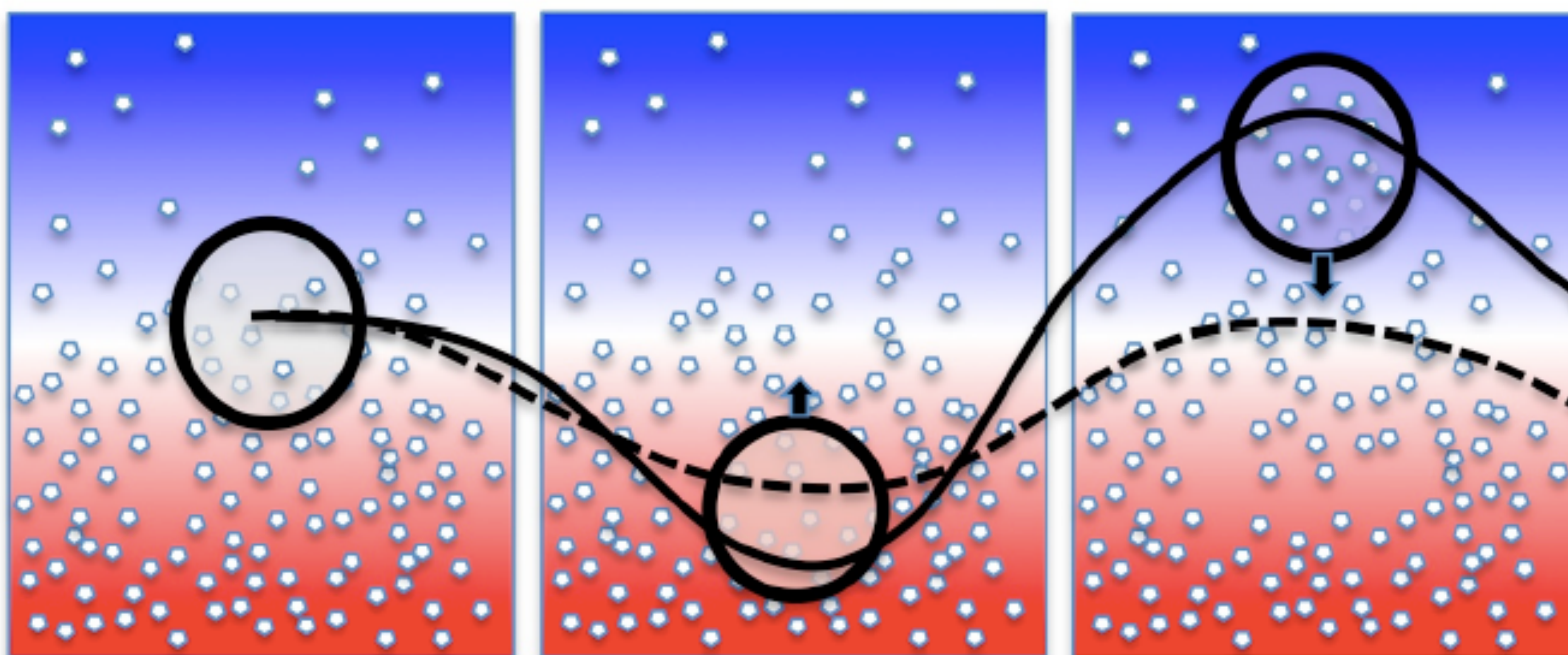
Double-diffusive processes in stellar astrophysics

Pascale Garaud
Department of Applied Mathematics
UC Santa Cruz

Lecture 3:
ODDC (semiconvection) in stars

Recap

Basic linear instability mechanism



Recap: Linear theory

The necessary condition for linear instability depends on the **inverse density ratio**

$$R_0^{-1} = \frac{\beta C_{0z}}{\alpha\left(T_{0z} - T_z^{ad}\right)} = \frac{\phi \nabla_\mu}{\delta(\nabla - \nabla_{ad})} = \frac{\text{Stabilizing composition stratification}}{\text{Destabilizing temperature stratification}}$$

Instability to ODDC occurs if

Threshold for overturning convection; Ledoux crit. →

$$1 < R_0^{-1} < \frac{\nu + \kappa_T}{\nu + \kappa_C} = \frac{\mathrm{Pr} + 1}{\mathrm{Pr} + \tau}$$

Fastest-growing modes have

- $k_z=0$ ("bungee" modes)
- $k_h \sim O(1)$ so wavelength $O(2\pi)d$
- Oscillation frequency ~ N

Stellar numbers

Typically:

- Non-degenerate regions of stars: Pr ~ 10^{-6}, $\tau \sim 10^{-7}$
- Degenerate regions of stars: Pr ~ 10^{-2}, $\tau \sim 10^{-3}$
- Basic instability size:

$$\sim 10d \sim 10\left(\frac{\kappa_T \nu}{N^2}\right)^{1/4} \sim 3\cdot 10^4 \left(\frac{\kappa_T}{10^7}\right)^{1/4}\left(\frac{\nu}{10}\right)^{1/4}\left(\frac{10^{-6}}{N^2}\right)^{1/4} \text{cm}$$

- Inverse density ratio R_0^{-1} varies substantially, and depends region in stars:
  - Near convective cores : R_0^{-1} ~ few – tens (Moore & Garaud 2016)
  - In detached semiconvective zones: R_0^{-1} can be >> 1

Question: how much mixing does this instability really cause?

Mixing by ODDC : it's complicated

- By contrast with fingering convection, ODDC tends to mix both heat and composition.
- The majority of stellar evolution codes use simplistic prescriptions for heat transport by semiconvection, either assuming the region is fully adiabatic, or fully radiative, or some interpolation between one and the other regime.
- Mixing of composition on the other hand has been given more attention, with two competing schools of thought.

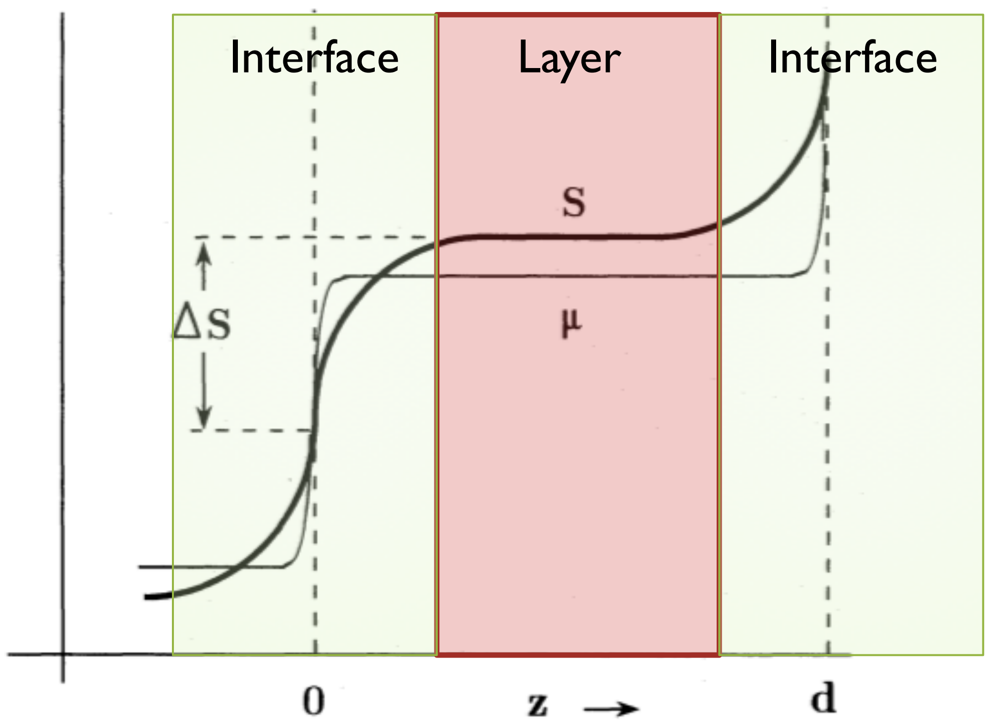
Langer et al. model for non-layered ODDC

- Derivation of Langer et al. (1983) model is quite similar to that of Ulrich (1972) and Kippenhahn et al. (1983) for fingering convection:
  - Assume that
    $$D_{semi} \propto \upsilon_{semi} l_{semi}$$
  - Assume that $l_{semi} \sim d$
  - Assume that $\upsilon_{fing} \sim \lambda d \sim \dfrac{\kappa_T}{d^2(R_0^{-1} - 1)} d$ (derivation analogous to fingering case)

- The diffusion coefficient then takes almost the same form as in the fingering case:
  $$D_{semi} = C_L \frac{\kappa_T}{R_0^{-1} - 1}$$
  where $C_L \sim O(1)$

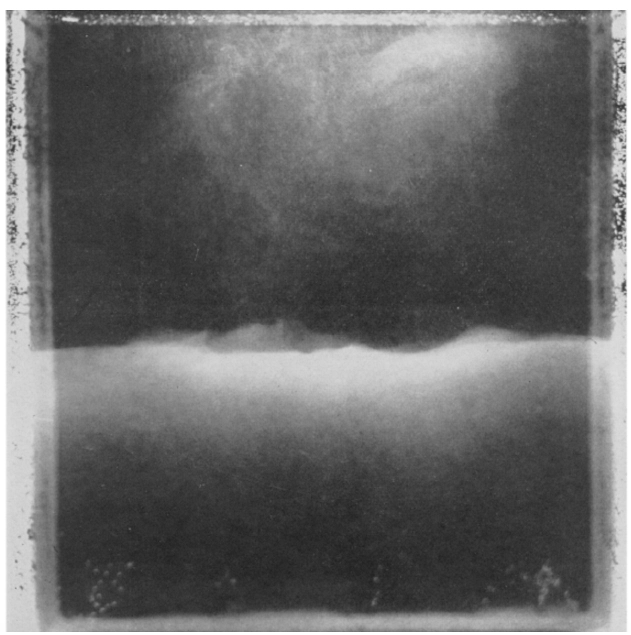
Spruit model for layered ODDC

- Spruit's (1992) model for semiconvection is radically different: assumes semiconvection takes the form of convective layers separated by diffusive interfaces (layered convection)



Why layered?

- Spruit's assumption are based on results from double-diffusive heat/salt laboratory experiments by e.g. Turner 1965; Shirtcliffe 1973; Linden & Shirtcliffe 1978; Huppert & Linden 1979, etc.
- These experiments typically start with (or sometimes naturally end up in) a layered state with properties similar to Spruit's assumptions.



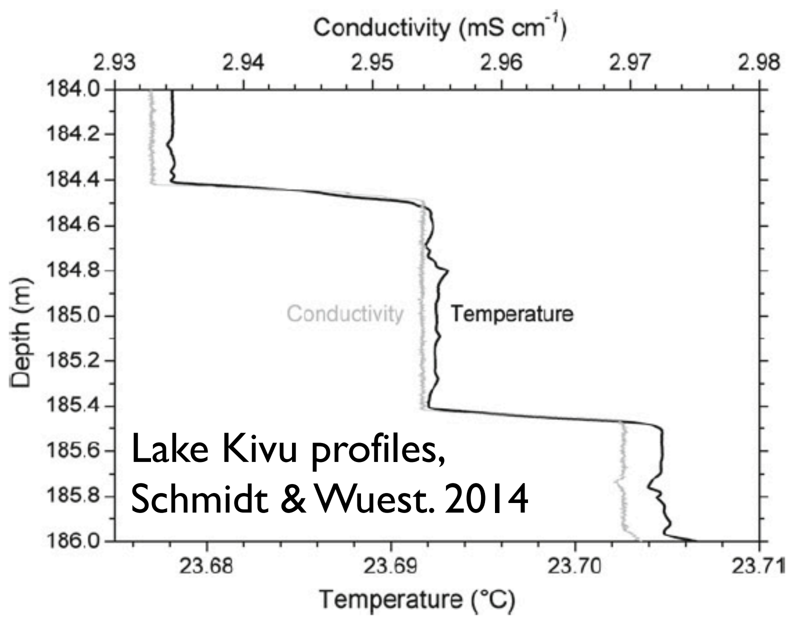
Fresh water, cooled from top

Salty water, heated from below

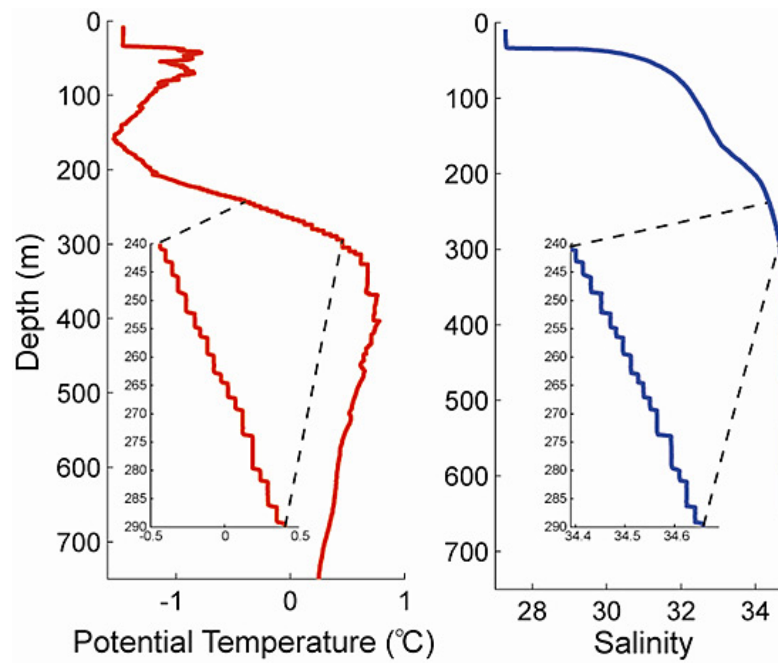
Turner 1965, Lab experiments, Pr = 7, τ =0.01

Why layered?

- Layered ODDC is also ubiquitously found in nature (Arctic, volcanic lakes), even when the basic stratification is actually linearly stable to ODDC.
- This has traditionally been attributed to a nonlinear branch of ODDC, and more recently, to the thermo-shear instability (Radko 2017).



Lake Kivu profiles,
Schmidt & Wuest. 2014



Arctic profiles, Timmermans et al. 2008

Models for layered ODDC

- Following Spruit's 1992 model, various variants have been proposed, notably:
  - Spruit 1992, Spruit 2013
  - Leconte & Chabrier 2012 + later papers.
- All these models contain similar ingredients:
  - A model for transport of heat across the layers:

$$F_{h,semi} = -\rho c_p \left\langle wT \right\rangle = -\rho c_p (\mathrm{Nu}_T - 1)\kappa_T \left(\frac{dT}{dr} - \frac{dT_{ad}}{dr} \right) = \frac{\rho c_p T}{H_p} (\mathrm{Nu}_T - 1)\kappa_T \left(\nabla - \nabla_{ad} \right)$$

where

$$\mathrm{Nu}_\mathrm{T} - 1 = C_{TBD} \mathrm{Ra}_*^a \quad \text{where} \quad \mathrm{Ra}_* = \frac{\alpha g \left| \frac{dT}{dr} - \frac{dT_{ad}}{dr} \right| H_L^4}{\kappa_T^2}$$

  - A model for transport across the (assumed diffusive) interfaces (Linden & Shirtcliffe 1978)

$$\frac{\beta \left\langle wC \right\rangle}{\alpha \left\langle wT \right\rangle} = \sqrt{\frac{\kappa_C}{\kappa_T}}$$

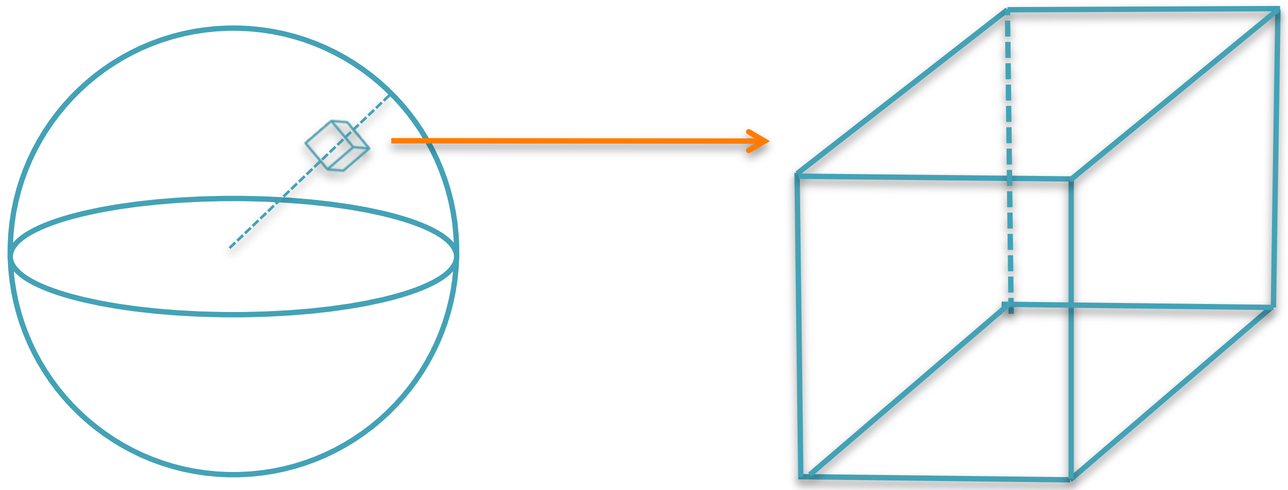
Direct numerical simulations (DNSs) of ODDC

Mathematical modeling

Model considered is same as before:

- Assume **background** temperature or salinity profiles are linear (constant gradients T_{0z}, C_{0z})
- Let $T'(x,y,z,t) = zT_{0z} + \tilde{T}(x,y,z,t)$ and $C'(x,y,z,t) = zC_{0z} + \tilde{C}(x,y,z,t)$
- Assume that all **perturbations** are triply-periodic in domain (L_x,L_y,L_z)

- This enables us to study the phenomenon with little influence from boundaries.



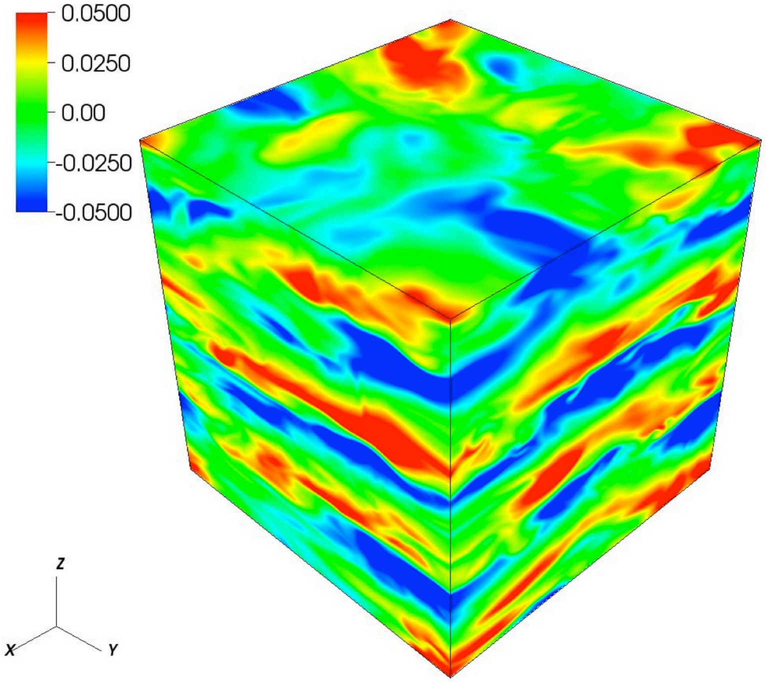
Rosenblum et al. 2011; Mirouh et al. 2012

- Rosenblum et al. (2011) discovered both outcomes are possible for ODDC at stellar parameters:
  - Layered ODDC for low inverse density ratio (more unstable)
  - Non-layered ODDC for high inverse density ratio (less unstable)

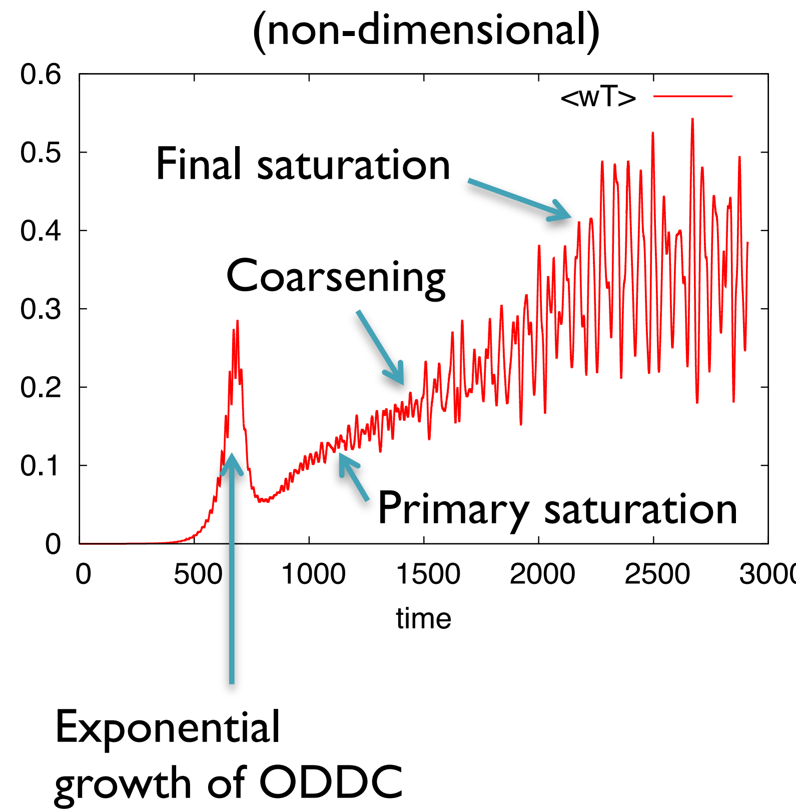
- The problem was more systematically studied by Mirouh et al. 2012, Wood et al. 2013, and Moll et al. 2016.

Layered vs. non-layered

- For high inverse density ratio, ODDC instability saturates into a weakly turbulent field of small-scale gravity waves, that later coarsen before finally settling into weakly turbulent larger-scale waves.



$$\mathrm{Pr} = \tau = 0.01 \quad R_0^{-1} = 5$$



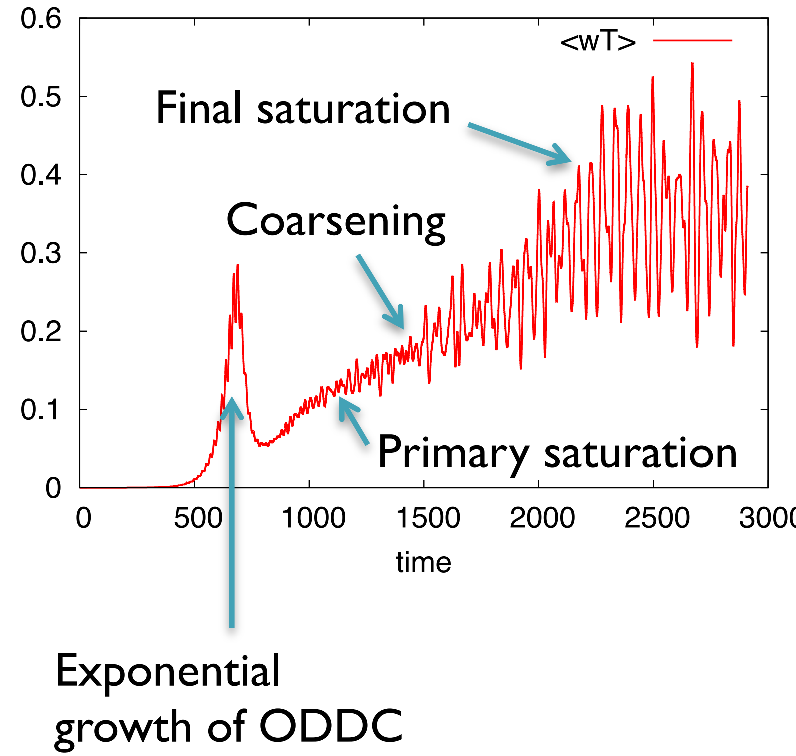
Aside: Sign of the fluxes

- By contrast with the fingering case, both temperature and compositional fluxes are always *positive* in ODDC.

$$\langle wT \rangle > 0, \quad \langle wC \rangle > 0$$

(Derivation similar to fingering case)

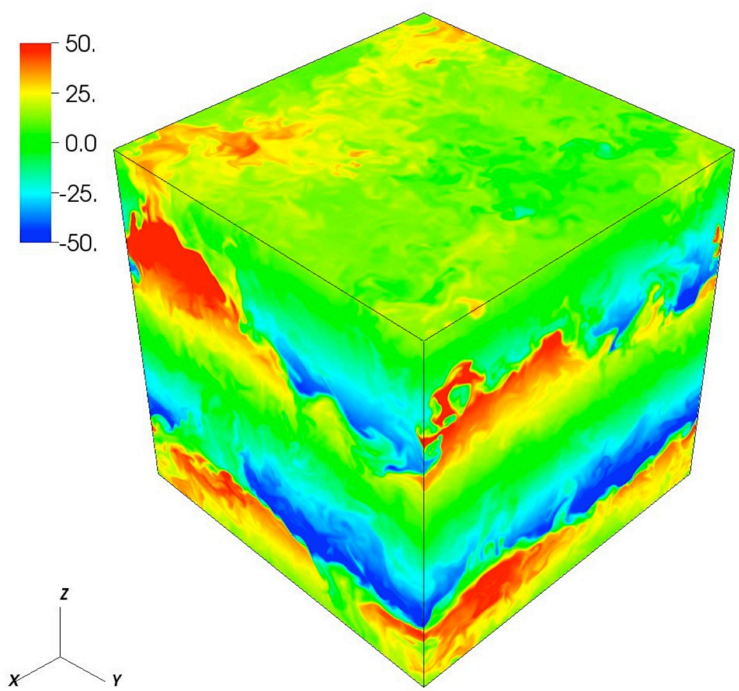
(non-dimensional)

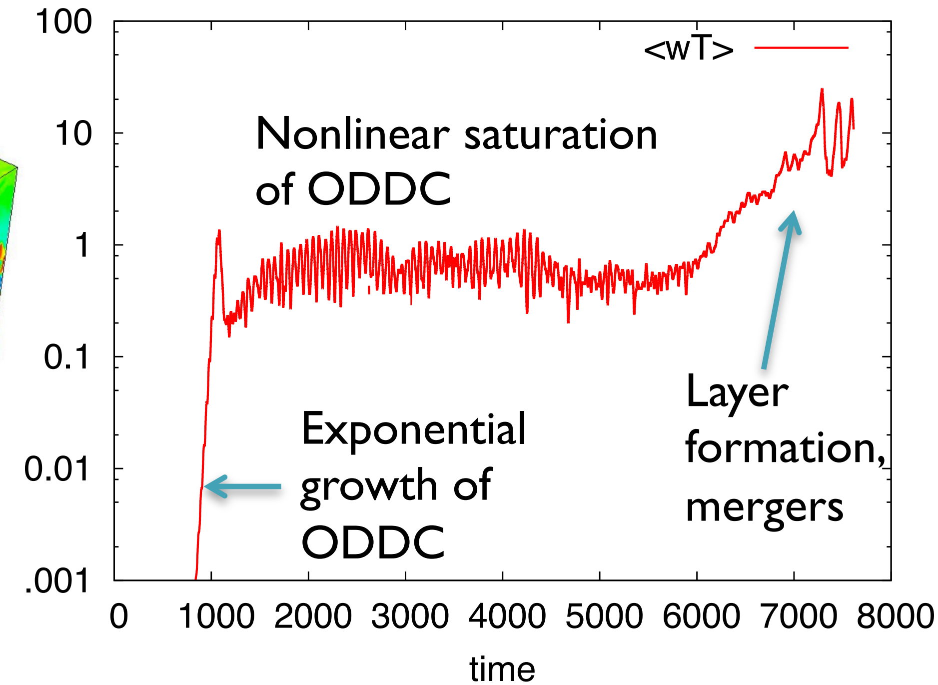


Layered vs. non-layered

- For low inverse density ratio cases (more unstable), layers systematically form. Transport (of T, C) increases significantly when they do.

(non-dimensional)

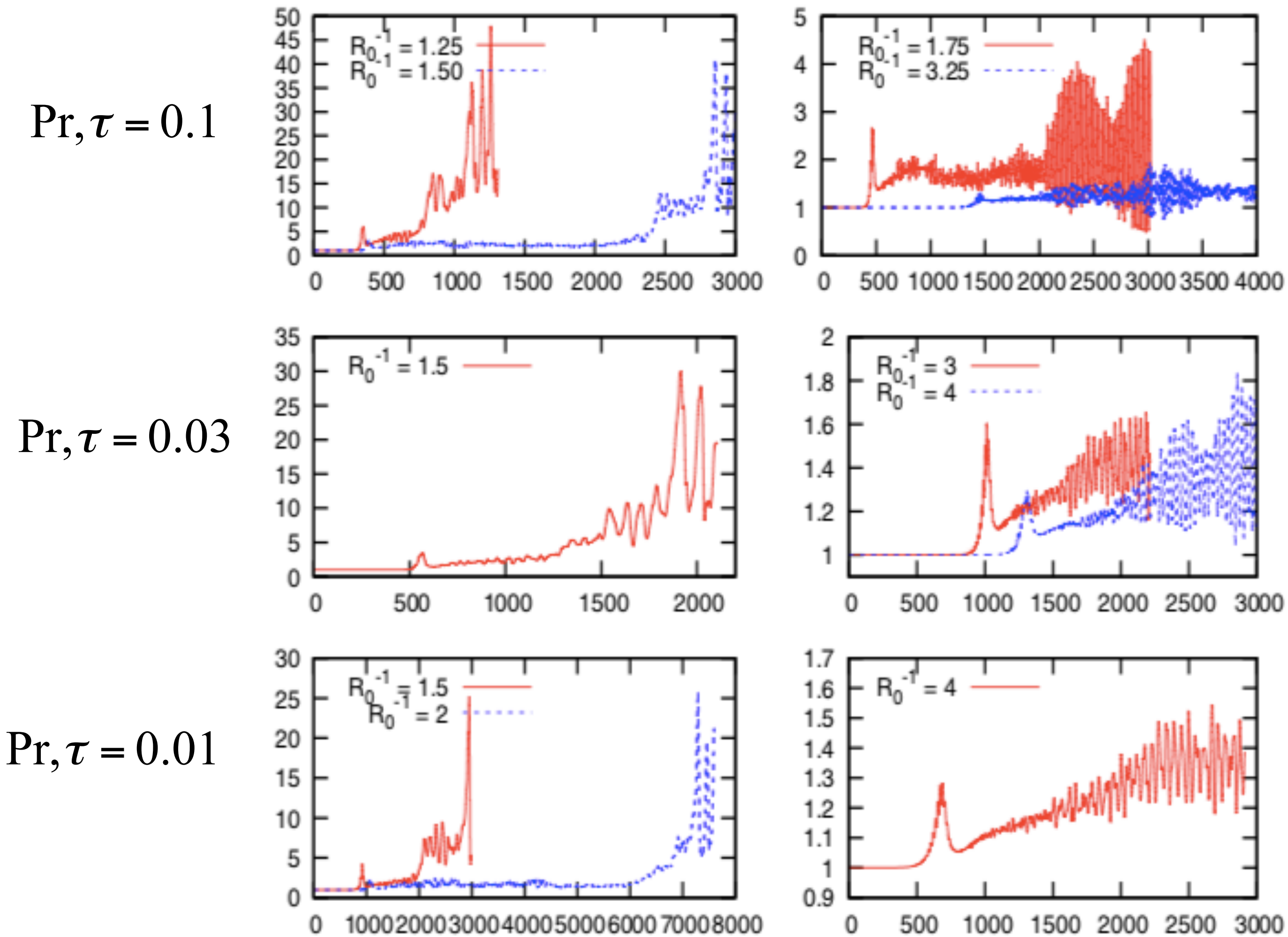




$$\mathrm{Pr} = \tau = 0.01 \quad R_0^{-1} = 2$$

Layered vs. non-layered

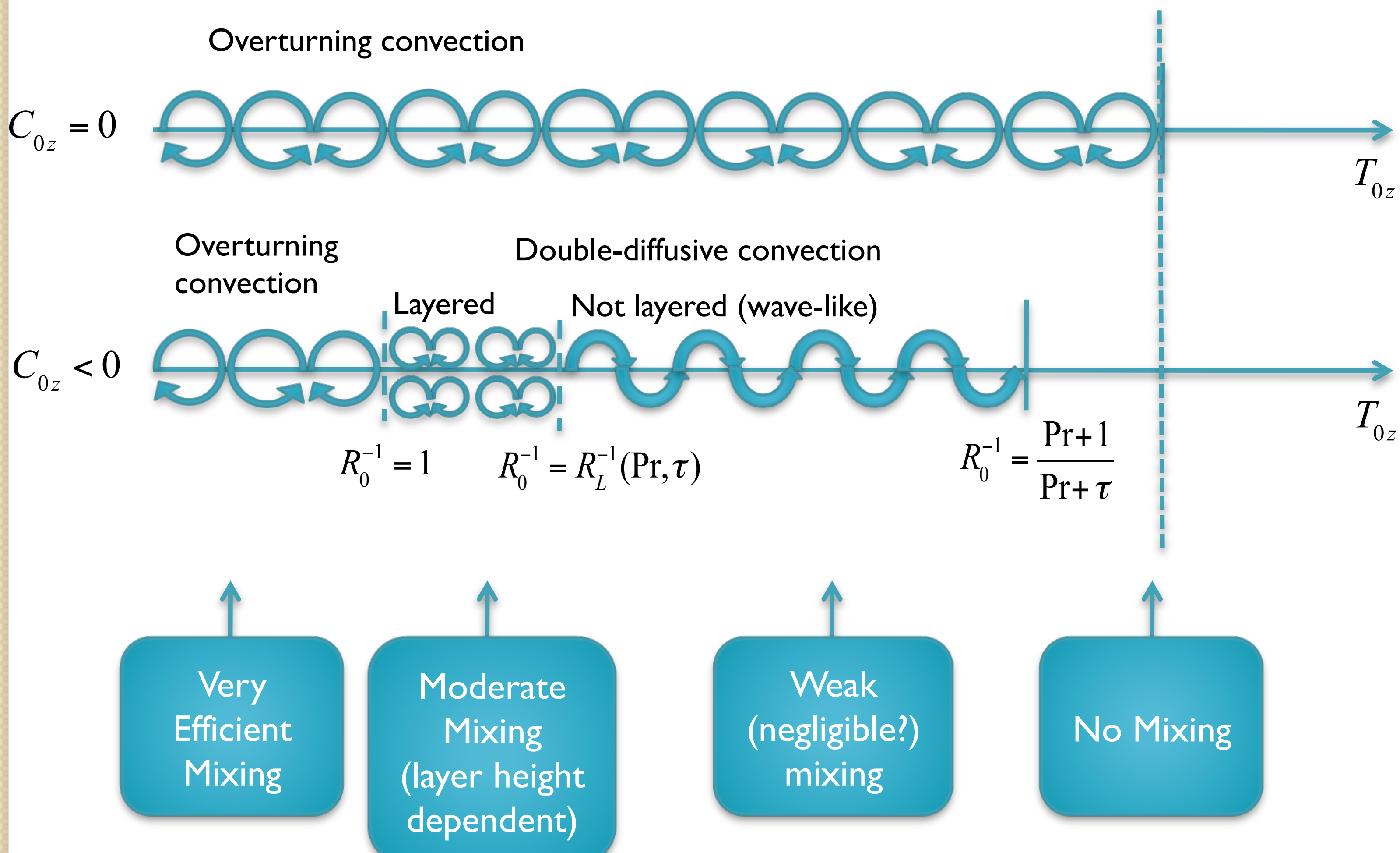
$$Nu_T = (D_T + \kappa_T)/\kappa_T$$



Layered vs. non-layered

Schematically:

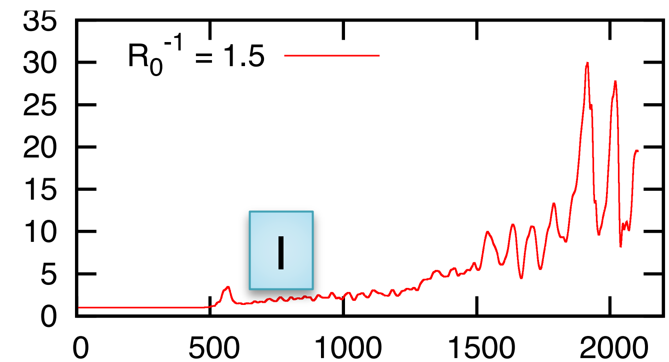
Rosenblum et al. 2011

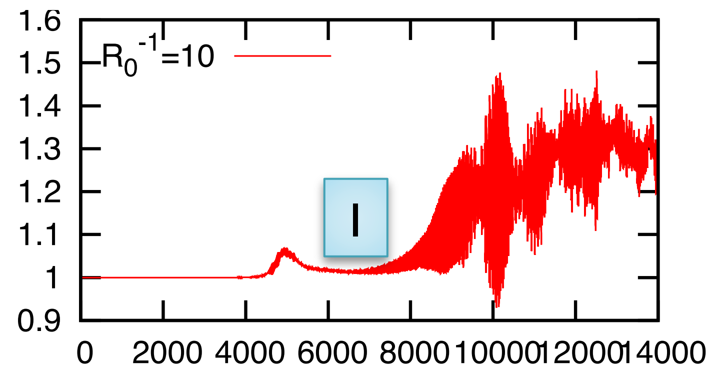


Questions to be answered.

Questions:

1. What causes the saturation of the primary instability? What is the level of saturation?





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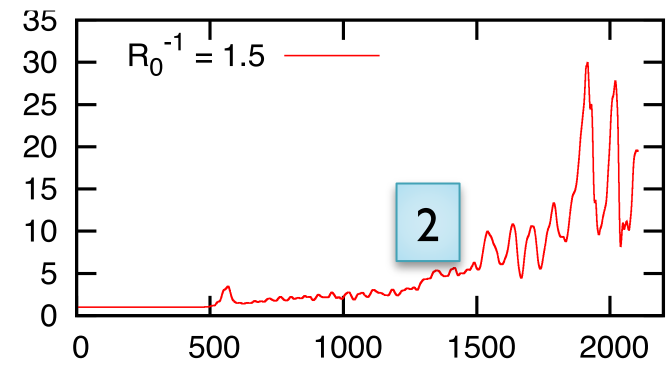
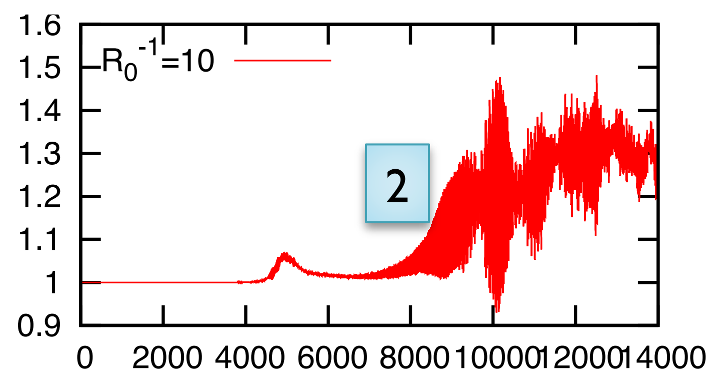
2. Why do layers and larger-scale gravity waves form? Where is the transition between layered/ non-layered ODDC?





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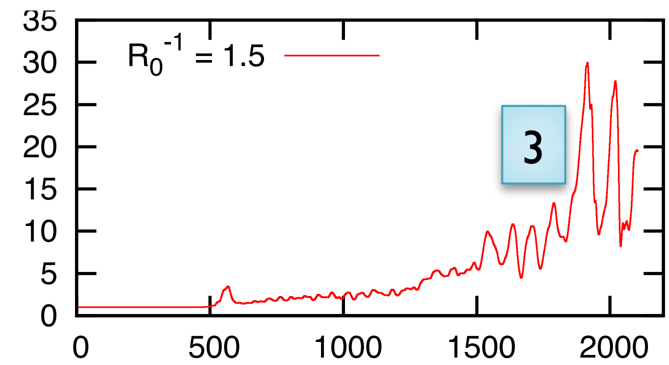

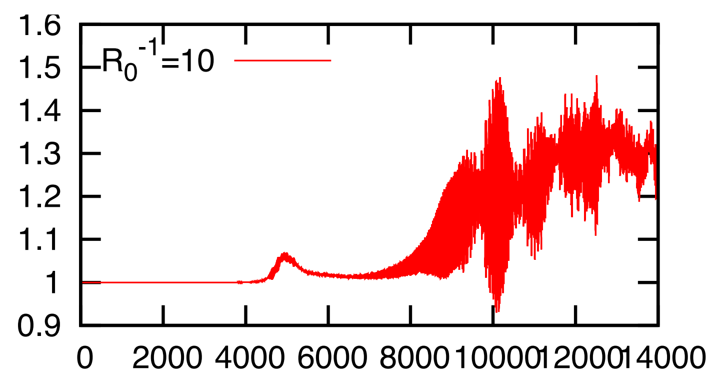

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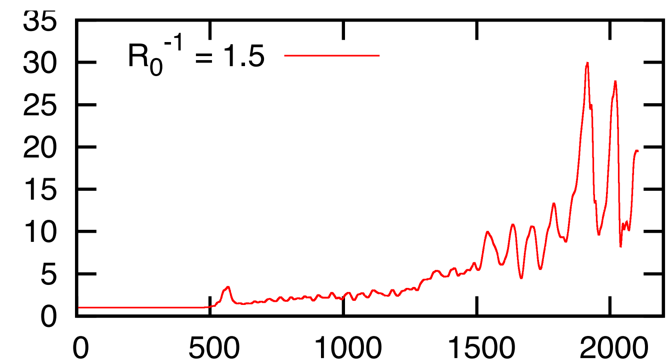
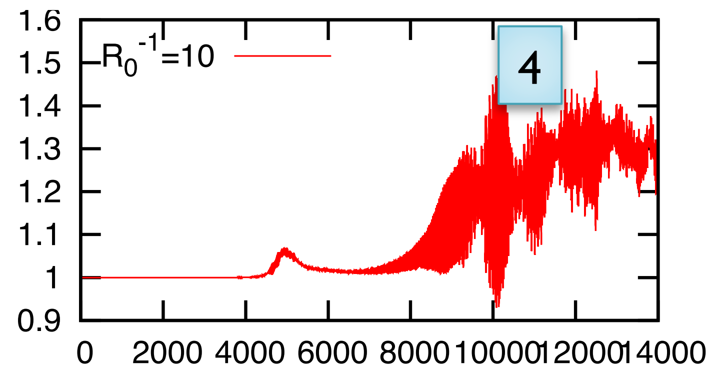




Saturation of the primary instability

- The saturation of the primary instability sets the transport properties of homogeneous ODDC, which then control the development of large-scale instabilities (recall the fingering case).
- Transport in that phase can be measured as we did in the fingering case.

$$\mathrm{Nu}_C = \frac{-\kappa_C C_{0z} + \langle wC \rangle}{-\kappa_C C_{0z}} = 1 + \tau R_0^{-1} \langle \hat{w} \hat{C} \rangle$$

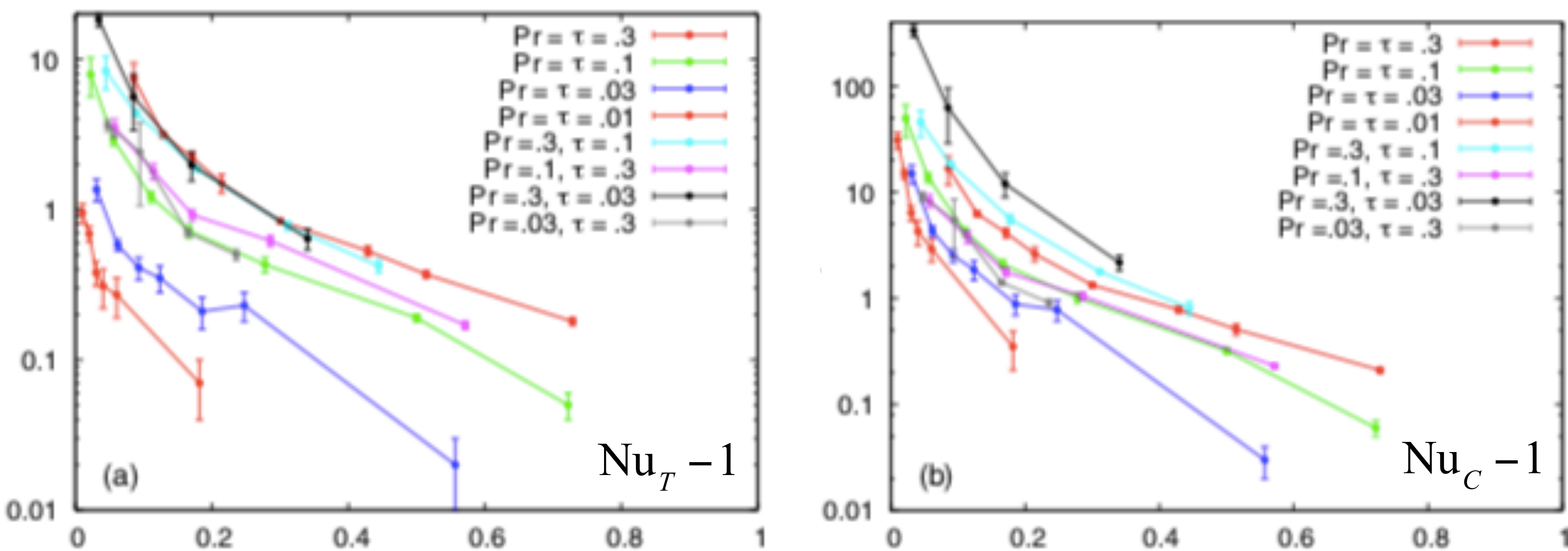
$$\mathrm{Nu}_T = \frac{-\kappa_T (T_{0z} - T_z^{ad}) + \langle wT \rangle}{-\kappa_T (T_{0z} - T_z^{ad})} = 1 + \langle \hat{w} \hat{T} \rangle$$

↓

Note that this is Nusselt for potential temperature, not temperature

Saturation of the primary instability

- Turbulent transport properties of small-scale homogeneous ODDC (prior to layer/large-scale wave formation) decrease as inverse density ratio increases (system becomes more stable).

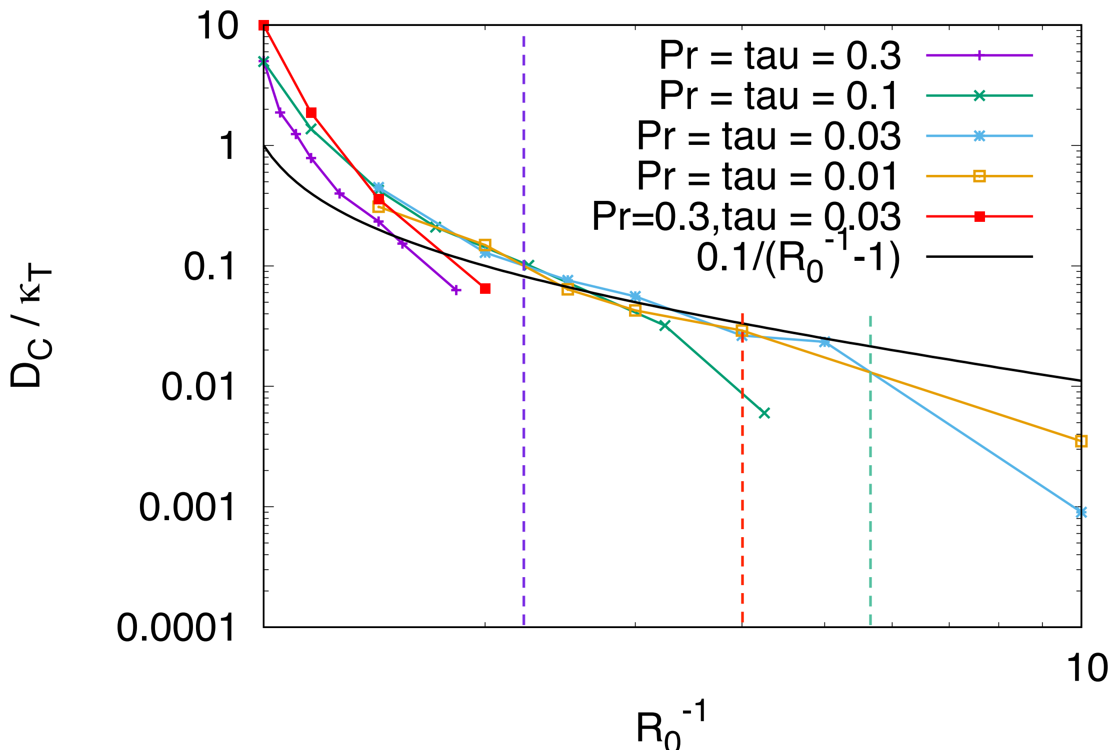


$$r = \frac{R_0^{-1} - 1}{R_0^{-1} - \dfrac{\mathrm{Pr}+1}{\mathrm{Pr}+\tau}}$$

$$r = \frac{R_0^{-1} - 1}{R_0^{-1} - \dfrac{\mathrm{Pr}+1}{\mathrm{Pr}+\tau}}$$

Saturation of the primary instability

- No simple theory has been very successful so far in predicting the fluxes for small-scale homogeneous ODDC.
- Langer et al. (1983) model is not too bad, but misses
  - stabilization of ODDC for R_0^{-1} greater than $(Pr+1)/(Pr+\tau)$
  - additional dependence on Pr or τ



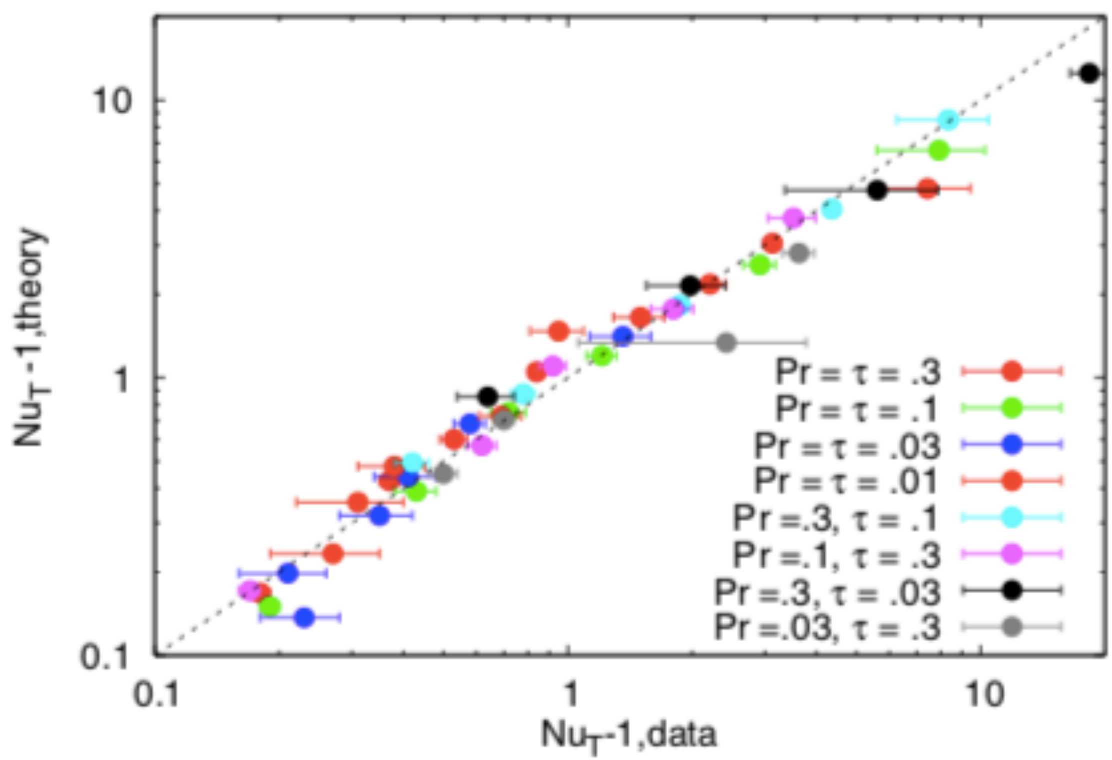
From Langer et al.,

$$D_C = C_L \frac{\kappa_T}{R_0^{-1} - 1}$$

Saturation of the primary instability

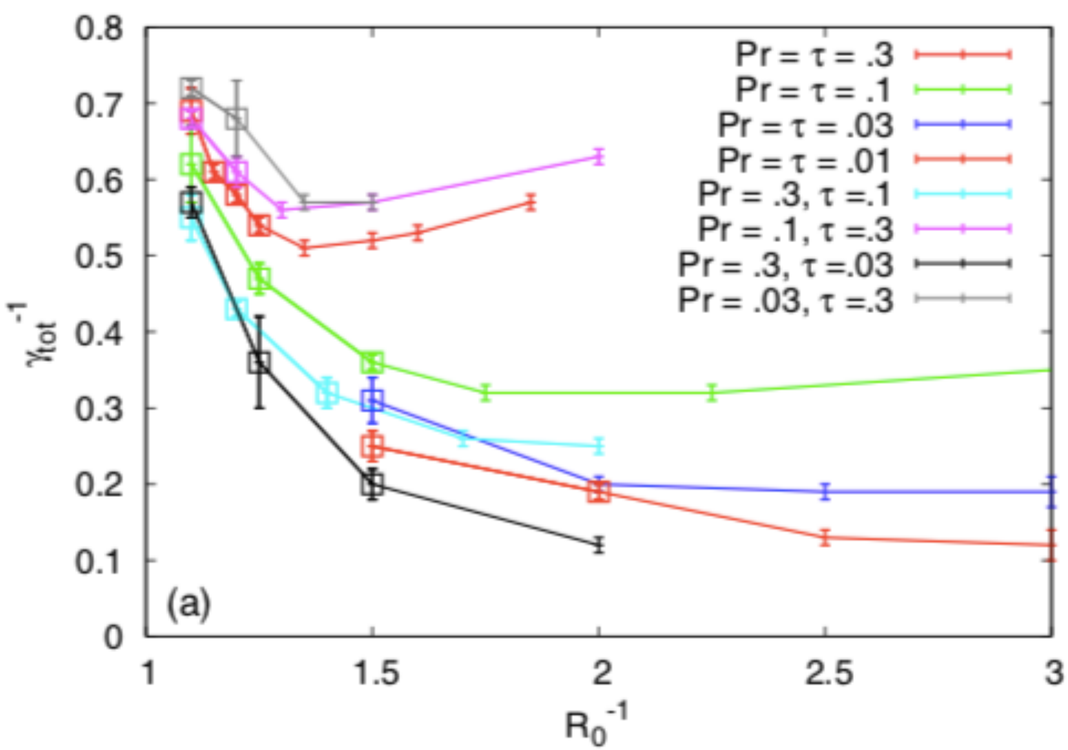
- Mirouh et al. (2012) proposed an *empirical* parametrization of their data which seems to be better:

$$\mathrm{Nu}_T - 1 = 0.75\left(\frac{\mathrm{Pr}}{\tau}\right)^{1/4} \frac{1-\tau}{R_0^{-1}-1}\,\frac{R_c^{-1}-R_0^{-1}}{R_c^{-1}-1}$$



Saturation of the primary instability

- More interestingly, the inverse flux ratio $\gamma_{tot}^{-1} = F_{C,tot} / F_{T,tot}$ is not a monotonous function of the inverse density ratio.
- Cases where inverse flux ratio decreases strongly with inverse density ratio all become layered. Others do not.



Symbol size marks ultimate outcome of simulations:

- Small : no layers
- Large : layers

This hints at the role of the γ instability!

Layering instabilities in ODDC

- Layering instabilities in ODDC are exactly the same as in fingering convection. To see this, consider the mean field equations, and use a horizontal average:

$$\frac{1}{\Pr}\left(\frac{\partial \bar{\mathbf{u}}}{\partial t} + \nabla \cdot \mathbf{R}\right) = -\nabla \bar{p} + \left(\bar{T} - \bar{C}\right)\mathbf{e}_z + \nabla^2 \bar{\mathbf{u}}$$

$$\frac{\partial \bar{T}}{\partial t} + \nabla \cdot \mathbf{F}_T \pm \bar{w} = \nabla^2 \bar{T}$$

$$\rightarrow \quad \frac{\partial \bar{T}}{\partial t} = -\nabla \cdot \mathbf{F}_{T,\mathrm{tot}}$$

$$\frac{\partial \bar{C}}{\partial t} + \nabla \cdot \mathbf{F}_C \pm \frac{\bar{w}}{R_0} = \tau \nabla^2 \bar{C}$$

$$\rightarrow \quad \frac{\partial \bar{C}}{\partial t} = -\nabla \cdot \mathbf{F}_{C,\mathrm{tot}}$$

Horizontally-invariant large-scale structures do not care about the sign of the stratification. Their formation mechanism is identical, and the condition for instability is

$$\frac{d\gamma_{tot}}{dR} < 0 \Leftrightarrow \frac{d\gamma_{tot}^{-1}}{dR^{-1}} < 0$$

Layering instabilities in ODDC

- Recall that growth rate of γ-mode is the solution of

$$\Lambda^2 + \Lambda k^2 \left[A_{\mathrm{Nu}}(1 - R_0\gamma_0^{-1}) + \mathrm{Nu}_0(1 - A_\gamma R_0) \right] - k^4 A_\gamma \mathrm{Nu}_0^2 R_0 = 0$$

with

$$\mathrm{Nu}_0 = \mathrm{Nu}(R_0) \qquad \gamma_0 = \gamma_{tot}(R_0)$$

$$A_{\mathrm{Nu}} = R_0 \left. \frac{d\mathrm{Nu}}{dR} \right|_{R=R_0} \qquad A_\gamma = R_0 \left. \frac{d\gamma_{tot}^{-1}}{dR} \right|_{R=R_0}$$

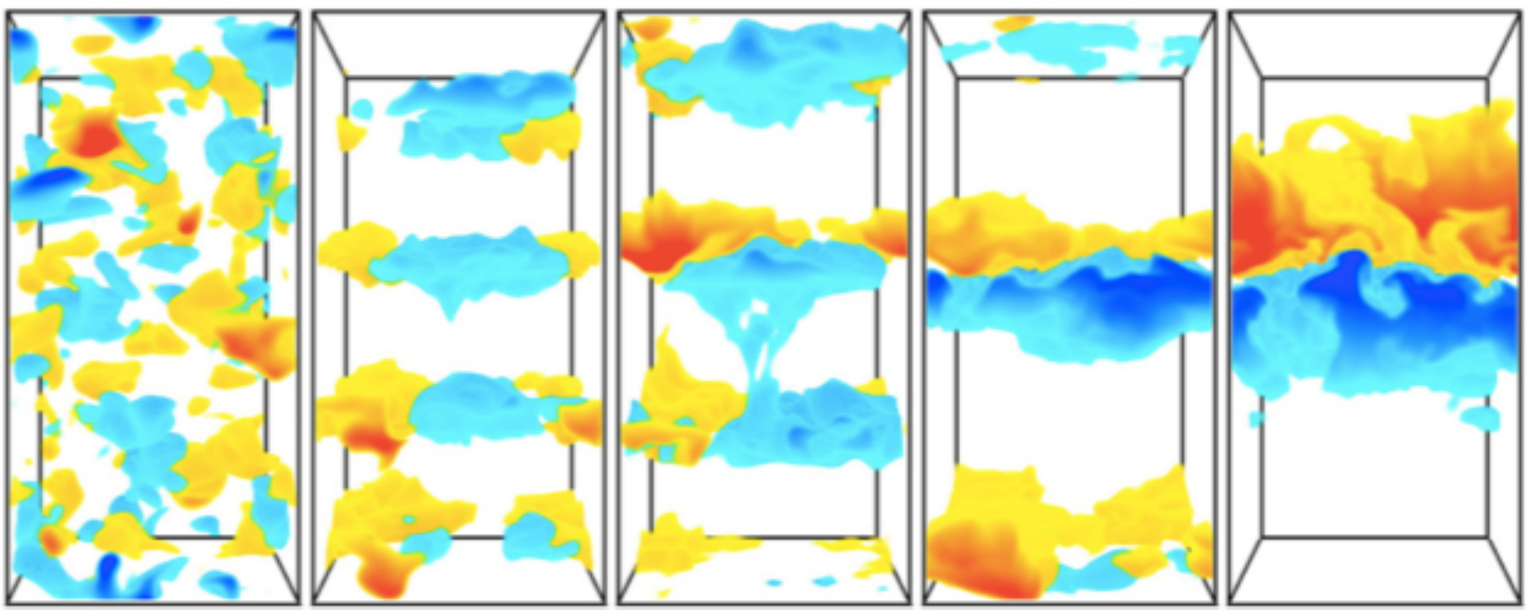
These can be computed from the data shown earlier

- To compute the predicted growth rate, we simply evaluate these constants for a given R_0 from the fluxes measured at primary saturation, and pick a wavenumber k.

Layering instabilities in ODDC

- Let's test the theory against data.
- Consider the simulation of Rosenblum et al. 2011, in which 4 layers are observed to form.

$$R_0^{-1} = 1.2, \quad \Pr = \tau = 0.3$$



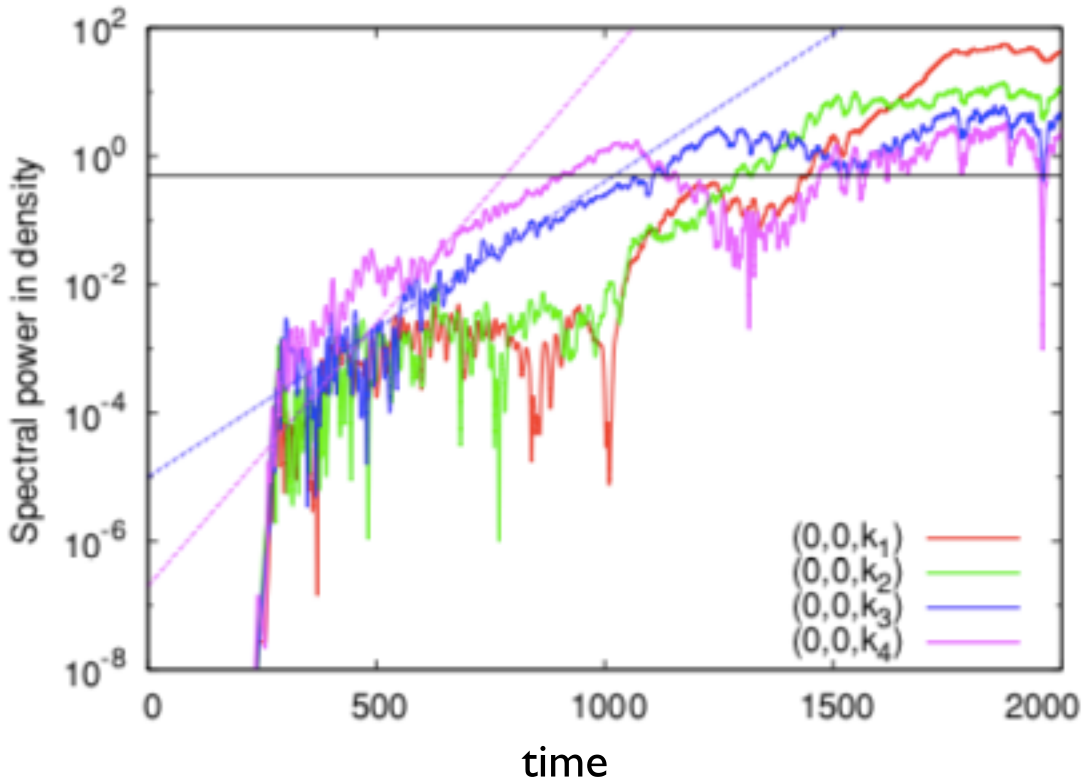
Layering instabilities in ODDC

- For, this simulation

$$\mathrm{Nu}_0 = 3.4 \qquad \gamma_0^{-1} = 0.55$$

$$A_{\mathrm{Nu}} = 12.9 \qquad A_\gamma = 0.45$$

$$\rightarrow \frac{\Lambda(k)}{k^2} = 0.47$$



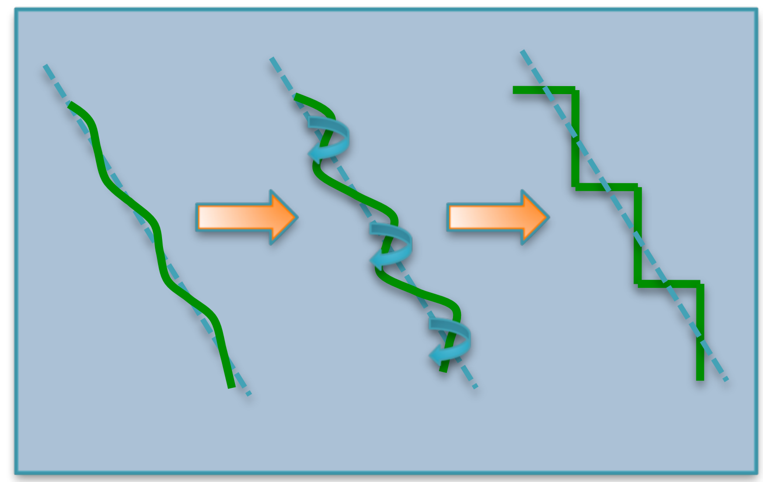
Sufficiently large-scale modes grow at the predicted rate!

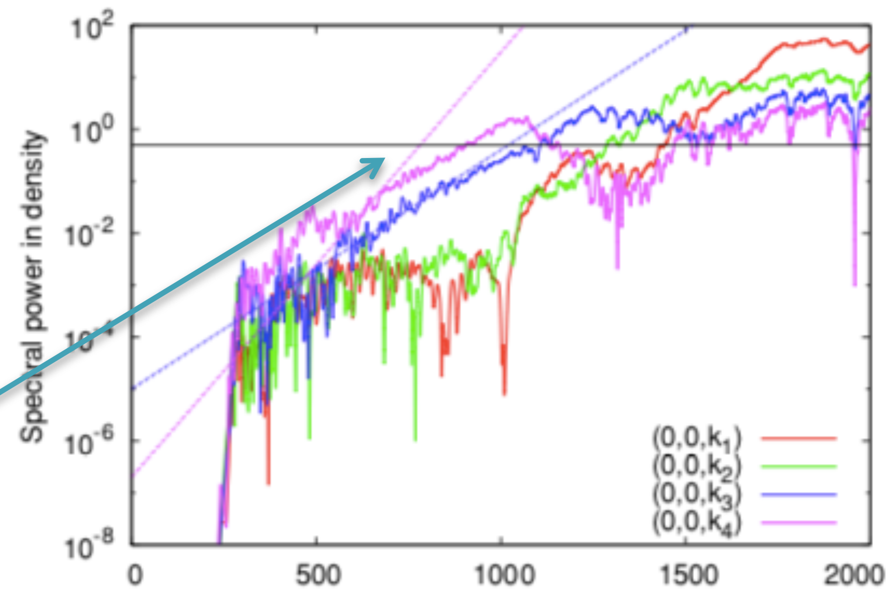
Layering instabilities in ODDC

The layering instability triggers staircase formation

- Modes of instability are horizontally invariant, vertically sinusoidal perturbations in temperature/composition/density.
- The mode overturns into a staircase when amplitude is large enough to cause density inversion. This amplitude can also be predicted analytically:

$$|\hat{\rho}(k)|^2 = \left(\frac{1 - R_0^{-1}}{2k}\right)^2$$





Layer formation (so far)

- The layering instability can quantitatively explain why/when/how fast layers form (or do not form)
- The theory has an ultraviolet catastrophe so the actual number of layers that emerge is largest possible one for which MFT still valid.
  - Layers rapidly merge after formation anyway, so initial number is not very important.
- Applying the theory to determine when layers form requires, however, knowledge of the small-scale fluxes (of both Nu_T and γ)

No simple theory !

- We can use empirical model of Mirouh et al. (2012) for Nu_T

$$\mathrm{Nu}_T - 1 = 0.75\left(\frac{\mathrm{Pr}}{\tau}\right)^{1/4} \frac{1-\tau}{R_0^{-1}-1} \frac{R_c^{-1} - R_0^{-1}}{R_c^{-1}-1}$$

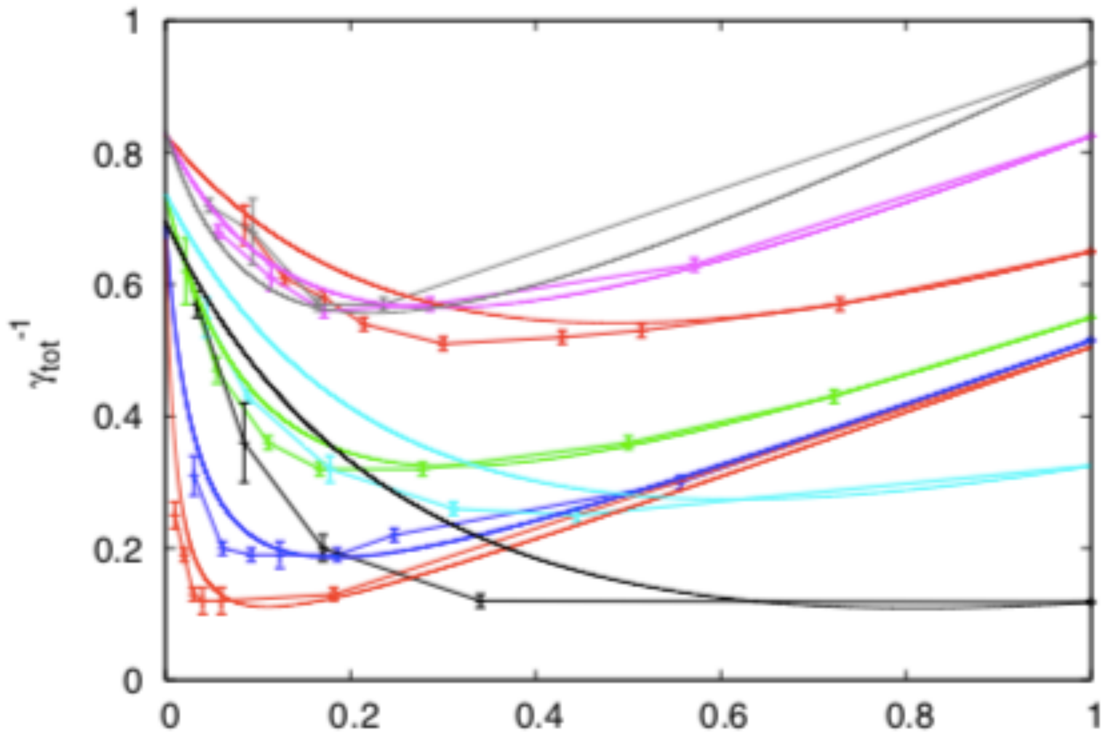
- For the flux ratio, use linear theory of Schmitt 1979, adapted to ODDC:

$$\gamma_{tot}^{-1} = \frac{\tau R_0^{-1} + \langle \hat{w}\hat{C} \rangle}{1 + \langle \hat{w}\hat{T} \rangle} = \frac{\tau R_0^{-1} + \frac{\langle \hat{w}\hat{C} \rangle}{\langle \hat{w}\hat{T} \rangle}(\mathrm{Nu}_T - 1)}{\mathrm{Nu}_T}$$

with $\frac{\langle \hat{w}\hat{C} \rangle}{\langle \hat{w}\hat{T} \rangle} = \frac{\langle \hat{w}^2 \rangle}{\langle \hat{w}^2 \rangle} \frac{\mathrm{Re}\left(\frac{R_0^{-1}}{\lambda + \tau k_h^2}\right)}{\mathrm{Re}\left(\frac{1}{\lambda + k_h^2}\right)}$ since

$$\lambda \hat{T} - \hat{w} = -k_h^2 \hat{T}$$
$$\lambda \hat{C} - R_0^{-1}\hat{w} = -\tau k_h^2 \hat{C}$$

Prediction for gamma

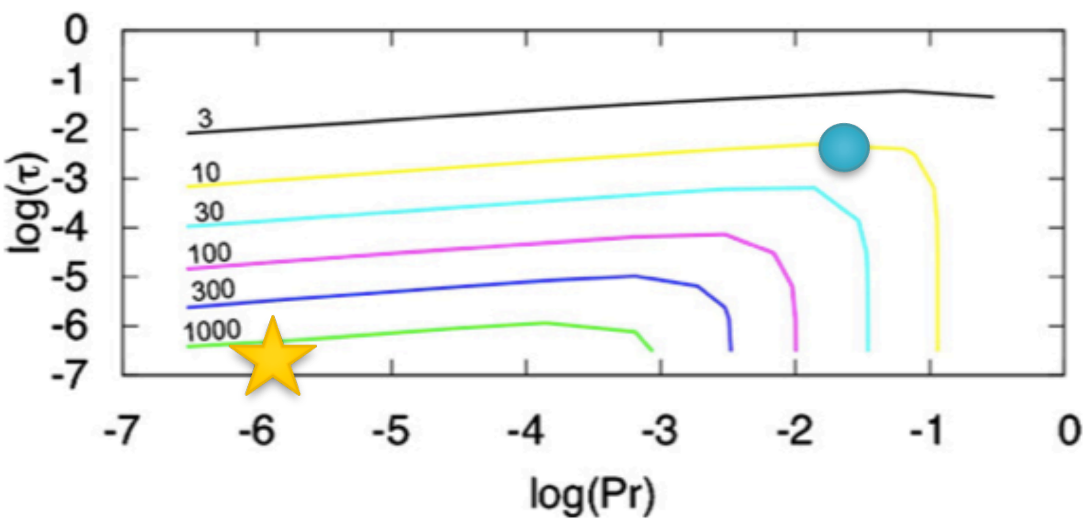


$$r = \frac{R_0^{-1} - 1}{R_0^{-1} - \frac{\Pr + 1}{\Pr + \tau}}$$

This model provides a reasonably good estimate for $\gamma_{tot}^{-1}(R_0^{-1})$

Layering in stars

- By finding where $\gamma_{tot}^{-1}(R_0^{-1})$ decreases we can chart the region of parameter space where spontaneous layering is expected



- For stellar parameters, we expect layering up to $R_0^{-1} \sim 1000$.
- For comparison, typical values in semiconvective zones close to stellar cores are closer to ~10 : layering is always expected there!

Questions to be answered.

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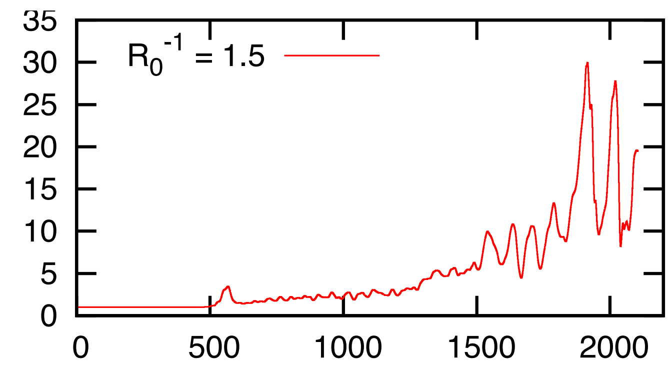
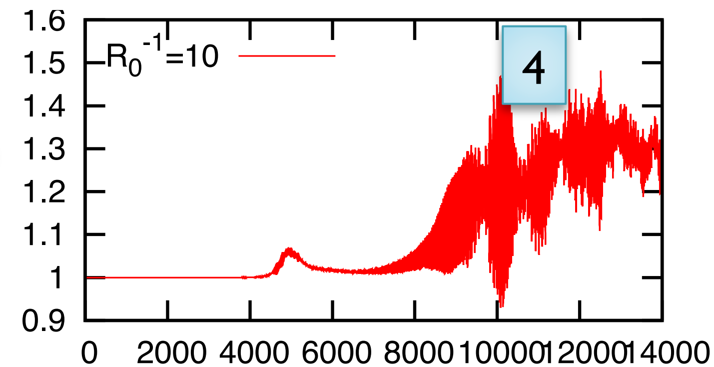


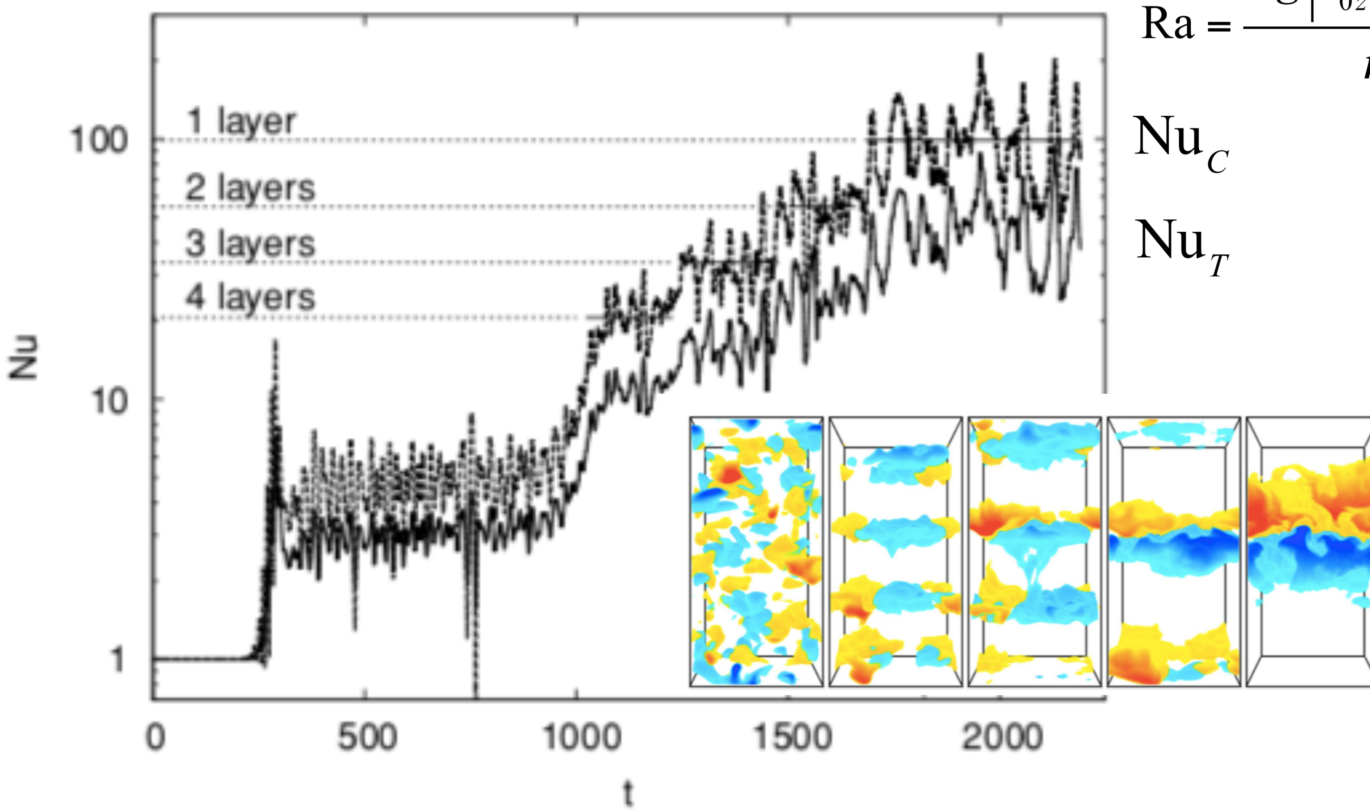


Transport in layered convection

Transport rate depends on layer height

- Rosenblum et al. 2011 established that layered convection transport rate depends on layer height, suggested $Nu_T \sim Ra^{1/3}$

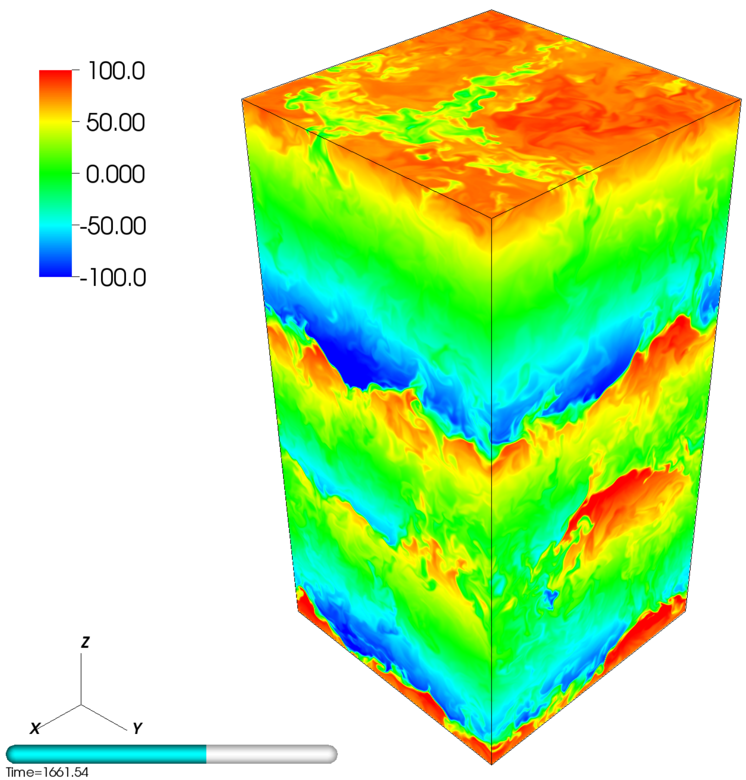
$$Ra = \frac{\alpha g \left| T_{0z} - T_z^{ad} \right| H^4}{\kappa_T \nu}$$

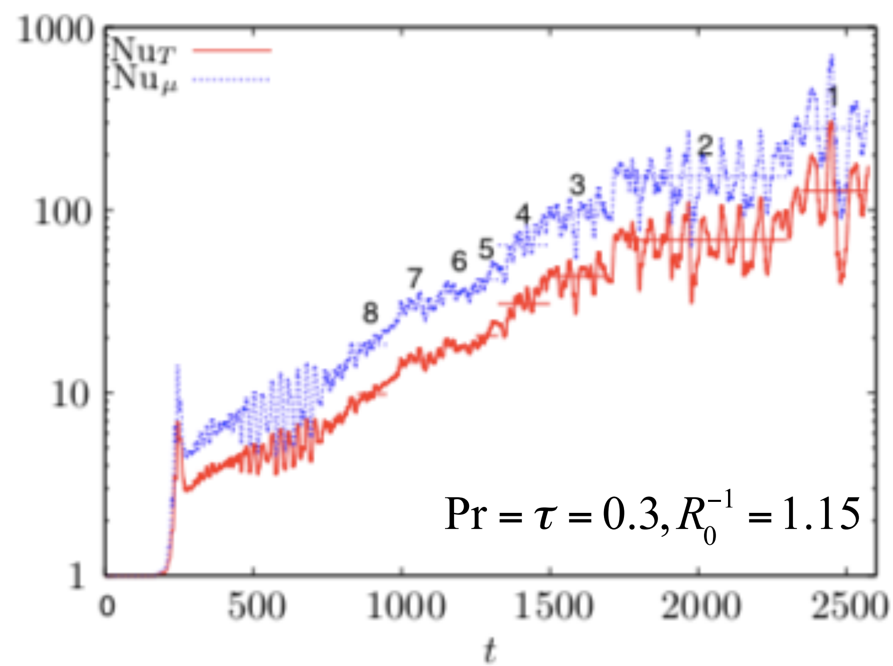


Transport rate depends on layer height

- This was studied more systematically by Wood et al. 2013, who confirmed Rosenblum et al finding that

$$\text{Nu-1} \propto \text{Ra}_*^{1/3} \quad \text{where } \text{Ra}_* = \frac{\alpha g \left| T_{0z} - T_z^{ad} \right| H^4}{\kappa_T^2}$$





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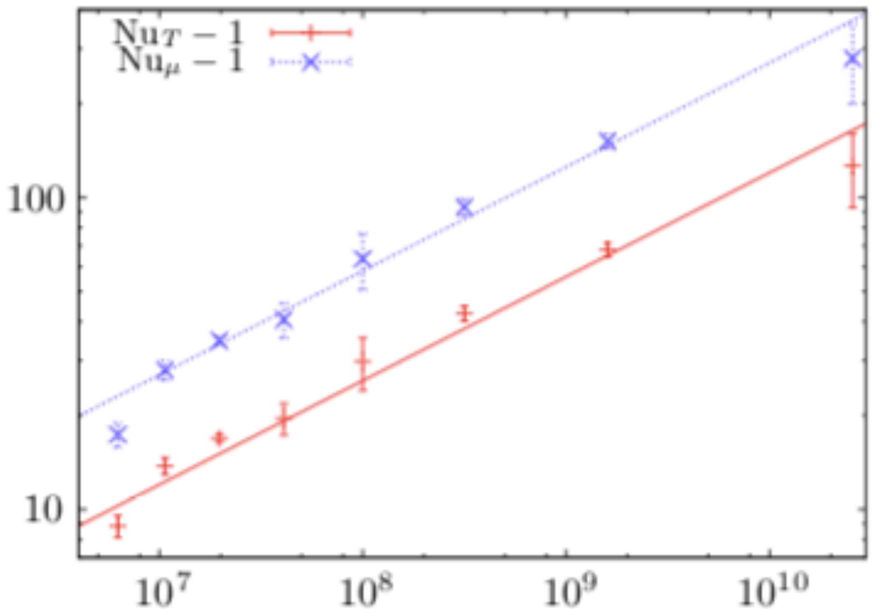
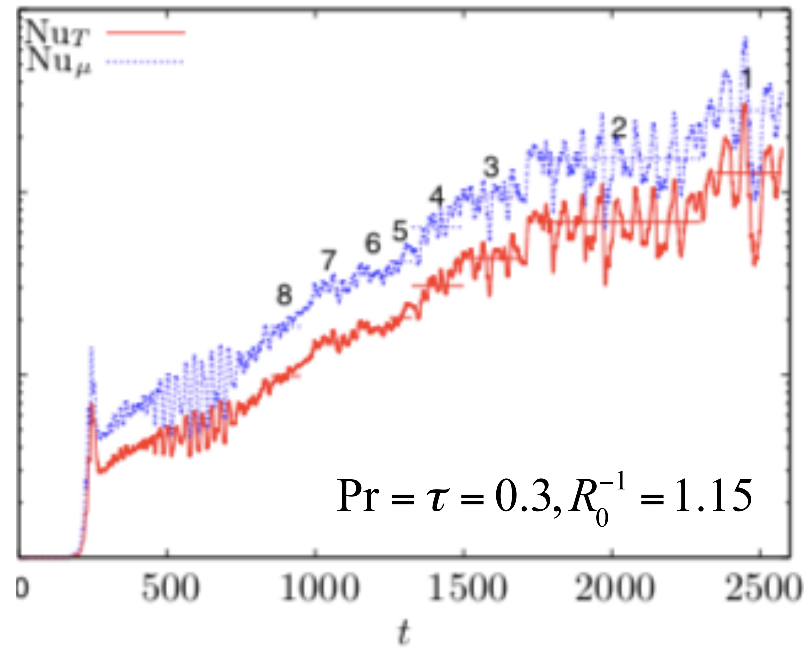


$$\mathrm{Ra}_L = \frac{\alpha g \left| T_{0z} - T_z^{ad} \right| H^4}{\kappa_T \nu} = \frac{\mathrm{Ra}_*}{\mathrm{Pr}}$$

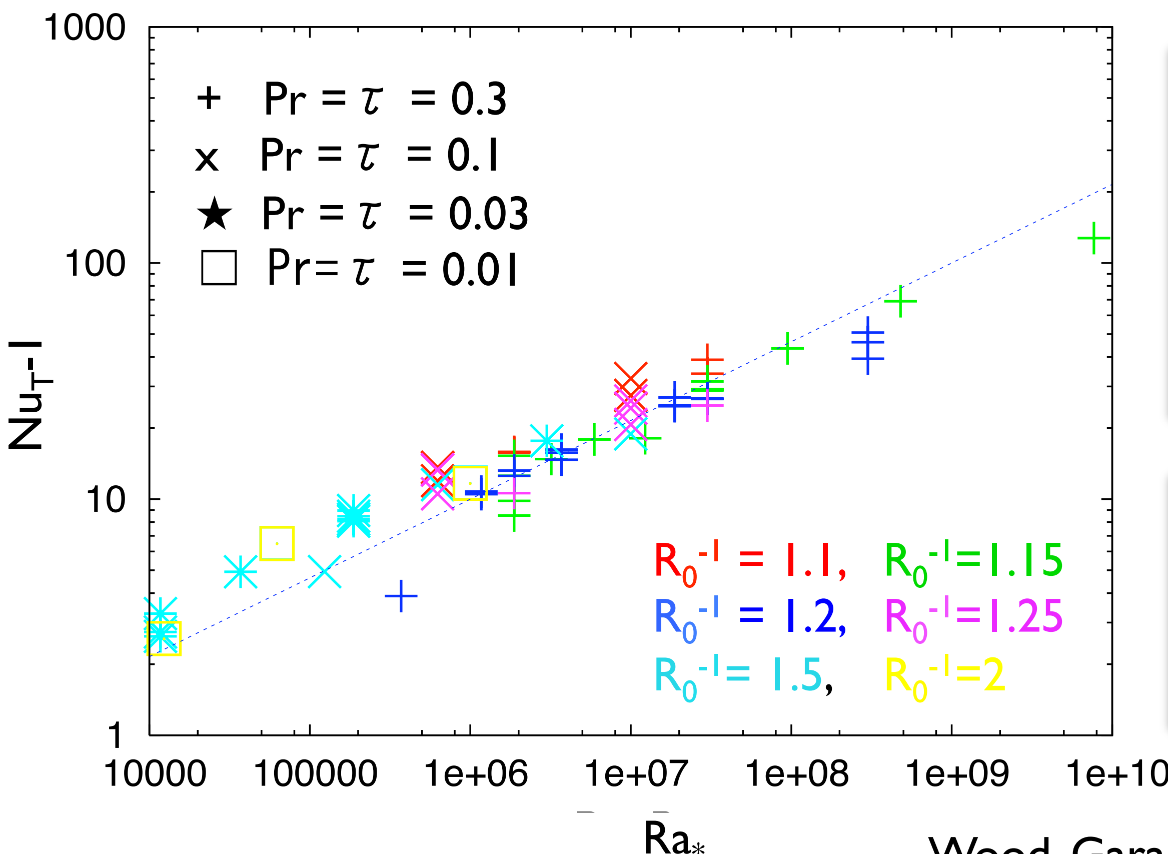


Caveat: This scaling could break once the layer height approaches a pressure scaleheight.

Layered convection

- Dependence on other parameters is less clear:

$$\mathrm{Nu}_T - 1 \approx f(\tau, R_0^{-1}) \cdot \mathrm{Ra}_*^{1/3}$$



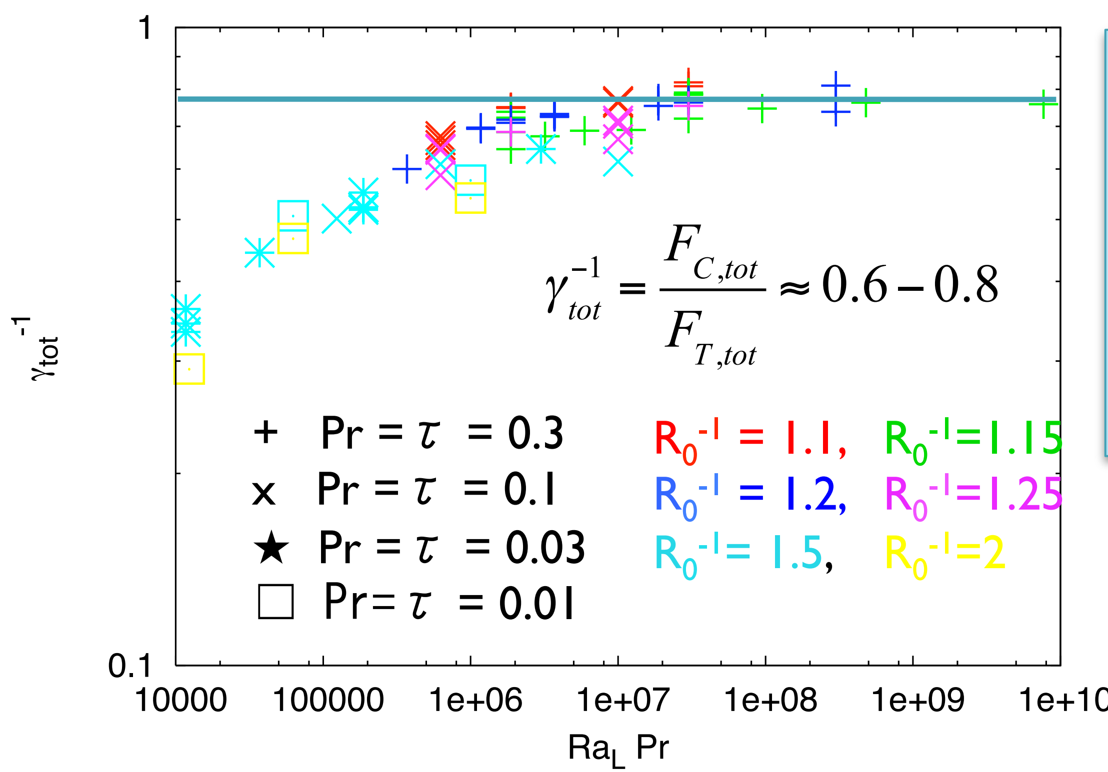
Scaling as in Rayleigh-Benard convection for wall-bounded flows but with turbulent boundary layer

Open questions: What is the dependence on τ, R_0^{-1}

Wood, Garaud & Stellmach (2013)

Layered convection

For well-separated layers, the compositional transport properties in layered convection seems to be "well" explained assuming that a more-or-less constant order-unity flux ratio

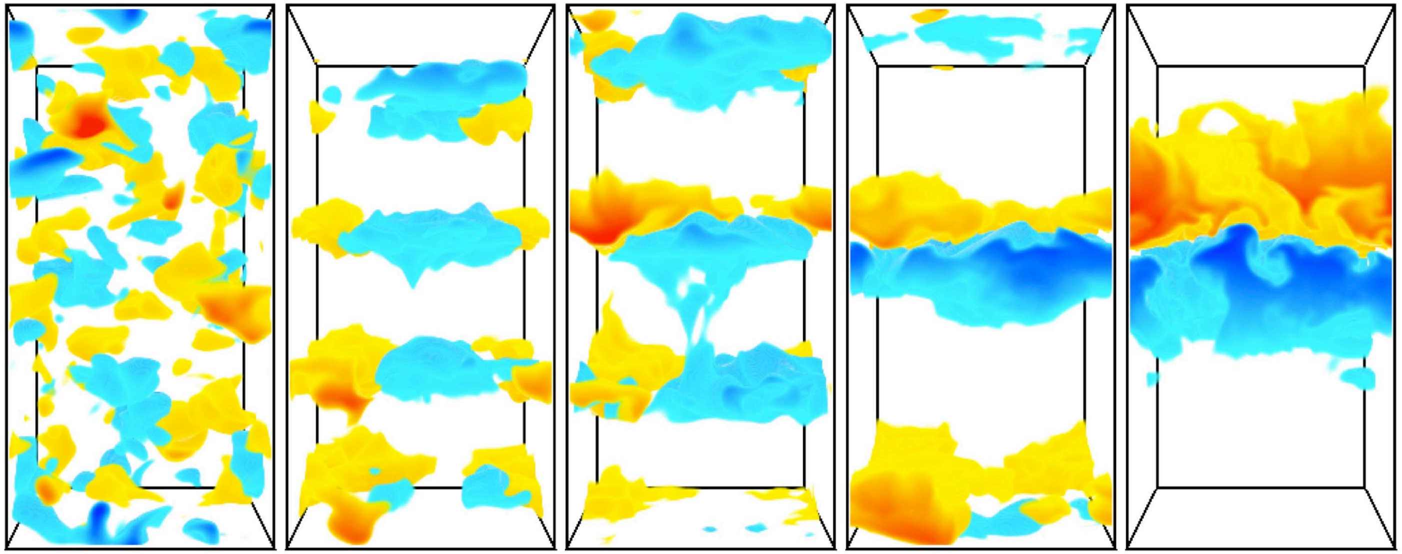


Scaling different from high-Pr case, where interfaces are laminar. Results are *not* consistent with Spruit or Leconte & Chabrier models.

Wood, Garaud & Stellmach (2013)

Layered convection

But what is the ultimate layer height? *Layers always appear to merge until a single one remains…*



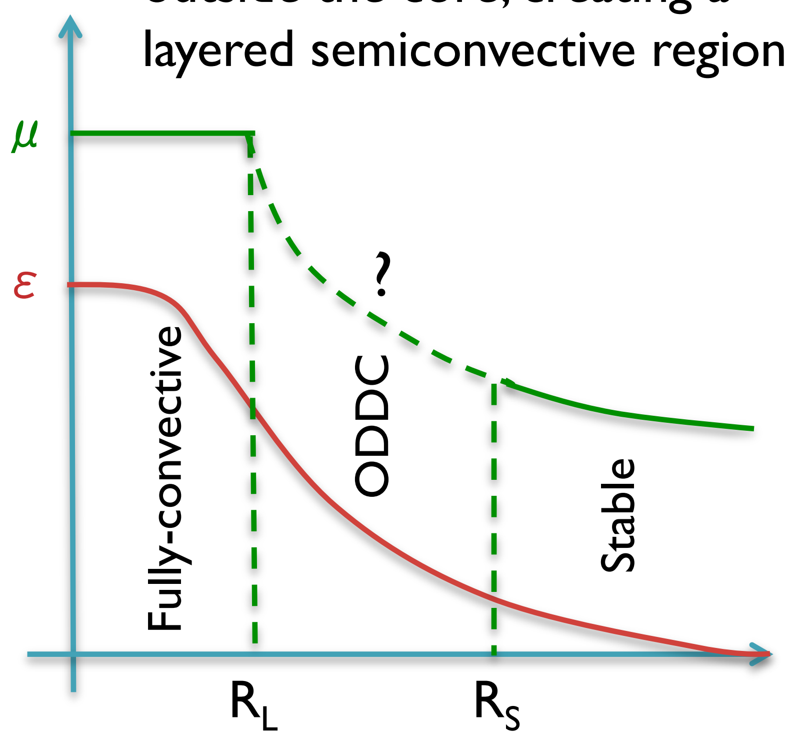
Open questions:

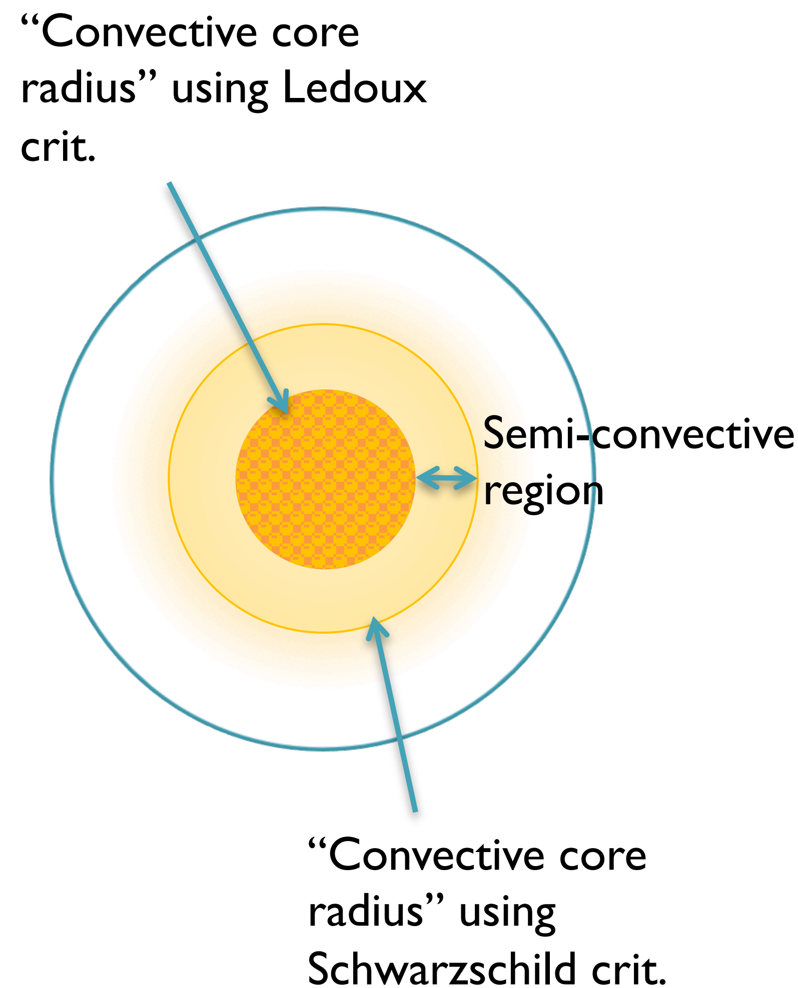
- What controls the merger dynamics of a staircase?
- Do layers eventually stop merging or not?

Rosenblum, et al. (2011); Wood et al. (2013)

Semiconvection in intermediate-mass stars.

- In intermediate mass stars, the answer to that question may not matter.
- Recall that for stars > $1.2M_{sun}$ some burning takes place outside the core, creating a layered semiconvective region.



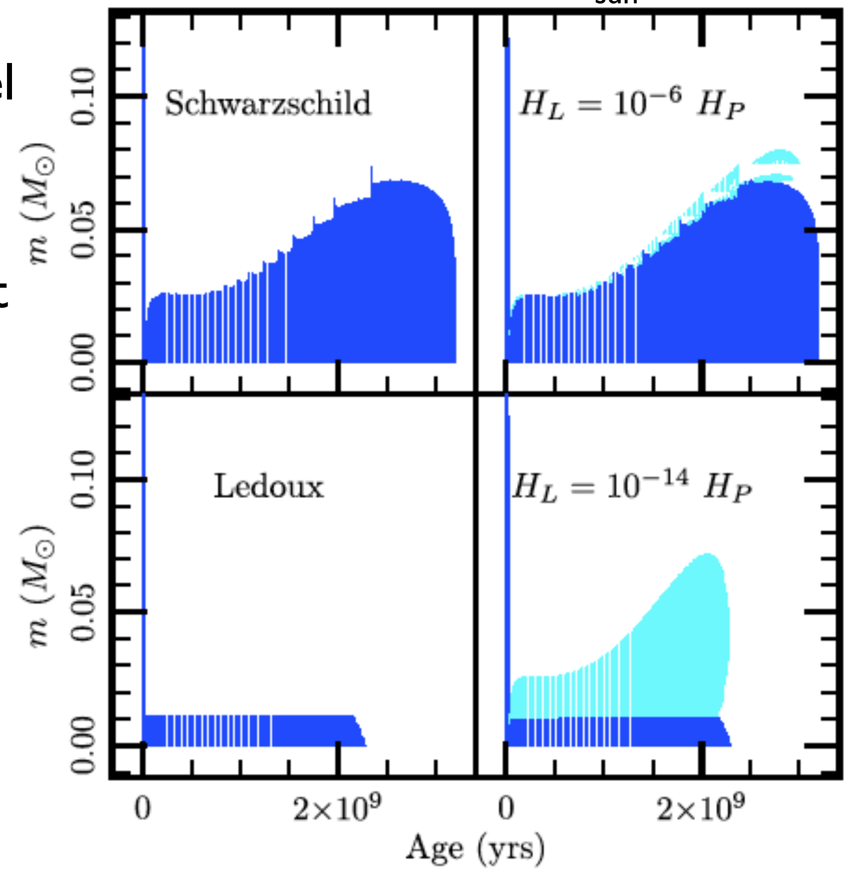


Semiconvection in intermediate-mass stars.

- Moore & Garaud 2016 implemented Wood et al. model for layered semiconvection in MESA.
- Outcome is very similar to that using Schwarzschild criterion unless H_L is *ridiculously* small.
- Mixing across a semiconvective region is so efficient that it rapidly becomes mixed & fully convective!

Ignoring semiconvection altogether, and merely using the Schwarzschild criterion is just as good !

Evolution of 1.3 M_{sun} star



Moore & Garaud, 2016

Semiconvection in intermediate-mass stars.

- Results are consistent with, e.g. Silva Aguirre et al. 2013 , Deheuvels et al. 2016, who find convective cores in MS stars are even larger than core size predicted using Schwarzschild criterion:
  - Ledoux criterion is not relevant
  - Need overshoot beyond Schwarzschild criterion radius

STELLAR AGES AND CONVECTIVE CORES IN FIELD MAIN-SEQUENCE STARS:
FIRST ASTEROSEISMIC APPLICATION TO TWO *KEPLER* TARGETS

V. Silva Aguirre[1,2,3], S. Basu[4], I. M. Brandão[5], J. Christensen-Dalsgaard[1,3], S. Deheuvels[4,6,7], G. Doğan[8], T. S. Metcalfe[9], A. M. Serenelli[3,10], J. Ballot[6,7], W. J. Chaplin[1,3,11], M. S. Cunha[5], A. Weiss[2], T. Appourchaux[12], L. Casagrande[13], S. Cassisi[14], O. L. Creevey[15], R. A. García[3,16], Y. Lebreton[17,18], A. Noels[19], S. G. Sousa[5], D. Stello[20], T. R. White[20], S. D Kawaler[21], and H. Kjeldsen[1]

In the case of Dushera, we have made the first direct detection of a convective core in a *Kepler* main-sequence target. Use of fits to the ratios r_{010} and r_{02} allow us to discard outliers and determine the characteristics of the star with an excellent accuracy. The size of the central mixed region is estimated to be ∼2.4% of the photospheric acoustic radius, showing that mixing beyond the formal Schwarzschild convective boundary exists. In this study the extra mixing has been modeled using

Semiconvection in intermediate-mass stars.

- Results are consistent with, e.g. Silva Aguirre et al. 2013 , Deheuvels et al. 2016, who find convective cores in MS stars are even larger than core size predicted using Schwarzschild criterion:
  - Ledoux criterion is not relevant
  - Need overshoot beyond Schwarzschild criterion radius

Measuring the extent of convective cores in low-mass stars using *Kepler* data: toward a calibration of core overshooting

S. Deheuvels[1,2], I. Brandão[3,4], V. Silva Aguirre[5], J. Ballot[1,2], E. Michel[6], M. S. Cunha[3,4], Y. Lebreton[6,7], and T. Appourchaux[9]

By comparing the slope and mean value of the r_{010} ratios of the 24 selected targets to those of a grid of models computed with the CESAM2K code, we were able to establish that

– 10 of these targets are in the post main sequence and therefore do not possess convective cores;
– 13 targets are in the main sequence (the evolutionary status of the remaining target is uncertain) and among them eight stars have a convective core;
– the convective cores of these eight targets extend beyond the classical Schwarzschild boundary.

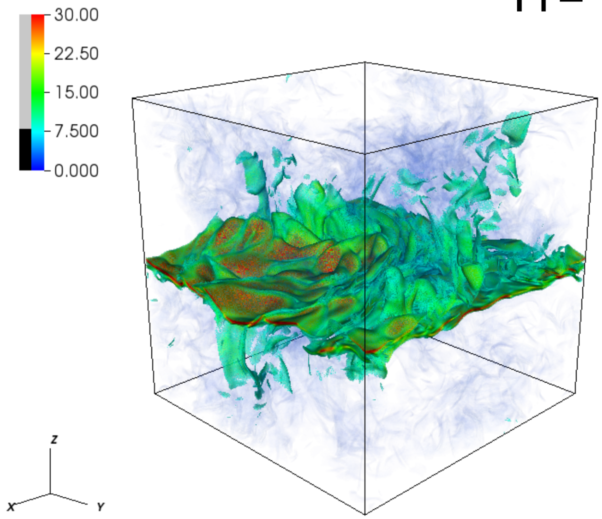
Layered convection without the γ - instability

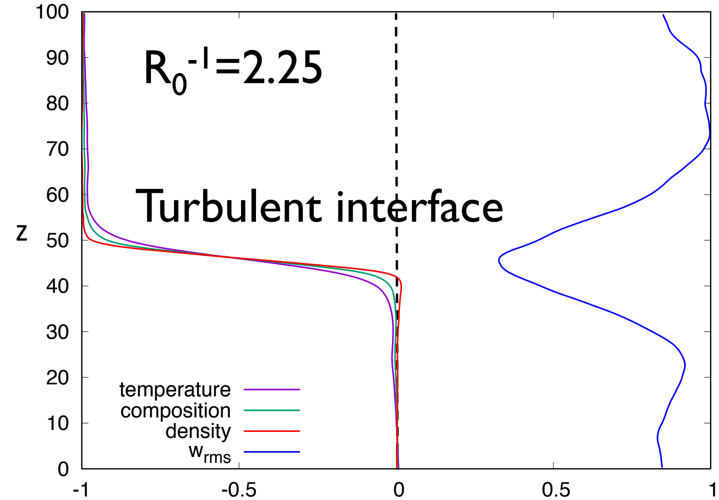
- When $R_0^{-1} > R_L^{-1}$, layers cannot spontaneously form. This could be the case, e.g.
  - in AGB stars with detached semiconvective zones,
  - in giant planets.

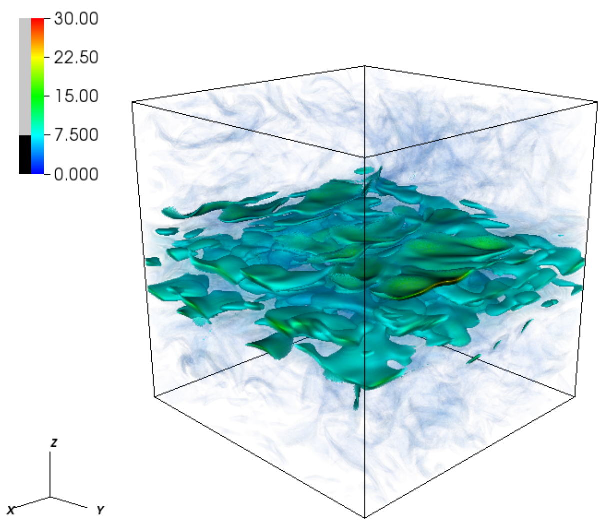
- However, if other physical processes exist to trigger layer formation, then layers can persist (cf. layers in the Arcic, in lakes), in some intermediate region of parameter space (Moll et al. 2017). This could be relevant in stars.

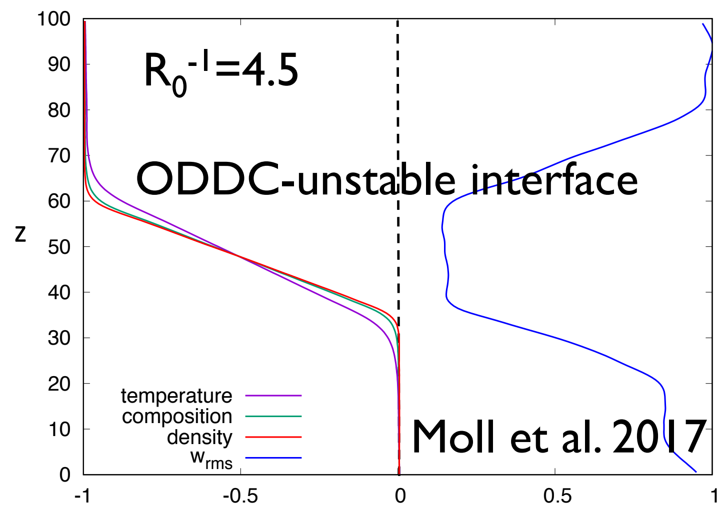
Layered convection without the γ - instability

Pr= τ =0.1. R_L^{-1} = 1.5, R_c^{-1}=5.5









Layered convection without the γ - instability

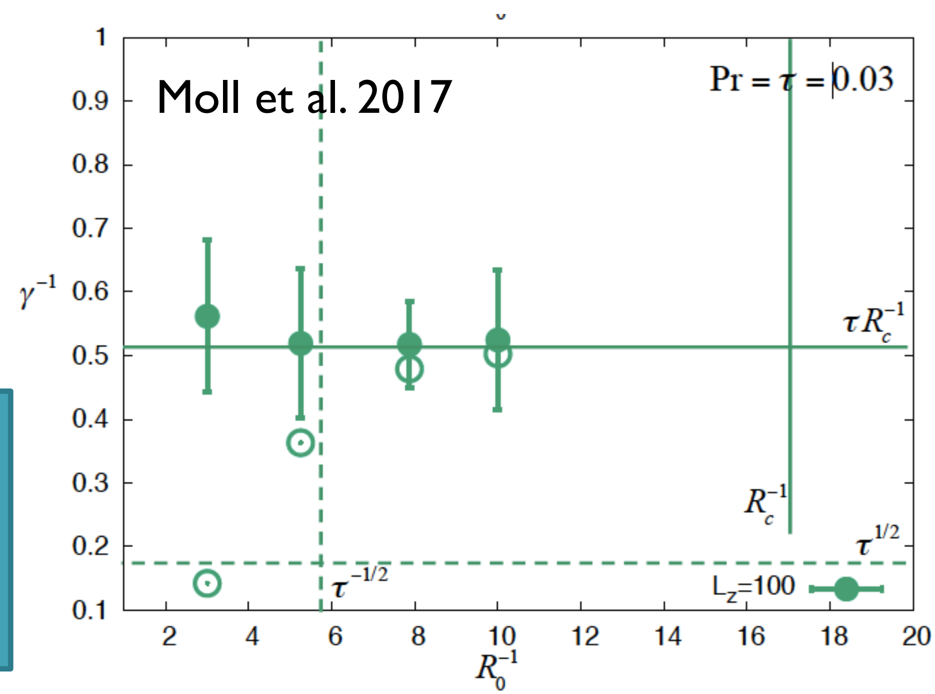
- In this regime, we find that $F_{h,semi} = -\rho c_p (\mathrm{Nu}_T - 1)\kappa_T \left(\frac{dT}{dr} - \frac{dT_{ad}}{dr} \right)$

where $\mathrm{Nu}_T - 1 = C_{TBD} \mathrm{Ra}_*^{1/3}$ where $\mathrm{Ra}_* = \frac{\alpha g \left| \frac{dT}{dr} - \frac{dT_{ad}}{dr} \right| H_L^4}{\kappa_T^2}$

- But $\frac{\beta \langle wC \rangle}{\alpha \langle wT \rangle} \approx \tau R_c^{-1} \neq \tau^{1/2}$

(consistent with non-diffusive Interfaces)

Even in this regime, compositonal transport can be quite efficient (if it is layered)!



Summary

- Stellar ODDC/semiconvection is often in the layered regime
- Transport of heat and composition is important in that regime, and depends on layer height:
- Non-dimensional turbulent temperature flux is :

$$\mathrm{Nu}_T - 1 \approx f(\tau, R_0^{-1}) \cdot \mathrm{Ra}_*^{1/3}$$

$$\text{where } f \approx 0.1, \quad \text{and} \quad \mathrm{Ra}_* = \frac{\alpha g \left| T_{0z} - T_z^{ad} \right| H_L^4}{\kappa_T^2}$$

- So dimensional turbulent heat flux is

$$F_{semi} = \rho c_p F_{T,semi} = \rho c_p (\mathrm{Nu}_T - 1)\kappa_T \left| \frac{dT}{dr} - \frac{dT_{ad}}{dr} \right|$$

- The dimensional turbulent compositional flux is

$$F_{C,semi} = \gamma_{tot}^{-1} \frac{\alpha}{\beta} F_{T,semi} \quad \text{where} \quad \gamma_{tot}^{-1} \sim 0.5$$

Summary

- Layer height is however unknown:
  - in all γ unstable cases, the layers merge until only one is left
  - Is this true in stars as well?
  - Or is there a finite semiconvective layer height?

- Maybe we do not care about the answer, since semiconvective transport is so efficient that the layer rapidly becomes fully convective.