1. For the dynamic model

\[ x_{k+1} = x_k - x_k^2 + 1 \]

(a) (5 points) Calculate the fixed point(s).

(b) (5 points) For each fixed point, denoted \( x_\ast \), use a calculator to compute the first 6 values of the orbit, starting from \( x_1 = x_\ast + 0.1 \). So, for each fixed point, you will write out a vector, with numeric values \( (x_1, ..., x_6) \), where \( x_1 = x_\ast + 0.1, x_2 = f(x_1), x_2 = f(x_2) \), and so on.

(c) (5 points) Based on the orbit calculated for each fixed point, decide on whether each fixed point is stable, stable and attractive, or unstable.
2. (10 points) In a sentence or two, define a dynamic model. Also, give an example of a dynamic system, from engineering or science, and describe why someone would be interested in a model of that system.

3. (5 points) Create a model that has no fixed points. Your answer should be in the form $x_{k+1} = f(x_k)$, and you provide the function $f$.

4. (5 points) Create a model that has infinitely many fixed points. Again, your answer should be in the form $x_{k+1} = f(x_k)$, and you provide the function $f$.

5. **Bonus**: (10 points) A dynamic model of a cruise-controlled car’s speed is

$$v_{k+1} = v_k + \frac{\Delta}{m}[-bv_k + u_k],$$

where the cruise controller is $u_k = c_1[v_{des} - v_k]$, $c_1 > 0$. Calculate the fixed point $v_*$ assuming constant desired speed $v_{des}$.