Performance of a single-input multiple-output decision feedback equaliser for polarisation mode dispersion compensation

Z. Zhu, H.R. Sadjadpour, R.S. Blum, P.A. Andrekson and T.J. Li

Abstract: To compensate the performance degradation induced by polarisation mode dispersion (PMD) in high-speed optical transmission, a single-input multiple-output (SIMO) decision feedback equaliser (DFE) technique is proposed to combat all orders of PMD-induced distortion. The scheme is based on a new SIMO PMD channel model which utilises information embedded in both polarisation states. The performance analysis of the proposed PMD SIMO DFE scheme is provided along with explicit expressions for the filter coefficients. The analysis results show that the new scheme provides a significant improvement over using a first-order optical compensator and conventional DFE.

1 Introduction

Polarisation-mode dispersion (PMD) becomes the capacity limiting factor for sufficiently long transmission distances and sufficiently high bit rates optical communications. In fact, deployed systems are rapidly approaching these limits. New signal processing techniques are needed to deal with the signal distortions introduced by PMD as well as by other impairments.

Polarisation-mode dispersion is caused by optical birefringence in fibres. The unintentional and random asymmetry of any optical fibre introduces a difference in the phase and group velocities of the two orthogonally polarised modes. The signal propagating along one polarisation moves slower or faster than the signal along the other polarisation. At the receiver, the shape of the signal is spread and distorted because of the differential group delay (DGD). Several PMD compensation techniques have been proposed in both the optical and electrical domains. Optical PMD compensators [1] correct the phase difference between the two polarisation modes using polarisation controllers and optical delays with feedback from the receiver.

The inter-symbol interference (ISI) caused by PMD is linear in the optical domain. Electrical compensators usually operate after the nonlinear square-law detector, therefore optical compensators are more effective than electrical ones because they process light directly without losing any information. Although electrical compensators can fully take advantage of the signal processing techniques developed in the electrical domain, unfortunately after the detection receiver (square-law detection), the polarisation phase information is lost in electrical domain [2] and the channel becomes a nonlinear channel. (The 'nonlinearity' used in this article is based on the signal processing definition discussed by A. Oppenheim and R. Schafer in 'Discrete-time signal processing' (Prentice Hall Inc, 1989) [3]. It is different than the 'power dependent nonlinearity' used in some optical literature.) In most of the previous research on signal processing techniques for PMD [4, 5], PMD optical channels were assumed to be linear, which is not an accurate assumption when a square-law detector is used in the receiver. This is one of the reasons that the conventional DFE or linear equaliser (LE) is not effective [6] as such schemes can only mitigate the first-order PMD [7]. In this article, we present a new approach in utilising electrical domain equalisers in the optical domain by modelling a PMD dominant fibre optic channel as a SIMO model.

Until very recently, research on electrical PMD compensation focused almost exclusively on the first-order PMD and assumed the PMD channels are single-input single-output (SISO) channels. The SISO assumption cannot resolve the two polarisation modes and therefore is a rough representation of the PMD channels which cannot represent high-order PMD. The proposed SIMO PMD model is able to reflect more accurately the interaction between the two polarisation modes which results in the DGD phase shift.

In this article, we propose a single-input multiple-output decision feedback equaliser (SIMO-DFE) scheme which compensates for all orders of PMD. In particular, we employ DFE-like equaliser on a new filtering structure that is motivated by the SIMO-PMD channel model and consequently, it is more effective. The new approach takes into consideration the fact that the output signals in the two polarisation modes are not linearly combined at the receiver. Using the recently developed MIMO signal processing techniques, the proposed equaliser detects the received signal optimally based on the outputs from both polarisation modes. The scheme can mitigate all orders of PMD and obtain significant performance gain over the conventional SISO DFE and first-order optical or electrical compensators.

doi:10.1049/iet-opt:20050080

Paper first received 3rd September 2005 and in revised form 28th June 2006

Z. Zhu, R.S. Blum, P.A. Andrekson and T.J. Li are with the Department of Electrical and Computer Engineering, Lehigh University, Bethlehem, PA, USA

H.R. Sadjadpour is with the Department of Electrical and Computer Engineering, University of California, Santa Cruz, CA, USA

P.A. Andrekson is with the Department of Microtechnology and Nanoscience, Chalmers University of Technology, Göteborg, Sweden

E-mail: zhenyu.k.zhu@gmail.com

The chapter is organised as follows: Section II proposes a SIMO channel model for PMD with its statistical characteristics. In Section III, a SIMO-DFE scheme based on this model is formulated. Optimal coefficients are derived and performance of the scheme is analysed with different operating conditions. Section IV compares the SIMO-DFE performance to two well-known PMD compensators, one using PSP transmissions and another using a nonlinear canceller. Section V draws the conclusion.

2 SIMO-PMD channel model

Based on the PMD vector concatenation rule and the principle states model [1, 7], a PMD-limited optical channel can be modelled using a series of linear birefringent elements which are sandwiched between polarisation adjustments. This model characterises all orders of PMD, not only the first-order. Signal propagation along the polarisation modes of a fibre is modelled in Jones space [7].

The baseband transmitted signal is assumed to be

$$f_{\text{in}}(t) = \sum_k s_k q(t - kT) \tag{1}$$

where $q(t)$ is the transmitted pulse and s_k is a binary input sequence. The transmitter modulates the signal by translating it to a laser carrier frequency f_c. The complex baseband representation of the electric field of the launched optical signal can be expressed as a two-dimensional vector of the orthogonal polarisation components as

$$\begin{bmatrix} m_1(t) \\ m_2(t) \end{bmatrix} = f_{\text{in}}(t) \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \mathrm{e}^{j\delta} \end{bmatrix} \tag{2}$$

where θ and δ define the input state of polarisation (SOP) along the polarisation axis. The launched SOP has a big impact on the resulting PMD distortion. When the input SOP is aligned with the fibre's principle states of polarisation (PSP), the first-order DGD can be minimised. This is called the PSP transmission method.

The transmitted signal is distorted by a PMD-limited fibre channel. The transfer function of the PMD channel can be expressed as transmission matrix $T(\omega)$

$$\begin{aligned} T(\omega) = U'(\alpha_N) & \begin{bmatrix} \mathrm{e}^{-j\tau_N \omega/2} & 0 \\ 0 & \mathrm{e}^{j\tau_N \omega/2} \end{bmatrix} \\ & \times U'(\alpha_{N-1}) \begin{bmatrix} \mathrm{e}^{-j\tau_{N-1} \omega/2} & 0 \\ 0 & \mathrm{e}^{j\tau_{N-1} \omega/2} \end{bmatrix} \\ & \times U'(\alpha_{N-2}) \ldots U'(\alpha_1) \begin{bmatrix} \mathrm{e}^{-j\tau_1 \omega/2} & 0 \\ 0 & \mathrm{e}^{j\tau_1 \omega/2} \end{bmatrix} U'(\alpha_0) \end{aligned} \tag{3}$$

where $U(\alpha)$ is the rotation matrix

$$\mathrm{U}(\alpha_i) = \begin{bmatrix} \cos(\alpha_i) & \sin \mathrm{e}(\alpha_i) \\ -\sin(\alpha_i) & \cos(\alpha_i) \end{bmatrix} \tag{4}$$

Birefringence element i has a delay of τ_i and α_i is the rotation angle between two adjacent retarders. Assuming no chromatic dispersion and polarisation-dependent loss, the optical field at the output of a PMD limited fibre can be written as

$$\begin{bmatrix} P^1(\omega) \\ P^2(\omega) \end{bmatrix} = T(\omega) \begin{bmatrix} M_1(\omega) \\ M_2(\omega) \end{bmatrix} = T(\omega) \begin{bmatrix} \cos\theta \\ \sin\theta \mathrm{e}^{-j\delta} \end{bmatrix} F_{\text{in}}(\omega) \tag{5}$$

where $M_1(\omega)$, $M_2(\omega)$ and $F_{\text{in}}(\omega)$ are the frequency domain representations of $m_1(t)$, $m_2(t)$ and $f_{\text{in}}(t)$, respectively. All the frequency domain representations are complex numbers which contain phase information.

It is clear that the model in (3) captures the frequency dependence of PMD, which represents first, and all higher orders components of PMD. The following proposed SIMO-DFE technique is designed based on this channel model and is capable to mitigate all-order PMDs. The transfer function in (3) causes a DGD between two orthogonal SOPs, that is, the PSPs, which results in the broadening of output pulse.

2.1 PMD as a SIMO Channel

Derived from (5), the PMD-limited single-mode optical channel can be formulated as the SIMO channel (single input two output) shown in Fig. 1. The SIMO channel is represented with discrete-time baseband notation hereafter. The single input signal $s(k)$ is first split into two orthogonal components $m_1(k)$ and $m_2(k)$ and then launched on two polarisation modes of the transmission fibre. It is well-known that for single-mode fibres, there always exist an orthogonal pairs of polarisation at the output of the fibre called the PSPs. On the fibre output, we denote the output signals on the two PSPs as $p^1(k)$ and $p^2(k)$, respectively. The signal paths between the $m_i(k)$ $(i = 1, 2)$ and $p^j(k)$ $(j = 1, 2)$ form four subchannels which are denoted by $h(i, j)$ in Fig. 1, where $h(i, j)$ will be represented using a discrete-time filter with an impulse response $(h_1^{i,j}, \ldots, h_w^{i,j})$. All four subchannels are linear, dispersive and noisy in the optical domain. $n^1(k)$ and $n^2(k)$ represent the noises on each PSP, which are assumed to be white and Gaussian distributed.

Using the complex baseband equivalent signal model, this SIMO-PMD channel can be expressed as

$$p^j(k) = \sum_{i=1}^{2} \sum_{l=1}^{w} h_l^{i,j} m_{k-l+1}^i + n_k^j \tag{6}$$

where $p^j(k)$ is the jth component of the polarisation vector which is the discrete time representation of $P^j(\omega)$, m_{k-1}^i is the ith input and $h_l^{i,j}$ is the channel impulse response between ih input and jth output for $i, j = 1$ and 2. The memory of the channel between ith input and jth output is assumed to be less than or equal to w and n_k^j is the noise in the jth component of the polarisation vector with a variance of σ_j^2. Using vector notation, (6) can be rewritten as

$$\boldsymbol{p}(k) = \sum_{l=1}^{w} \boldsymbol{h}_l \boldsymbol{m}_{k-l+1} + \boldsymbol{n}_k \tag{7}$$

where $\boldsymbol{h}_l$ is a 2×2 channel matrix and $\boldsymbol{m}_{k-l}$ is the 2×1 input vector at time $k - l$.

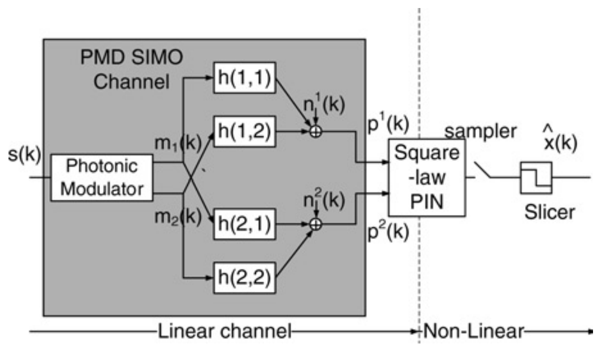

Fig. 1 *PMD complex baseband system model*

From (3), (4) and (6), the transfer function of the PMD-limited channel $T(\omega)$ can be simplified as

$$\boldsymbol{T}(\omega) = \begin{bmatrix} \boldsymbol{H}(1,1)(\omega) & \boldsymbol{H}(1,2)(\omega) \\ \boldsymbol{H}(2,1)(\omega) & \boldsymbol{H}(2,2)(\omega) \end{bmatrix} = \begin{bmatrix} \boldsymbol{T}1(\omega) & \boldsymbol{T}2(\omega) \\ -\boldsymbol{T}2(\omega)^* & \boldsymbol{T}1(\omega)^* \end{bmatrix} \tag{8}$$

where $\boldsymbol{H}(i, j)(\omega)$ is the Fourier transform of $(h_1^{i,j}, \ldots, h_w^{i,j})$.

At the receiver, a PIN detector converts the optical electric field into electrical current. The detected electrical signal is proportional to the power of the received optical signal

$$R(k) = |p^1(k)|^2 + |p^2(k)|^2 \tag{9}$$

After this square-law detector, the polarisation phase information between the two polarisation modes is lost. That phase information is important to mitigate the high-order PMD in electrical domain.

2.2 PMD-SIMO model validation

The PMD-SIMO model in (6) can be validated against two well-known PMD characteristics, that is, DGD's Maxwellian distribution and the proportional relationship between average DGD and square root of the fibre distance.

It is generally well accepted that DGD is not deterministic and thus we must resort to a statistical description. The probability density function (pdf) of DGD is Maxwellian distributed over an ensemble of fibre realisations at any fixed optical frequency. As shown in Fig. 2, the probability density function of the simulated fibre with 15 birefringent elements agrees reasonably well with the analytic Maxwellian prediction. The pdf of the DGD is generated by 10 000 fibre realisations with the same intrinsic PMD. The mismatch from the Maxwellian distribution appears to decrease if the number of the birefringence elements is increased.

The second important characteristic of PMD is that for a long fibre, the average DGD is proportional to the square root of fibre length, instead of the fibre length itself. In Fig. 3, 10 000 fibre realisations were generated. It can be seen that the average DGD along the fibre distance match very well with the analytic square root curve.

From these two results, it is clear that the SIMO channel model matches very well with the statistical characteristics of the real optical fibres. The analysis and the simulations in the following sections are based on this statistical PMD model.

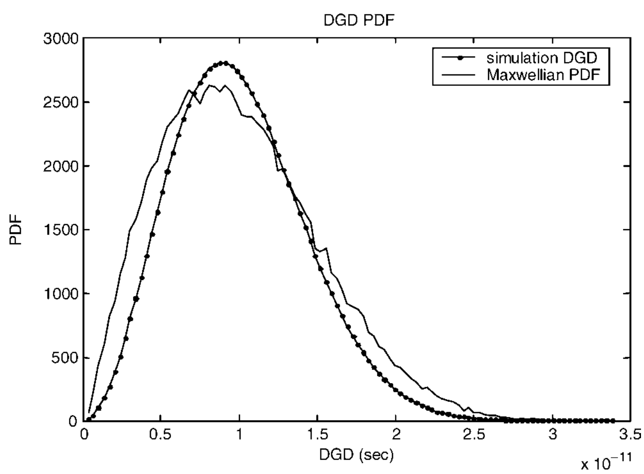

Fig. 2 *Maxwellian DGD pdf*

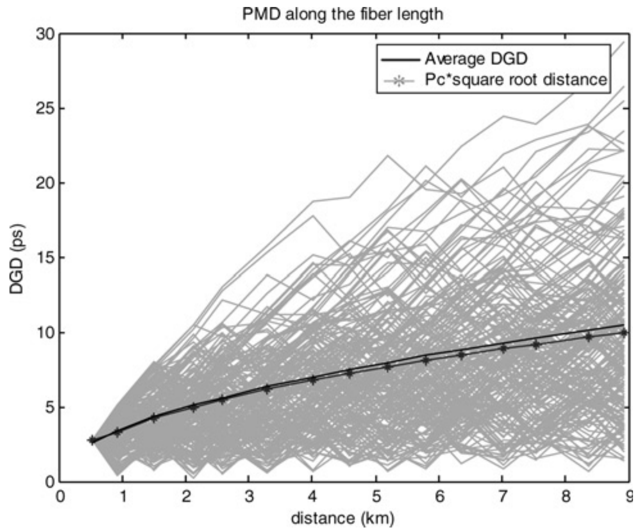

Fig. 3 *Mean DGD increases with the square root of distance*

3 SIMO-PMD decision feedback equaliser

The SIMO-PMD model indicates that the end-to-end PMD-limited optical channel is a dispersive and nonlinear channel in the electrical domain. The conventional equalisation methods, such as the MMSE linear equaliser or the MMSE-DFE are based on assuming linear ISI channels and therefore are not suitable for this kind of nonlinear channel. As an alternative, we propose a new SIMO DFE architecture to equalise the PMD channel as shown in Fig. 4. In a conventional DFE, a slicer is used as a decision element and the assumption is that the decision after the slicer is correct. By comparing the receiver structure of the PMD receiver with the structure of the conventional SISO DFE, we found that the PMD square-law detector combined with the decision element (the big grey box in Fig. 4) can be treated as a generalised slicer (the lower small grey box in Fig. 4), which is equivalent to the decision element in a conventional DFE. Prior to this generalised slicer, the PMD channel is linear and a new SIMO DFE can deal with it effectively.

In the proposed SIMO DFE, the output of a PMD channel is first split into two orthogonal SOP $q^j(k)$ ($j = 1, 2$) by a polarisation beam splitter (PBS). The input polarisation state of the PBS is assumed to be optimal. The optimal polarisation states are derived based on the characteristics of the fibres and the input polarisation state of the transmitter. The PBS has to be adjusted to align with the main axis of fibre during the training stage with low-rate data sequences. As shown in Fig. 4, two feedforward filters $\boldsymbol{FF}_1$ and $\boldsymbol{FF}_2$ are applied to $p^j(k)$ ($j = 1, 2$) in order to

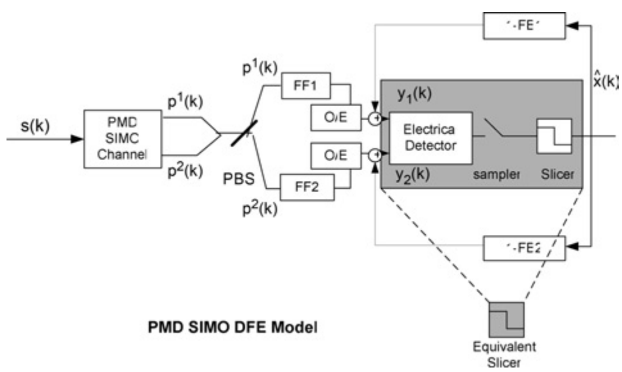

Fig. 4 *PMD SIMO DFE*

remove the post-cursor ISI and shape the channel output signal so that it is a causal signal. The feedback filters $\boldsymbol{FB}_1$, $\boldsymbol{FB}_2$ operate on the decision made by the slicer which follows the square-law detector. The square-law detector sums the squares of $y_1(k)$ and $y_2(k)$. In this system, the inputs to the detector are represented as $\boldsymbol{y}(k)$ which is described as

$$\boldsymbol{y}(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} \boldsymbol{FF}_1 \begin{bmatrix} p^1(k) \\ p^1(k-1) \\ \cdots \\ p^1(k-l_f+1) \end{bmatrix} - (\boldsymbol{E} - \boldsymbol{FB}_1) \begin{bmatrix} x(k) \\ x(k-1) \\ \cdots \\ x(k-l_b+1) \end{bmatrix} \\ \boldsymbol{FF}_2 \begin{bmatrix} p^2(k) \\ p^2(k-1) \\ \cdots \\ p^2(k-l_f+1) \end{bmatrix} - (\boldsymbol{E} - \boldsymbol{FB}_2) \begin{bmatrix} x(k) \\ x(k-1) \\ \cdots \\ x(k-l_b+1) \end{bmatrix} \end{bmatrix} \tag{10}$$

where $\boldsymbol{FF}_1$, $\boldsymbol{FF}_2$, $\boldsymbol{FB}_1$, $\boldsymbol{FB}_2$ and $\boldsymbol{E}$ are all row vectors. $\boldsymbol{FF}_1$ and FF_2 are the impulse responses of the two feedforward filters and their lengths are l_f. $\boldsymbol{FB}_1$ and $\boldsymbol{FB}_2$ are the impulse responses of the two feedback filters and their lengths are l_b. $\boldsymbol{E} = [1 \ 0 \ 0 \ \cdots \ 0]$ is a row vector of length l_b.

These feedforward filters are finite-length impulse response (FIR) filters, which can be implemented with high-bandwidth optical lattice FIR filters, as described in [8, 9]. The feedback filters can be implemented using electrical transversal FIR filters. The outputs of the feedforward filters are converted into electrical signals before being combined with the corresponding feedback filters outputs. The scheme provides a general framework to analyse and design equalisation schemes for PMD limited channels. The implementation complexity may not be optimal. This article only describes the new approach knowing that the complexity of this system should be reduced further in order to make this approach more practical. Future areas of improvement on this SIMO PMD compensation scheme include how to further reduce the implementation complexity without significantly impacting the performance.

3.1 SIMO DFE Coefficients Optimisation

In the following, we derive the optimal filter coefficients for the equalisers in Fig. 4 using some results from [10]. Both the minimum mean square error (MMSE) and the zero-forcing (ZF) solutions are formulated in [10]. The zero-forcing solution is a special case of the MMSE solution for the case when the SNR approaches infinity. In this section, the MMSE solution is derived, which subsumes the zero-forcing solution. The MMSE-LE (Linear Equaliser) and ZF-LE are also the special cases when only the feedforward filters are used and optimised while the feedback filters are removed (i.e. $FB_j = 1$).

Since the SNR of optical channels are normally high, the error propagation of DFE can be ignored as in other high SNR applications. Assuming that the previous decisions are always correct ($y_1(k)_{\text{true}} = y_2(k)_{\text{true}} = x(k)$), the vector error signal at the input to the square law is

$$\boldsymbol{e}(k) = \begin{bmatrix} \boldsymbol{FF}_1 \begin{bmatrix} p^1(k) \\ p^1(k-1) \\ \cdots \\ p^1(k-l_f+1) \end{bmatrix} - \boldsymbol{FB}_1 \begin{bmatrix} x(k) \\ x(k-1) \\ \cdots \\ x(k-l_b+1) \end{bmatrix} \\ \boldsymbol{FF}_2 \begin{bmatrix} p^2(k) \\ p^2(k-1) \\ \cdots \\ p^2(k-l_f+1) \end{bmatrix} - \boldsymbol{FB}_2 \begin{bmatrix} x(k) \\ x(k-1) \\ \cdots \\ x(k-l_b+1) \end{bmatrix} \end{bmatrix} = \boldsymbol{FFP} - \boldsymbol{FBX} \tag{11}$$

$$\text{where } \boldsymbol{FF} = \begin{bmatrix} FF_1 & E_F \\ E_F & FF_2 \end{bmatrix}; \ \boldsymbol{FB} = \begin{bmatrix} FB_1 \\ FB_2 \end{bmatrix}$$
$$\boldsymbol{P} = [p^1(k)p^1(k-1)\cdots p^1(k-l_f+1)p^2(k)p^2(k-1)\cdots p^2(k-l_f+1)]^{\mathrm{T}}$$
$$\boldsymbol{X} = [x(k)x(k-1)\cdots x(k-l_b+1)]^{\mathrm{T}} \tag{12}$$

where E_F is a zero-row vector $[0 \ 0 \ \cdots \ 0]$ of length l_f.

The MMSE measure of this multi-dimensional error random process is equal to the trace of the error auto-correlation matrix $\boldsymbol{R}_{ee}$ which is

$$\begin{aligned} \boldsymbol{R}_{ee} &= E[\boldsymbol{e}(k)\boldsymbol{e}(k)^*] \\ &= \boldsymbol{FF}^*\boldsymbol{R}_{pp}\boldsymbol{FF} - \boldsymbol{FF}^*\boldsymbol{R}_{px}\boldsymbol{FB} \\ &\quad - \boldsymbol{FB}^*\boldsymbol{R}_{xp}\boldsymbol{FF} + \boldsymbol{FB}^*\boldsymbol{R}_{xx}\boldsymbol{FB} \end{aligned} \tag{13}$$

where $\boldsymbol{R}_{pp}$ is the auto-correlation matrix of feedforward output vector Q, $\boldsymbol{R}_{px}$ is the cross-correlation matrix of p and input x, and $\boldsymbol{R}_{xx}$ is the auto-correlation matrix of input vector x. $\boldsymbol{FF}^*$ is the Hermitian transpose of $\boldsymbol{FF}$. In the following derivation, the channel responses $h(i, j)$ and the noise auto-correlation matrix, $\boldsymbol{R}_{nn}$ are assumed known.

By applying the Orthogonality Principle, it can be shown that the optimum feedforward and feedback filters are related by

$$\begin{aligned} &E[\boldsymbol{e}(k)\boldsymbol{P}^*(k)] = 0 \quad \text{(Orthogonality Principle)} \\ &\boldsymbol{FF}^*_{opt} = \boldsymbol{FB}^*_{opt}\boldsymbol{R}_{xp}\boldsymbol{R}^{-1}_{pp} \end{aligned} \tag{14}$$

From (13) and (14) we arrive at

$$\boldsymbol{R}_{ee} = \boldsymbol{FB}^*(\boldsymbol{R}^{-1}_{xx} + \boldsymbol{H} * \boldsymbol{R}^{-1}_{nn}\boldsymbol{H})\boldsymbol{FB} \tag{15}$$

The MMSE solution can be found as

$$\begin{aligned} &\min_{FB} \ \text{trace}(\boldsymbol{R}_{ee}) \\ &= \min_{FB} \ \text{trace}(\boldsymbol{FB}^*(\boldsymbol{R}^{-1}_{xx} + \boldsymbol{H} * \boldsymbol{R}^{-1}_{nn}\boldsymbol{H})\boldsymbol{FB}) \end{aligned} \tag{16}$$

$$\boldsymbol{FB}_{opt} = \boldsymbol{R}\Phi(\Phi^*\boldsymbol{R}\Phi)^{-1}\boldsymbol{C} \tag{17}$$

where $\boldsymbol{R} = (\boldsymbol{R}^{-1}_{xx} + \boldsymbol{H} * \boldsymbol{R}^{-1}_{nn}\boldsymbol{H})$ and

$$\Phi = \begin{bmatrix} I_{\Delta\times(\Delta+1)} \\ 0_{1\times(\Delta+1)} \end{bmatrix} \ \boldsymbol{C} = \begin{bmatrix} 0_{1\times\Delta} \\ 1 \end{bmatrix}$$

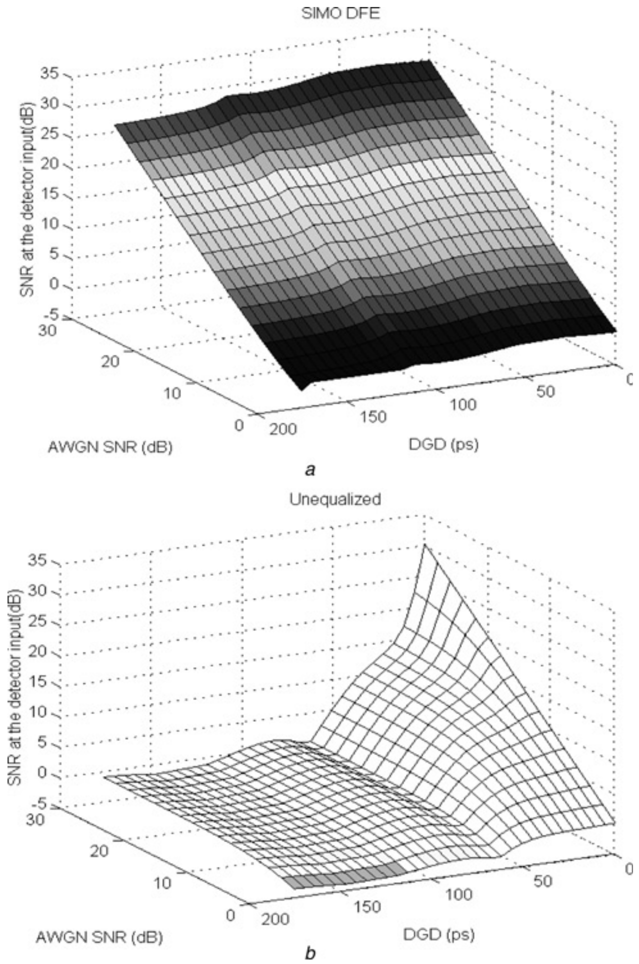

Fig. 5 *Performance comparison of PMD SIMO DFE based channel vs. uncompensated channel*

a Decision-point SNR of PMD SIMO DFE
b Decision-point SNR of unequalised PMD channels

Note that Δ, the decision delay of the DFE, is defined as the delay between the sample currently detected and the source symbol being generated. It is reasonable to assume that the input and noise processes are uncorrelated at different times. Then $\boldsymbol{R}_{xx}$ and $\boldsymbol{R}_{nn}$ can be represented by diagonal matrices.

3.2 PMD SIMO DFE performance analysis

The performance of the PMD SIMO DFE can be evaluated with the decision-point SNR, which is the signal-to-noise ratio measured at the symbol detector input. The SIMO decision point SNR is defined as an extension of the SISO receiver SNR [11] and is calculated as

$$\begin{aligned}\mathbf{SNR}_D &= \frac{\mathrm{trace}(E[\boldsymbol{x}(k)\boldsymbol{x}(k)^*]/(l_\mathrm{f}+w))}{\mathrm{trace}(E[\boldsymbol{e}(k)\boldsymbol{e}(k)^*])} \\ &= \frac{\mathrm{trace}(\boldsymbol{R}_{xx})}{(l_\mathrm{f}+w)\mathrm{trace}(\boldsymbol{R}_{ee})}\end{aligned} \tag{18}$$

The trace operation is equal to the arithmetic average of the eigenvalues of the corresponding auto-correlation matrix. The defined decision point SNR represents the average SNR of the two inputs to the detector ($y_1(k)$ and $y_2(k)$) on the two polarisation modes.

When the DFE filter coefficients are optimised based on the MMSE criteria as derived in (15) and (16), the optimal SNR is given by

$$\begin{aligned}\mathrm{SNR}_{D,opt} &= \frac{\mathrm{trace}(\boldsymbol{R}_{xx})}{(l_\mathrm{f}+w)\mathrm{trace}(\boldsymbol{R}_{ee,\ \min})} \\ &= \frac{\mathrm{trace}(\boldsymbol{R}_{xx})}{(l_\mathrm{f}+w)\mathrm{trace}(\boldsymbol{C}^*(\Phi^*\boldsymbol{R}\Phi)^{-1}\boldsymbol{C})}\end{aligned} \tag{19}$$

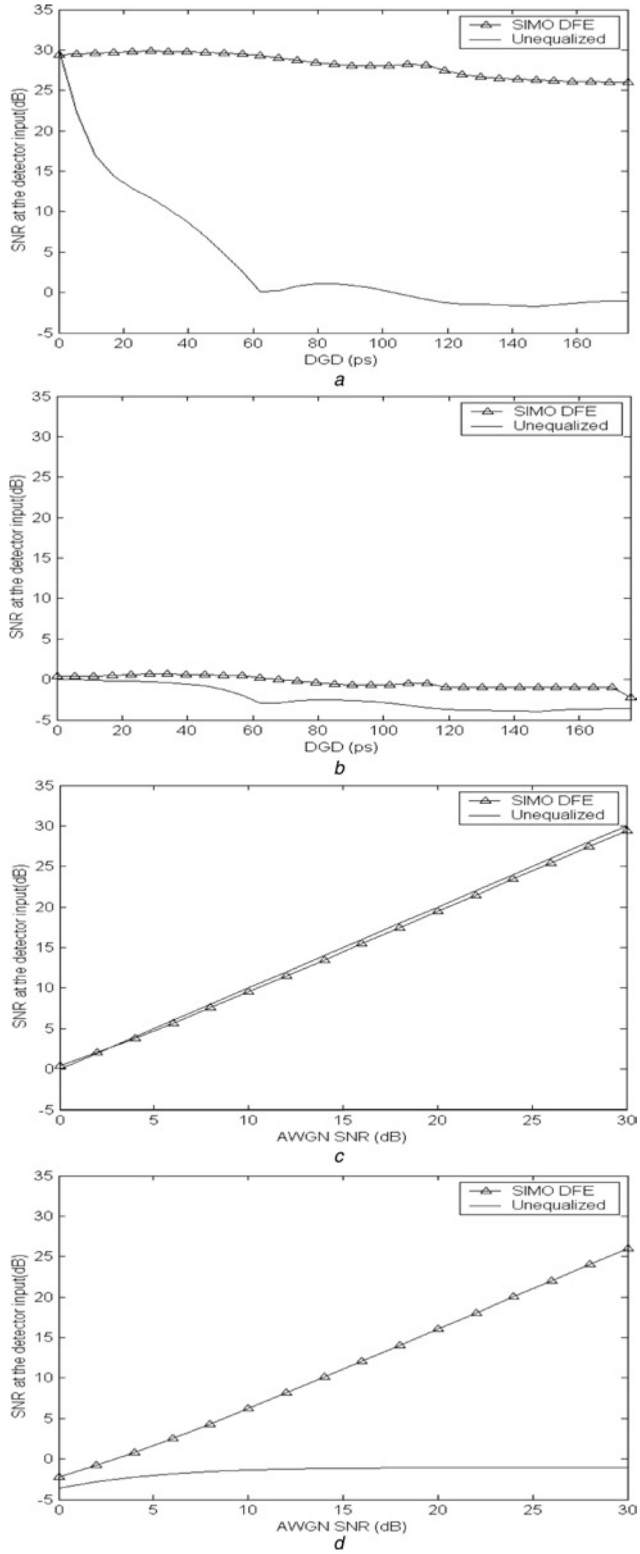

Fig. 6 *Performance of the PMC SIMO DFE at different channel conditions*

a Performance vs. DGD at high SNR
b Performance vs. DGD at low SNR
c Performance vs. SNR of channels with no PMD
d Performance vs. SNR at high DGD (160 ps) channels

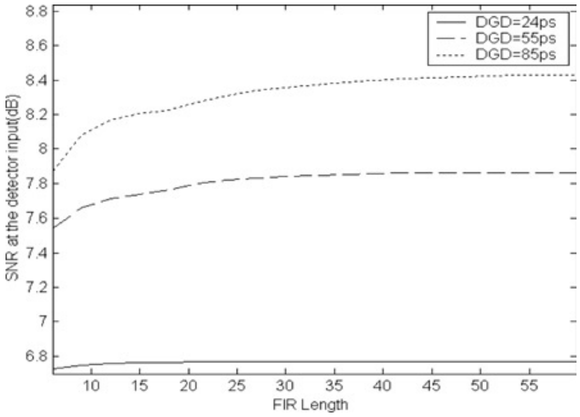

Fig. 7 *Finite length FIR's effect on PMD SIMO DFE's performance*

Hence, the performance of the SIMO DFE depends on several factors, including (1) the PMD SIMO channel impulse response, (2) the Gaussian noise on each polarisation mode level relative to the source signal power, and (3) the decision delay Δ. The SNR defined here is a baseband criteria which assumes the resulting optical channel is an Additive White Gaussian Noise (AWGN) channel after the equalisation.

The optimal decision-point SNR of the PMD SIMO DFE is shown in Fig. 5*a* with varied levels of DGD and different levels of Gaussian noise. It can be seen that the performance of the SIMO DFE is insensitive to the level of DGD for a fixed level of Gaussian noise.

Fig. 5*b* shows the decision-point SNR of unequalised PMD channels. Compared with the SIMO DFE, it is apparent that the SNR of unequalised PMD channels degrade dramatically when the DGD increases. When DGD exceeds 60 ps, the SNR of the unequalised PMD channel decreases to 0 dB.

Besides the distortion induced by PMD, the optical signal is corrupted by noise during transmission. In this article, the level of the optical noise is represented using optical SNR, which is defined as the ratio of transmit signal power over optical noise power in optical channels.

Fig. 6*a* depicts the performance of the SIMO DFE and unequalised PMD channel with different DGD values in a high optical SNR (optical SNR =30 dB) environment. The improvement of the SIMO DFE over the unequalised case is significant, especially for high DGD channels. The performance of the SIMO DFE is again relatively insensitive to the variation of the DGD. Fig. 6*b* shows the performance of the SIMO DFE and unequalised PMD channel in a low SNR (5 dB) environment. In this case, the equaliser provides little improvement because the noise dominates the SNR as opposed to the ISI introduced by PMD.

Fig. 6*c* demonstrates the performance of the SIMO DFE and the unequalised PMD channel in a zero PMD channel. The equaliser performs the same as the un-equalised case, which is expected. However in a high DGD (160 ps) case as depicted in Fig. 6*d*, the performance degradation is apparent for un-equalised channels. The SIMO DFE's SNR increases proportionally with the improved Gaussian SNR in both high and low DGD cases.

3.3 *Effect of finite length in SIMO DFE*

The proposed PMD SIMO DFE can be implemented with finite impulse response (FIR) filters. Fig. 7 shows the variation of the receiver SNR with the number of feed-forward filter taps for several realisations of fibres with different DGDs. The number of feedback filter taps is fixed. Fig. 7 shows that the receiver SNR is relatively insensitive to the filter lengths. For the high DGD channels, receiver SNR varies more significantly with FIR length than for the low DGD channels.

4 Comparison with other PMD compensators

The proposed scheme is compared to the two other most-frequently used types of PMD compensators, that is, PSP transmission, which is an optical method, and a Non-linear Canceller, which is an electrical method. For the simulated PMD channel model, 15 sections of birefringent elements are used with fixed, but different lengths to avoid DGD spectrum periodicity. The angles of the coupling modes are randomly generated with uniform distributions. The channels used in analysis and the simulations are generated randomly based on the statistical PMD model described in Section 2. However, they are not the average results from hundreds of fibre realisations. We believe the performance presented in this article accurately presents the general behaviour of our algorithm since the equaliser is designed to eliminate all PMD caused ISI, therefore it is not sensitive to the PMD channel response.

A typical long-haul optical transmission system is simulated similar to [1]. 10 Gb/s, 2^{15}-1 PRBS signals are used as inputs to the model, which are Gaussian pulse shaped with pulse width of 50 ps and then RZ modulated before being

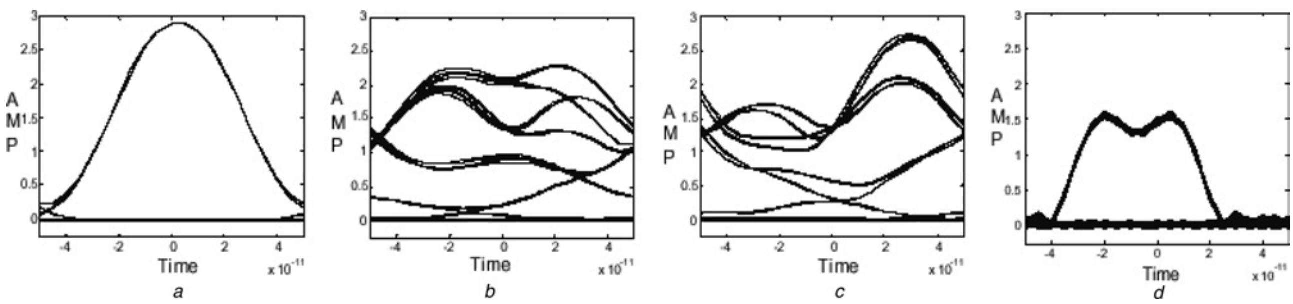

Fig. 8 *Receive signal eye diagrams*

a Channel without PMD
b Uncompensated PMD channel
c PMD channel w/PSP transmission
d PMD SIMO DFE

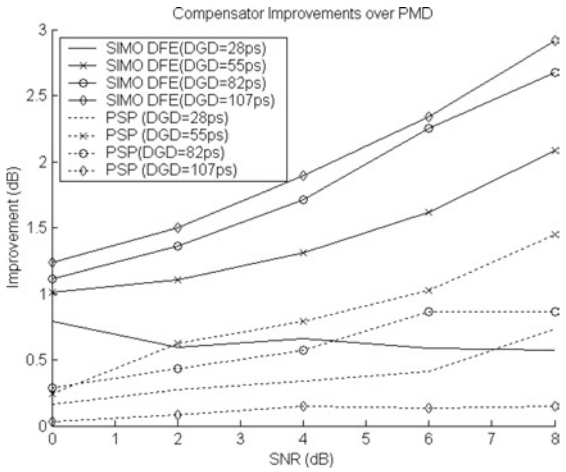

Fig. 9 *Performance improvements w.r.t. uncompensated PMD channel*

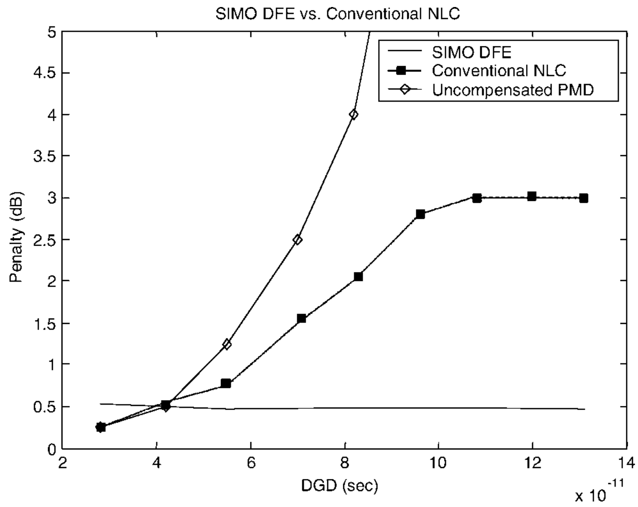

Fig. 10 *Comparison with conventional nonlinear canceller*

launched by the laser. The simulated laser carrier wavelength is 1550 nm. The signals are modulated with a random input SOP. A 10 km PMD-limited optical channel is simulated with DGD ranging from 20 to 120 ps. No polarisation dependent loss or Chromatic Dispersion (CD) is considered in the model. A direct-detection receiver with an integrate-and-dump detector is used.

Fig. 8 shows the eye diagrams of the received signals from different schemes when DGD is 107 ps. The signal eye is almost closed in the uncompensated systems (Fig. 8*b*). PSP transmission method helps to open the eye a little but not significantly (Fig. 8*c*). The SIMO DFE opens the eye completely with only a small amount of SNR loss (Fig. 8*d*).

For the performance comparison, a PSP transmission technique is also simulated as a typical optical first-order compensator. The basic idea of PSP transmission is adjusting the input SOP to align with the PSP based on the feedback from the receiver. The results are based on perfect knowledge of PSP at the transmitter.

Fig. 9 shows the performance improvement of the proposed SIMO DFE and the PSP transmission method over uncompensated PMD channels. The vertical axis represents the amount of performance improvement the SIMO DFE and PSP can provide compared to uncompensated PMD channel with the same amount of noise. The performance improvements are measured using the difference between the received SNR of the compensated system and the same quantity for the uncompensated system. It is shown that for a wide range of DGDs (28 ps to 107 ps), the SIMO DFE provides improvement over the PSP compensator with perfect knowledge of PSP at the transmitter. It is also noticed that when DGD is higher, the SIMO DFE provides larger gain whereas the PSP compensator's improvement is reduced. This is because of the fact that high-order PMD has a larger impact on channel performance when DGD is high. PSP cannot compensate it since PSP is only a first-order compensator. On the other hand, the SIMO DFE can handle PMD of all orders and therefore provides larger gain when high-order PMD is significant.

The performance of the proposed PMD SIMO DFE (solid curve) is also compared to that of a nonlinear canceller (NLC) [5] (dashed curve) implemented in electrical domain (see Fig. 10). The NLC is effectively a DFE which assumes the PMD channel without considering the effects of square-law detector. The performance is measured in terms of the SNR penalty of the received signal. It can be found from the figure that the performance of the proposed SIMO DFE is significantly better than that of a NLC. The major reason for the improvement is that the proposed SIMO DFE is based on a more appropriate channel model. The conventional DFE assumes the PMD ISI is linearly added to the received signal after direct detection, which is not accurate. The results also show the SIMO DFE's performance is relatively insensitive to the DGD level because it is able to cancel ISI more effectively so that the PMD-induced ISI becomes a less-dominant factor compared to the noise.

5 Conclusions

In this article, we proposed and statistically validated a new SIMO PMD channel model. Motivated by this more accurate model, we propose an SIMO DFE scheme that provides excellent performance for SIMO PMD channels. The proposed SIMO DFE can effectively mitigate the ISI caused by all orders of PMD. It provides performance improvement over first-order PMD compensator such as PSP and conventional DFE. The article has provided a new framework. Further research is required to find more practical solutions under the proposed SIMO DFE framework for PMD compensation.

6 Acknowledgments

This material is based on research supported by the Air Force Research Laboratory under agreement No. FA9550-06-1-0041 and by the National Science Foundation under Grant No. CCR-0112501.

7 References

1 Sunnerud, H., Xie, C., Karlsson, M., Samuelsson, R., and Andrekson, P.A.: 'A comparison between different PMD compensation techniques', *J. Lightwave Technol.*, 2002, **20**, (3), pp. 368–378
2 Lima, A.O., Adali, T., Lima, I.T., and Menyuk, C.R.: 'Polarization diversity and equalization for PMD mitigation in optical communication systems'. Proc. IEEE Inter., Conf., Acoustics, Speech, and Signal Processing, (ICASSP), May 2002, vol. 3, pp. 13–17
3 Oppenheim, A., and Schafer, R.: 'Discrete-time Signal Processing' (Prentice-Hall Inc, 1989)
4 Winters, J.H., and Gitlin, R.D.: 'Electrical signal processing techniques in long-haul fiber-optic systems', *IEEE Trans. Commun.*, 1990, **38**, pp. 1439–1453

5 Poirrier, J., Gnauck, A., and Winters, J.: 'Experimental nonlinear cancellation of polarization-mode dispersion'. Proc., Optical Fiber Communication Conf., 2000, vol. 3, pp. 7–10
6 Bulow, H., and Thielecke, G.: 'Electronic PMD mitigation – from linear equalization to maximum-likelihood detection'. Proc. Optical Fiber Communication Conf., WAA3, 2001, pp. 1–3
7 Kaminow, I.P., and Tingye, Li. (Eds.): 'Optical fiber telecommunications IV-B: systems and impairments' (Academic Press, April 2002, 1st edn.)
8 Horst, F., Germann, R., Bapst, U., Wiesmann, D., Offrein, B.J., and Bona, G.L.: 'Compact tunable FIR dispersion compensator compact tunable FIR dispersion compensator', *IEEE Photonics Technol. Let.*, 2003, **15**, (11), pp. 1570–1572
9 Cusick, T.A., Iezekiel, S., Miles, R.E., Sales, S., and Capmany, J.: 'Synthesis of all-optical microwave filters using mach–zehnder lattices', *IEEE Trans. Microwave Theory and Tech.*, 1997, **45**, (8), pp. 1458–1462
10 Al-Dhahir, N., and Sayed, A.H.: 'The finite-length multi-input multi-output MMSE-DFE', *IEEE Trans. Signal Process.*, 2000, **48**, (10), pp. 2921–2936
11 Cioffi, J.M., Dudevoir, G.P., Eyuboglu, M.V., and Forney, G.D.: 'MMSE decision feedback equalisers and coding – part I: equalization results', *IEEE Trans. Commun.*, 1995, **43**, pp. 2582–2594