

Multiscale Principal Components Analysis for Image Local Orientation Estimation *

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Abstract

This paper presents an image local orientation estimation method, which is based on a combination of two already well-known techniques: the principal component analysis (PCA) and the multiscale pyramid decomposition. The PCA analysis is applied to find the Maximum Likelihood (ML) estimate of the local orientation. The proposed technique is shown to enjoy excellent robustness against noise. We present both simulated and real image examples to demonstrate the proposed technique.

1 Introduction

Image local orientation estimation plays an important role in many computer vision and image processing tasks such as edge detection, image segmentation, and texture analysis. The 2-D local orientation estimation is also directly related to optical flow estimation, which is the generalization of orientation in a 3-D space/time volume.

Several techniques for local orientation estimation have been proposed in the past. Perona [2] extended the idea of anisotropic diffusion to orientation maps. Bigun et al. [3] posed the problem as the least-squares fitting of a plane in the Fourier transform domain. Another set of techniques is based on steerable filters [6], but they are often limited in precision and generalization. Wilson et al. [8] developed a multiscale orientation estimation approach, which is closely related to the one we propose here. But our method of combining the PCA and multiscale is novel, more efficient, and will give more robust results.

Almost all the established local orientation estimation techniques are based on the analysis of the local gradient field of the image. But the local gradients are

very sensitive to noise, making the estimate of local orientation directly from these rather unreliable. So there is often a need for some local or even global process to 'smooth' the estimate. However, by smoothing one gives up localization. A mechanism with both noise robustness and feature localization is needed.

The estimation method presented here is based on the Principal Components Analysis (PCA) [4], which at its core uses the Singular Value Decomposition (SVD) of the ensemble of gradient vectors in a local neighborhood to find the dominant orientation. We develop this PCA-based method in a multiscale framework, which effectively enforces smoothness. Another benefit of the proposed method is the adaptive compromise between the resolution of the orientation field and its relative accuracy.

In Section 2, the PCA analysis of local orientation will be introduced. We will also show that this method results in the ML estimate of the local orientation. In Section 3, we will develop the multiscale implementation of the PCA-based method. Section 4 contains the experimental results, in both simulated images and real images. In Section 5 we present our conclusions.

2 PCA Analysis of local orientation

Principal Components Analysis is used for computing the dominant vectors representing a given data set and also provides an optimal basis for minimum mean-squared reconstruction of the given data. It is also sometimes referred to as the Karhunen-Loeve Transform [4]. The computational basis of PCA is the calculation of the Singular Value Decomposition (SVD) of the data matrix, or equivalently the eigen-decomposition of the data covariance matrix. Here we describe the method in terms of the SVD.

Let us assume that in the image of interest $f(x, y)$, the orientation field is piecewise constant. Under this

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assumption, the gradient vectors in a block should *on average* be orthogonal to the dominant orientation of the image pattern. So orientation estimation can be formulated as the task of finding a unit vector \vec{a} , to maximize the average of angles θ_i between \vec{a} and gradient vectors $\vec{g}_i = \nabla f(x_i, y_i)$, $i = 1, \dots, n$, (within a local window) [1], equivalently to minimize:

$$\sum_{i=1}^n (\vec{a}^T \vec{g}_i)^2 = \vec{a}^T \sum_{i=1}^n (\vec{g}_i \vec{g}_i^T) \vec{a} = \vec{a}^T C \vec{a} \quad (1)$$

where

$$C = \begin{bmatrix} \sum_{i=1}^n g_x^{(i)} g_x^{(i)} & \sum_{i=1}^n g_x^{(i)} g_y^{(i)} \\ \sum_{i=1}^n g_x^{(i)} g_y^{(i)} & \sum_{i=1}^n g_y^{(i)} g_y^{(i)} \end{bmatrix}$$

subject to $\|\vec{a}\| = 1$, where $g_x^{(i)}$ and $g_y^{(i)}$ are derivatives in x and y direction, respectively.

This problem can be solved using Lagrange multipliers and it can be shown that the unit vector \vec{a} minimizing $\vec{a}^T C \vec{a}$ is the eigenvector of C corresponding to the smallest eigenvalue, or equivalently, the singular vector corresponding to the smallest singular value of G , defined in (3). Note that the SVD method is numerically more stable and efficient.

From the theory of Directional Statistics [5], for a bivariate Gaussian distributed random vector field, the direction of the vectors will have von Mises distribution, with the probability density function (PDF):

$$f(\theta; \mu, k) = \frac{1}{2\pi I_0(k)} e^{k \cos(\theta - \mu)} \quad (2)$$

where

$$I_0(k) = \frac{1}{2\pi} \int_0^{2\pi} e^{k \cos(\theta)} d\theta$$

In the task of local orientation estimation, we are concerned with the 'axes' of orientation, which means the vectors \vec{a} and $-\vec{a}$ define the same orientation. For Gaussian distributed random vectors (in our case, the gradient vectors), the axes of the vectors will have Bingham distribution. In the 2-D case, this is just the *2-wrapped* von Mises distribution, which means, given the PDF of von Mises distribution $f(\theta)$, the PDF of Bingham distribution $f^*(\theta) = f(2\theta)$ [5]. From [1], for Bingham distributed unit vectors, the ML estimate of the dominant direction equals to the first singular vector \vec{v}_1 from the SVD of the data vector matrix (defined in equation 3). This shows that by assuming the gradient vectors to have Gaussian distribution, the PCA method will give the ML estimate.

The first step of the PCA-based method is the computation of the gradient map for the whole image.

This can be accomplished by a variety of differencing operators, which ultimately can and should be optimized. Denote the local estimate of the gradient of image $f(x, y)$ at point (x_k, y_k) by:

$$\nabla f(k) = \nabla f(x_k, y_k) = [\partial f(x_k, y_k)/\partial x, \partial f(x_k, y_k)/\partial y]^T$$

In order to estimate the local orientation, we divide the gradient field into local blocks (overlapped or non-overlapped). For each block, group the gradients into an $N \times 2$ matrix G as follows:

$$G = \begin{bmatrix} \nabla f(1)^T \\ \nabla f(2)^T \\ \vdots \\ \nabla f(N)^T \end{bmatrix}. \quad (3)$$

The last step is to compute the SVD of matrix G :

$$G = USV^T \quad (4)$$

where U is orthogonal and $N \times N$, representing each vector's 'contribution' to the corresponding singular vector; S is $N \times 2$, representing the energy in the dominant directions; and V is orthogonal and 2×2 , in which the first column v_1 represents the dominant orientation of the gradient field. By rotating v_1 by 90 degrees, we have the dominant orientation in the image block.

The difference between the singular value s_1 and s_2 can be used as a measure of accuracy or dominance of the estimate. However, because $s_1 - s_2$ is an energy dependent measure, the quantity

$$R = \frac{s_1 - s_2}{s_1 + s_2} \quad (5)$$

will be more suitable for this task [3].

Note that the above quantity is bounded between 0 and 1, and is related to the condition number k of matrix G by:

$$R = \frac{k - 1}{k + 1} \quad (6)$$

The PDF of the condition number of an $N \times 2$ independent and identically distributed (*i.i.d*) white Gaussian matrix is derived in [7]. From this we can get the PDF of R for an $N \times 2$ *i.i.d* white Gaussian matrix (shown in Figure 1):

$$p(R) = 4(N - 1)R \frac{(1 - R^2)^{N-2}}{(1 + R^2)^N} \quad (7)$$

We can use this PDF in a significance test [11] to distinguish between a pure noise image and an image

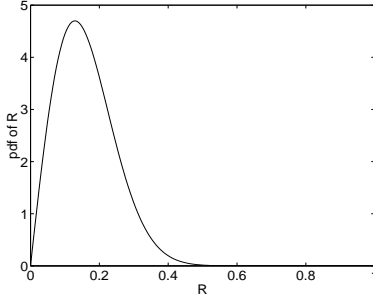


Figure 1: PDF of R (N=16)

with orientation pattern. First we can set a *significance level* threshold R^* . For any given image block, we can perform the PCA-based estimation, if the R is less than R^* , it is very likely that the corresponding image block is only pure white noise and contains no dominant orientation.

3 Multiscale Implementation

Multiscale signal and image analysis have been investigated for some time with applications in data compression, edge detection, and segmentation, etc. Wilson et al. [8] used a multiscale approach in orientation estimation. The method we propose here is similar to Wilson’s method, but our method combines PCA and multiscale, using the dominance measure R as the propagation weight. The combination of these techniques makes our method very efficient and is shown to be more robust.

As introduced in [9], multiscale estimation can be described within the framework of Kalman filtering across scales:

$$s[n] = A[n]s[\gamma n] + B[n]w[n] \quad (8)$$

$$x[n] = H[n]s[n] + v[n] \quad (9)$$

where $s[n]$ is the signal in the current layer; $s[\gamma n]$ is the signal in the parent layer (coarser resolution); $x[n]$ is the measurement in the current layer; $w[n]$ and $v[n]$ are the innovation signal and measurement noise in current layer, respectively. The covariance matrix of $v[n]$ is denoted as $C[n]$.

In the task of local orientation estimation, the signal $s[n]$ is the local dominant orientation (or equivalently, the local dominant gradients). The signal in the current layer can be modeled as the corresponding signal from parent layer (up-sampled to fit the size) plus an innovation vector. And the measurement is simply the signal vector plus a noise vector. So we can simplify the Kalman filter equations as :

$$s[n] = s[\gamma n] + w[n] \quad (10)$$

$$x[n] = s[n] + v[n] \quad (11)$$

The MMSE estimate can be written in the form of recursive linear combination:

$$\hat{s}[n] = \hat{s}[\gamma n] + K[n](x[n] - \hat{s}[\gamma n]) \quad (12)$$

where $K[n]$ is the Kalman gain matrix:

$$K[n] = M[n|\gamma n](C[n] + M[n|\gamma n])^{-1} \quad (13)$$

$M[n|\gamma n]$ is the covariance matrix of the estimation error.

There are three major steps in our multiscale PCA-based method:

1. From the computed gradient field of the given image, build up a gradient pyramid (Gaussian pyramid).
2. On each layer of the gradient pyramid, divide the gradient field into local blocks (overlapped or non-overlapped), and on each block, use the PCA-based method to estimate the local orientation.
3. Propagate the estimates from coarser layer to finer layer (as described in Equation 12), all the way to the finest resolution.

In step 3, the optimal propagation weight is given by (13) in terms of the covariance of estimation error and the covariance of noise. But these quantities are unknown or hard to determine. Wilson et al. [8] used the ‘average energy’ as propagation weights, which is not directly related to the variance of orientation. In our approach, we assume the innovation vector $w[n]$ and the measurement noise $v[n]$ are both zero-mean white Gaussian. Then both $M[n|\gamma n]$ and $C[n]$ will be diagonal matrices and the Kalman gain matrix $K[n]$ can be simplified as a scalar quantity $k[n]$ times the identity matrix, where $k[n]$ can be approximated as:

$$k[n] \sim \frac{\sigma_{\gamma n}^2}{\sigma_{\gamma n}^2 + \sigma_n^2} \quad (14)$$

where $\sigma_{\gamma n}^2$ and σ_n^2 are variances of orientation on parent layer and current layer, respectively. Note that in the PCA-based method, the dominance measure R can be viewed to be *inverse proportional* to the variance σ^2 . Thus we elect to define the propagation weight in terms of R , which is easy to compute as a byproduct of the SVD. Our experiments show that this weight will give more robust estimates than Wilson’s approach. We have:

$$\hat{s}[n] = \hat{s}[\gamma n] + \frac{R[n]}{R[n] + R[\gamma n]}(x[n] - \hat{s}[\gamma n]) \quad (15)$$

4 Experimental Results

In order to test our method’s stability in the presence of noise, we generate a test image with single known sinusoidal orientation pattern and apply zero mean Gaussian white noise to it (Figure 2). Observe the change of the average estimation error with increasing noise variance. We compare the multiscale method with the single-scale method, both using PCA-based method to estimate the local orientation. For both methods, we compare the results of overlapped-neighbor-block and non-overlapped-neighbor-block. We also compare the result with Wilson’s multiscale method. The results are shown in Figure 3.

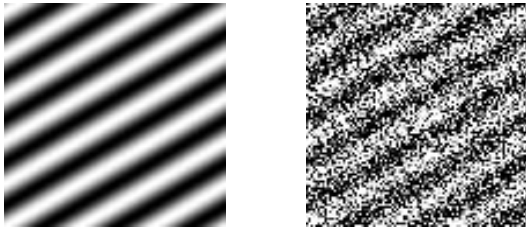


Figure 2: Left:clean test image(value range: 0-1).Right:noisy test image(noise variance: 0.6)

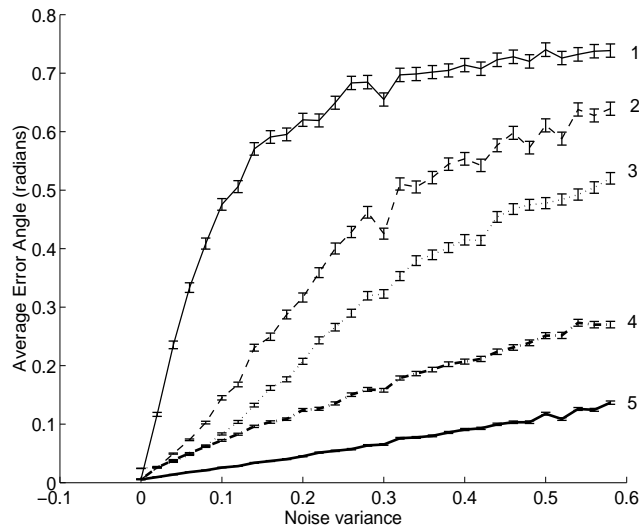


Figure 3: Multiscale VS. Single scale (1: PCA/Single scale, non-overlap; 2:PCA/Single scale, overlap; 3:Wilson’s multiscale method; 4:PCA/Multiscale, non-overlap; 5:PCA/Multiscale, overlap;)

From Figure 3 we can see that the multiscale methods work much better than single scale methods; the methods with overlapped local blocks are more robust

than methods with non-overlapped blocks; and Wilson’s multiscale method is not as robust as the method proposed here.

The multiscale method’s robustness to noise can be shown in another experiment. The test image in Figure 4 has diverse orientations with different contrast. Gaussian white noise is applied in the right half. From the estimated orientation map we can see (Figure 5 and Figure 6), the single scale method is quite messy in the noisy half, while the multiscale method works well on both sides.

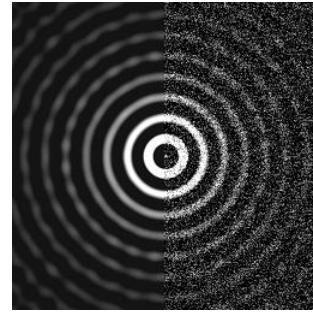


Figure 4: Test image

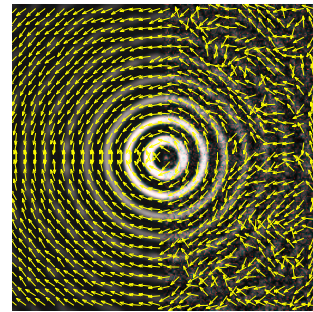


Figure 5: Orientation map of Figure 4, Single scale method

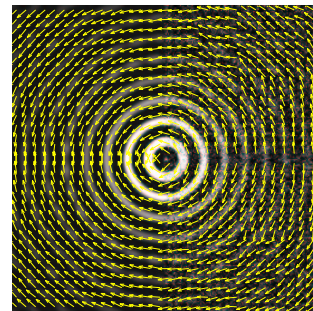


Figure 6: Orientation map of Figure 4, Multiscale method

Finally, as an example of the performance of the proposed method on a real image, the orientation estimation result for a real fingerprint image is shown in Figure 8.



Figure 7: Fingerprint image

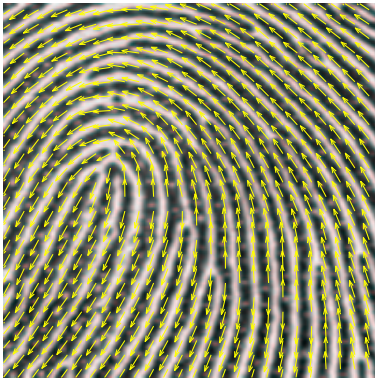


Figure 8: Orientation map of Figure 7

5 Conclusion

The proposed orientation estimation method works well in terms of both robustness and accuracy. The PCA-based estimation gives the optimal (Maximum Likelihood) estimation of the local dominant orientation. The multiscale framework helps in noise rejection and balancing localization and accuracy.

We point out that this multiscale PCA-based method can also be developed for other applications, e.g. corner detector. The PCA method is used in the Harris corner detector [10]. The multiscale PCA scheme presented here can be used to detect corners at different scales.

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