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PUMPING OF MAGNETIC FIELDS BY TURBULENT PENETRATIVE CONVECTION

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ABSTRACT

A plausible scenario for solar dynamo action is that the large-scale organized toroidal magnetic field is generated by the action of strong radial shear at the base of the solar convection zone, whereas the weaker poloidal field is regenerated by cyclonic convection throughout the convection zone. We show, using high-resolution three-dimensional numerical simulations, that the required transport of magnetic field from the convection zone to the overshoot region can be achieved on a convective rather than diffusive timescale by a pumping mechanism in turbulent penetrative compressible convection. A layer of magnetic field initially placed in the convection zone is swept down by strong sinking plumes, locally amplified, and deposited in the stable region at the base of the convection zone, despite the opposing action of magnetic buoyancy. The rate of transport is insensitive to the strength of the initial imposed field.

Subject headings: convection — MHD — Sun: interior — Sun: magnetic fields

1. INTRODUCTION

One of the central problems of magnetic field generation in the Sun is to explain the origin of the strong magnetic field that emerges at the solar surface to form active regions. It is now widely believed that such activity derives from a strong organized toroidal magnetic field generated in a layer just below the base of the solar convection zone (Spiegel & Weiss 1980; Weiss 1994). Helioseismology has revealed the existence of a large radial shear in this region as the angular velocity profile changes from being largely constant on radial lines throughout the convective envelope to nearly solid body rotation in the deeper radiative interior (e.g., Thompson et al. 1996). This thin layer of shear, known as the tachocline, appears to lie within the stable region below the convection zone, and vigorous mixing may occur in the upper part of this shear layer due to the overshooting of convective motions.

The current paradigm for dynamo action (modeled by Parker 1993) is that the crucial process of generating strong toroidal fields takes place in this overshoot region, whereas the weaker poloidal field is regenerated within the convection zone by the action of cyclonic turbulence. Recent mean-field dynamo models (Charbonneau & MacGregor 1997) have shown that this model can bypass the suppression of poloidal field generation by mean magnetic fields (α-quenching; see, e.g., Vainshtein & Cattaneo 1992; Cattaneo & Hughes 1996) and therefore produce field strengths similar to those inferred from observations. Moreover, such models can also give rise to modulation of the basic solar cycle and recurrent grand minima (Tobias 1996; Beer, Tobias, & Weiss 1998). The model relies on efficient mechanisms for transporting the toroidal field upward from the overshoot region into the convection zone and then returning the weaker poloidal field to the overshoot region. In the mean-field *Ansatz*, this transport is modeled by turbulent diffusion of the magnetic fields representing a crude parametrization of the physical effects that are in operation.

Here we utilize three-dimensional numerical experiments to investigate a mechanism for the transport of magnetic fields from the convection zone into the layer of convective overshoot. Previously, the turbulent transport of magnetic fields has been considered in either idealized two-dimensional MHD calculations (Tao, Proctor, & Weiss 1998) or as an incidental component of rotating dynamo calculations (Nordlund et al. 1992; Brandenburg et al. 1996). The former approach shows that flux can be expelled from a region of turbulence into a quiescent region by turbulent eddies, although the timescale for the expulsion is too long for efficient pumping to occur. We investigate here whether a more realistic convection zone model including effects of stratification and penetration can lead to expulsion of the magnetic field on a convective rather than a diffusive timescale.

2. THE MODEL PROBLEM

We are interested in examining the fate of magnetic fields that are introduced into a convection zone involving highly turbulent and fully compressible three-dimensional flows. We study such magnetoconvection by utilizing an idealized two-layer planar configuration involving an upper convectively unstable layer placed above a layer of stable stratification. Such an approach has been used previously to study penetrative convection in which motions overshoot from the convectively unstable layer into the stable domain (e.g., Hurlburt et al. 1994; Brummell, Clune, & Toomre 1998). We shall extend such work to include magnetic fields and study the transport of that field, while it in turn feeds back on the motions through the Lorentz force. Here the field is artificially imposed rather than self-consistently generated by dynamo action within the flow. This simplification is used because it allows us to investigate the transport of magnetic fields of various strengths (and net fluxes) in isolation.

We utilize a nondimensional notation, similar to that of Brummell, Hurlburt, & Toomre (1996), derived from a polytropic static state, in which length is measured in units of the depth of the convective layer and the sound crossing time is the unit of time. We consider a Cartesian domain that is horizontally periodic in $0 \leq x \leq x_m$ and $0 \leq y \leq y_m$, with $0 \leq z \leq z_m$ (z-axis downward). The two-layer configuration is imposed by a piecewise polytropic stratification (in the absence of convection) such that the upper layer ($0 \leq z \leq 1$) is convectively unstable and the lower layer ($1 \leq z \leq z_m$) is stable. The physics is simplified by employing a perfect gas, and the specific heats c_p and c_v, shear viscosity $\mu = \nu\rho$, electrical conductivity η, and gravitational acceleration are all assumed constant. The temporal evolution of the velocity $\boldsymbol{u} = (u, v, w)$, the magnetic field $\boldsymbol{B} = (B_x, B_y, B_z)$, and the total pressure p_t, density ρ, and tem-

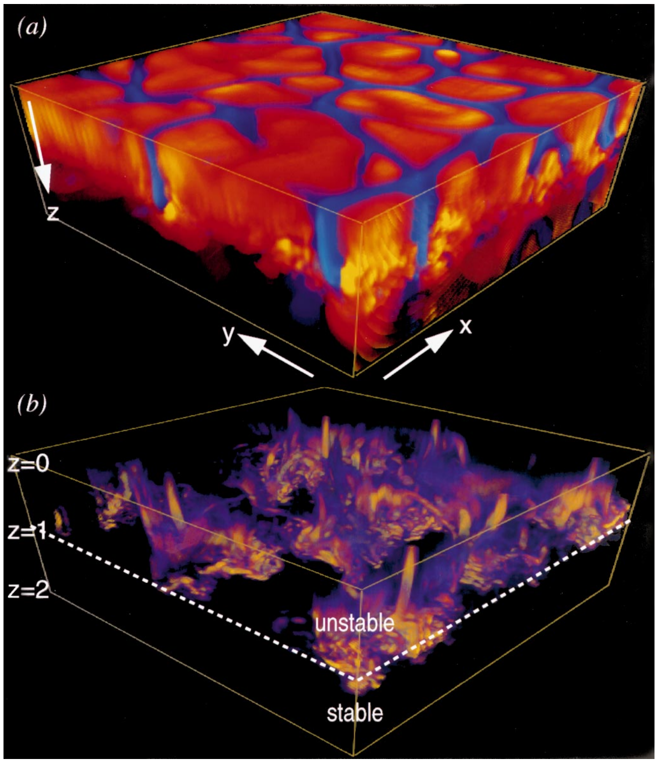

FIG. 1.—Volume renderings of the turbulent penetrative convection used for the initial state. (*a*) Vertical velocity rendered with red-yellow as upflowing fluid and blue as down. (*b*) Enstrophy ranging from weak (black, translucent) to strong (purple-yellow-white, opaque). Coherent downflowing enstrophy structures penetrate into the stable layer.

perature T are then governed by

$$\partial_t\rho + \boldsymbol{\nabla}\cdot(\rho\boldsymbol{u}) = 0, \tag{1}$$

$$\partial_t(\rho\boldsymbol{u}) + \boldsymbol{\nabla}\cdot(\rho\boldsymbol{uu} - \alpha\boldsymbol{BB}) = -\nabla(p_t) + \rho\theta(m+1)\hat{z} + P_r C_k\left[\nabla^2\boldsymbol{u} + \frac{1}{3}\nabla(\boldsymbol{\nabla}\cdot\boldsymbol{u})\right], \tag{2}$$

$$\partial_t T + \boldsymbol{\nabla}\cdot(\boldsymbol{u}T) + (\gamma - 2)T\boldsymbol{\nabla}\cdot\boldsymbol{u} = \gamma C_k\rho^{-1}\boldsymbol{\nabla}\cdot[\kappa(z)\nabla T] + \zeta C_k\alpha(\gamma-1)\rho^{-1}|\boldsymbol{\nabla}\times\boldsymbol{B}|^2 + V_\mu, \tag{3}$$

$$\partial_t\boldsymbol{B} = \boldsymbol{\nabla}\times(\boldsymbol{u}\times\boldsymbol{B}) + C_k\zeta\nabla^2\boldsymbol{B}, \tag{4}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0, \tag{5}$$

and

$$p_g = \rho T, \quad p_t = p_g + \alpha|\boldsymbol{B}|^2/2, \quad \alpha = P_r\zeta Q C_k^2. \tag{6}$$

Here p_g is the gas pressure; C_k is the thermal dissipation parameter; $P_r = \nu/C_k$ is the Prandtl number, and $\zeta = \eta/C_k$ is the magnetic Prandtl number (both evaluated in the upper layer; note that they take different values in the lower layer, but $P_r C_k$ and $C_k\zeta$ are constant throughout the domain); V_μ represents viscous heating; and γ is the ratio of specific heats. For the present calculations, $P_r = \zeta = 0.1, \gamma = 5/3, x_m = y_m = 6$, and $z_m = 2$. The unstable layer possesses an initial temperature and density contrast of 11, achieved by setting $\theta = 10$ and the polytropic index $m = 1$, and thus effects of compressibility are substantial. The conductivity function $\kappa(z)$ is so chosen that the lower stable layer has a polytropic index of 9, and thus fairly rapid deceleration is expected as the motions overshoot into a region of strong subadiabatic stratification (the relative stability parameter S as defined in Hurlburt et al. 1994 is equal to 15). The superadiabatic gradient, which is related to the vigor of the convection, is measured by the Rayleigh number (a function of C_k; see Brummell et al. 1996) evaluated at the middle of the unstable layer, and here $R_a = 4.9 \times 10^5$. The Chandrasekhar number $Q \propto B_0^2$ serves to measure the strength of the imposed magnetic field B_0, and several values are considered.

We impose stress-free, impermeable velocity conditions at the upper and lower boundaries of the domain where the magnetic field vanishes and enforce a constant temperature on the upper surface and a fixed heat flux through the lower one, so that

$$\partial_z u = \partial_z v = w = \boldsymbol{B} = 0, \text{ at } z = 0, z_m; \tag{7}$$

$$T = 0, \text{ at } z = 0; \quad \partial_z T = \theta, \text{ at } z = z_m. \tag{8}$$

The general dependence of the solutions on the parameters described above, including the effects of more complicated magnetic boundary conditions and the addition of rotation, is discussed by Tobias et al. (1998). The equations are solved numerically using a hybrid pseudo-spectral/finite difference scheme (a poloidal/toroidal magnetic extension of that used by Brummell et al. 1996 optimized for massively parallel architectures) with typical spatial resolution of 128 × 128 × 192.

3. TURBULENT PUMPING

We take as initial conditions a vigorously time-dependent penetrative solution, as shown in Figure 1, that has attained statistically steady properties in the absence of any magnetic fields. The convection involves isolated strong downdrafts penetrating into the stable layer with weaker upward return flows (Brummell et al. 1998). This asymmetry between the upward and downward flows is due to the highly stratified nature of the convection (Hurlburt, Toomre, & Massaguer 1984). The solution is then modified by imposing a thin horizontal layer of magnetic field aligned in the y-direction and positioned between $z = 0.55$ and 0.60 in the convection zone, with the field strength determined by the parameter Q. The local density is decreased to account for the additional magnetic pressure so keeping the total pressure continuous. Hence, the fluid inside the magnetic layer is buoyant relative to its surroundings and would, in the absence of the convective motions, be unstable, forming magnetic mushrooms in two dimensions (Cattaneo & Hughes 1988) and magnetic arches that rise as loops of flux in three dimensions (Matthews, Hughes, & Proctor 1995). The competition between the buoyant rise and other convective transport mechanisms is the subject of this paper. One might expect that for a strong field the magnetic buoyancy effects will dominate, whereas a weak field might be more passively advected by the convection. However, as we will now show, this does not appear to be the case.

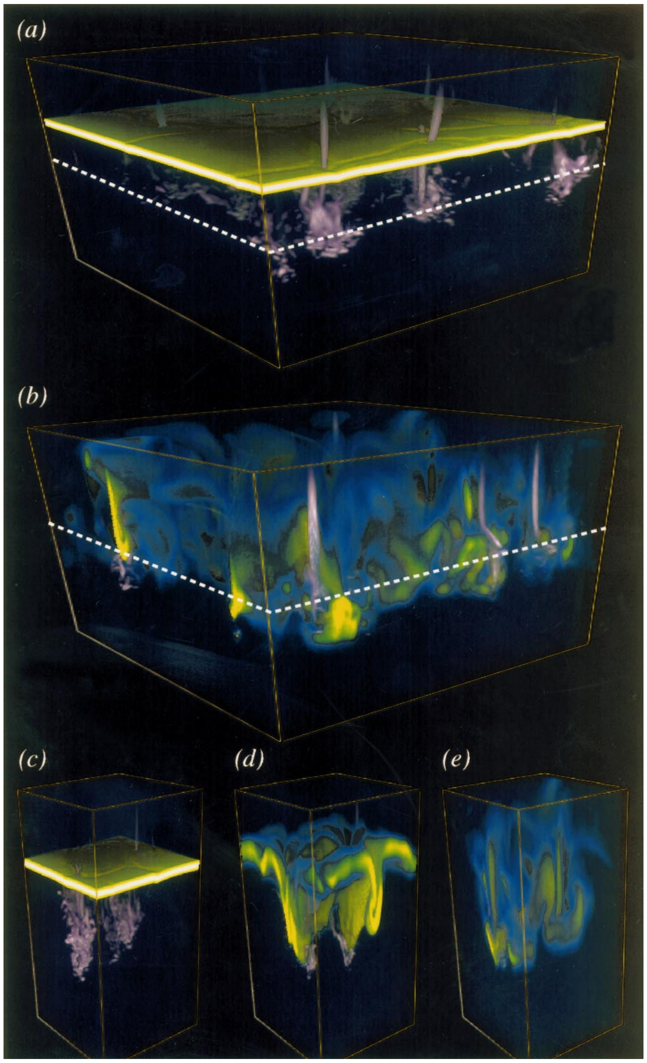

FIG. 2.—Combined volume renderings of enstrophy (purple-white) and of magnetic energy (blue-green-yellow), in which high values appear as opaque and bright. (*a*) The initial configuration with a layer of magnetic field inserted in the unstable convection zone. (*b*) A later time, showing concentrations of the magnetic field in the stable region. (*c*–*e*) Volume renderings for a subvolume of the full domain centered around a coherent downflow. The strong plumes pump magnetic fields downward and amplify them by local dynamo action.

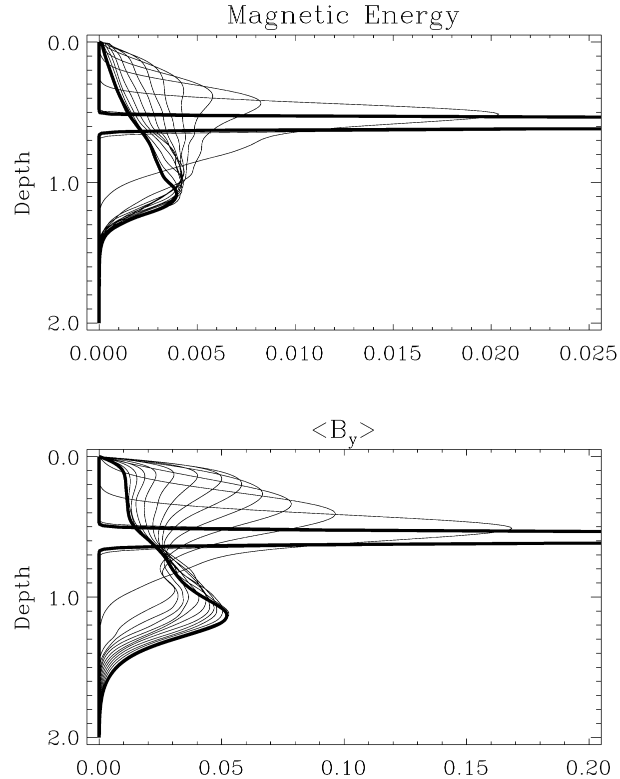

FIG. 3.—Plots of the horizontal average of magnetic energy (*top*) and $\langle B_y \rangle$ (*bottom*) as functions of z and time. The initial and final states (corresponding to Figs. 2*a* and 2*b*) are shown as thicker lines. Both magnetic energy and flux are transported downward with time into the stable region.

An example of a simulation revealing the turbulent pumping of the magnetic field is shown in Figure 2 for $Q = 10$, presenting volume renderings of the magnetic energy density ($B^2/2$) and enstrophy (the vorticity squared) as the calculation progresses. Fields are assigned a color and opacity according to their strength. Magnetic energy ranges from bright yellow for strong fields to greens and blue tones for weak; strong enstrophy appears white-purple. Figure 2*a* shows the initial configuration containing a relaxed penetrative solution with the imposed magnetic layer. The most vigorous motions are concentrated into coherent downflows that emanate from an upper cellular surface network, traverse the unstable layer, and penetrate into the stable layer where they are rapidly braked by the stable stratification. These structures are coherent in both space and time. Figure 2*b* exhibits the fields 7 time units (sound crossing times) after these initial conditions. A redistribution of the magnetic energy can clearly be seen. Magnetic energy now exists throughout the convectively active domain, with the strongest field (bright yellow) concentrated at the base of the downflow structures below the unstable-stable layer interface in the overshoot region. The latter panels of Figure 2 show details of this process. A subsection of the domain of extent ($0.25x_m$, $0.25y_m$, z_m) centered around a couple of strong plumes is exhibited in Figure 2*c*. After an initial rise of the magnetic flux due to buoyancy, the magnetic layer begins to interact with the strong convection (Fig. 2*d*). The magnetic field is entrained by the local downdrafts, wrapped up by the local vorticity, and dragged down toward the base of the convection zone. Such motions may have a local dynamo effect since the field is stretched and therefore amplified (Nordlund et al. 1992; Brandenburg et al. 1996). When the downflows leave the unstable region, decelerate, and break up in the overshoot layer, further dynamo action may occur owing to the strong resultant vorticity there. The net result of both the advection and the local dynamo action is a transport of magnetic energy from the convection zone into the overshoot region, i.e., the magnetic field has been "pumped" from the convection zone into the stably stratified region.

Figure 3 (*top*) shows more clearly that there has been a net transport of magnetic energy from the convection zone to the overshoot region. It displays the horizontal average of the magnetic energy as a function of depth at various times. The maximum magnetic energy initially centered around $z = 0.575$ can be seen to rise at first and then to subsequently migrate toward a more steady position below $z = 1$. Figure 3 (*bottom*) shows the corresponding z-dependence of $\langle B_y \rangle$ as the calculation progresses and confirms that there is also a transfer of flux into the overshoot region. Flux is clearly lost through the top of the computational domain as it is carried upward by the upflows and buoyancy, but a significant amount is pumped down into the overshoot region. Simulations repeated with boundary conditions that allow no flux to leave show that flux first accumulates at the upper boundary due to magnetic buoyancy but is then removed by the pumping mechanism.

The crucial finding is that magnetic flux is transported to the overshoot region by the strong plumes of the turbulent flow, and therefore the pumping of the field occurs on a convective rather than a diffusive timescale. Indeed, the 7 time units between Figures 2*a* and 2*b* correspond to less than a typical convective overturning time. This result persists as the strength of the magnetic field is varied. Figure 4 shows the average magnetic energy at the interface between the stable and unstable regions ($z = 1$) for two significantly different values of the Chandrasekhar number, $Q = 10$ and 10^4. It can be seen that the evolution of the field is strikingly similar in each case: a significant pumping event (around $t = 4.5$) is even evident in both simulations and is not suppressed in the strong-field case. The initial steep rise of these curves again demonstrates that the time taken for the field to reach the overshoot region is short in both cases. This figure appears to indicate that the rate of pumping depends dominantly on the properties of the convection.

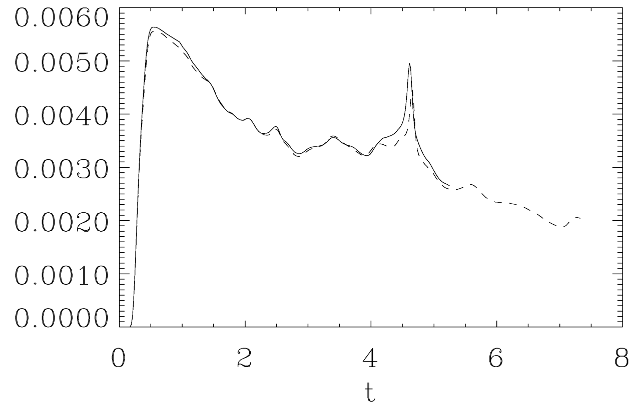

FIG. 4.—Plot of the evolution of the nondimensional magnetic energy at $z = 1$ for $Q = 10$ (*dashed line*) and $Q = 10{,}000$ (*solid line*). It is apparent that these results are relatively insensitive to Q.

4. DISCUSSION

We have found that efficient transport of magnetic fields (or flux) from the convection zone to the overshoot region is a natural consequence of three-dimensional turbulent penetrative convection. The strong coherent downflows advect flux and create strong fields by local stretching. This flux is then deposited in the reasonably quiescent overshoot region where it may be stored. The competing effect of magnetic buoyancy means that some flux is lost through the top of the computational domain. However, a significant amount of flux is still expelled from the convection zone into the overshoot region. Moreover, we have shown that the rate of this expulsion is not sensitive to the strength of the initially imposed magnetic field. In these turbulent simulations, the convective flows are sufficiently strong that the field is advected in a surprisingly passive manner.

The transport of poloidal field from the convection zone into the tachocline is an important part of solar interface dynamo models (e.g., Parker 1993). Our studies show that this crucial component does indeed operate in an efficient manner. Since the flux is pumped and locally stretched by downward plumes, the field has a tendency to align itself with a preferred orientation. Once this field has reached the overshoot region, it may be amplified by the large radial shear of the tachocline into a strong toroidal field. The fact that the field has a large component aligned with the downflows when it is pumped may be important for the formation of a strong coherent toroidal field with a preferred direction. Moreover, the turbulence may also confine this large toroidal field. Similar calculations (see Tobias et al. 1998) in which the magnetic layer is initially imposed below the convection zone show that only the strongest field rises and the majority is pinned in the overshoot region. Strong magnetic field can form compact structures, either via interactions with the overshooting convection or by instabilities (Matthews et al. 1995), and subsequently rise, although the layer as a whole may be contained. In the tachocline, the field may continue to be sheared until it reaches a critical strength that allows flux to escape from the overshoot region. Such a mechanism would select strong fields in isolated structures to emerge at the solar surface. Calculations to investigate the critical field strength for a flux tube to rise through convection as a coherent structure are currently being performed.

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