol{b}$ are called Golay complementary pairs [24] over $Z_H^N$ if $C_{\boldsymbol{a}}(u) + C_{\boldsymbol{b}}(u) = 2N\delta(u)$, where $H$ is the alphabet size and $\delta(\cdot)$ is the Kronecker function. Any sequence with this property is called a Golay sequence. Now assume $a_i = \mathrm{j}^{x_1^i}$ and $b_i = \mathrm{j}^{x_2^i}$ for $0 \leq i \leq N-1$. Let $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ be a Golay complementary pair in $Z_4^N$. Based on the definition of Golay complementary pairs and using (8), we can show [10] that $P_{\boldsymbol{a}}(t) + P_{\boldsymbol{b}}(t) = 2N$. Since $P_{\boldsymbol{a}}(t)$ and $P_{\boldsymbol{b}}(t)$ are positive real numbers, then $P_{\boldsymbol{a}}(t) \leq 2N$. The mean envelope power is $N$, and therefore the PMEPR is at most 2 (the proof was given in [10]). Comparing this bound with a sequence that does not have any autocorrelation properties (PMEPR $\leq N$), we see that Golay sequences have small PMEPRs.

Reed–Muller codes can be defined in terms of Boolean functions. These codes provide good error correction properties as long as the block length is not too large [25]. Their minimum distance is lower than that of BCH codes. The decoding of Reed–Muller codes is relatively simple using majority logic circuits. The $r$th order binary Reed–Muller code of length $2^m$ $RM(r, m)$ is constructed by the monomials in the Boolean function of degree $r$ or less. For the binary data, Golay sequences are cosets of the first-order Reed–Muller code within the second-order Reed–Muller code. This was first introduced in [10]. For general non-binary signals ($M=2^n$), these can be generalised for Reed–Muller codes [10] based on the results in [26].

In [10], it was shown that one can use Golay-like sequences with low PMEPR where $|S_a(t)| = U \geq \sqrt{2N}$, where the lower bound is for Golay sequences and larger values for Golay-like sequences. This will allow us to have a trade-off between the block code rate and the PMEPR.

## 5 Computation of PMEPR upper bounds for $M$-QAM sequences constructed with QPSK and BPSK Golay-like sequences

For the QPSK constellation, we showed that the PMEPR of a single Golay sequence is at most 2. For the $M$-QAM constellation, an analogous result is given below for Golay-like sequences.

*Theorem 1:* For any sequence $\boldsymbol{x} \in \boldsymbol{Z}_4^N$, let $\boldsymbol{z}=\boldsymbol{x}+\boldsymbol{2}$ denote the sequence given by $z_i = x_i + 2$ for $i = 0, 1, \cdots, N-1$. Then

(i) If $\boldsymbol{x}$ and $\boldsymbol{y}$ are Golay-like sequences then

$$P_{y,\underbrace{x+2,\cdots,x+2}_{\frac{n-3}{2}},x}(t) \leq 4.5U$$

(ii) If

$$x_{\frac{n-3}{2}} \text{ and } x_{\frac{n-5}{2}}$$

form a Golay complementary pair and

$$y, x_0, \cdots, x_{\frac{n-7}{2}}$$

are Golay-like sequences and not necessarily Golay complementary pairs, then

$$P_{y,x_0,x_1,\cdots,x_{\frac{n-3}{2}}}(t) \leq \left(\frac{U}{\sqrt{2}} + \sqrt{5N2^{n-4}} + \sqrt{2}(2^{\frac{n-5}{2}} - 1)U\right)^2$$

(iii) If $y, x_0, \cdots, x_{\frac{n-3}{2}}$ are Golay-like sequences and not necessarily Golay complementary pairs, then

$$P_{y,x_0,x_1,\cdots,x_{\frac{n-3}{2}}}(t) \leq \left(\sqrt{2} \times 2^{\frac{n-1}{2}} - \frac{\sqrt{2}}{2}\right)^2 U^2$$

*Proof:* Recall from (12) that

$$P_{y,\underbrace{x+2,\cdots,x+2}_{\frac{n-3}{2}},x}(t) = \left| \left(\frac{\sqrt{2}}{2}\right) S_y(t) + 2^{\frac{n-3}{2}}(\sqrt{2}) S_x(t) + \sum_{i=0}^{\frac{n-5}{2}} 2^{i_1}(\sqrt{2}) S_{x+2}(t) \right|^2 \tag{13}$$

By direct computation

$$S_{x+2}(t) = -S_x(t)$$

Thus

$$\begin{aligned} P_{y,\underbrace{x+2,\cdots,x+2}_{\frac{n-3}{2}},x}(t) &= \left| \left(\frac{\sqrt{2}}{2}\right) S_y(t) + 2^{\frac{n-3}{2}}(\sqrt{2}) S_x(t) + \sum_{i=0}^{\frac{n-5}{2}} 2^{i_1}(\sqrt{2})(-S_x(t)) \right|^2 \\ &= \left| \left(\frac{\sqrt{2}}{2}\right) S_y(t) + (\sqrt{2}) S_x(t) \right|^2 \end{aligned} \tag{14}$$

We now can use the Golay-like sequence equality $P_a(t) = |S_a(t)|^2 = U^2$ and the triangle inequality to conclude that

$$P_{y,\underbrace{x+2,\cdots,x+2}_{\frac{n-3}{2}},x}(t) \leq 4.5U^2 \tag{15}$$

which proves part (i)

To prove part (ii), suppose that

$$x_{\frac{n-3}{2}} \text{ and } x_{\frac{n-5}{2}}$$

are a Golay complementary pair. It follows from the proof of part (i) that

$$S_{x_{\frac{n-3}{2}}+2}(t) = -S_{x_{\frac{n-3}{2}}}(t)$$

Furthermore, from (12) we have

$$\begin{aligned} &P_{y,x_0,x_1,\cdots,x_{\frac{n-5}{2}},x_{\frac{n-3}{2}}}(t) = \\ &\left| \left(\frac{\sqrt{2}}{2}\right) S_y(t) + \sum_{i_1=0}^{\frac{n-3}{2}} 2^{i_1}(\sqrt{2}) S_{x_{i_1}}(t) \right|^2 \\ &= \left| \left(\frac{\sqrt{2}}{2}\right) S_y(t) + 2^{\frac{n-5}{2}}(\sqrt{2}) S_{x_{\frac{n-5}{2}}}(t) + 2^{\frac{n-3}{2}}(\sqrt{2}) S_{x_{\frac{n-3}{2}}}(t) + \sum_{i_i=0}^{\frac{n-7}{2}} 2^{i_1}(\sqrt{2}) S_{x_{i_1}}(t) \right|^2 \\ &= \left| \left(\frac{\sqrt{2}}{2}\right) S_y(t) + 2^{\frac{n-5}{2}}(\sqrt{2}) C + 2^{\frac{n-3}{2}}(\sqrt{2}) B + A \right|^2 \\ &\leq \left( \left| \left(\frac{\sqrt{2}}{2}\right) S_y(t) \right| + \left| 2^{\frac{n-3}{2}}(\sqrt{2}) B + 2^{\frac{n-5}{2}}(\sqrt{2}) C \right| + |A| \right)^2 \end{aligned} \tag{16}$$

where

$$A = \sum_{i_1=0}^{\frac{n-7}{2}} 2^{i_1}(\sqrt{2}) S_{x_{i_1}}(t), \quad B = S_{x_{\frac{n-3}{2}}}(t) \text{ and } C = S_{x_{\frac{n-5}{2}}}(t)$$

It can be easily shown that

$$\left| 2^{\frac{n-3}{2}}(\sqrt{2}) B + 2^{\frac{n-5}{2}}(\sqrt{2}) C \right| \leq \sqrt{5N2^{n-4}} \tag{17}$$

All the elements in $A$ are Golay-like sequences.

$$|A| \le \sum_{i_1=0}^{\frac{n-7}{2}} \left|2^{i_1}(\sqrt{2})S_{x_{i_1}}(t)\right| \le$$

$$U \times \sum_{i_1=0}^{\frac{n-7}{2}} 2^{i_1}(\sqrt{2}) = \sqrt{2}\times\left(2^{\frac{n-5}{2}}-1\right)U \qquad (18)$$

Combining (18) and (17) with (16) and assuming $S_y(t)$ is a Golay-like sequence will prove (ii).

$$P_{y,x_0,x_1,\cdots,x_{\frac{n-3}{2}}}(t) \le \left(\frac{U}{\sqrt{2}} + \sqrt{5N2^{n-4}} + \sqrt{2}\left(2^{\frac{n-5}{2}}-1\right)U\right)^2 \qquad (19)$$

If all the sequences are Golay sequences, then this upper bound will be

$$\left(1+\sqrt{5\times 2^{n-4}}+2\times\left(2^{\frac{n-5}{2}}-1\right)\right)^2 N$$

To prove part (iii), we recall from (12) that

$$P_{y,x_0,x_1,\cdots,x_{\frac{n-3}{2}}}(t) = \left|\left(\frac{\sqrt{2}}{2}\right)S_y(t) + \sum_{i_1=0}^{\frac{n-3}{2}} 2^{i_1}(\sqrt{2})S_{x_{i_1}}(t)\right|^2$$

By the triangle inequality we have

$$P_{y,x_0,x_1,\cdots,x_{\frac{n-3}{2}}}(t) \le \left(\left|2^{\frac{n-3}{2}}(\sqrt{2})S_{x_{\frac{n-3}{2}}}(t)\right| + \cdots + \left|(\sqrt{2})S_{x_0}(t)\right| + \left|\left(\frac{\sqrt{2}}{2}\right)S_y(t)\right|\right)^2$$

Considering both $|S_x(t)|^2$ and $|S_y(t)|^2$ are equal to $U^2$, we have

$$P_{y,x_0,x_1,\cdots,x_{\frac{n}{2}-1}}(t) \le \left(\sum_{i_1=0}^{\frac{n-3}{2}} 2^{i_1}(U)(\sqrt{2}) + (U)\left(\frac{\sqrt{2}}{2}\right)\right)^2$$

$$= \left(\sqrt{2}\times 2^{\frac{n-1}{2}} - \frac{\sqrt{2}}{2}\right)^2 U^2 \qquad (20)$$

This completes the proof. □

In order to derive an upper bound on the PMEPR of the $M$-QAM Golay-like sequences constructed above, we prove the following theorem.

*Theorem 2*: Let $y$ and $x_i$s $(1\le i\le (n-3)/2)$ be independent sequences of length $N$ with equi-probable elements such that $E(S_{x_i}(t)S_{x_j}(t))=0$ and $E(S_y(t)S_{x_i}(t))=0$. Then the mean envelope power is

$$P_{av} = \left(\frac{1}{2}+\frac{2}{3}(2^{n-1}-1)\right)N$$

*Proof*:

$$P_{av} = E\left(\left|\left(\frac{\sqrt{2}}{2}\right)S_y(t) + \sum_{i_1=0}^{\frac{n-3}{2}} 2^{i_1}(\sqrt{2})S_{x_{i_1}}(t)\right|^2\right)$$

$$= \frac{1}{2}E\left|S_y(t)\right|^2 + \sum_{i_1=0}^{\frac{n-3}{2}} (2^{i_1}\sqrt{2})^2 E\left|S_{x_{i_1}}(t)\right|^2$$

$$= \left(\frac{1}{2}+\frac{2}{3}(2^{n-1}-1)\right)N \qquad (21)$$

$P_{av}$ is derived based on the assumption of independence between these sequences since they are generated from statistically independent uncoded data and the fact that $E|S_{x_i}(t)|^2 = E|S_y(t)|^2 = N$. □

*Theorem 3*: Let $A\subseteq Z_4^N$ and $B\subseteq Z_2$ be the set of Golay-like sequences. Then the PMEPR for $Z := B\times A^1\times\cdots\times A^{\frac{n-3}{2}}$ is bounded by

$$\frac{\left(\sqrt{2}\times 2^{\frac{n-1}{2}} - \frac{\sqrt{2}}{2}\right)^2 U^2}{\left(\frac{1}{2}+\frac{2}{3}(2^{n-1}-1)\right)N}$$

provided that $B\times A^1\times\cdots\times A^{\frac{n}{2}-1}$ is used for equi-probable $M$-QAM OFDM transmission.

*Proof:* The result can be derived directly from part (iii) of theorem 1 and theorem 2.

Figure 1 shows the PMEPR bound for non-square $M$-QAM Golay sequences constructed from a combination of BPSK and QPSK Golay sequences. As the constellation size of the $M$-QAM sequence increases, the PMEPR reaches a maximum value of 6.

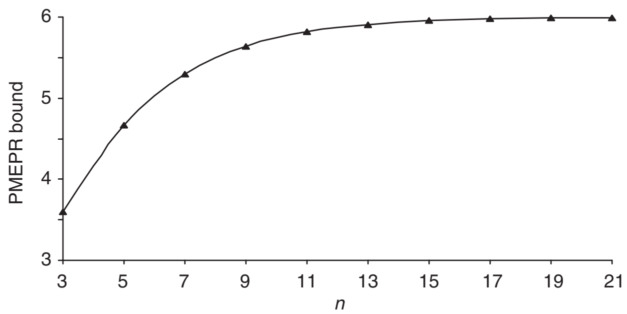


**Fig. 1** *PMEPR bound for M-QAM (with n odd) constructed by a combination of BPSK and QPSK Golay sequences*

## 6 Conclusion

In this paper, we have proposed technique to construct a non-square $M$-QAM sequence from a combination of BPSK and QPSK sequences. If these QPSK and BPSK sequences are Golay (or Golay-like) sequences derived from Reed–Muller codes [10], then the constructed $M$-QAM sequence has a low PMEPR which is suitable for OFDM systems.

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