2 EoS

⇒ $$e = e(p, v) = e(p, \rho)$$ Equation of State. (EoS)

⇒ EoS depends on the particular gas under study.

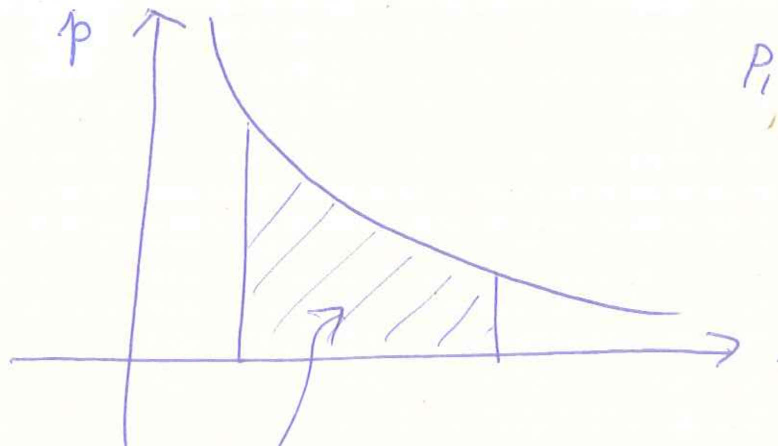
⇒ EoS governs a system in thermodynamic equilibrium that can be described by the basic thermodynamic variables

$$\begin{cases} p \text{ (pressure)}, \ \& \\ v \text{ (specific volume, } = \frac{1}{\rho}) \end{cases}$$

⇒ A family of states in thermodynamic equilibrium may be described by a curve in the p-v plane, at a particular value of temperature T.

⇒ p-v-T system.

→ For example, Boyle's Law (for ideal gas) can be given as a hyperbola in the p-v plane at any given T:



$P_1V_1 = P_2V_2$

$v = \frac{V}{M} = \frac{1}{\rho}$

area = work

⇓

(unit check): pressure × volume

$= \frac{\text{force}}{\text{area}} \times \text{volume}$

= force × Length

= **work** (or energy).

⇒ In p-v-T system, one can write

(i) $T = T(p, v)$, or

(ii) $p = p(T, v)$, or

(iii) $v = v(T, p)$.

⇒ For thermally ideal gases (or perfect gas), where the interaction of fluid particles is only perfect elastic collision and there are No intermolecular forces, one has the simple expression called the "ideal gas law";

$$\boxed{p = R\rho T} \left(\Leftrightarrow \; pv = RT\right),$$

where R is a gas constant depending on the particular gas under study.

→ **First law of Thermodynamics**

for non-adiabatic system states !

(Rmk)

Def: Adiabatic process:
One in which no heat (Q) is added to or taken out from the system

Def: Reversible process
One in which no dissipation takes place (i.e, no heat diffusion, no fluid viscosity, no magnetic diffusivity)

Def. Isentropic process
One which is both Adiabatic & Reversible

the change in internal energy (Δe) in a process is given by

$$\Delta e = \Delta W + \Delta Q$$

ΔW: work done on the system

ΔQ: heat transmitted to the system.

⇒ Mathematically,

$$de = dW + dQ. \quad \text{DE.}$$

⇒ Note that the <u>work done</u> in <u>expanding</u> the volume v by dv is $p\,dv$.

⇒ In a reversible system,

$$dW = -p\,dv$$

⇒ Note here we have "−" sign to indicate $-dv$, which is <u>compressing</u> (or decreasing) the volume, that is, the process of <u>work done</u> in the system

⇒ 1st Law of Thermodynamics

$$dQ = de + p\,dv$$

⇒ The fourth form of the EoS can be derived, which is called "the caloric eos";

(i) $e = e(p, v)$, in (p–v–e system)

(ii) $p = p(v, e)$

(iii) $v = v(e, p)$.

⇒ The calorically ideal gas Eos;

$$e = \frac{pv}{\gamma - 1} = \frac{p}{\rho(\gamma - 1)}$$

Quick Summary

Thermal Eos ; $p = R\rho T \Leftrightarrow pv = RT$

Caloric Eos ; $e = \frac{pv}{\gamma - 1} = \frac{p}{\rho(\gamma - 1)}$.

→ ① They both are closely related,
② Both are necessary for a complete description of thermodynamics of a system.

③ For the Euler Eqns,
one only needs a <u>caloric EOS</u>,
unless temperature T is needed
for some other purposes.

~~[illegible]~~

④ Combining the two, one also gets

$$e = \frac{RT}{\gamma - 1}, \quad \text{~~For~~}$$