Contributions to the Theory of Practical QBF Solving

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http://www.cse.ucsc.edu/~avg/

http://www.cse.ucsc.edu/~avg/Papers/
These slides are qpup-trans.pdf

http://www.cse.ucsc.edu/~avg/ProofChecker/
Software directory, contains QdpllexpSimple.tar.
Overview of Topics in QBF Solving

Exponential Case for Usual Clause Learning Procedure

Shaky Proposal for QBF Pseudo-Unit Propagation (QPUP)

Observations on Pure Literals

- Treat Existential Pure Literals as Assumptions
- Treat Universal Pure Literals as Universal Reductions

Depth-Monotonic Literals
Exponential Case for Usual Clause Learning Procedure

Assume outermost existential 41 is true, implying 11 and 12 at innermost scope.

Now 55 is tailing, allowing 9 and 10 to be implied.

• In each four-literal clause the two negative existential literals “block” the universal literal.

• After they are falsified by unit-clause propagation, the universal literal can be reduced, yielding a new implied existential literal.

This pattern continues until \([2, 1]\) is falsified.
Exponential Case for Usual Clause Learning Procedure, Part 2

Learning Scheme

- Try to resolve out most recently assigned (i.e., trail latest) existential.
- If tautology, resolve out innermost quantifier scope (max qdepth).

Walk through shows 11 and 12 get resolved out $2^k$ times.
### Exponential Case for Usual Clause Learning Procedure, Part 3

Running times in seconds on $qdllexp$ family

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<th>21</th>
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<td>3</td>
<td>5</td>
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Running times in seconds on a tougher version of $qdllexp$ family

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<td>33</td>
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<td>267</td>
</tr>
</tbody>
</table>
An Alternative: QBF Pseudo-Unit Propagation

\[
\begin{align*}
\text{qpup}(12) &= [12, 41] \\
\text{qpup}(11) &= [11, 41] \\
\text{qpup}(10) &= [10, 41] \\
\text{qpup}(9) &= [9, 41] \\
\ldots \\
\text{qpup}(\bot) &= [41]
\end{align*}
\]

Last is the learned clause.

In general, the learned clause has negations of some of the assumptions.
Making QPUP Practical: a Fuzzy idea

- Find a safe UIP literal.
- Treat assignments at lower decision levels as assumptions.
- Make latest assumption the safe UIP literal.
- Do QPUP from there through the falsified clause.

*Safe* means: Since the UIP will be in all derived clauses it should not block any universal reductions.

The *most recent* existential assumed literal is a safe UIP.

Complications:
- Unit clauses with large qdepth (very inner scopes)
- Decision levels with Universal assumed literals.
- Existential pure literals
- Universal pure literals
- Universal “implied” literals — from unit cubes
- Oh No! *Dependency Schemes.*
**Existential Pure Literals**

These are *not* logically implied from the assumptions.

So, treat as a new assumption.

However, *never* let it be the UIP literal for learning

- Pretend it was assigned at a lower decision level; choose something else.

**Theorem**

If $e$ is existential pure based on original clauses and . . .

If $\overline{e}$ is in a learned clause, say $C$, then . . .

then $C - \overline{e}$ is also logically implied by the original formula (as restricted at the time that $e$ became pure).

An existential pure literal cannot have a *quadrangle dependency* on any universal literal, so it can move scopes without changing the truth value of the formula.
**Universal Pure Literals**

These are *not* logically implied from the assumptions.

So, treat as a universal reductions (i.e., clause by clause).

**Justification:**
No existential literal can have a *quadrangle dependency* on any universal pure literal, so the universal pure literal can “sink” to innermost scope without changing the truth value of the formula.
Depth-Monotonic Literals

See the proceedings.
Conclusion

Theory is a lot easier than implementation.

*Useful* theory should make implementation easier.