Global Image Editing Using the Spectrum of Affinity Matrices
(Invited Paper)

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***Abstract*—In this work we introduce a new image editing tool, based on the spectrum of a global filter computed from image affinities. Recently, we have shown that the global filter derived from a fully connected graph representing the image, can be approximated using the Nyström extension [1]. This filter is computed by approximating the leading eigenvectors of the filter. These orthonormal eigenfunctions are highly expressive of the coarse and fine details in the underlying image, where each eigenvector can be interpreted as one scale of a data-dependent multiscale image decomposition. In this filtering scheme, each eigenvalue can boost or suppress the corresponding signal component in each scale. Our analysis shows that the mapping of the eigenvalues by an appropriate polynomial function endows the filter with a number of important capabilities, such as edge-aware sharpening, denoising and tone manipulation.**

***Index Terms*—Image Editing, Image Filtering, Nyström Extension**

I. Introduction

Edge-aware filtering is a key tool in image processing, computer vision and graphics. In most of the existing methods the underlying image is decomposed into piecewise smooth and detail layers. Then, a variety of applications, such as tone mapping, edge editing and edit propagation are developed based on this type of decomposition [2]–[6].

The optimal edge-aware filter coarsens details of the image, yet the principal edges are ideally not altered. Several non-linear (data-dependent) operators such as the bilateral filter [7], [8] have been used for this task. Chen et al. [9] used the bilateral filter by progressive increment of the spatial and range width of the Gaussian for building a pyramid of image layers. In a similar iterative approach, the bilateral filter is applied successively on the coarsened image while decreasing the range width [10]. In all of these methods edges are preserved by the gradual change in the tuning parameters of the bilateral kernel. However, the kernel weights have to be recomputed in every iteration.

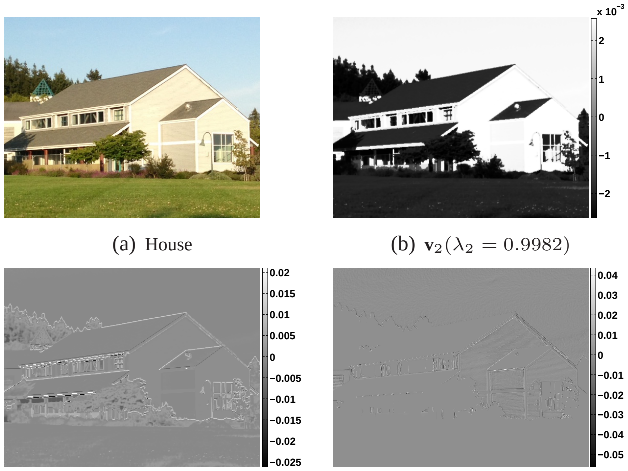
Almost all existing edge-aware methods use the same general idea: *Using a local operator decompose image into base layer and detail layer and then manipulate each layer separately and recombine to reach the desired edit*. There are two main problems with this approach:

- Since noise is always an unavoidable part of our imaging systems, boosting the detail layer usually worsens the signal-to-noise-ratio (SNR). Even with today's mega-pixel images, the trade off between sharpness and SNR is still a bottleneck. Increasing exposure time will result in higher SNR, but more blurry image. On the other hand, a small aperture leads to sharper but noisier images.
- While it is always desirable to treat similar edges of an image in the same way, the existing local filters have irregular behaviors when handling edges with slightly different brightness and gradient profiles. In other words, global structure among similar edges are usually ignored by the low-level feature vectors associated with each pixel. Even with all the edge-aware operators in hand, performing local adjustments to pixels and then *evenly* propagating the edit to the similar regions *all* across the image has proved to be a challenging task.

To alleviate the first problem, some methods have been proposed that build on the classic linear unsharp masking. Adaptive unsharp masking [11] controls the contrast enhancement to happen in texture areas and avoids noise magnification by leaving relatively smooth regions unchanged. A hierarchical framework based on Non-Local Means (NLM) kernel [12] is proposed in [13] where the noise removal is applied first as a separate step and then the detail layers are extracted.

To mitigate the second problem, there have been some efforts to interactively propagate the edits to regions with similar appearance. Recently, the sparse optimization formulation is used to provide stroke-based editing workflows with propagative tonal adjustments [6], [14], [15]. Using an edge-aware energy minimization method, the tonal adjustment imposed by the user is interpolated to the pixels with similar luminance. Farbman et al. [5] also proposed an edit propagation method based on the concept of *diffusion distances* which can measure closeness of pixels on a manifold space. By approximating a diffusion map built upon this high-dimensional similarity measure, the input adjustments can propagate to nearby pixels on the manifold.

In our framework, the two above-mentioned shortcomings of the existing methods are tackled at the same time. Our image filter is global in the sense that all the node (pixel) pairs on the graph (image) are directly connected to each other. As it was shown in [1], the eigen-decomposition of the corresponding symmetric, doubly-stochastic filter matrix can



(a) House (b) $\mathbf{v}_2(\lambda_2 = 0.9982)$

(c) $\mathbf{v}_{10}(\lambda_{10} = 0.9188)$ (d) $\mathbf{v}_{200}(\lambda_{200} = 0.0332)$

Fig. 1. Some leading eigenvectors computed from the luminance channel of the House image using less than 0.2% of the pixels.

be approximated using the Nyström extension. The obtained eigenvectors are very informative of the similar regions and edge information of the image (Fig. 1). More specifically, the approximated eigenfunctions enable us to employ diffusion distance for propagating the same manipulation over pixels belonging to the similar regions. Having the spectrum of the filter, noise suppression and detail enhancement become much easier by mapping the spectrum of the filter using a polynomial function with a few parameters (sliders) to tune. Our experimental results show that this strategy reduces the halo artifacts around principle edges, avoids the common noise magnification problem and can interactively propagate the user's edit across the intended similar regions with ease. In contrast to [5], [6], [14], [15], our approach does not require the solution of a complex optimization problem to achieve this effect.

In what follows, a description of global filter and its eigen-decomposition are given in Section II. Then, our detail manipulation strategy is explained in Section III. After discussing our experimental results for different applications of the proposed scheme, this report is concluded in Section V.

II. The Global Filter

Our filtering framework is based on non-parametric regression in which a kernel function K_{ij} measures the similarity between the input samples y_i and y_j, where $\widehat{y}_i$ denotes the i-th output pixel. The NLM kernel [12] is a very popular data-dependent filter in which the photometric similarity is captured in a patch-wise manner:

$$K_{ij} = \exp\left\{\frac{-\|\mathbf{y}_i - \mathbf{y}_j\|^2}{h^2}\right\}, \tag{1}$$

where $\mathbf{y}_i$ and $\mathbf{y}_j$ are patches centered at y_i and y_j, respectively. The non-parametric framework yields a global filter description as follows:

$$\widehat{\mathbf{y}} = \begin{bmatrix} \mathbf{w}_1^T \\ \mathbf{w}_2^T \\ \vdots \\ \mathbf{w}_n^T \end{bmatrix} \mathbf{y} = \mathbf{W}\mathbf{y}, \tag{2}$$

where the i-th row of the matrix $\mathbf{W}$ has the corresponding normalized weights as:

$$\mathbf{w}_i^T = \frac{1}{\sum_{j=1}^{n} K_{ij}}[K_{i1}, K_{i2}, \ldots, K_{in}]. \tag{3}$$

A solution for reducing the computational burden of this global scheme is proposed in [1] where instead of computing each element of the filter $\mathbf{W}$, some sample rows (or columns) of the filter are exactly computed and used to approximate the remaining rows. In practice, exact computation of the filter matrix is avoided by instead approximating the leading eigenvectors [1]. This gives an approximate of the symmetric, positive definite $\mathbf{W}$ based on the row-rank assumption of the similarity (affinity) matrix $\mathbf{K}$. Having m leading eigen-decomposition elements of the data-dependent filter $\mathbf{W}$ as:

$$\mathbf{W} = \mathbf{V}\mathbf{S}\mathbf{V}^T \approx \mathbf{V}_m\mathbf{S}_m\mathbf{V}_m^T = \mathbf{W}_m, \tag{4}$$

where $\mathbf{V}_m = [\mathbf{v}_1, ..., \mathbf{v}_m]$ denotes the leading orthonormal eigenvectors and $\mathbf{S}_m = \text{diag}[\lambda_1, ..., \lambda_m]$ contains the leading eigenvalues in decreasing order $0 \leq \lambda_m \leq ... < \lambda_1 = 1$. Global features of the underlying image can be represented in these approximated eigenvectors. This has been shown in Fig. 1 where some eigenvectors with different indices are illustrated. As can be seen, various features of the image are represented by these basis functions. Eigenvectors with lower indices contain principal edges and corresponding eigenvectors of larger indices represent texture regions. As can be seen, these features are globally separated in each eigenmode. This suggests that each corresponding eigenvalue can manipulate these features in the image. This is discussed in the following section.

III. Multiscale Detail Manipulation

Our global multiscale filtering process is illustrated in Fig. 2. Fig. 2(a) depicts the multiscale decomposition and reconstruction where the input image $\mathbf{y}$ is layered to k detail layers $\mathbf{y}_{d^i}$ and one basic smooth layer $\mathbf{y}_{s^k}$ such that:

$$\mathbf{y} = \mathbf{y}_{d^1} + \ldots + \mathbf{y}_{d^k} + \mathbf{y}_{s^k} \tag{5}$$

The edited image $\widehat{\mathbf{y}}$ can be computed by weighting each layer and adding the components back together:

$$\widehat{\mathbf{y}} = \alpha_1\mathbf{y}_{d^1} + \ldots + \alpha_k\mathbf{y}_{d^k} + \alpha_{k+1}\mathbf{y}_{s^k} \tag{6}$$

Replacing the orthonormal eigen-decomposition of the filter $\mathbf{W}_m$ in the above, the equivalent filter is (Fig. 2(b)):

$$\begin{aligned} \widehat{\mathbf{y}} &= \alpha_1(\mathbf{I} - \mathbf{W}_m)\mathbf{y} + \alpha_2\mathbf{W}_m(\mathbf{I} - \mathbf{W}_m)\mathbf{y} + ... \\ &+ \alpha_k\mathbf{W}_m^{k-1}(\mathbf{I} - \mathbf{W}_m)\mathbf{y} + \alpha_{k+1}\mathbf{W}_m^k\mathbf{y} \\ &\approx \mathbf{V}_m f(\mathbf{S}_m)\mathbf{V}_m^T\mathbf{y} = \widehat{\mathbf{W}}_m\mathbf{y} \end{aligned} \tag{7}$$

where the function f has the following effect on each eigenvalue λ_j:

$$\begin{aligned} f(\lambda_j) &= \alpha_1 + (\alpha_2 - \alpha_1)\lambda_j + (\alpha_3 - \alpha_2)\lambda_j^2 + ... \\ &+ (\alpha_k - \alpha_{k-1})\lambda_j^{k-1} + (\alpha_{k+1} - \alpha_k)\lambda_j^k \end{aligned} \tag{8}$$

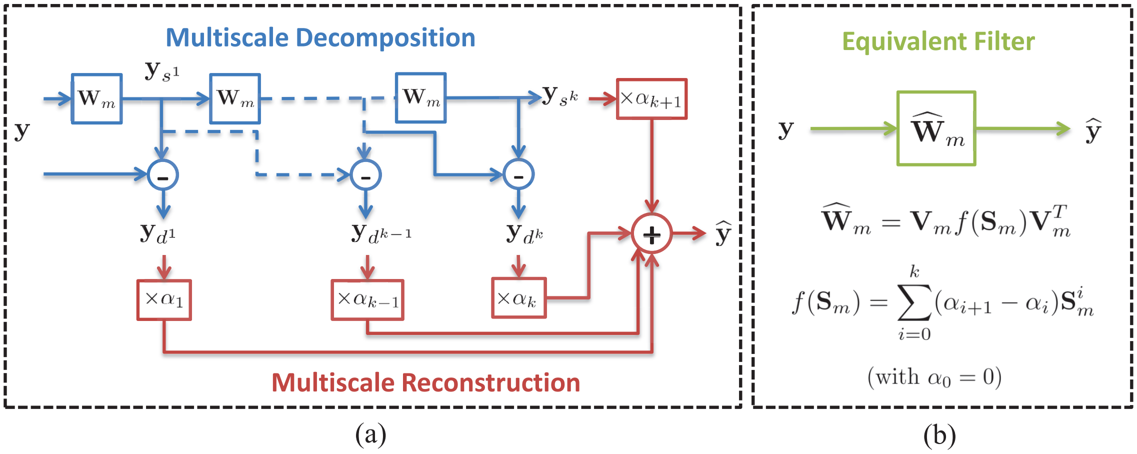


Fig. 2. (a) Multiscale decomposition: The low-pass filter $\mathbf{W}_m$ is used to extract detail layers $\mathbf{y}_{d^i}$. Multiscale reconstruction: Weighting each layer with α_i and adding them together. (b) The equivalent filter of the process given in (a) can be interpreted as the band-pass $\widehat{\mathbf{W}}_m$ in which the eigenvalues are a polynomial function of the low-pass filer's eigenvalues.

This is a special polynomial with $\alpha_0 = 0$, $f(0) = \alpha_1$ and $f(1) = \alpha_{k+1}$. The two coefficients α_1 and α_{k+1} correspond to the first detail layer and the basic smooth image, respectively. Examples of this filtering scheme are represented in following.

IV. Experiments

Performance of the proposed scheme is evaluated in this section. Fig. 3 gives a visual comparison of the proposed filters in Section III. As can be seen, applying filter $\widehat{\mathbf{W}}_m$ can boost the contrast and details of the image in fine, medium and coarse scales. Another interesting application of this filter is propagation of a particular editing parameters to a group of similar pixels. Using the diffusion map concept [16], squared diffusion distance of the i-th and j-th pixel is measured as:

$$D_{ij}^2(t) = \sum_{l=2}^{m} \lambda_l^{2t} (v_{il} - v_{jl})^2 \tag{9}$$

where t denotes the diffusion parameter and v_{il} denotes the i-th entry of the l-th eigenvector. We employ a simple gaussian function to embed this squared distance into $[0, 1]$ interval:

$$\mathbf{M}_{is}(t) \equiv \exp(-D_{is}^2(t)) \tag{10}$$

where s refers to the pixels of a selected region by user. Having such distance map enables us to easily propagate input edits to the similar regions on image. Using the computed mask, it can be seen that the sharpened region is propagated to the similar pixels in Fig. 4. Comparing results of the global editing and propagated edit in Fig 4, we can see that the main edges of the image are preserved and there is almost no halo effect on them. Also, existing noise in image regions with low SNR is no longer boosted.

V. Conclusion

Our contribution to the existing research work is as follows: Our framework handles noise naturally, because the image is projected onto the data adapted basis obtained from affinity weights. In other words, the noise is separated from the underlying signal components by projecting the image onto the approximated leading eigenvectors. The proposed scheme is able to deliver a filtering tool with various capabilities. As part of our future work, we will explore other editing applications of the global filter.

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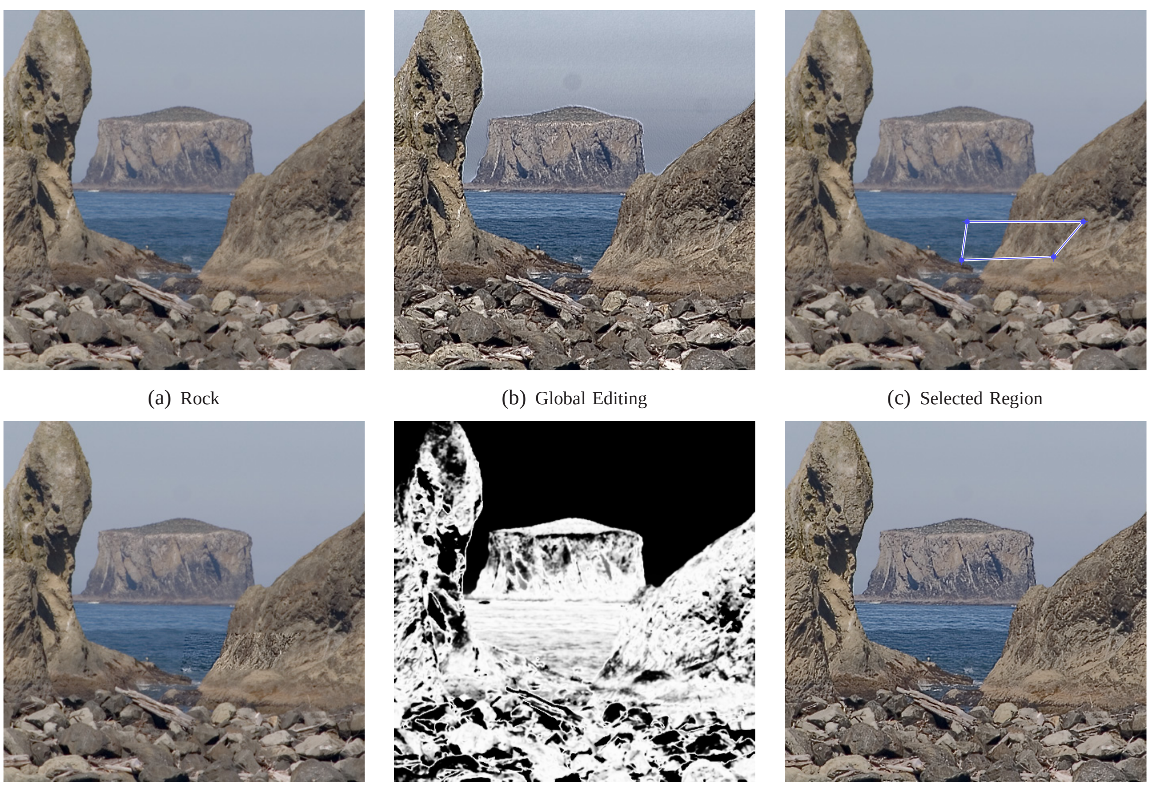
(a) Flower (b) Change of Contrast (c) Fine Scale Details

(d) Medium Scale Details (e) Coarse Scale Details (f) Combination

Fig. 3. Contrast and detail manipulation of the flower image. (b) $\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 1, \alpha_4 = 1.5$, (c) $\alpha_1 = 5, \alpha_2 = 1, \alpha_3 = 1, \alpha_4 = 1$, (d) $\alpha_1 = 1, \alpha_2 = 10, \alpha_3 = 1, \alpha_4 = 1$, (e) $\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 15, \alpha_4 = 1$, (f) $\alpha_1 = 3, \alpha_2 = 5, \alpha_3 = 10, \alpha_4 = 1.1$.



(a) Rock (b) Global Editing (c) Selected Region

(d) Edited Region (e) Propagation Mask (f) Propagated Edit

Fig. 4. Detail propagation of the rock image. (b) $\alpha_1 = \alpha_2 = \alpha_3 = 5, \alpha_4 = 1$, (c) Region of interest selected by user (d) Edited region by $\alpha_1 = \alpha_2 = \alpha_3 = 5, \alpha_4 = 1$, (e) Propagation mask for t=50 (f) Propagated edit of the selected region based on the propagation mask.