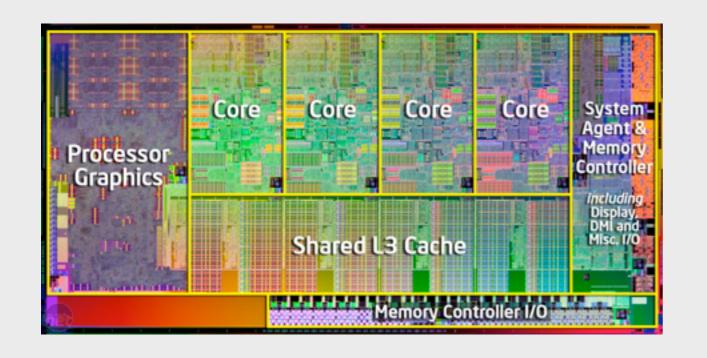
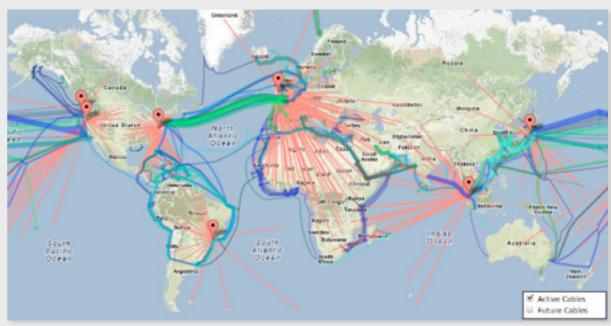
LVars:

Lattice-based Data Structures for Deterministic Parallel and Distributed Programming

Lindsey Kuper Indiana University

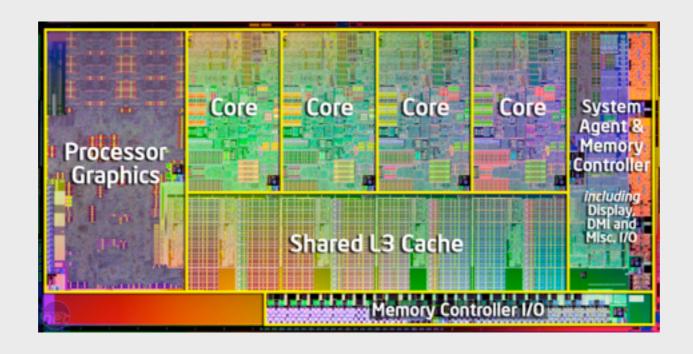
Intel Labs · Santa Clara, CA March 21, 2014



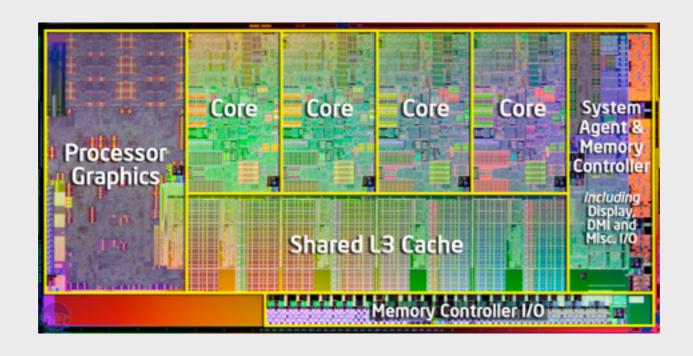


Parallel systems

Distributed systems



Deterministic Parallel Programming



(observably)
Deterministic Parallel Programming



data Item = Book | Shoes | ...



```
data Item = Book | Shoes | ...
p :: IO (Map Item Int)
```

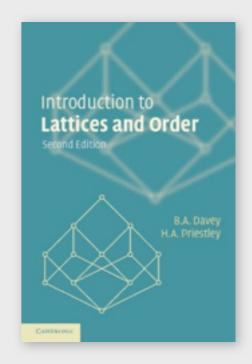


```
data Item = Book | Shoes | ...

p :: IO (Map Item Int)
p = do cart <- newIORef empty</pre>
```

```
data Item = Book | Shoes | ...

p :: IO (Map Item Int)
p = do cart <- newIORef empty</pre>
```





```
data Item = Book | Shoes | ...

p :: IO (Map Item Int)
p = do cart <- newIORef empty</pre>
```



















bash

```
landin:lvar-examples lkuper$ make map-ioref-data-race
 ghc -02 map-ioref-data-race.hs -rtsopts -threaded
 [1 of 1] Compiling Main
                                                                                                                                                                                                 ( map-ioref-data-race.hs, map-ioref-data-race.o )
Linking map-ioref-data-race ...
while true; do ./map-ioref-data-race +RTS -N2; done
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```
data Item = Book | Shoes | ...
p :: IO (Map Item Int)
p = do cart <- newIORef empty</pre>
       a1 <- async (atomicModifyIORef cart</pre>
                (\m -> (insert Book 1 m, ())))
       a2 <- async (atomicModifyIORef cart</pre>
                (\m -> (insert Shoes 1 m, ())))
       res <- async (do waitBoth a1 a2
                          readIORef cart)
       wait res
```

deterministic

deterministic...now

deterministic...now...we hope

```
deterministic...now...we hope
```

```
p :: Par Det (IMap Item Int)
p = do
    cart <- newEmptyMap
    fork (insert Book 1 cart)
    fork (insert Shoes 1 cart)
    return cart

main = print (runParThenFreeze p)</pre>
```

deterministic by construction [FHPC '13, POPL '14]







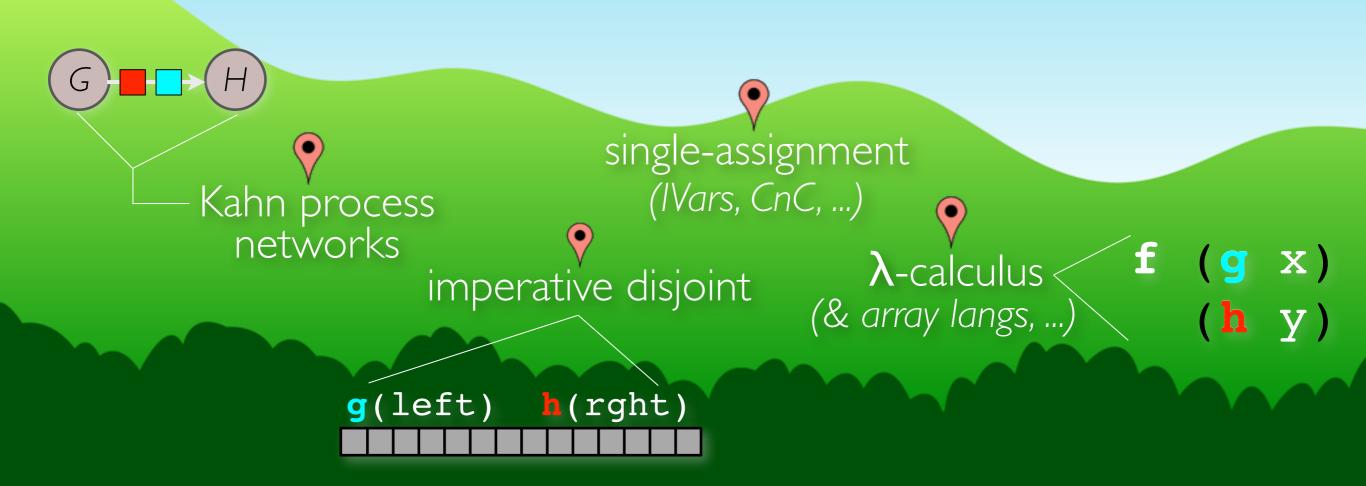


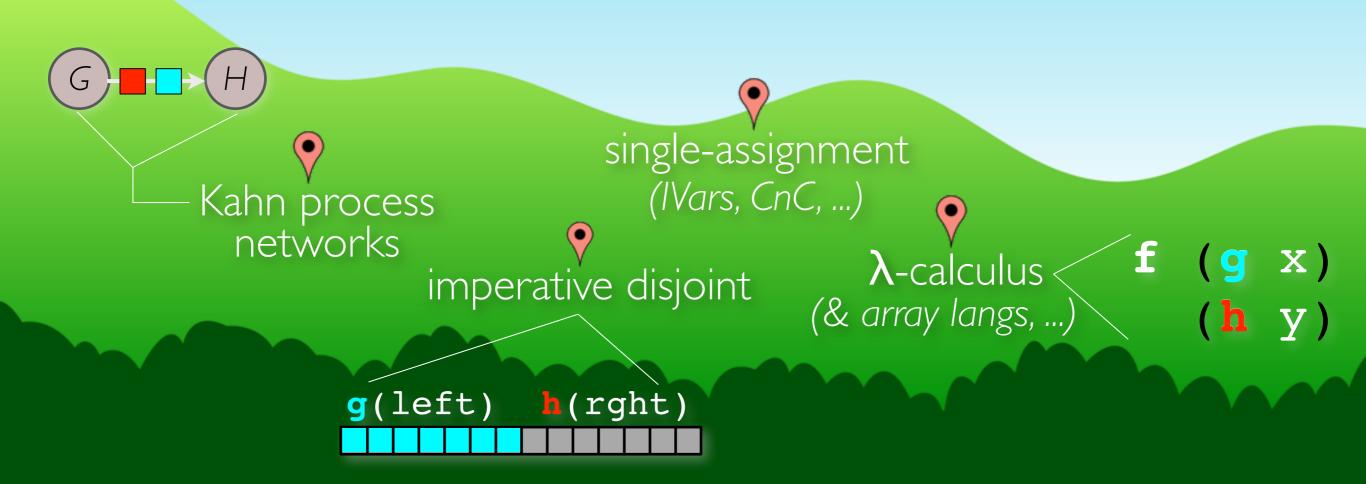


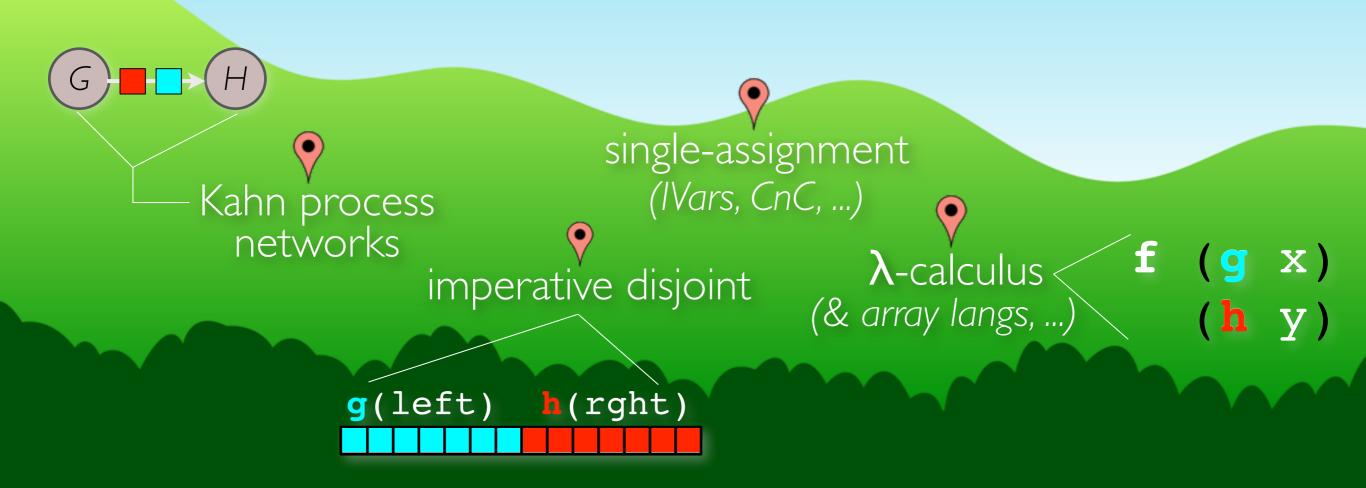


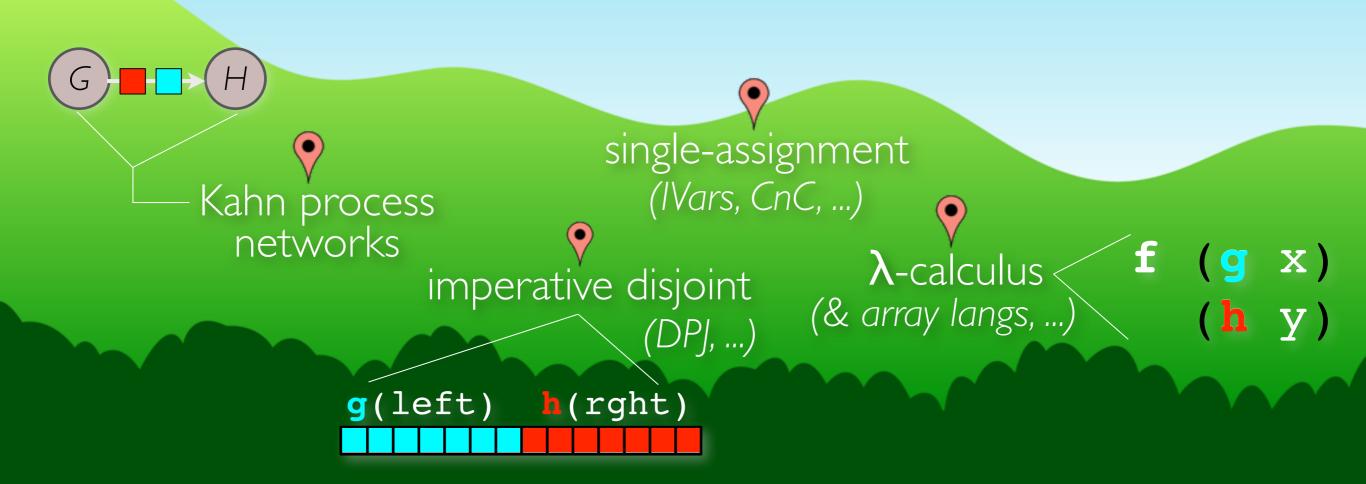


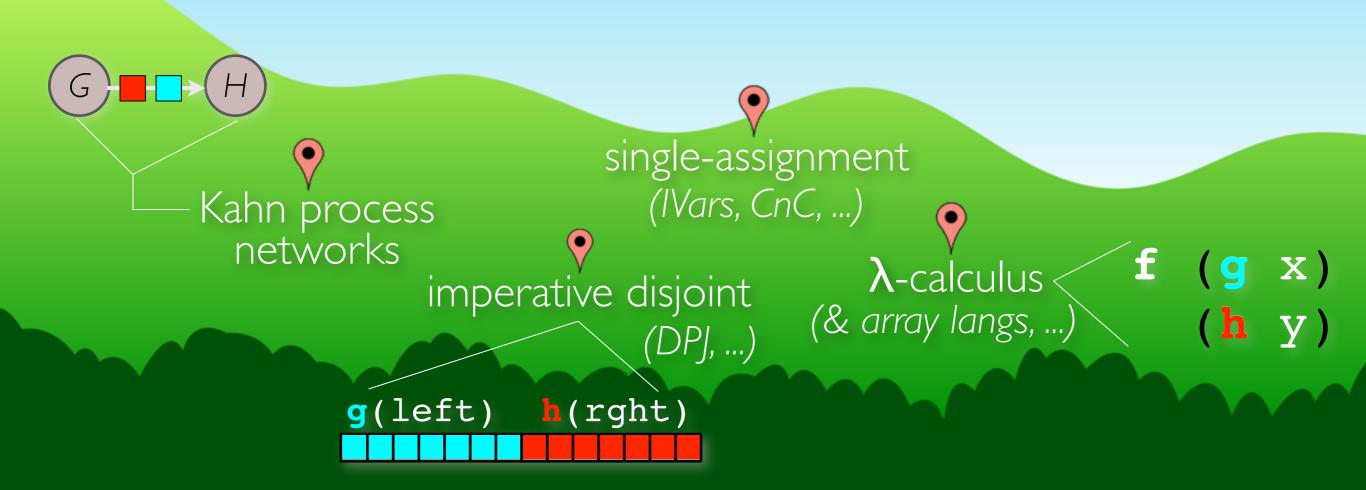




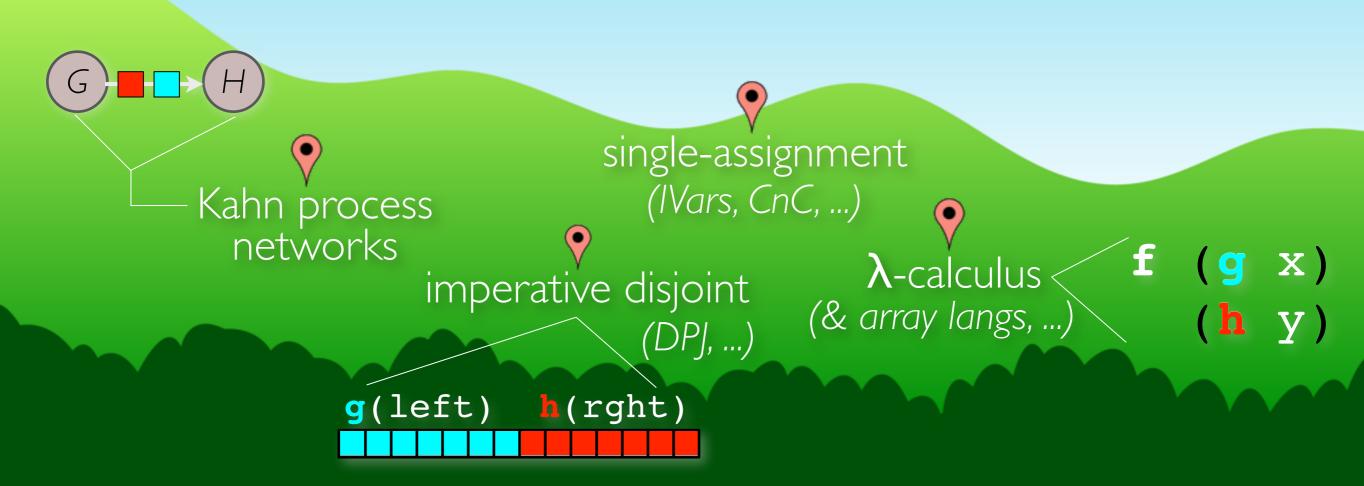








Can we generalize and unify these points on the map?



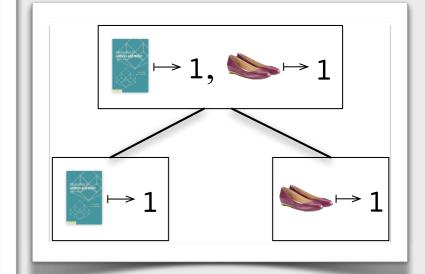
Can we generalize and unify these points on the map? Yes!





IVars: single writes, blocking (but exact) reads [Arvind et al., 1989]





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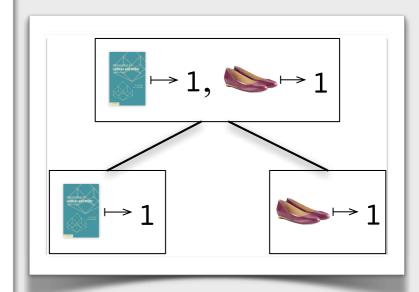


 \mapsto 1, \mapsto 1

IVars: single writes, blocking (but exact) reads [Arvind et al., 1989]

LVars: multiple least-upper-bound writes, blocking threshold reads
[FHPC '13]



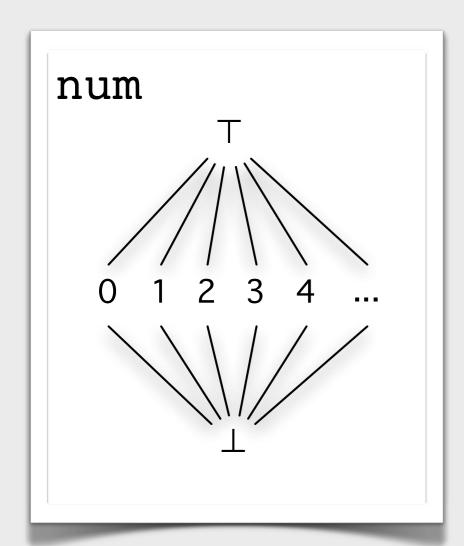


IVars: single writes, blocking (but exact) reads [Arvind et al., 1989]

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[FHPC'13]



^{*} actually a bounded join-semilattice



```
Raises an error, since 3 \sqcup 4 = T

do

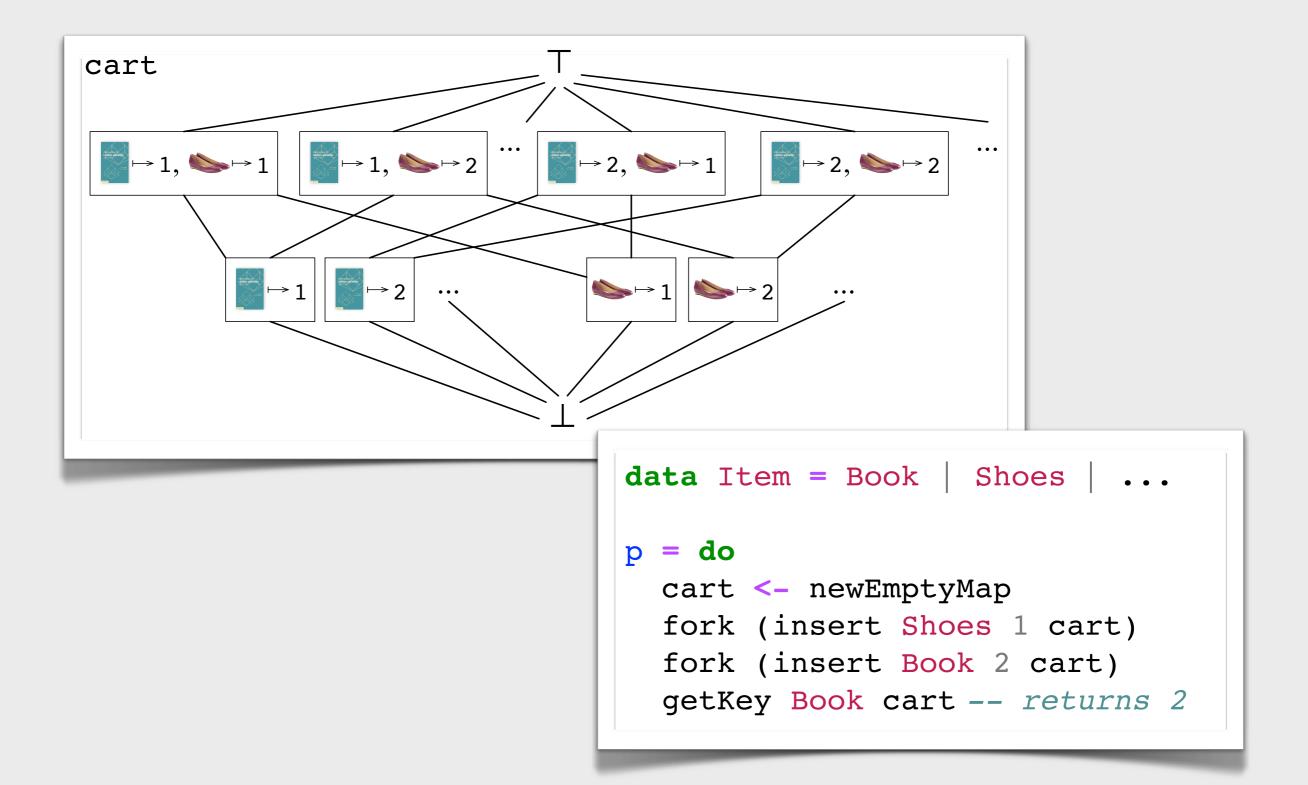
fork (put num 3)
fork (put num 4)
```

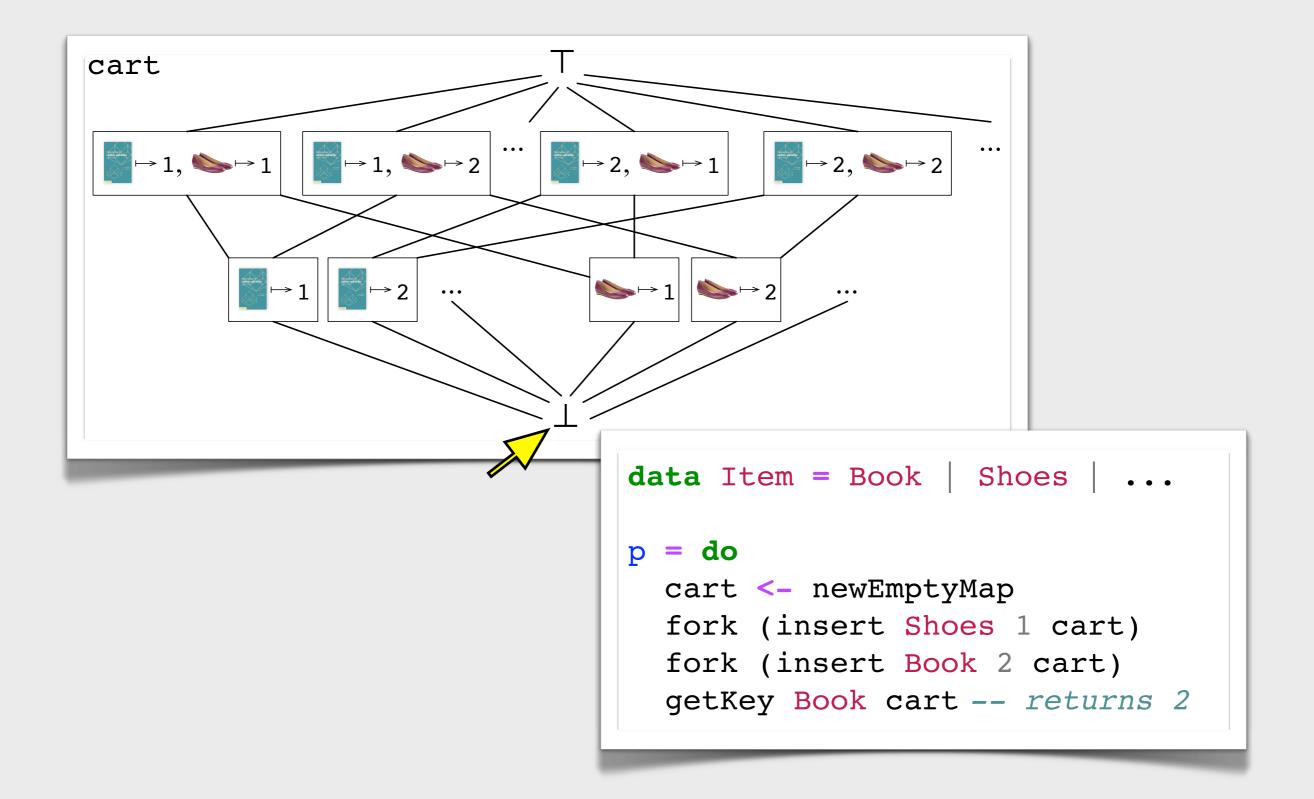
```
Works fine, since 4 \sqcup 4 = 4

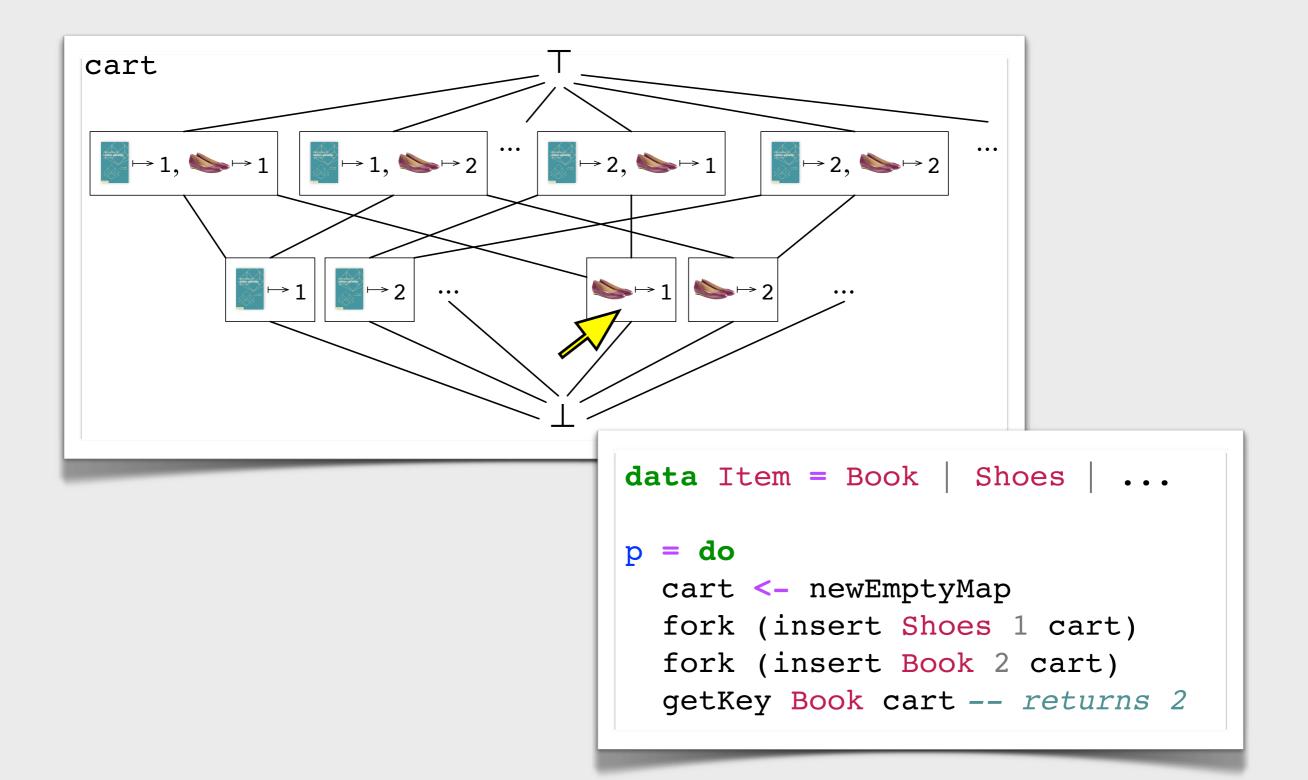
do

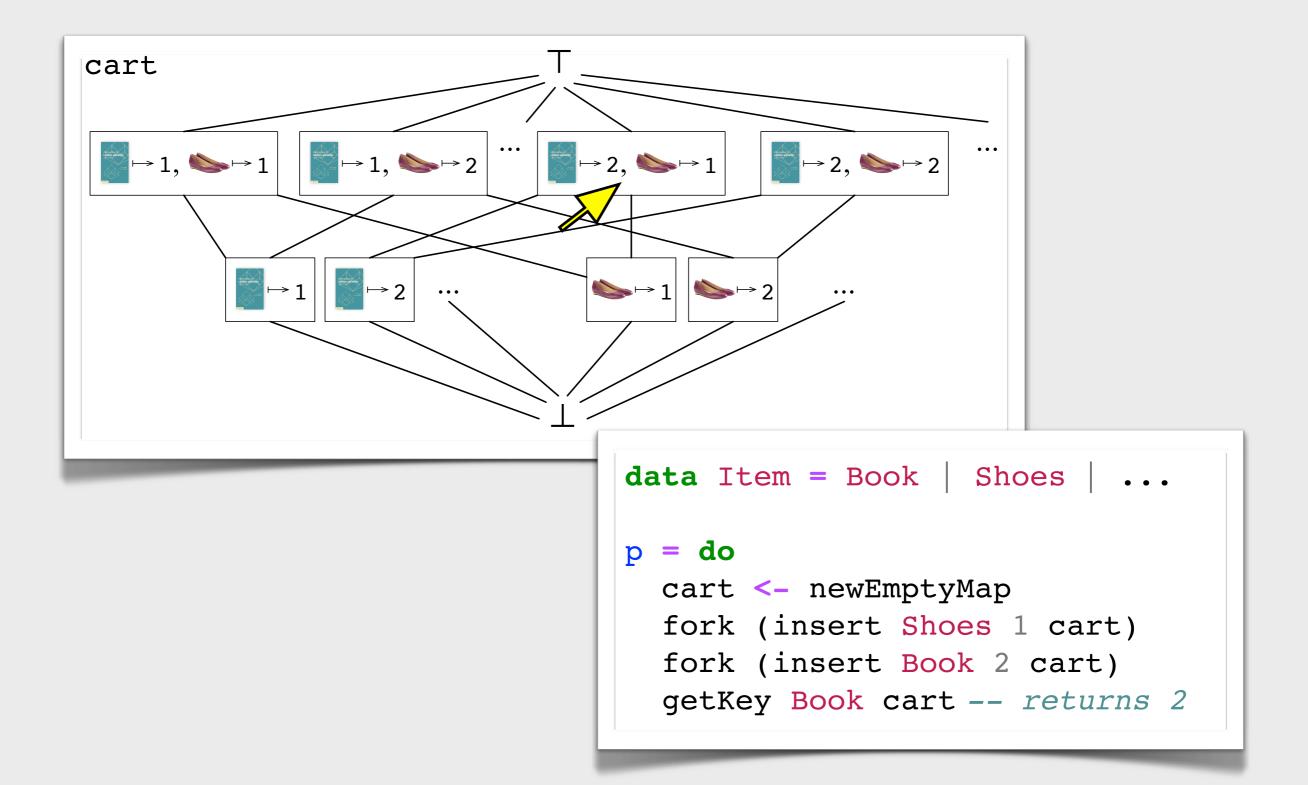
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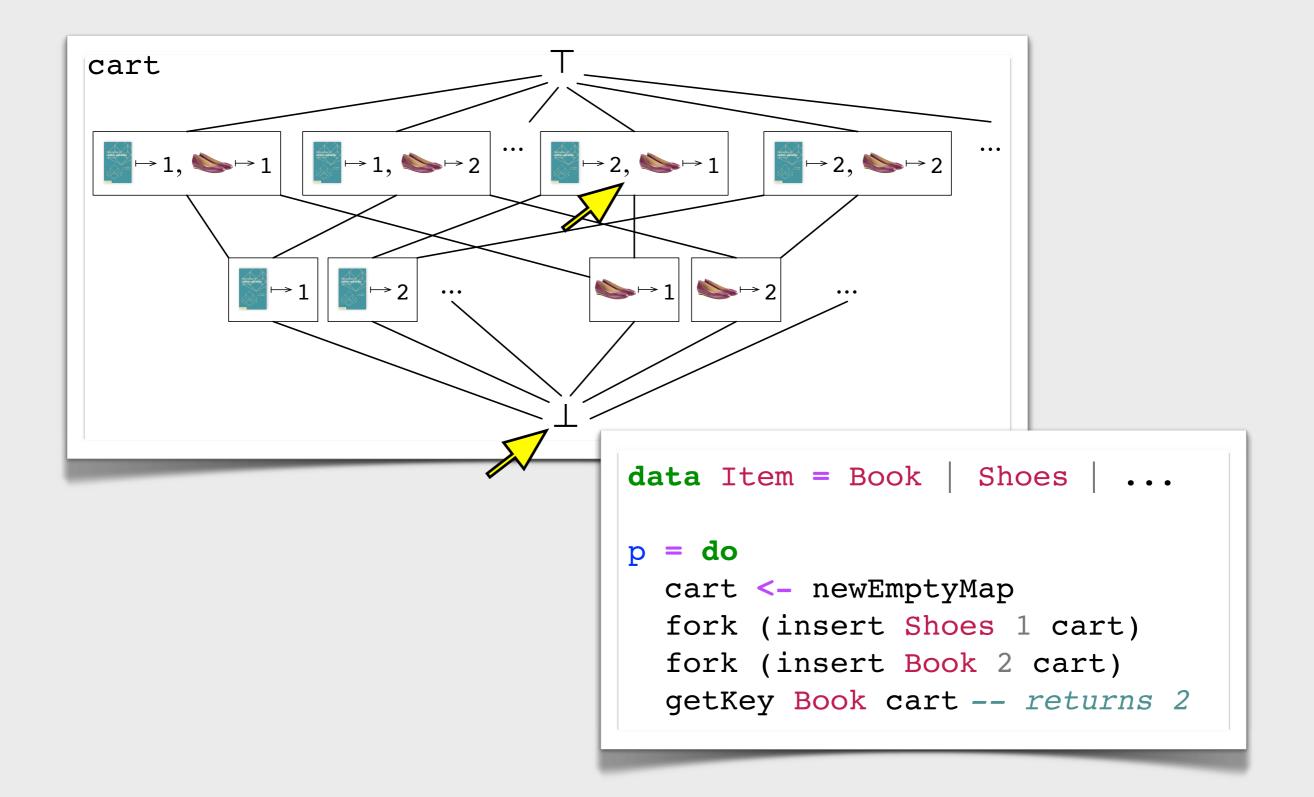
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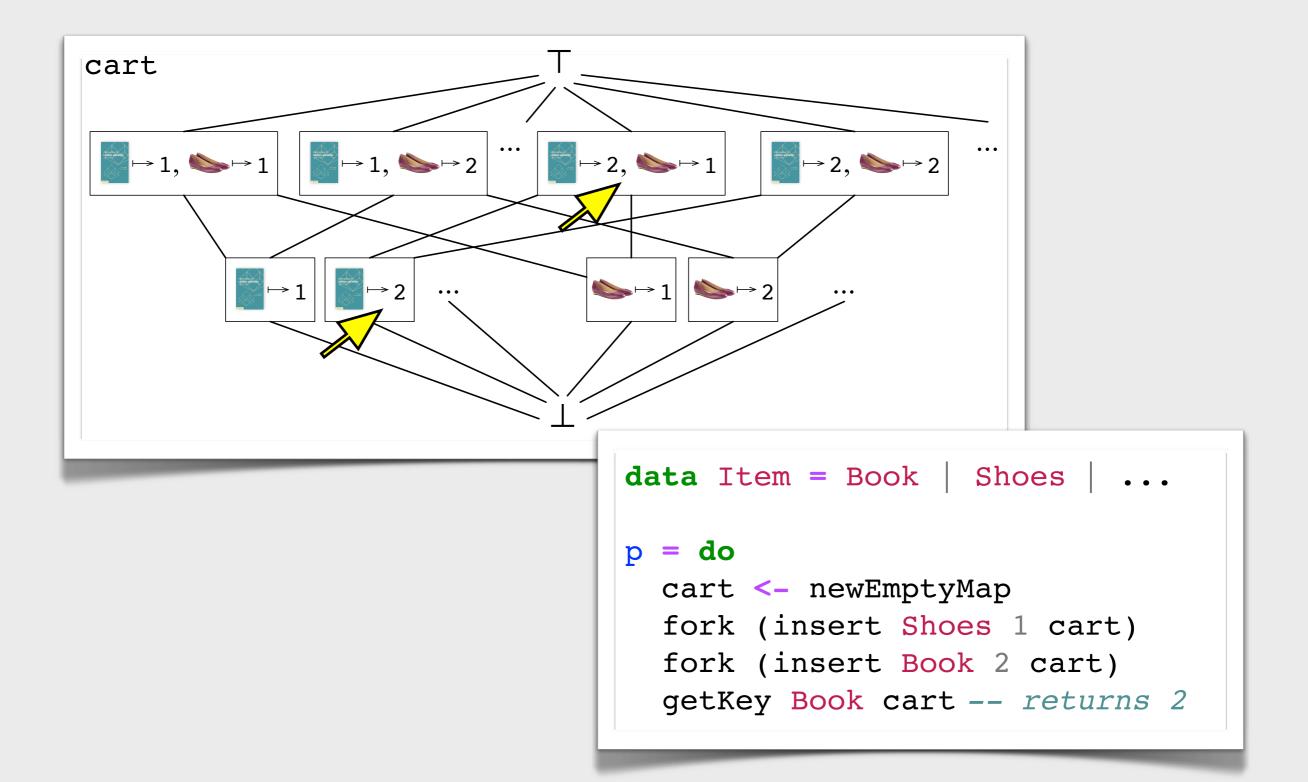


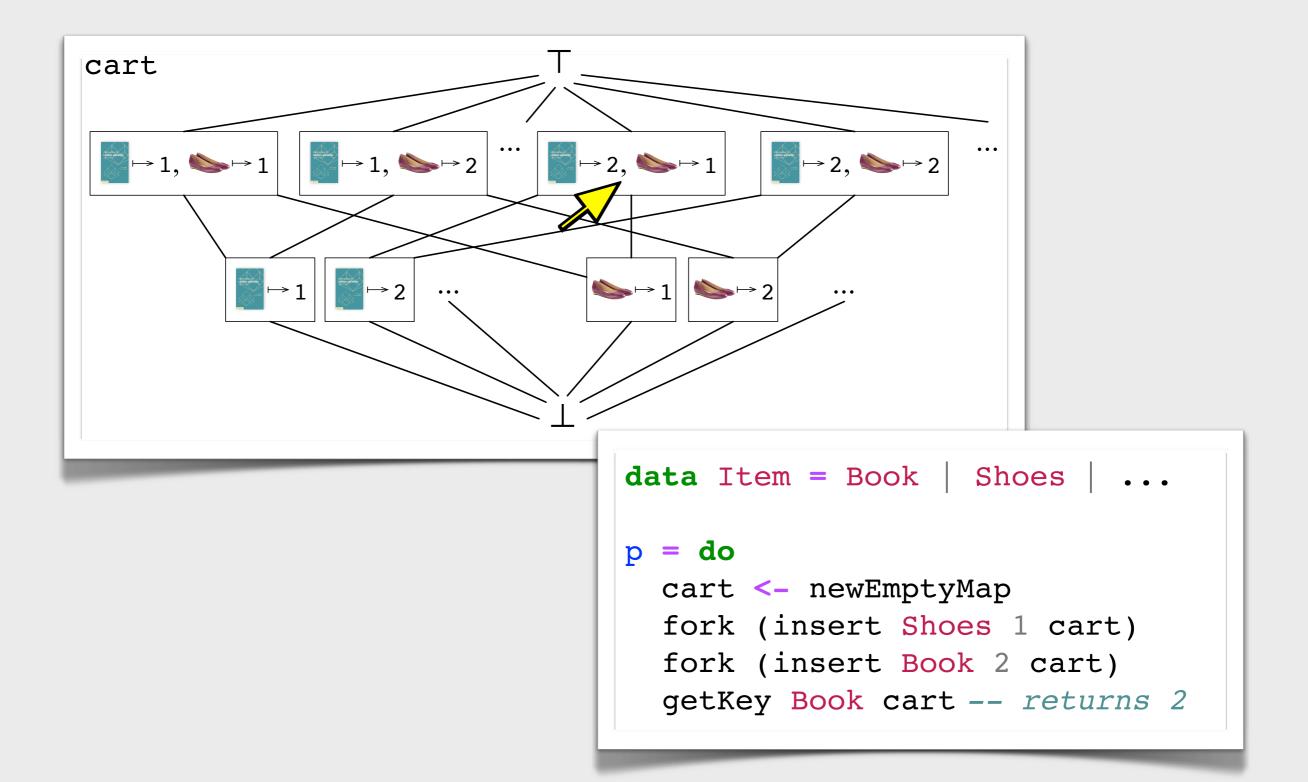


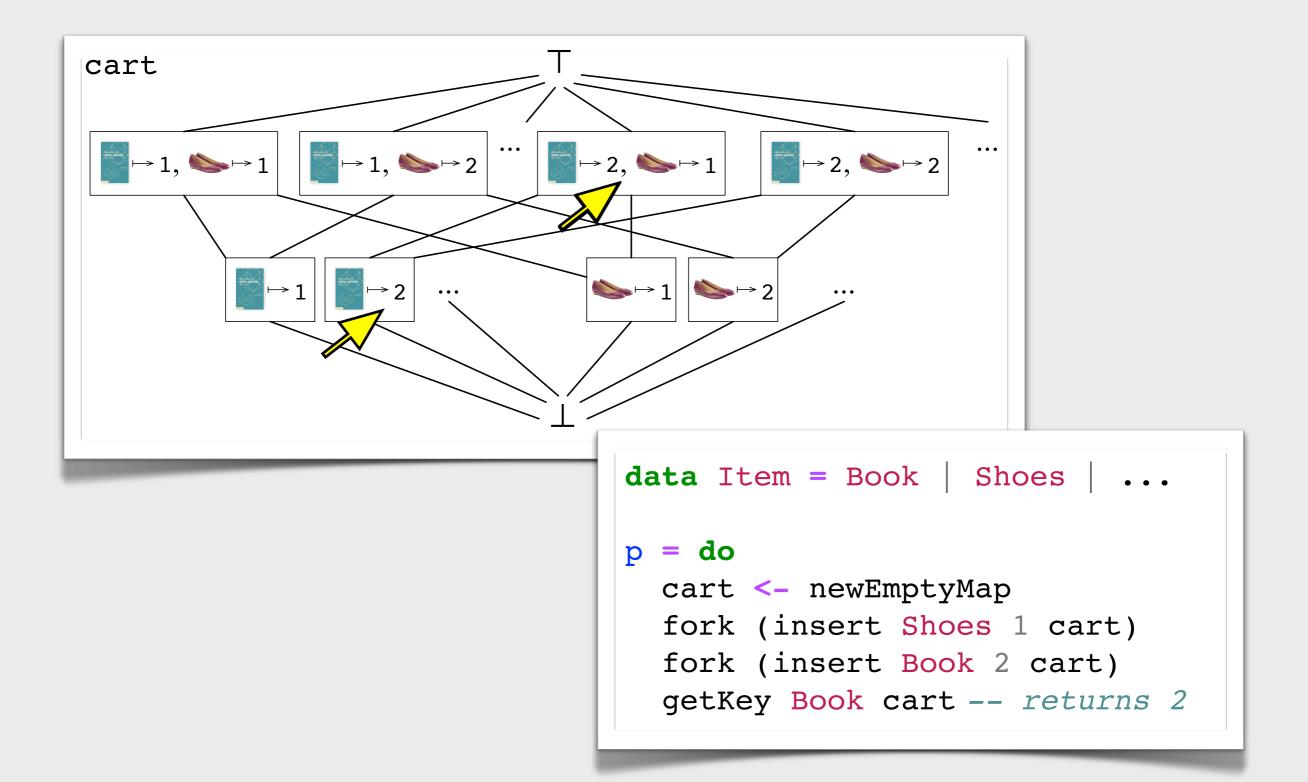


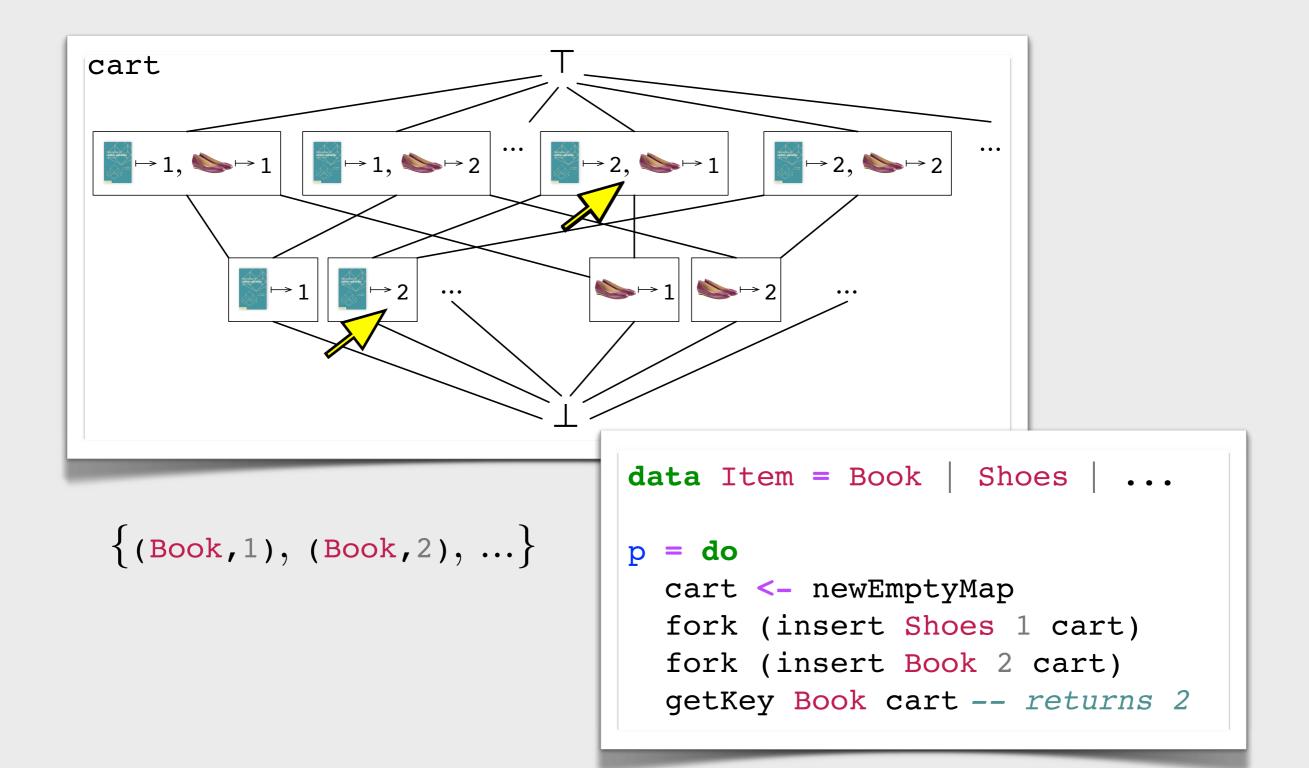


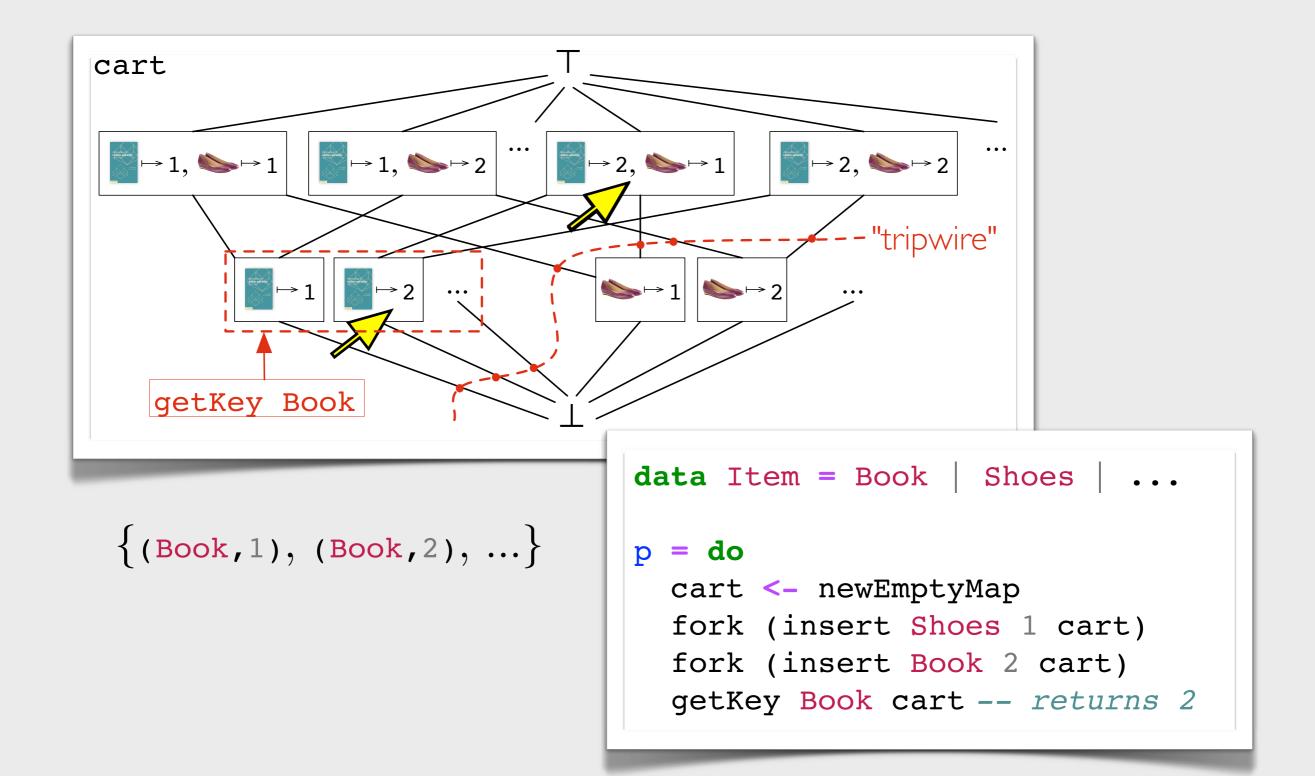


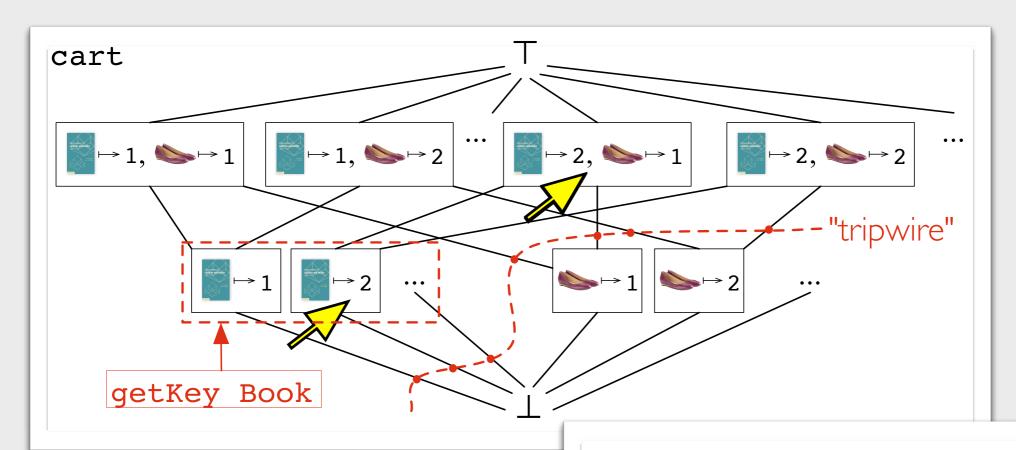










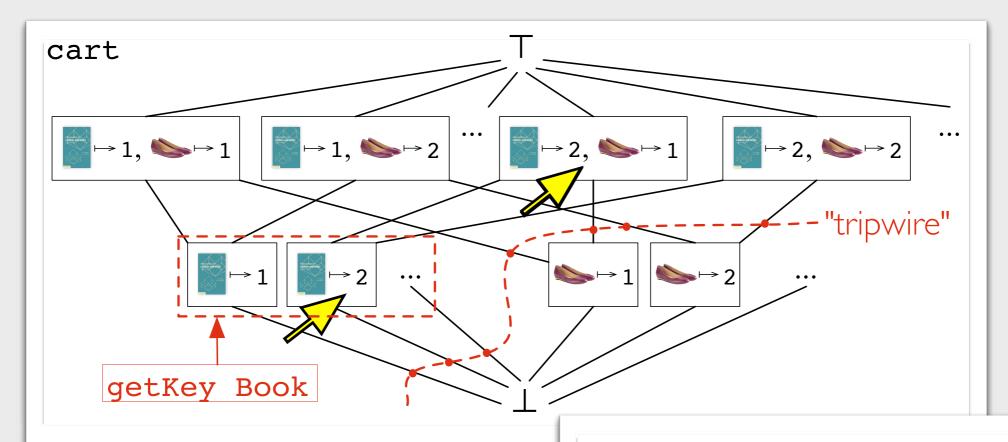


```
\{(Book, 1), (Book, 2), ...\}
```

The threshold set must be pairwise incompatible

```
data Item = Book | Shoes | ...

p = do
    cart <- newEmptyMap
    fork (insert Shoes 1 cart)
    fork (insert Book 2 cart)
    getKey Book cart -- returns 2</pre>
```



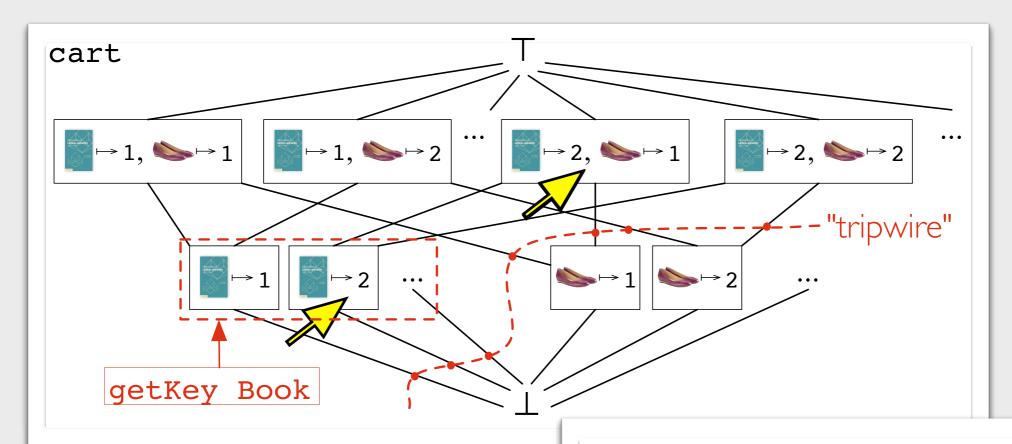
```
\{(Book, 1), (Book, 2), ...\}
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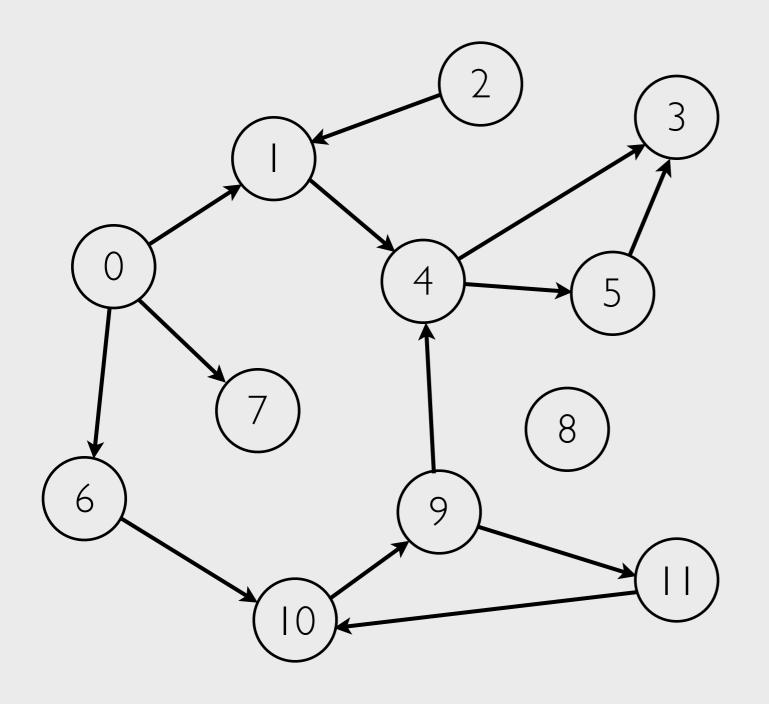
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proof obligation

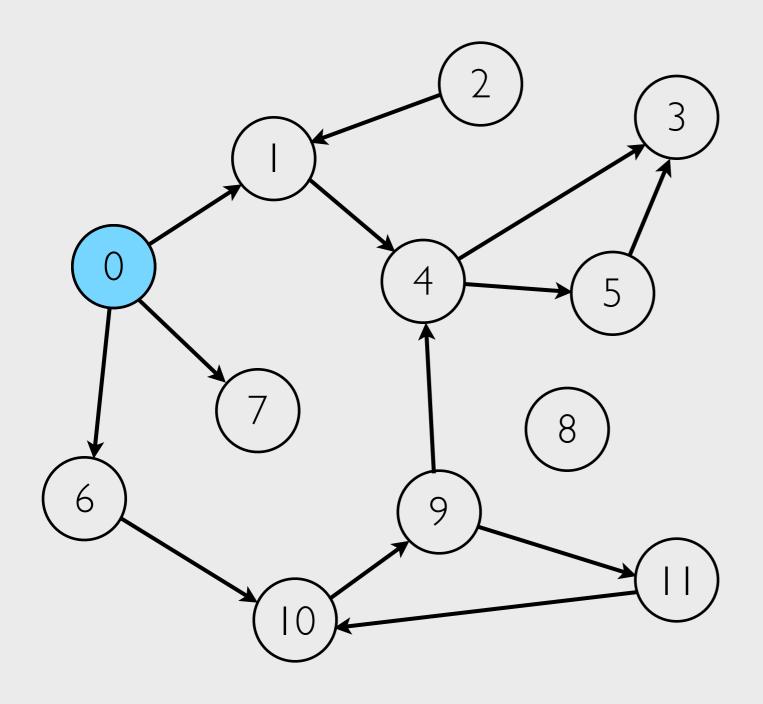
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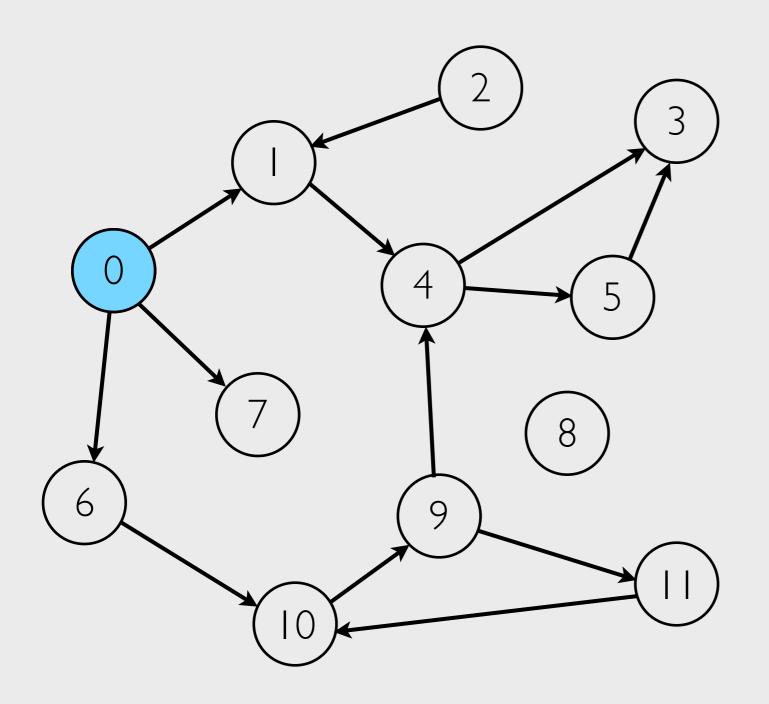
client guarantee

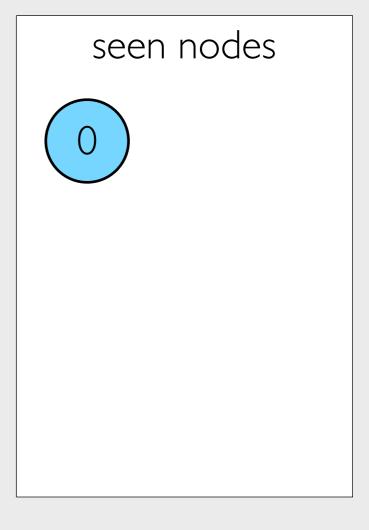


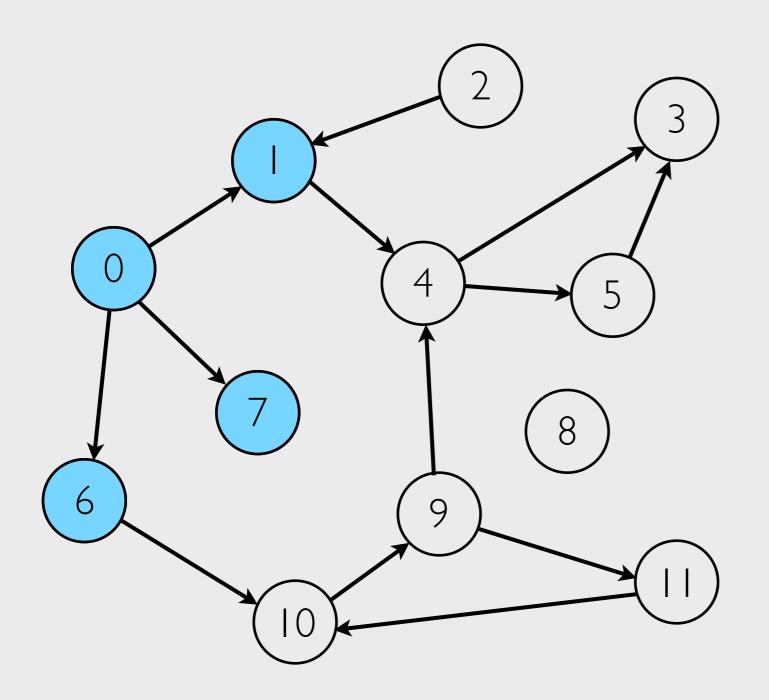
seen nodes

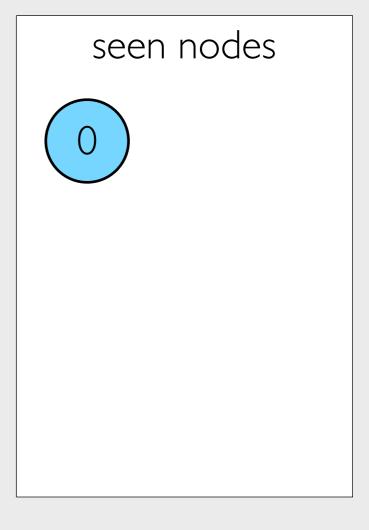


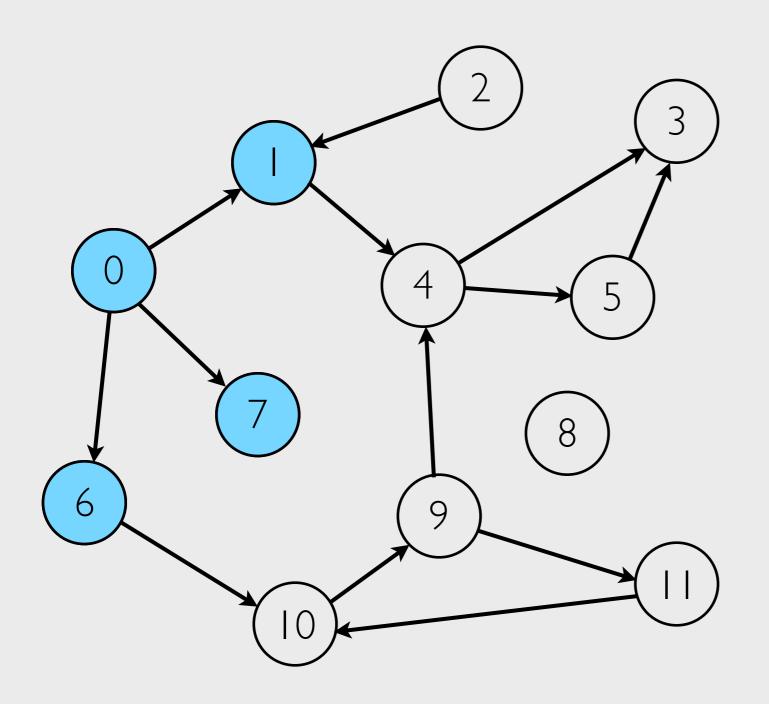
seen nodes

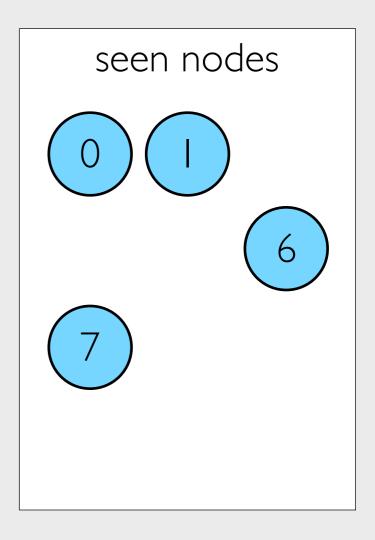


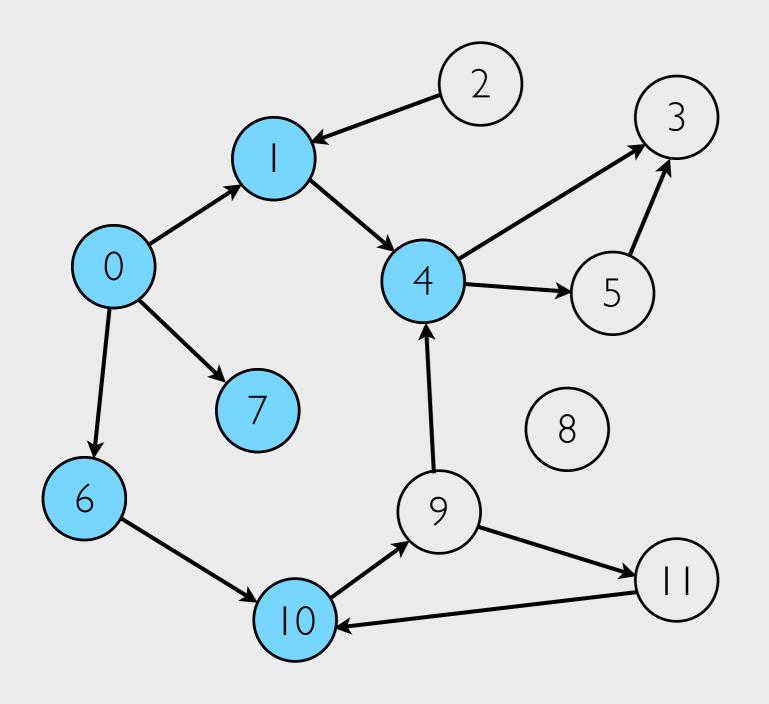


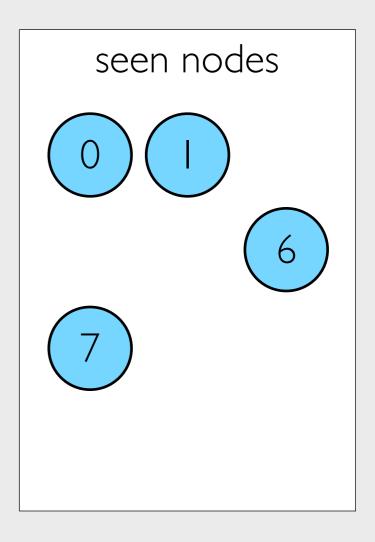


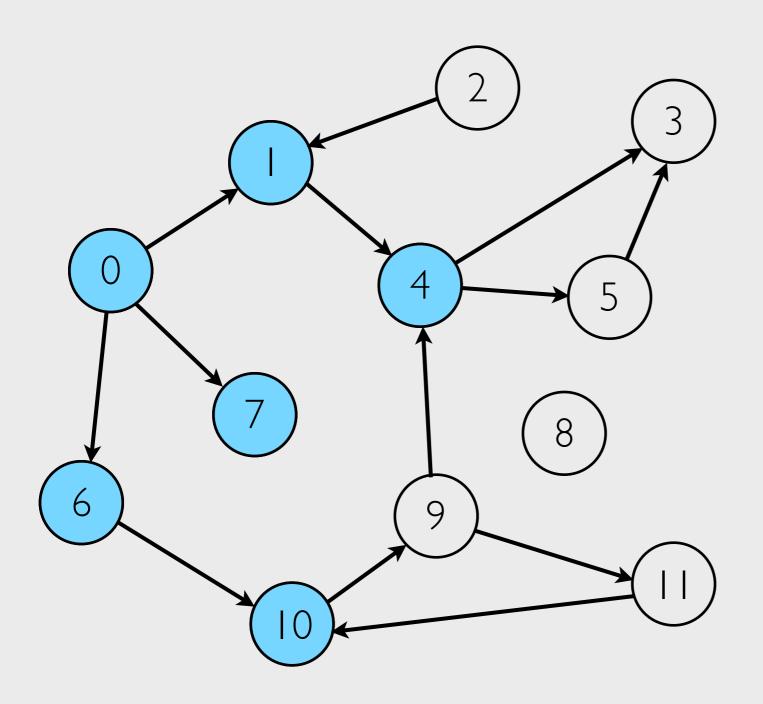


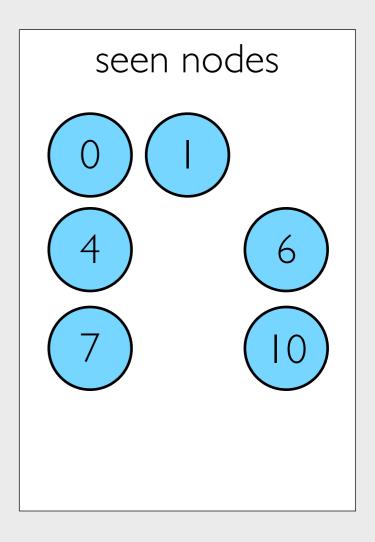


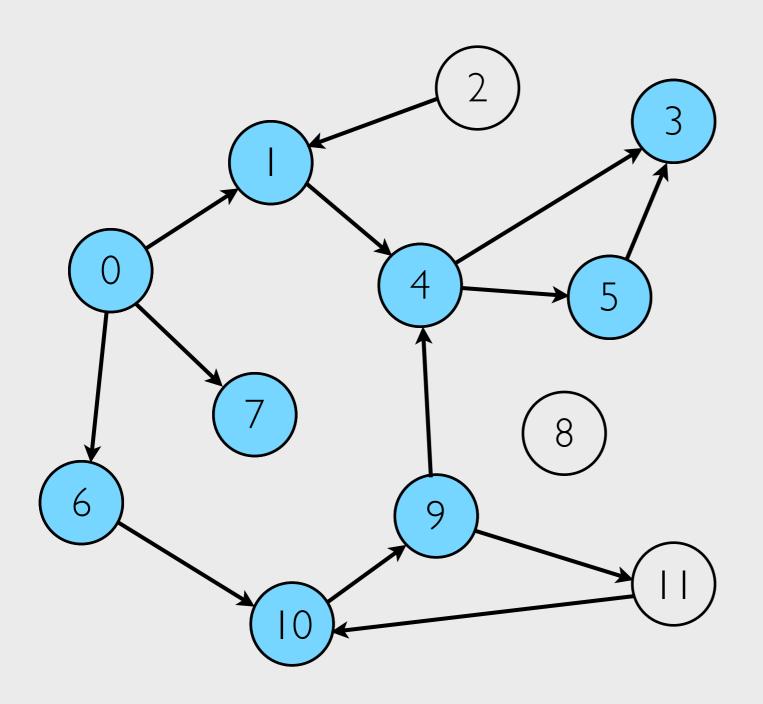


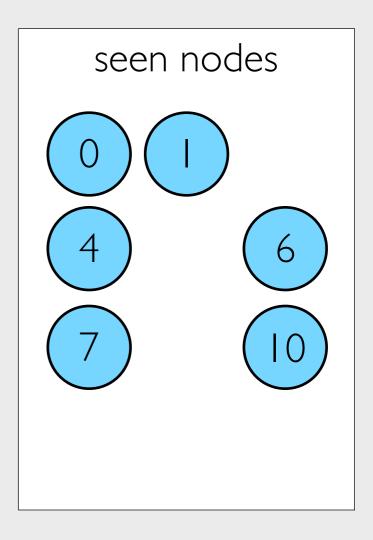


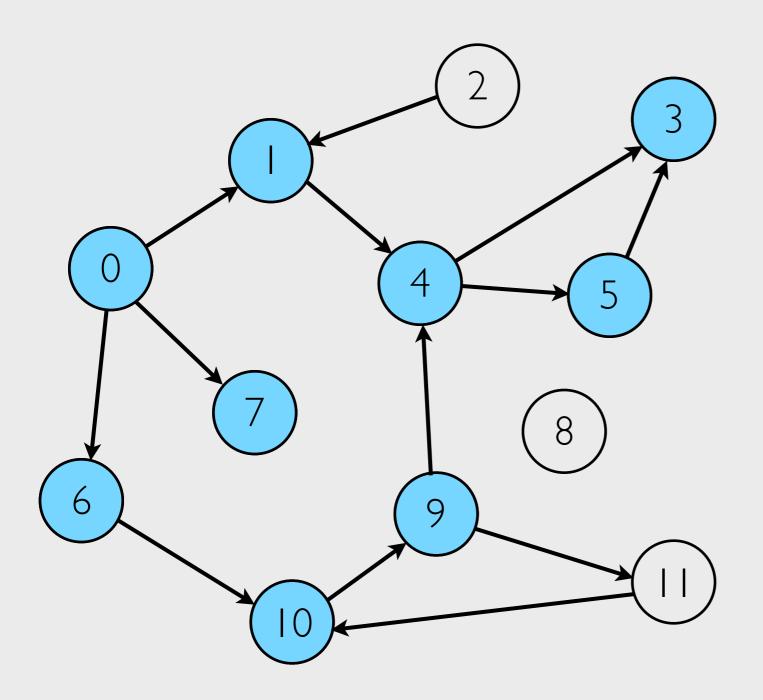


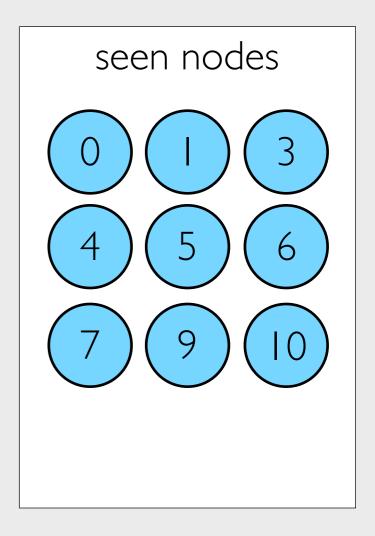


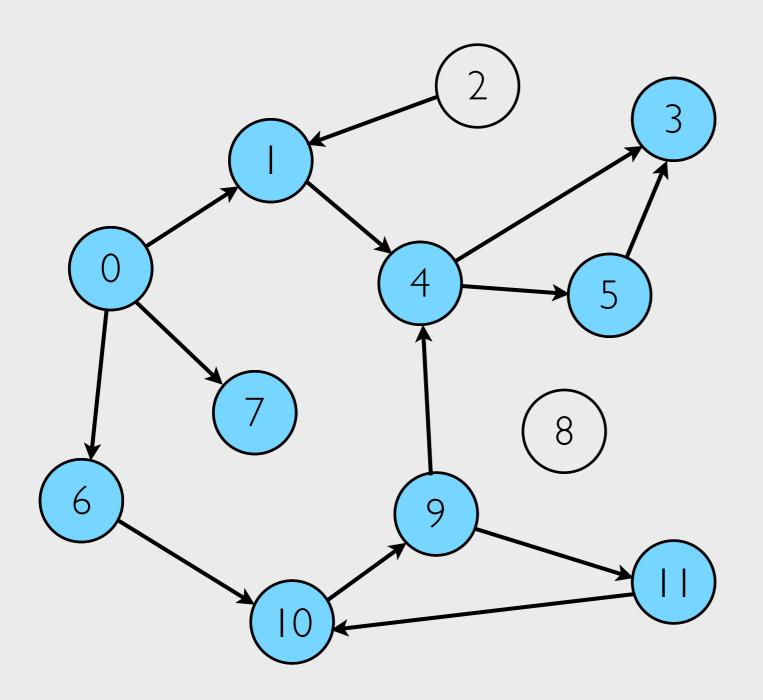


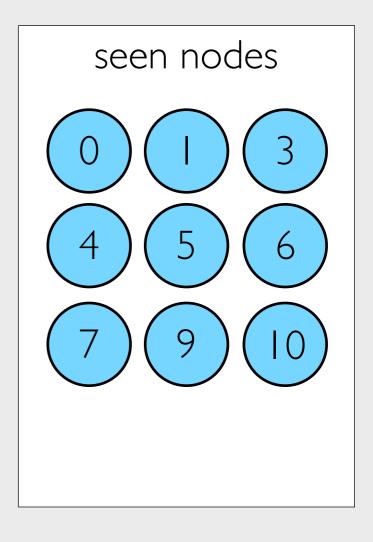


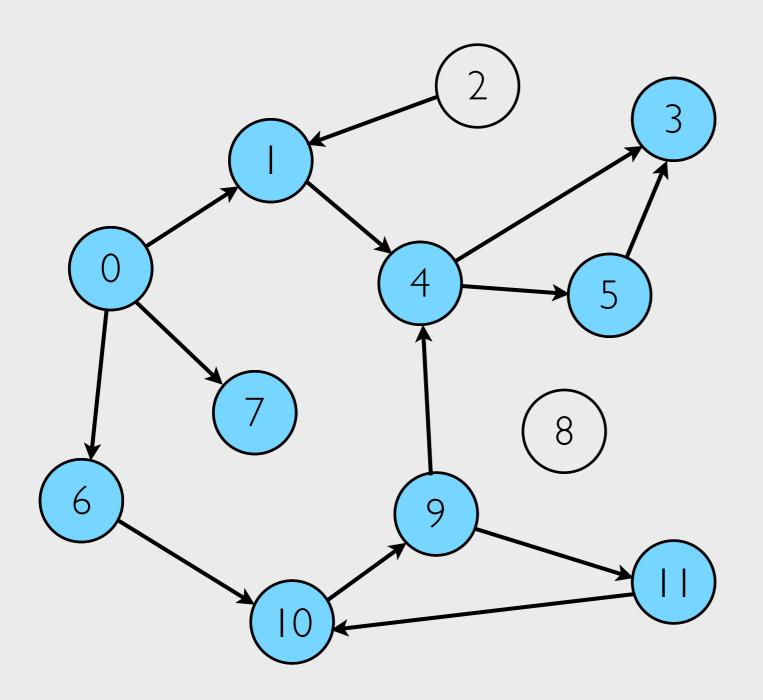


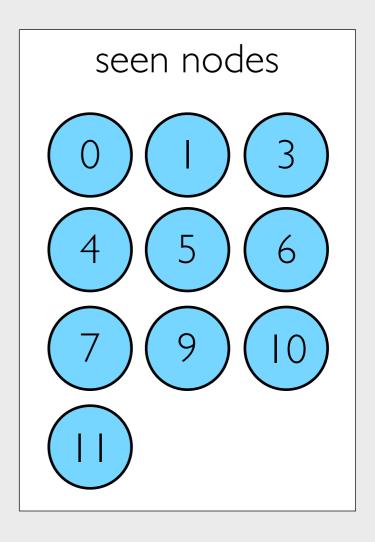


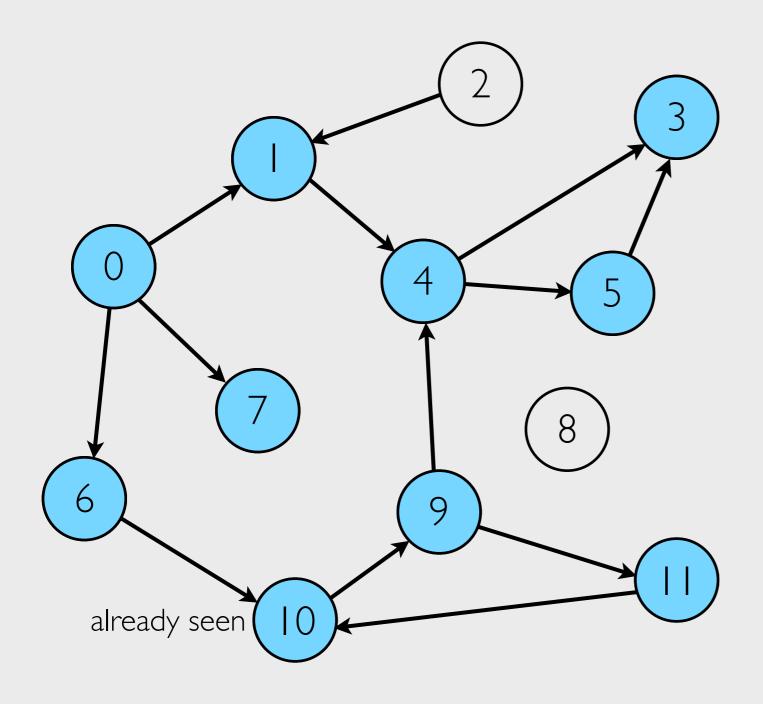


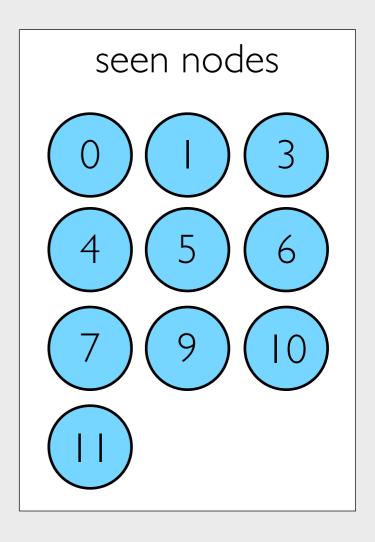


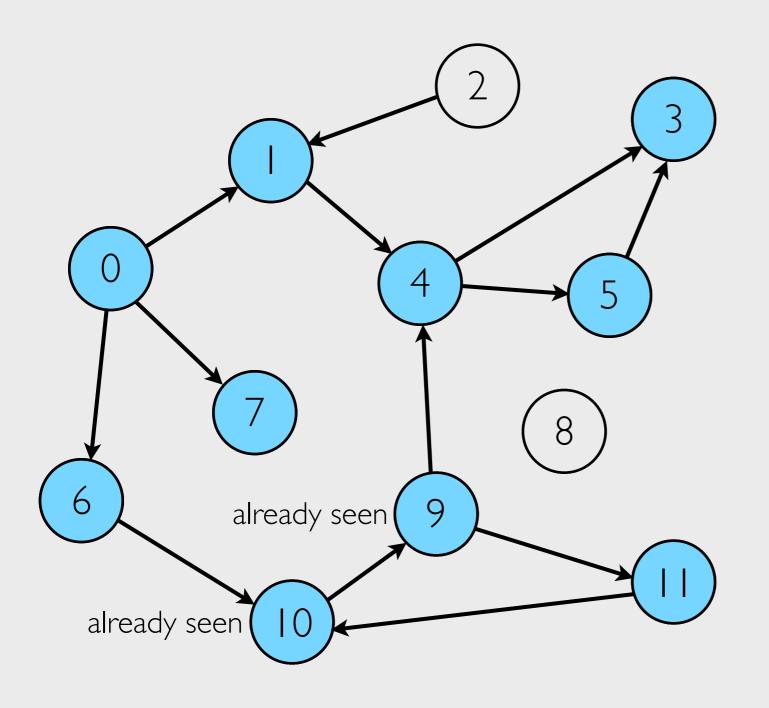


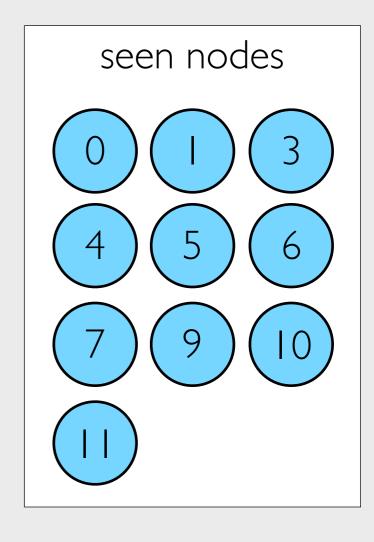


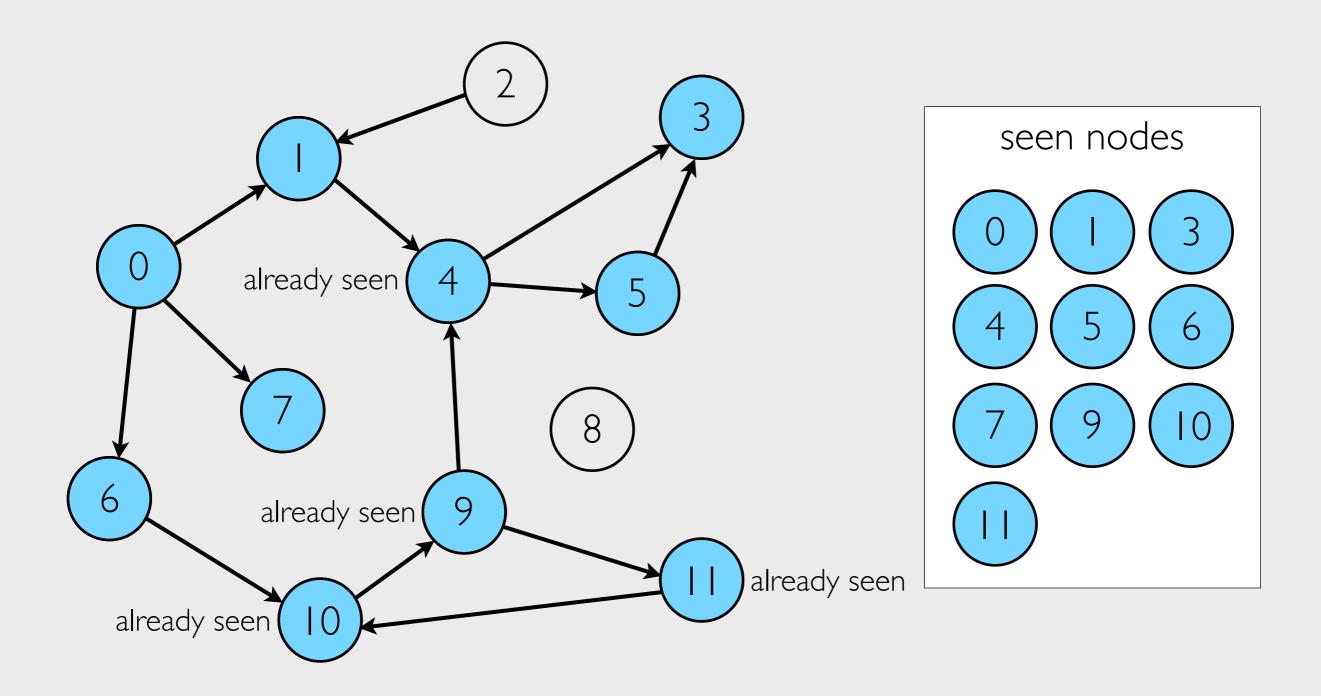


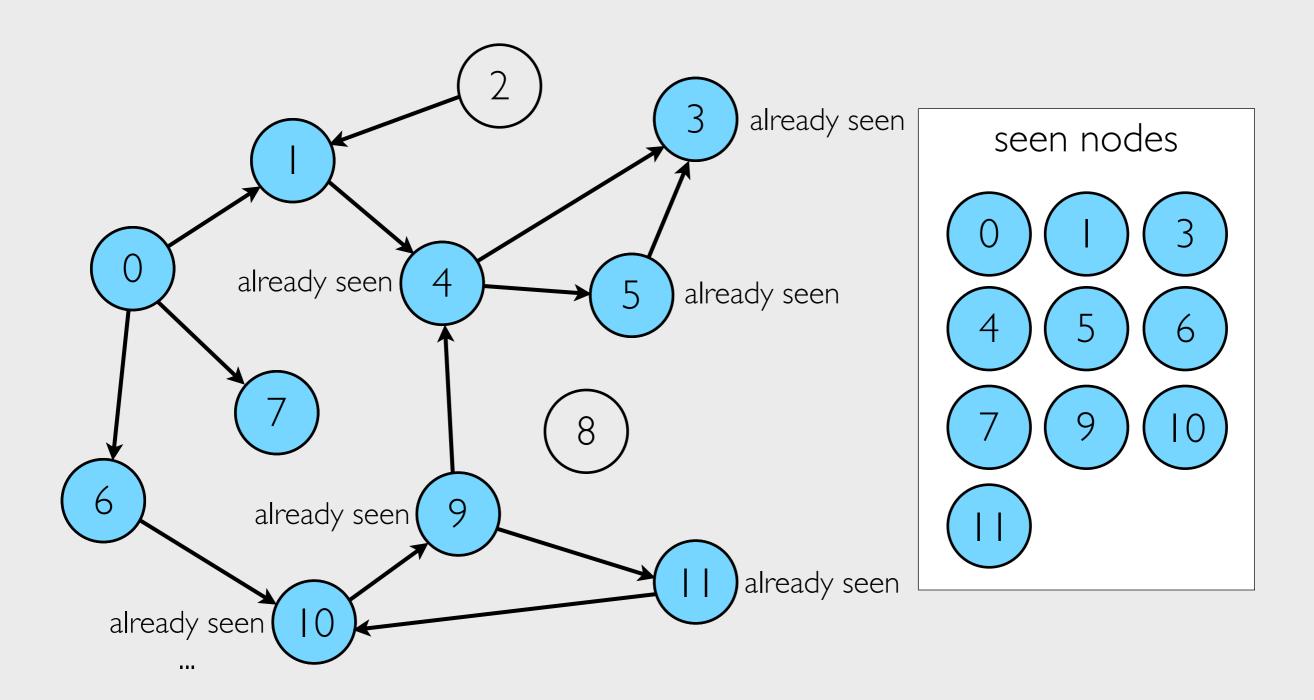


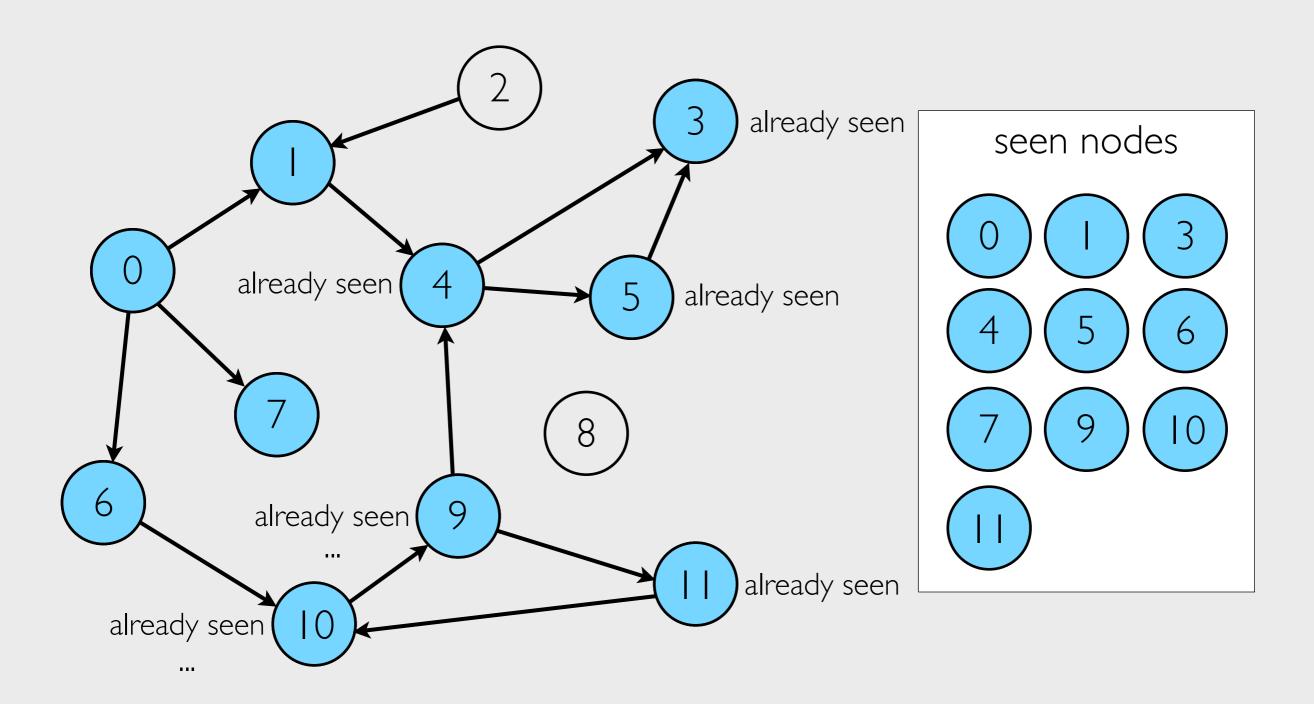


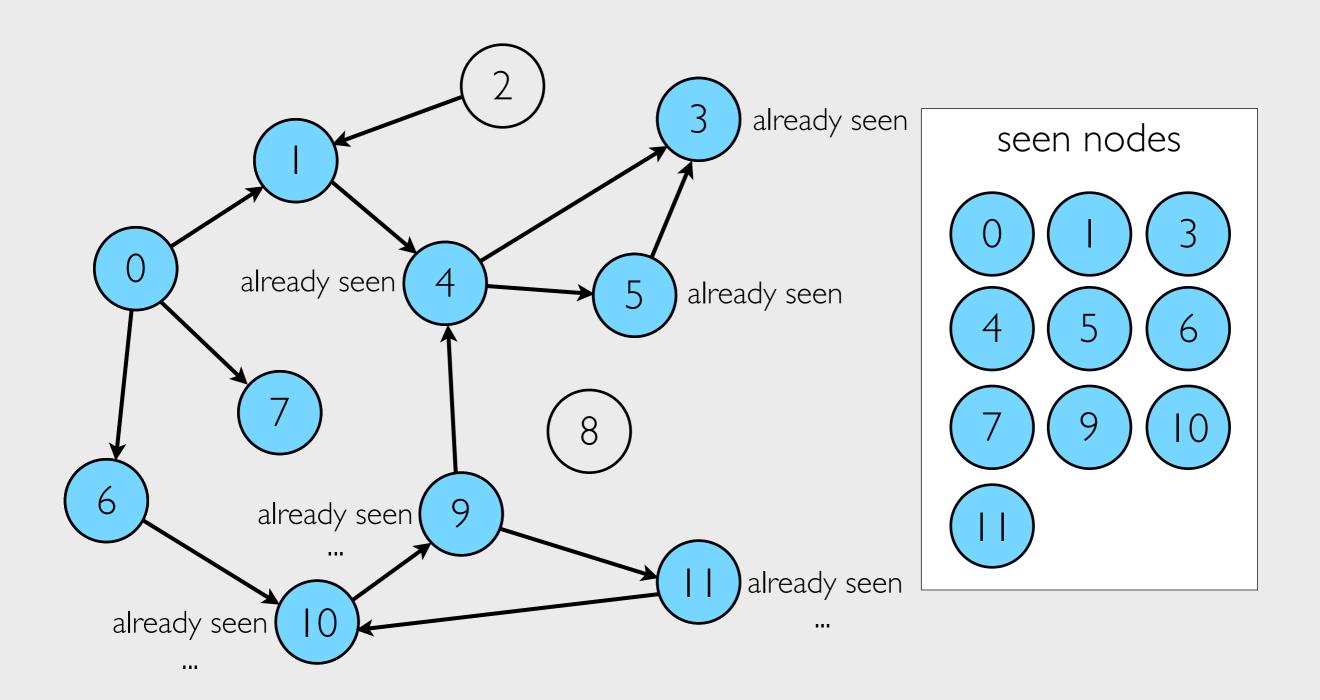




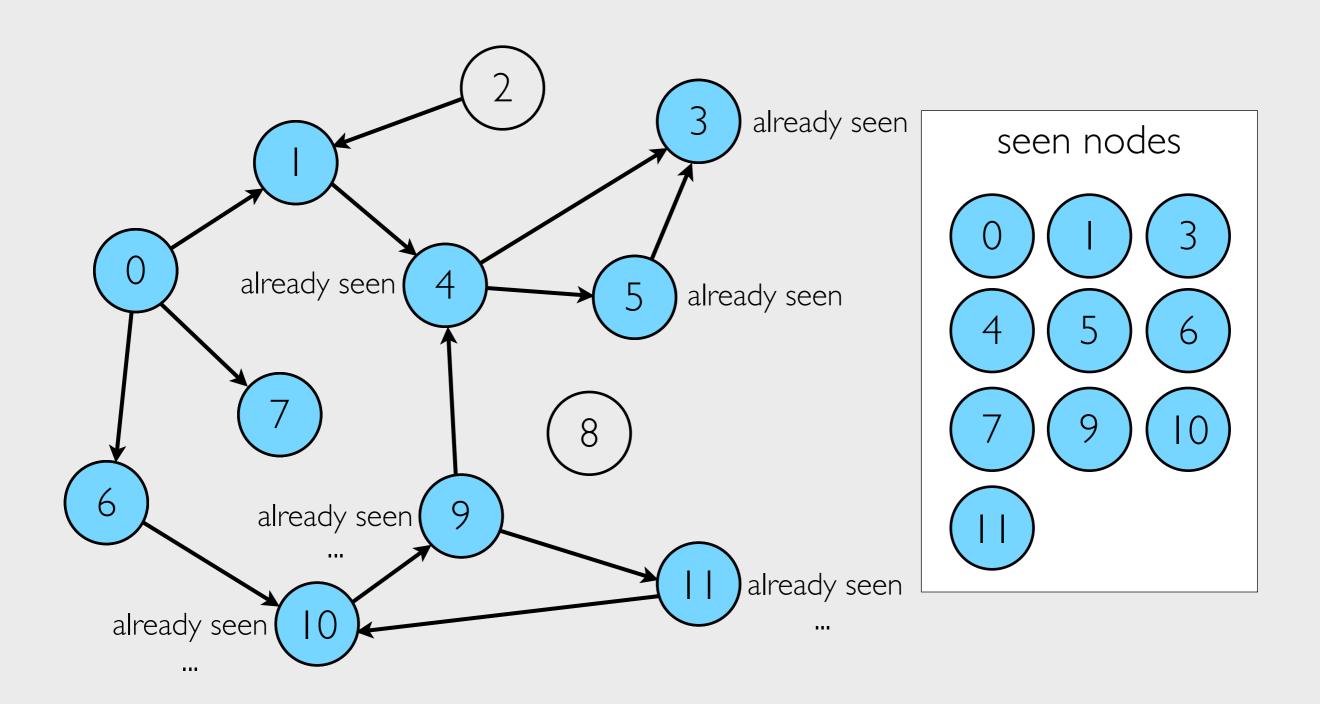


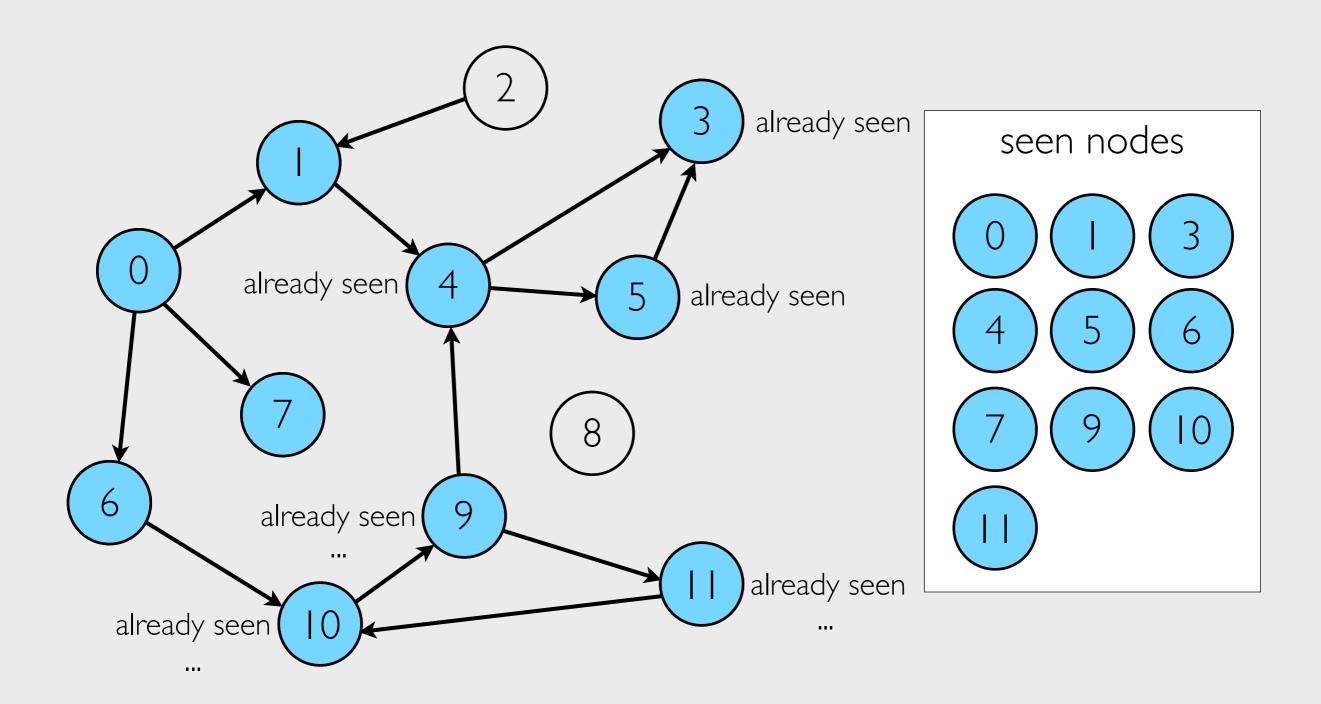


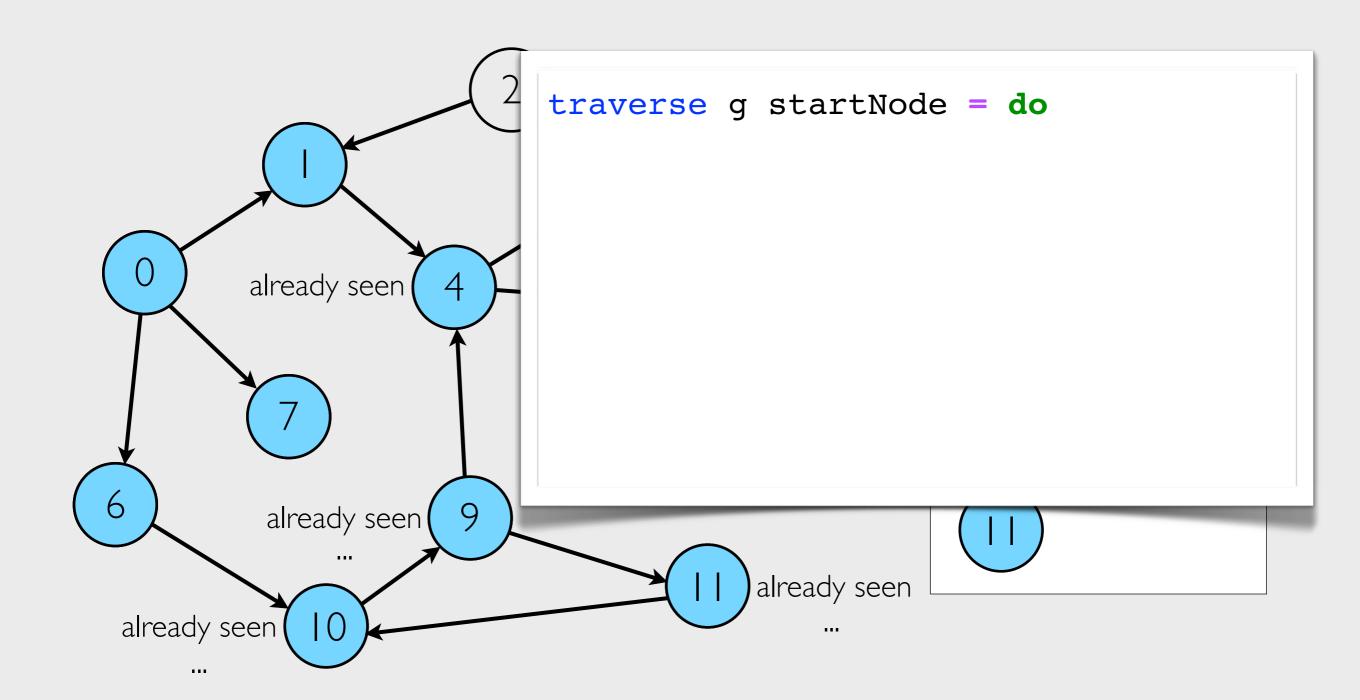


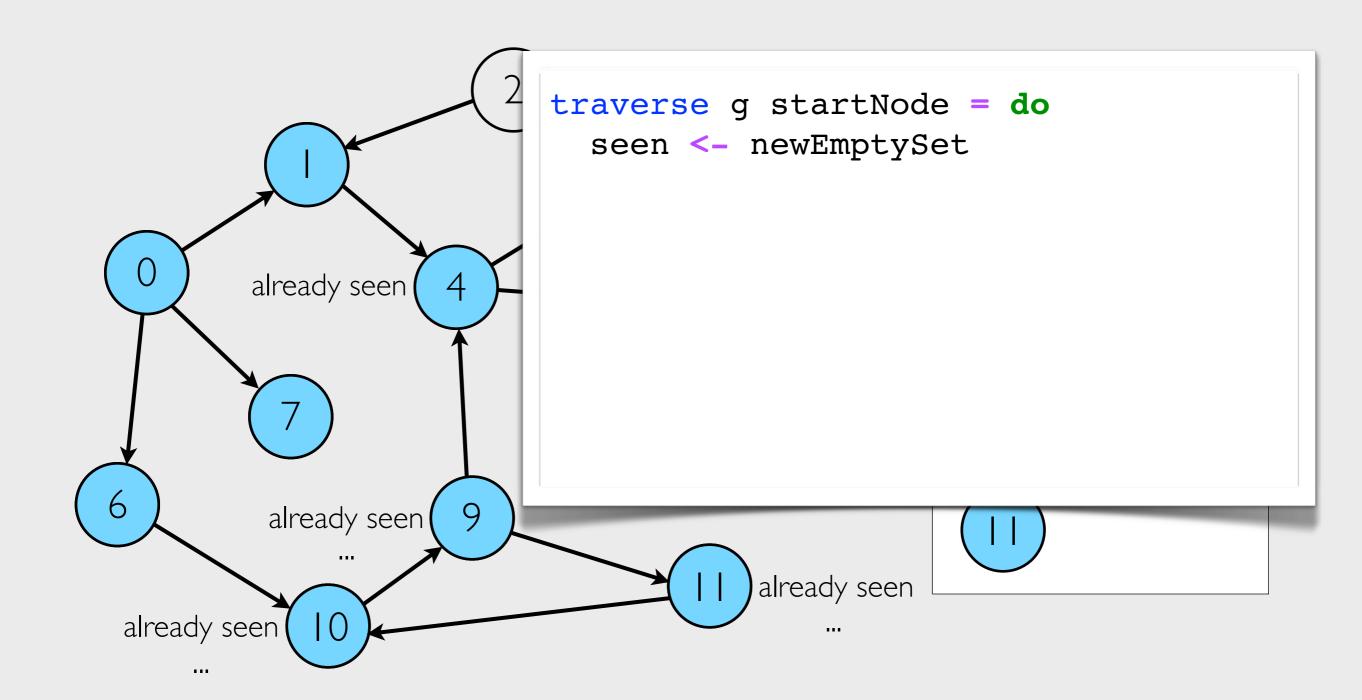


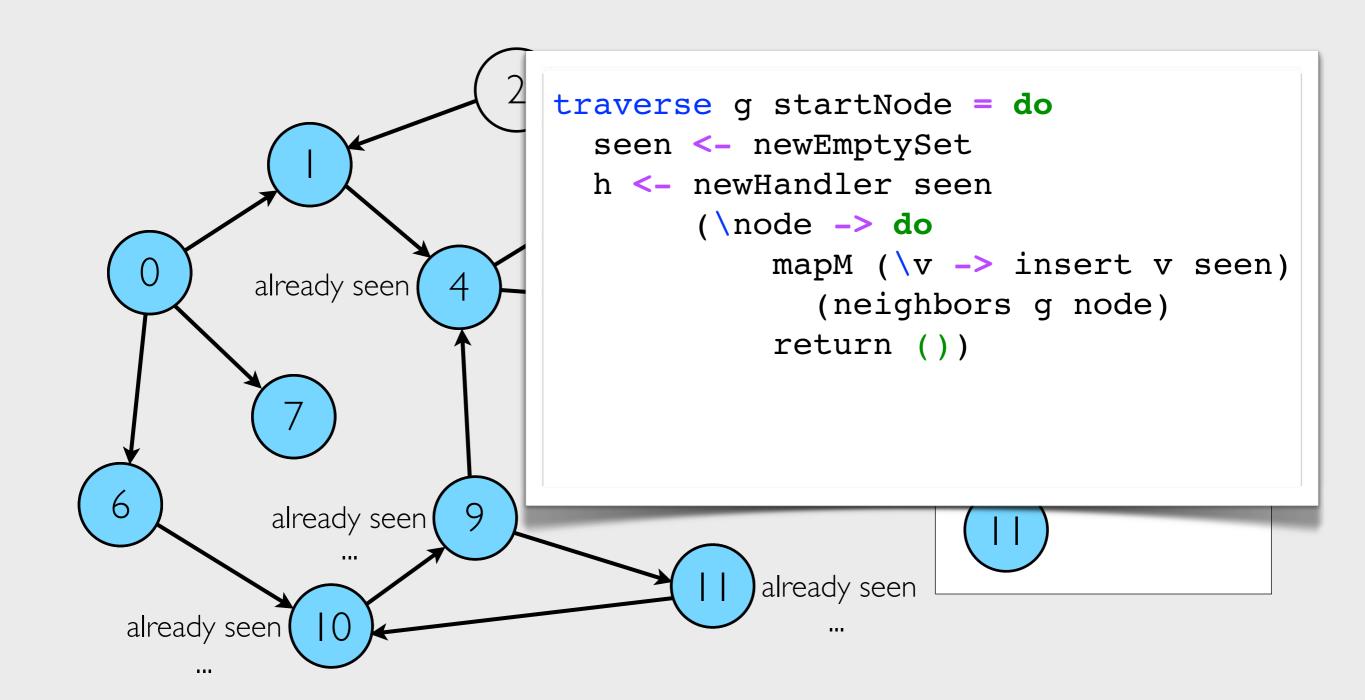
Events are updates that change an LVar's state

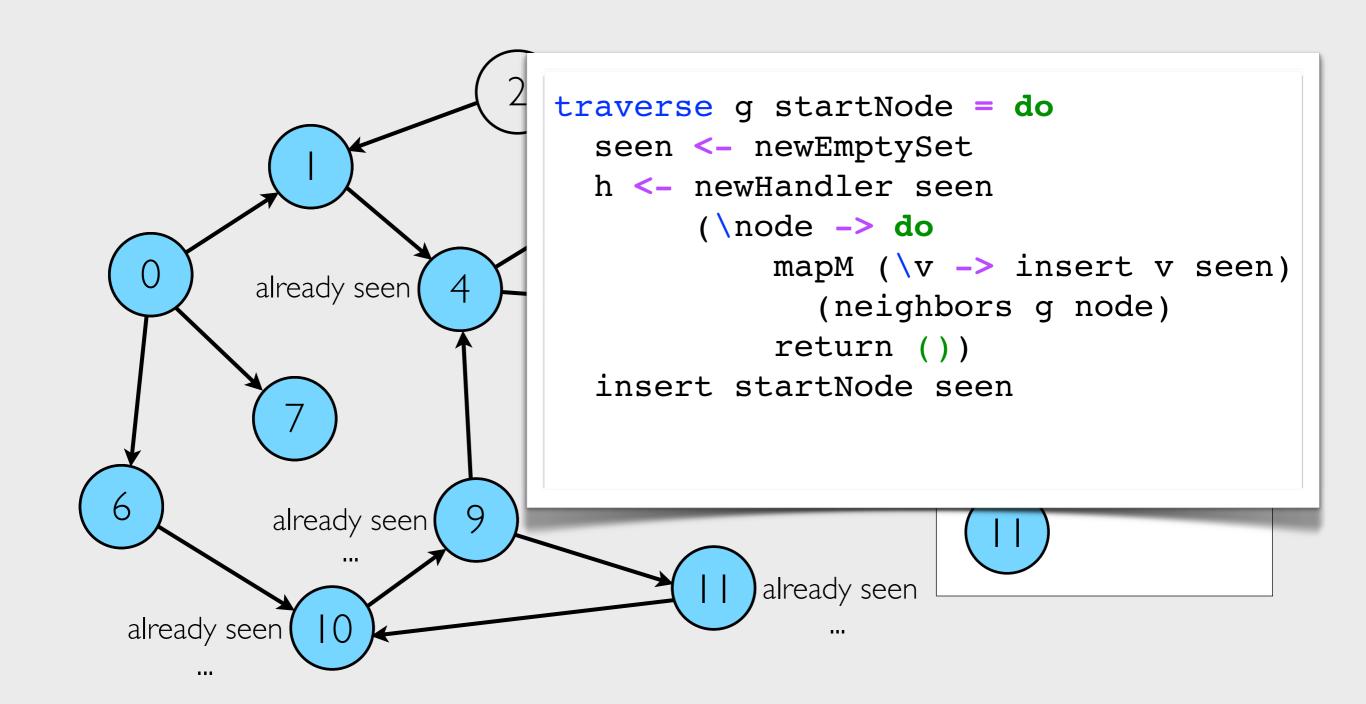








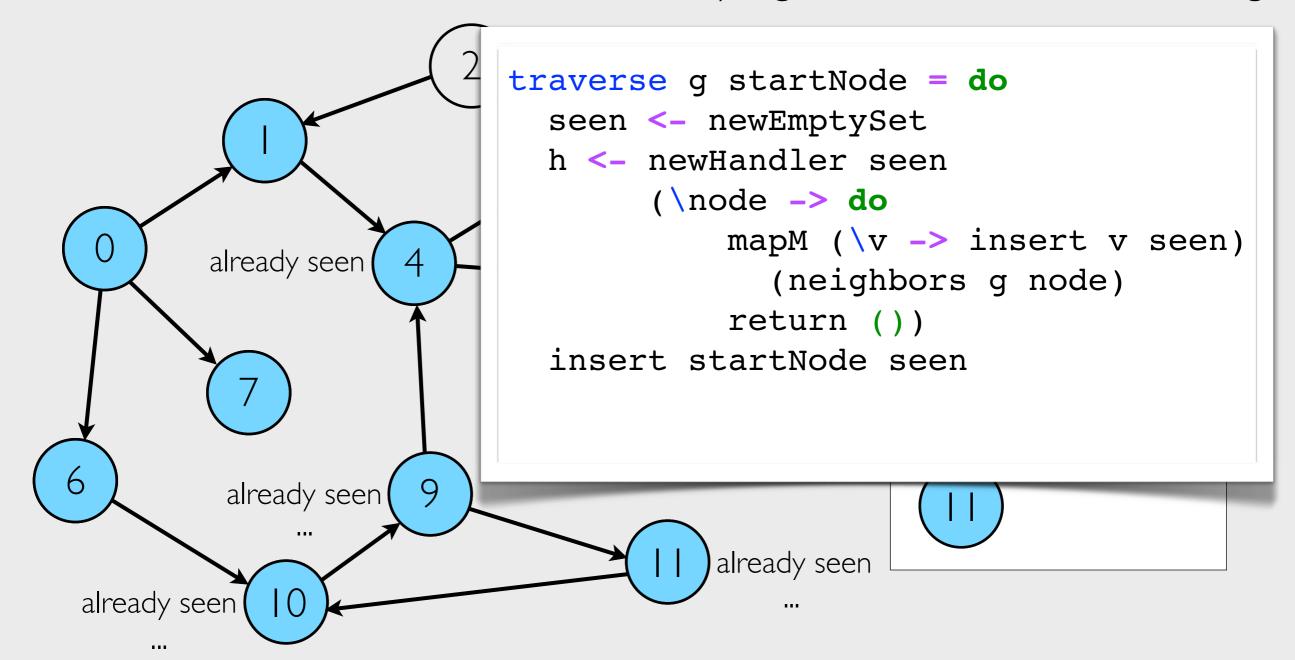




Events are updates that change an LVar's state

Event handlers listen for events and launch callbacks in response

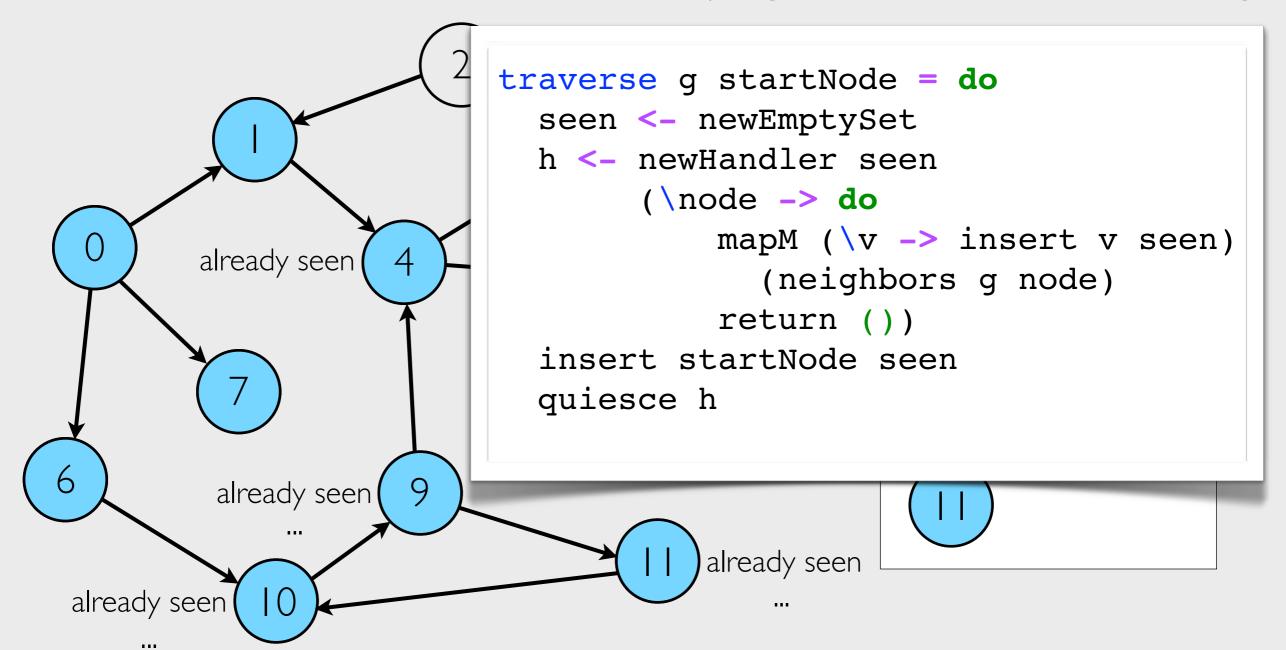
quiesce blocks until all callbacks launched by a given handler are done running



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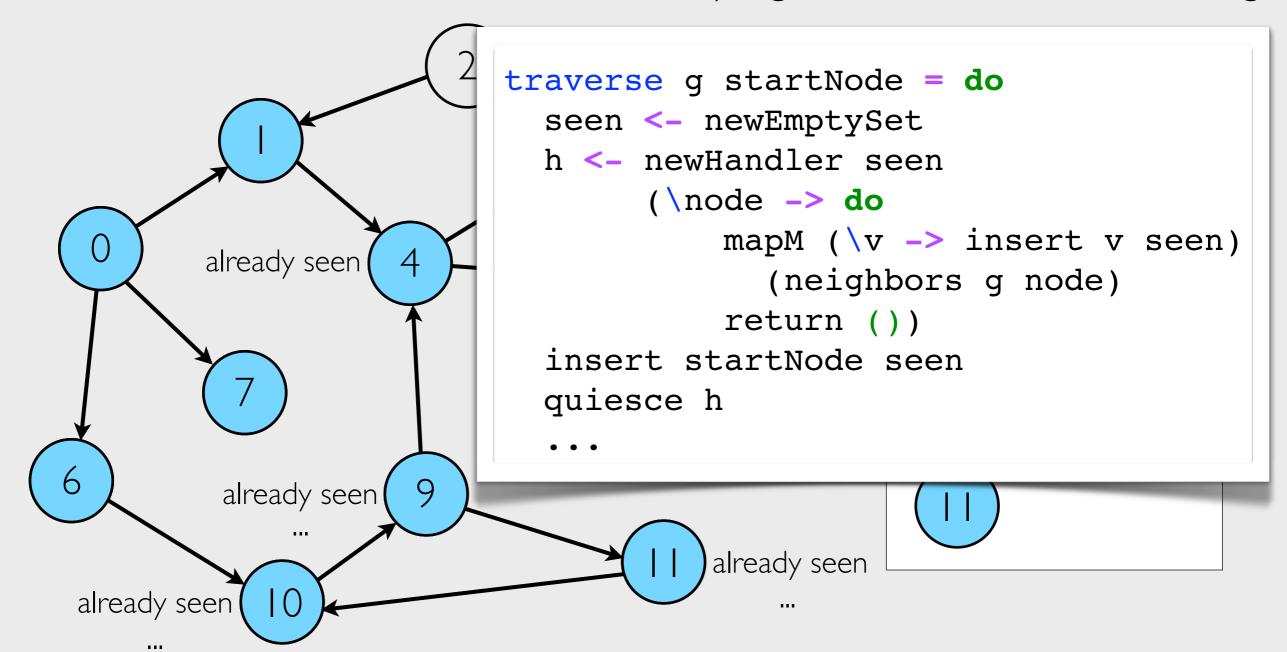
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Attempts to write to a frozen LVar raise a write-after-freeze exception

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```

- 1. $\sigma' = \sigma''$ up to a permutation on locations π , or
- 2. $\sigma' = \text{error } or \, \sigma'' = \text{error.}$

[POPL'|4] nsert v seen)

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[(Book,1),(Shoes,1)] or error.



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Let the system handle this for us: — runParThenFreeze

a Haskell library for parallel programming with LVars



a Haskell library for parallel programming with LVars

LVar operations run in Par computations

a Haskell library for parallel programming with LVars

LVar operations run in Par computations Lightweight threads

a Haskell library for parallel programming with LVars

LVar operations run in Par computations

Lightweight threads

Par computations indexed by effect level

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p :: Par Det (IMap Item Int)
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a Haskell library for parallel programming with LVars

LVar operations run in **Par** computations Lightweight threads

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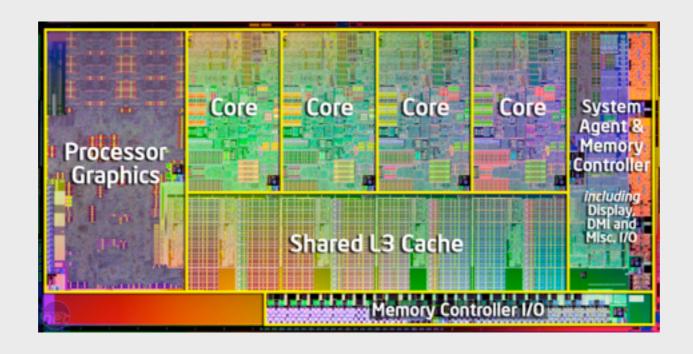
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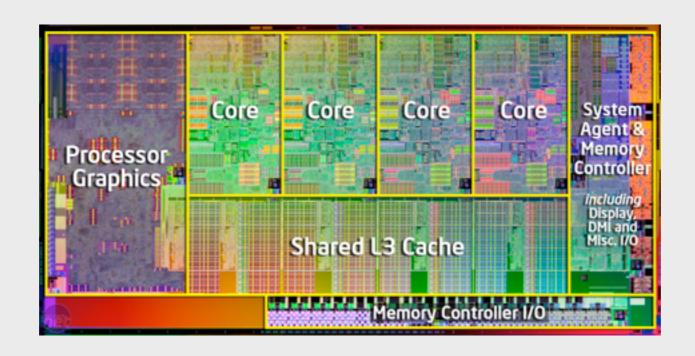
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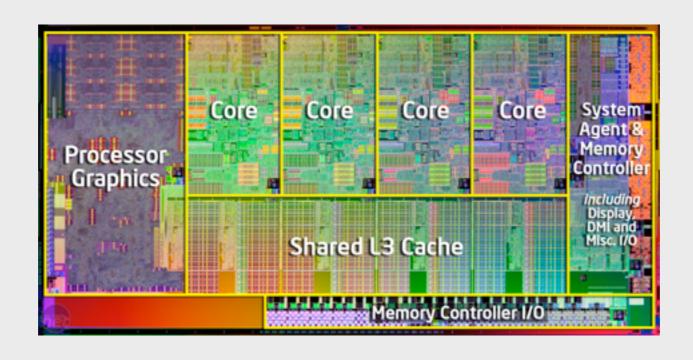
hackage.haskell.org/package/lvish



Deterministic Parallel Programming

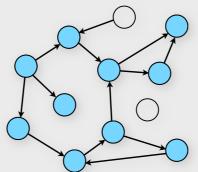


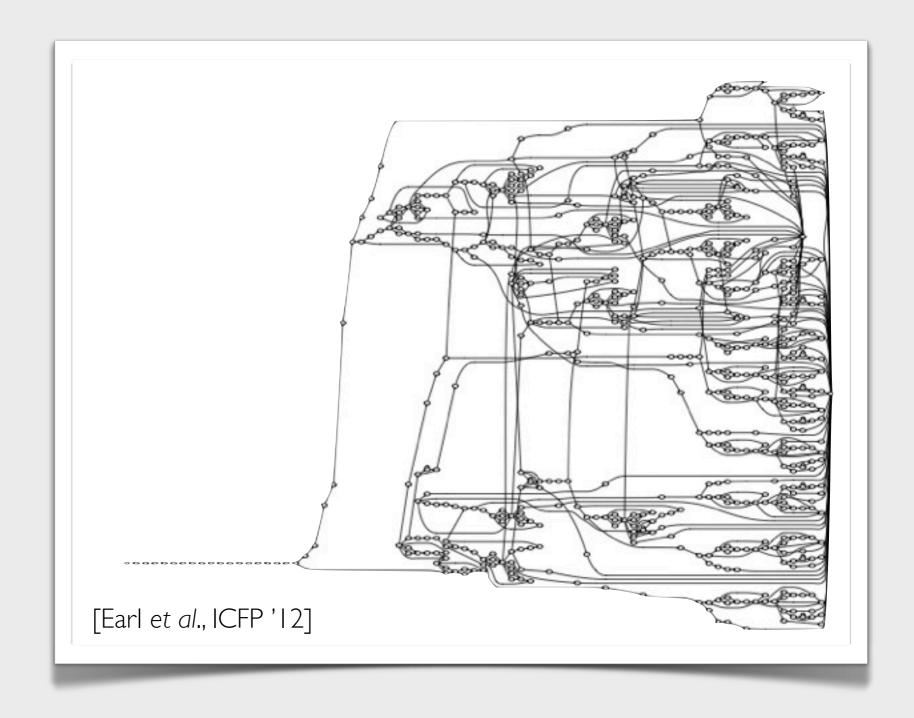
(observably)
Deterministic Parallel Programming



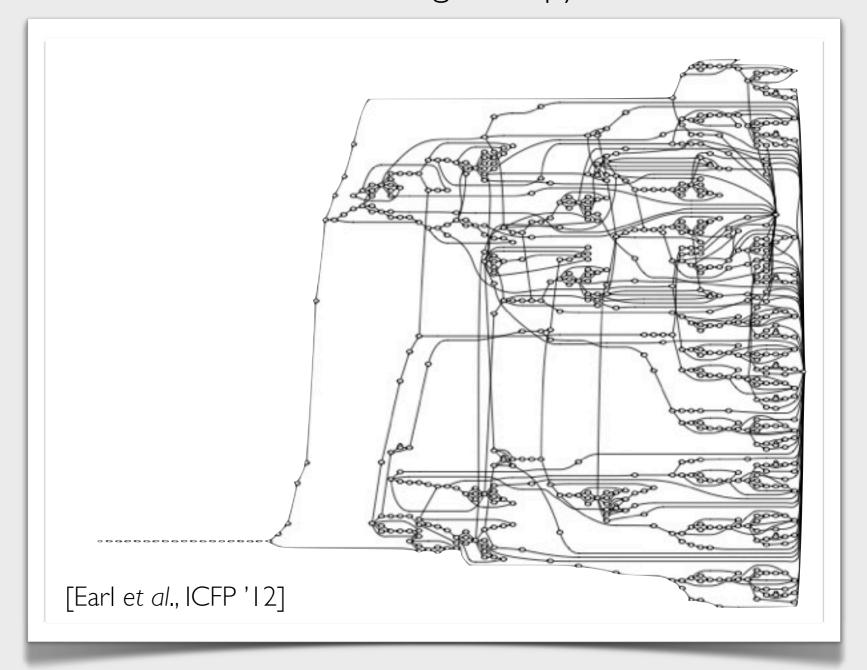
(observably) (irregular)

Deterministic Parallel Programming





b up to 20x speedup, even on one core, from not having to copy data



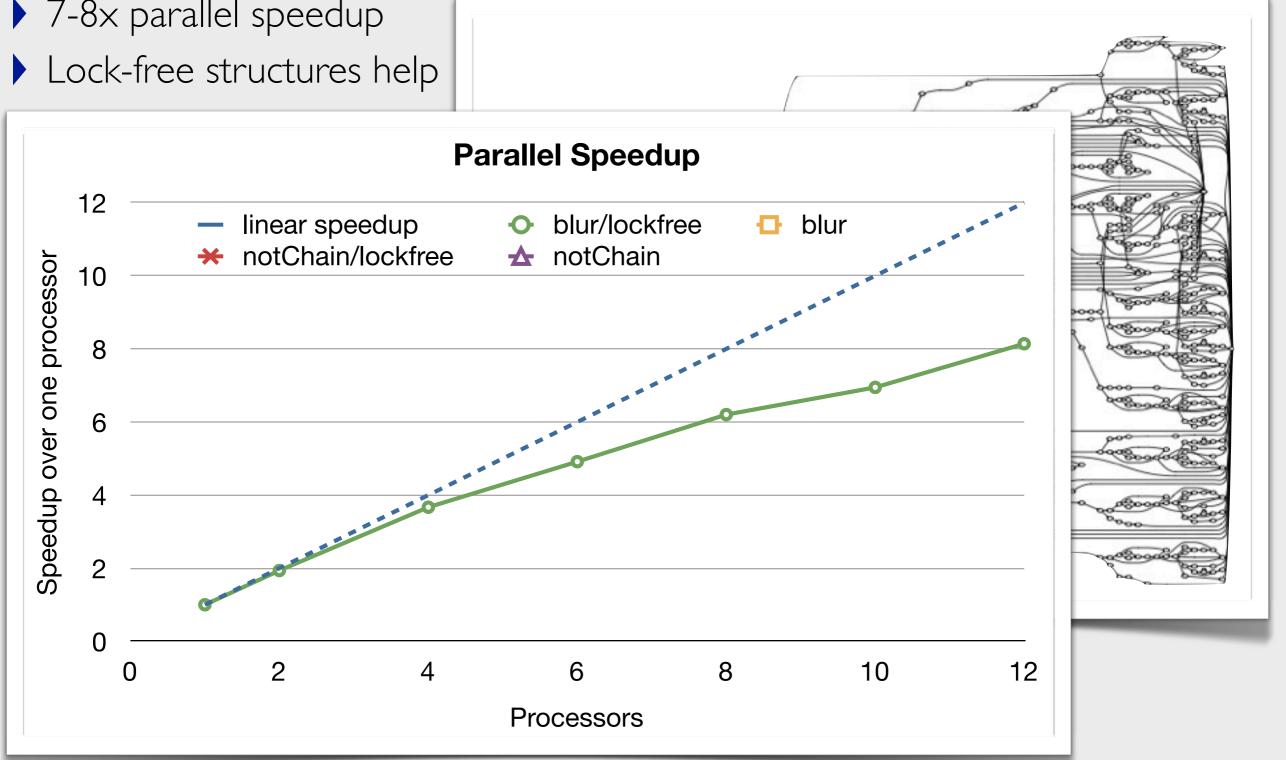
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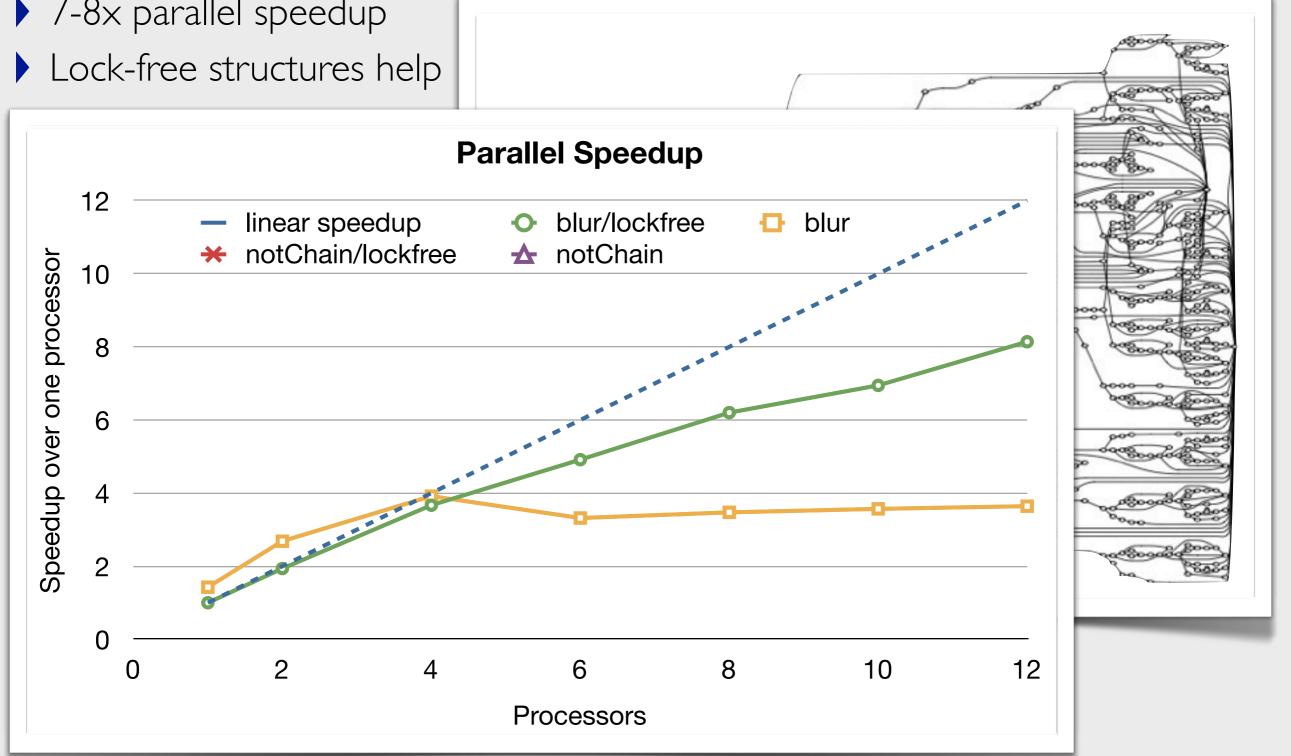
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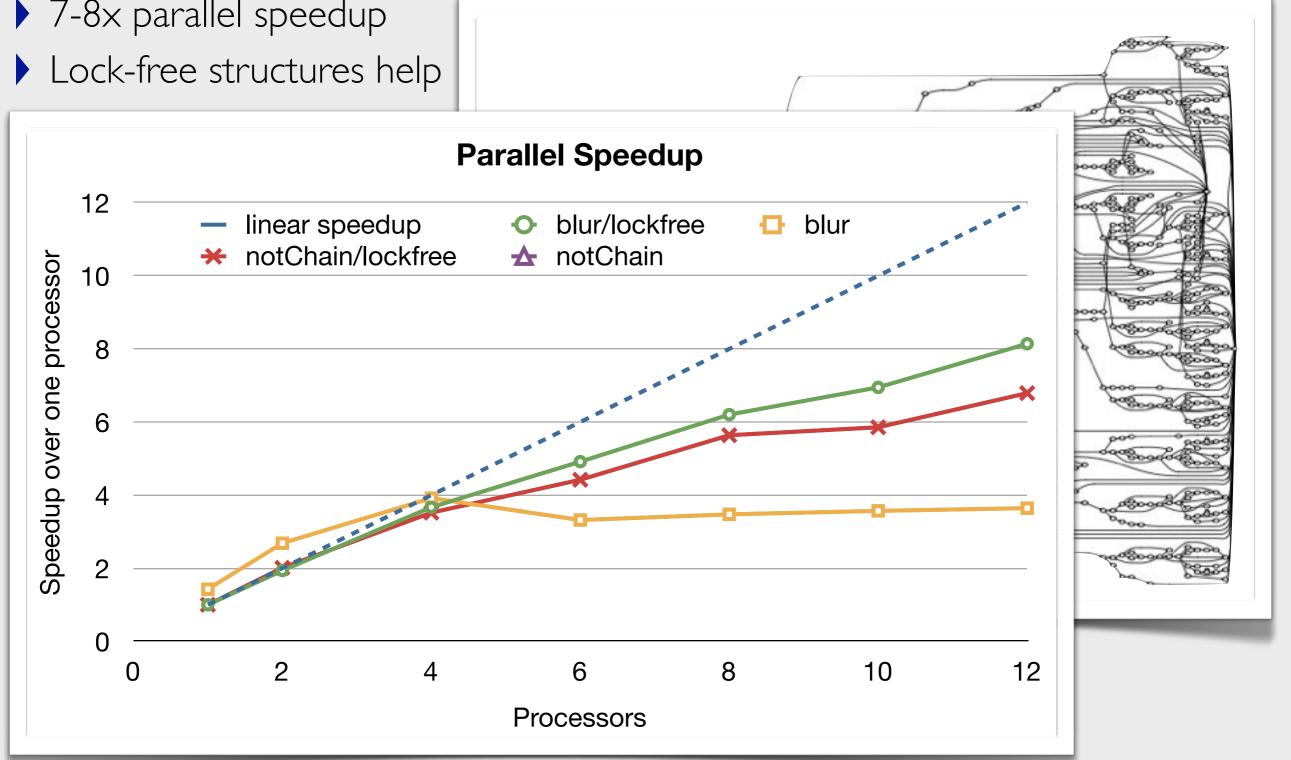
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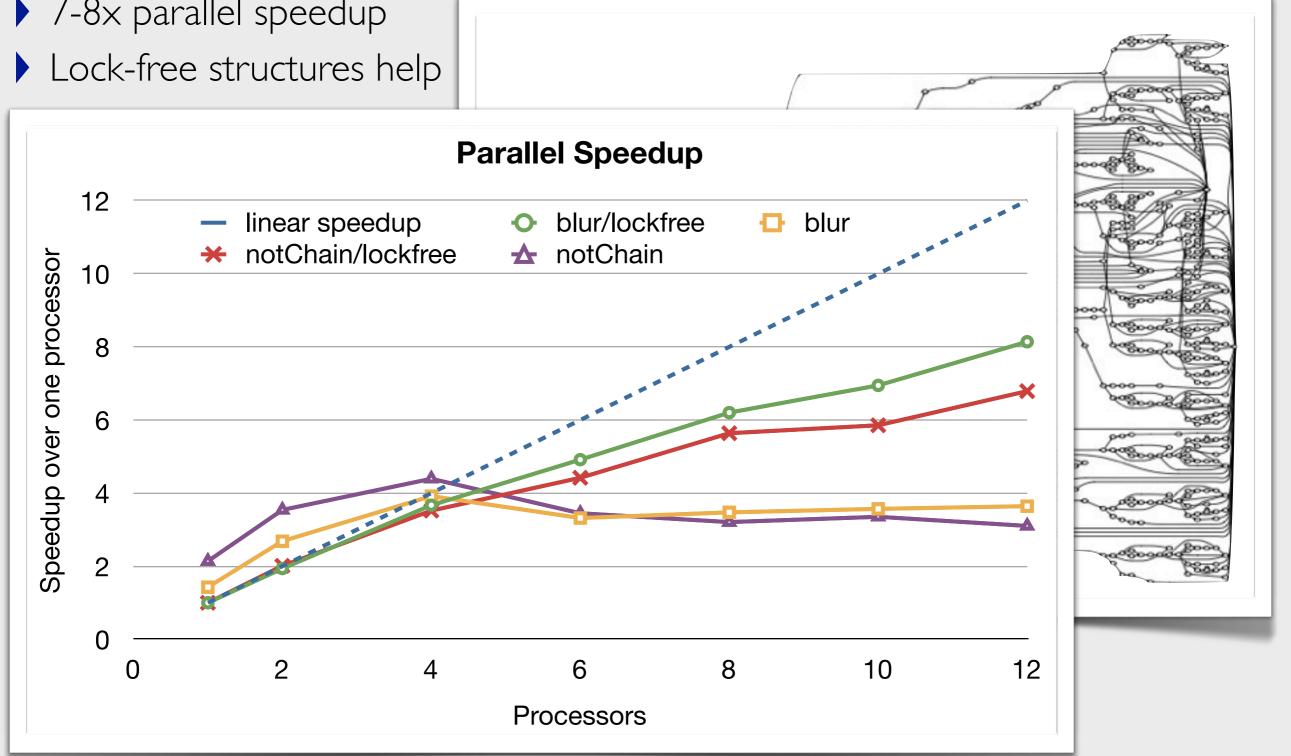
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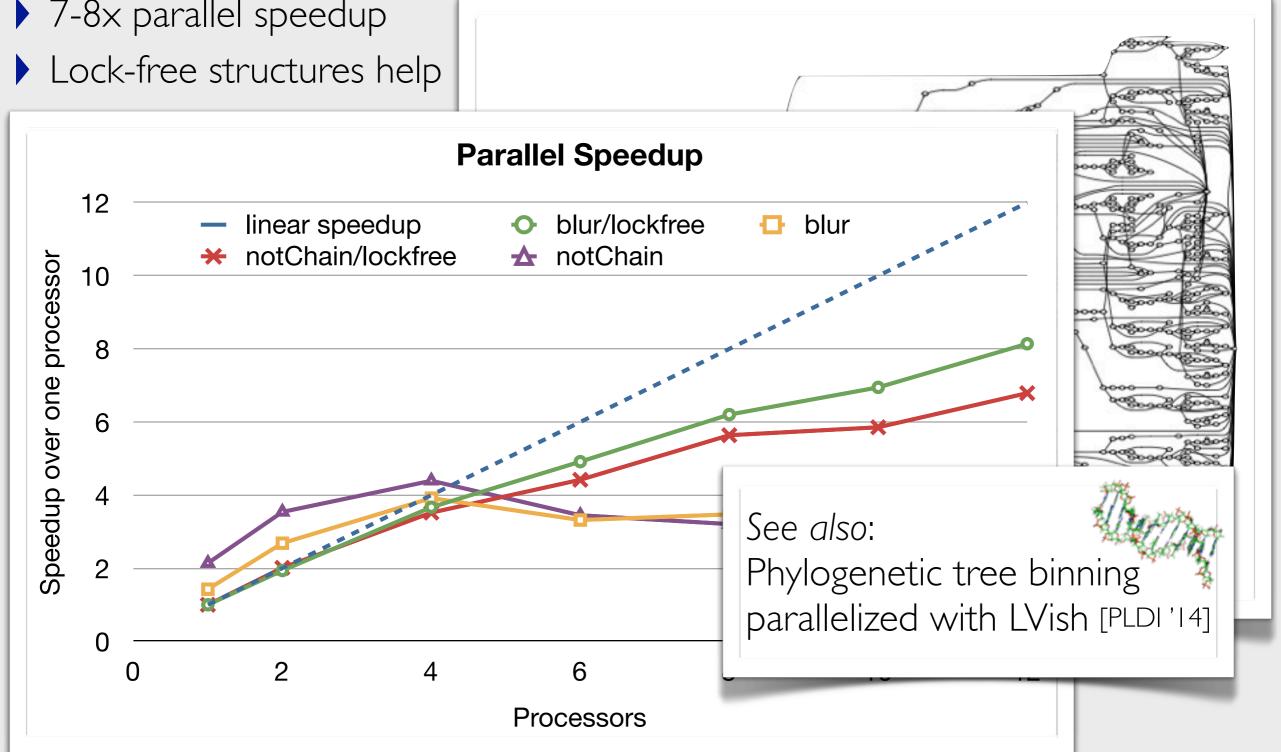
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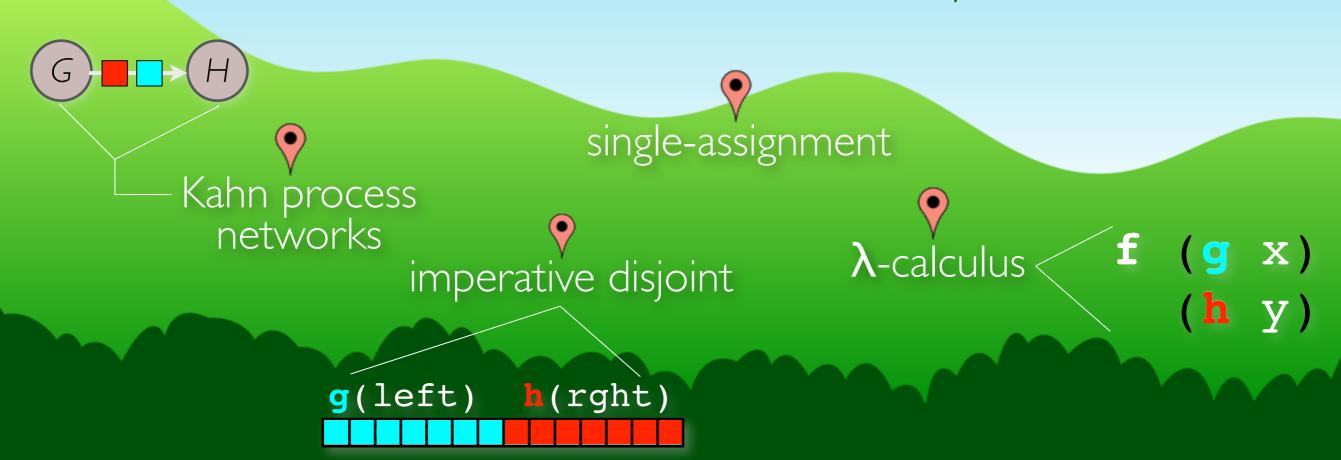


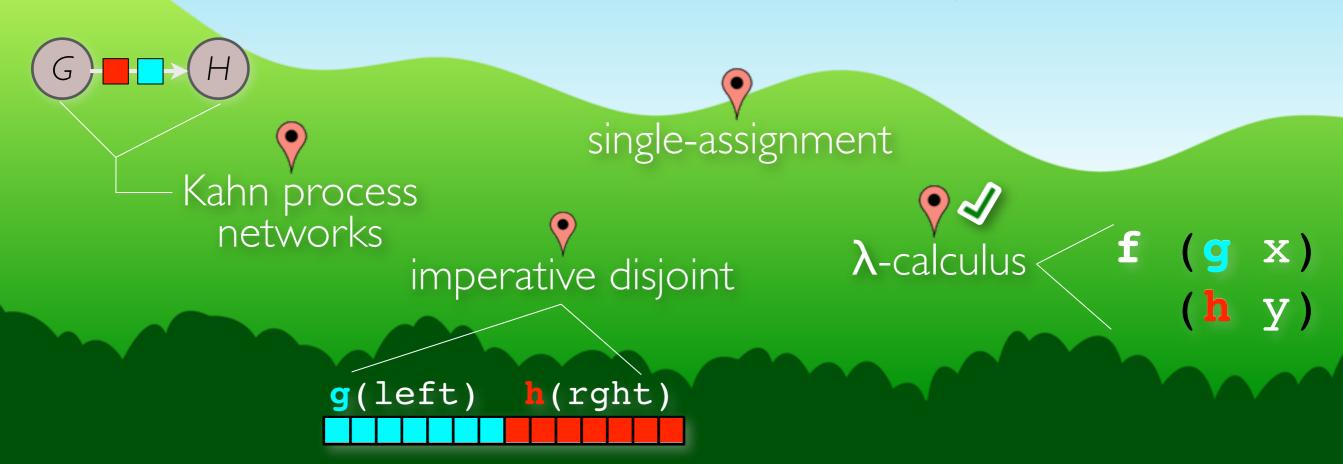
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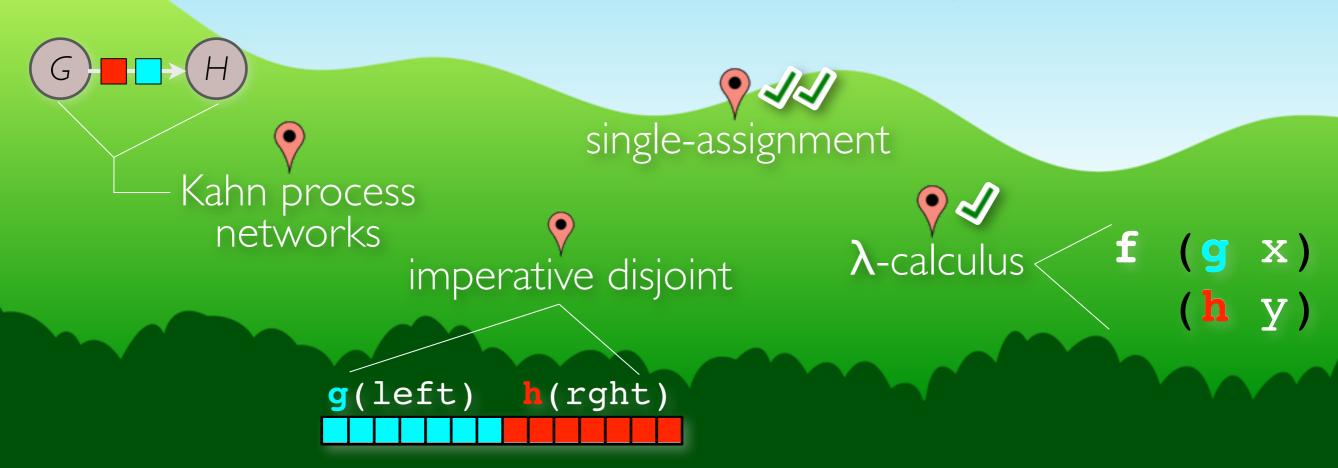


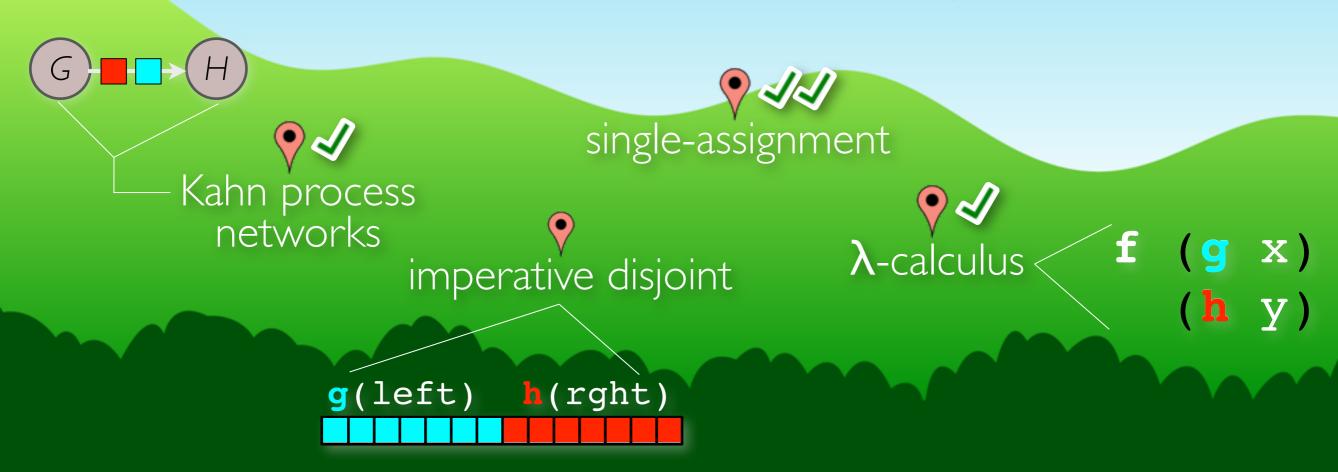
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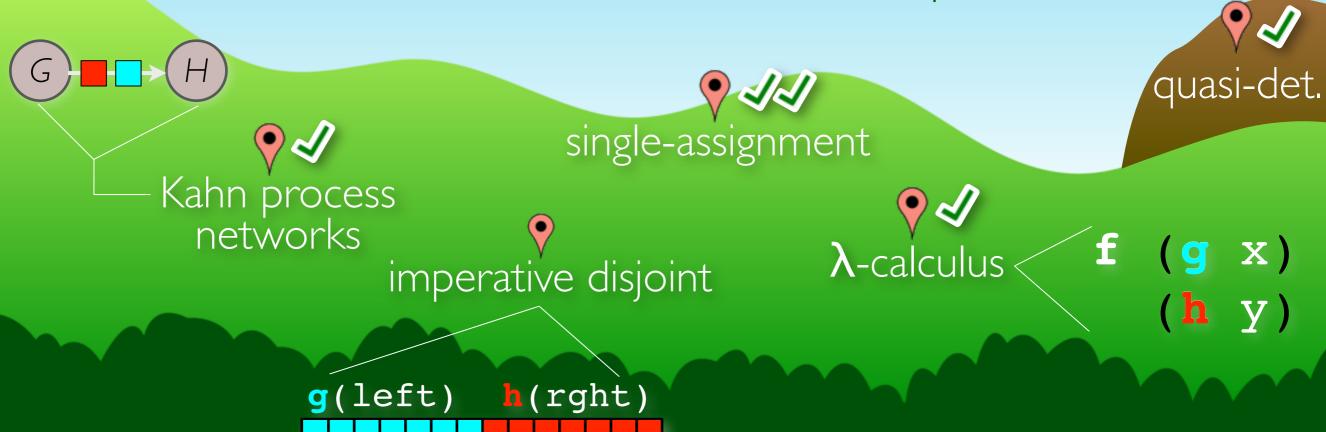






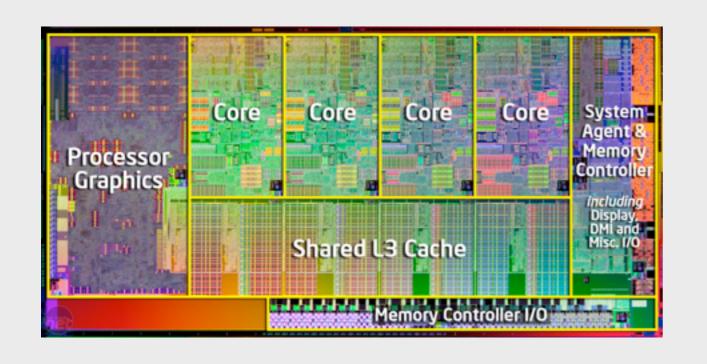


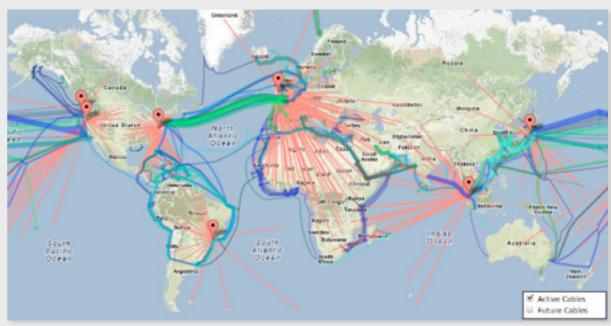












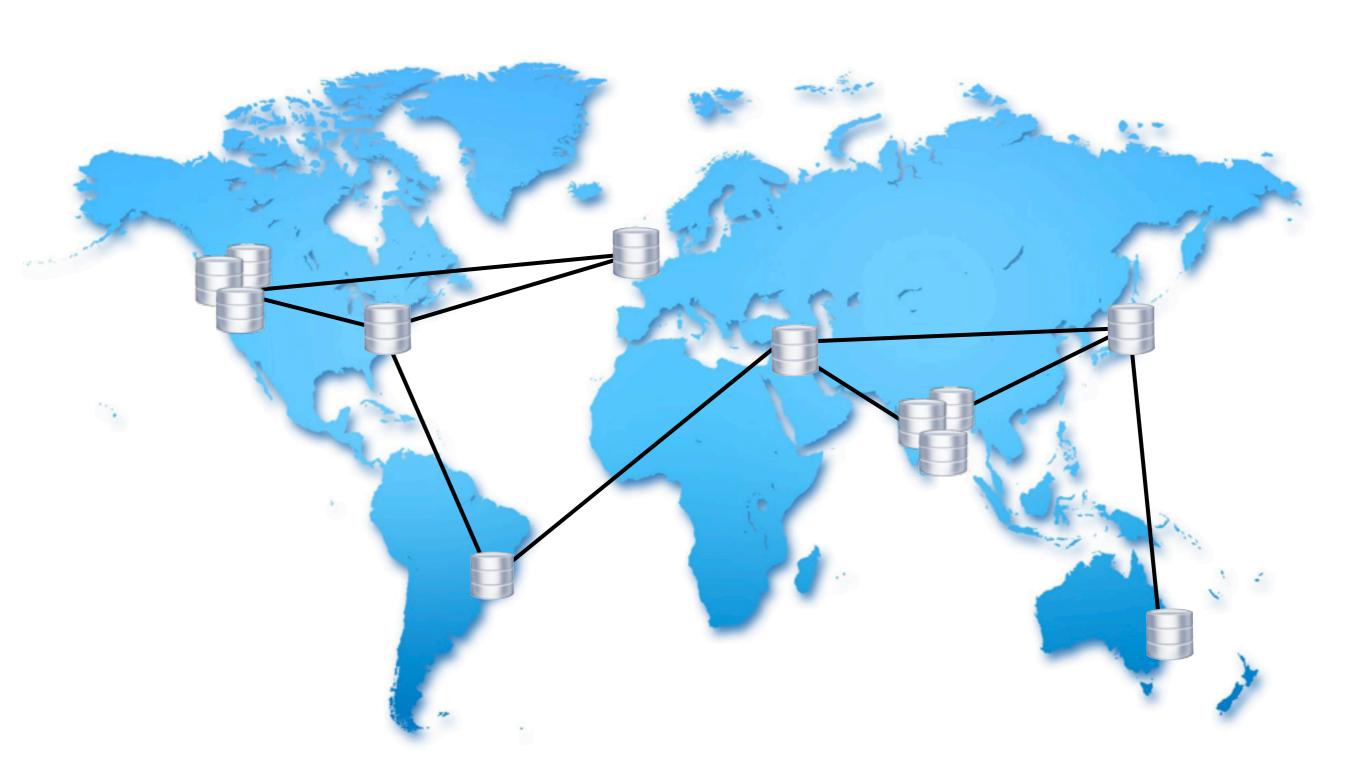
Parallel systems

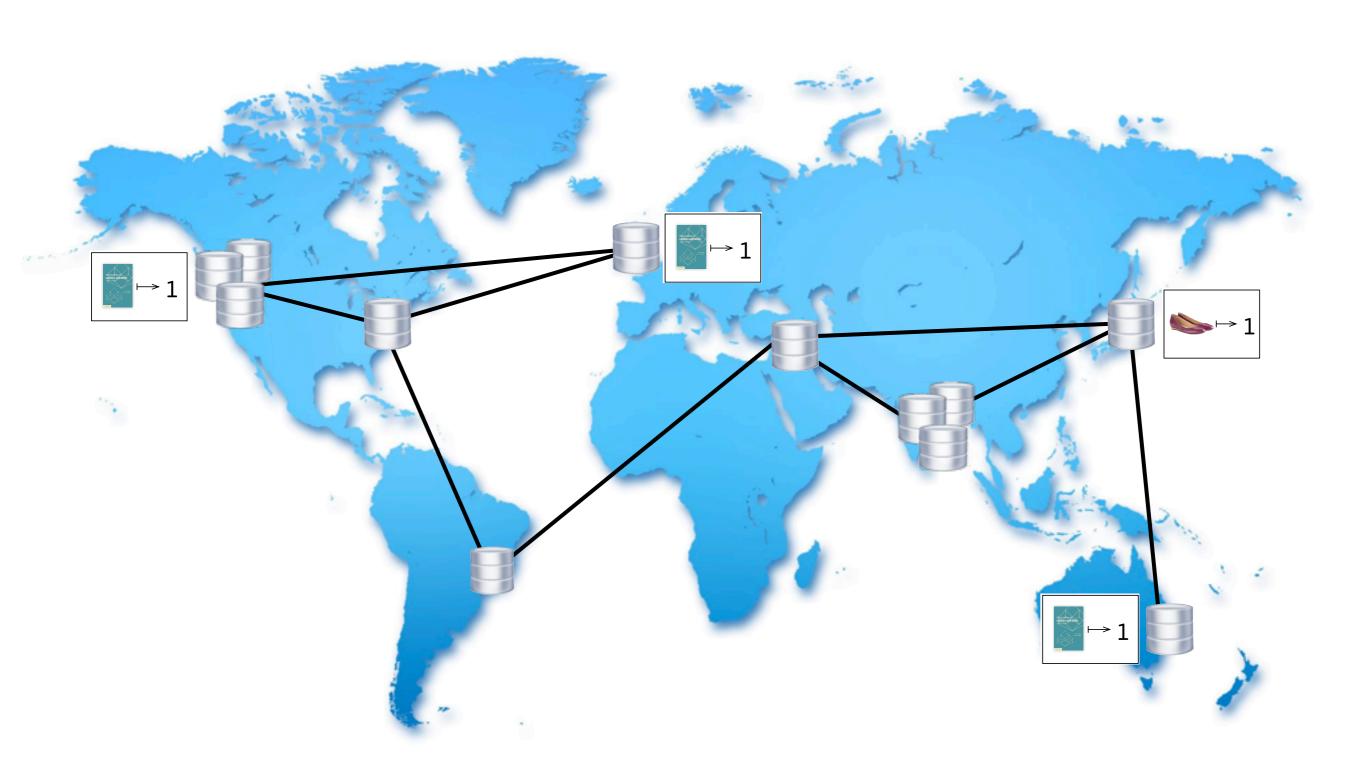
Distributed systems

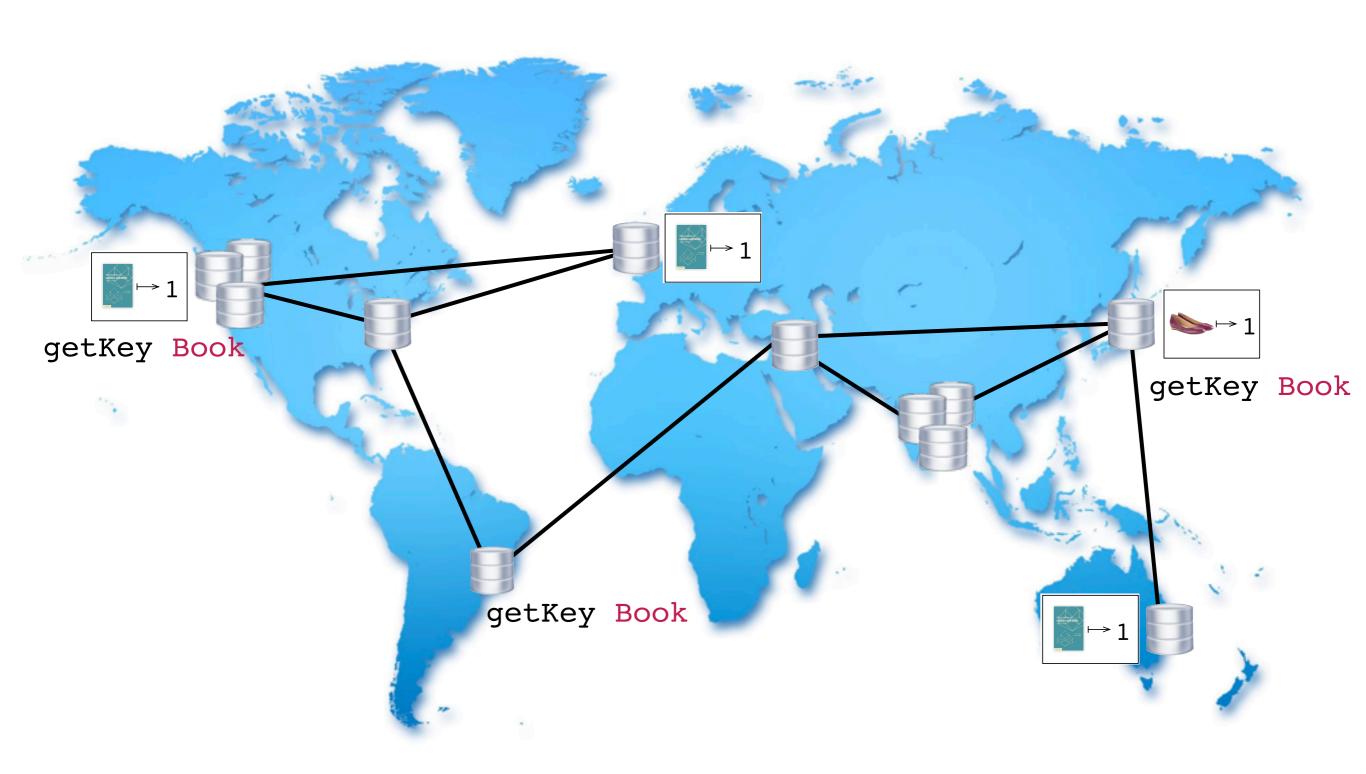


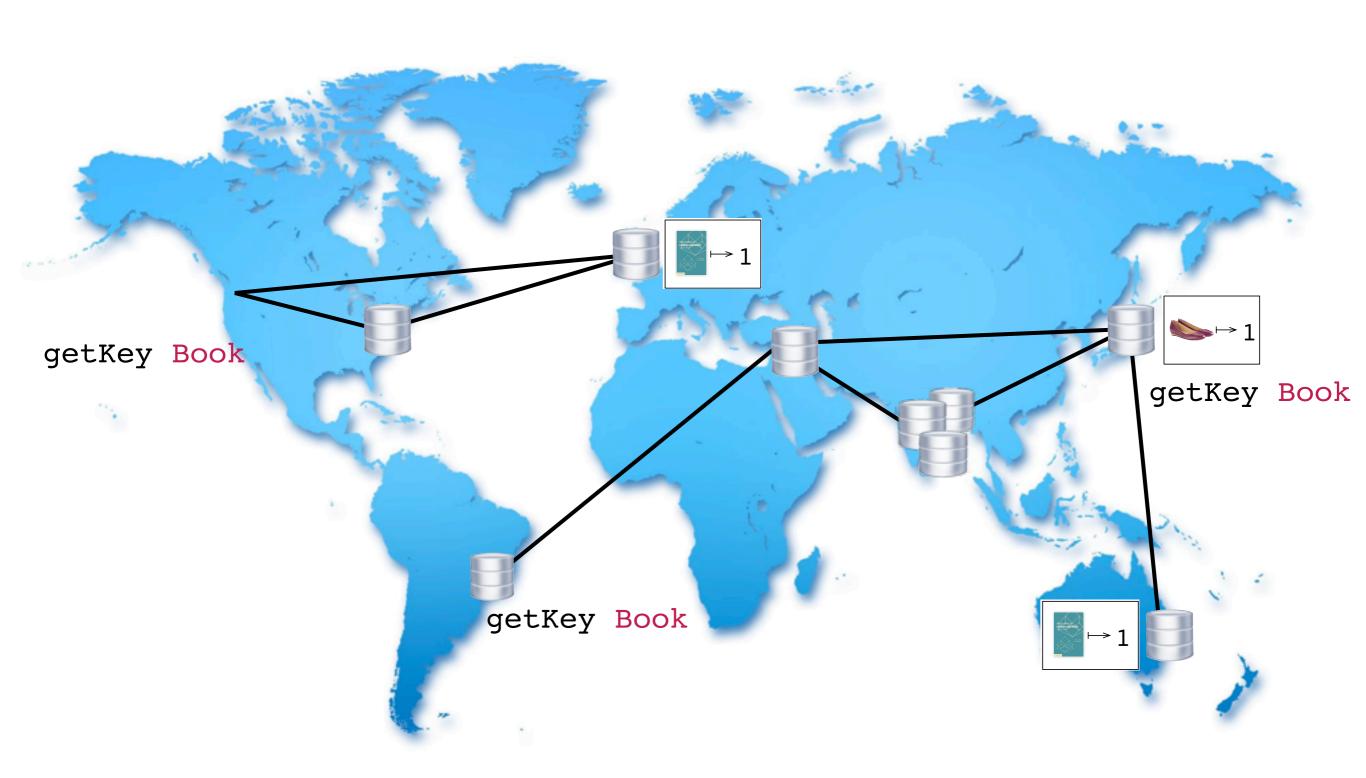
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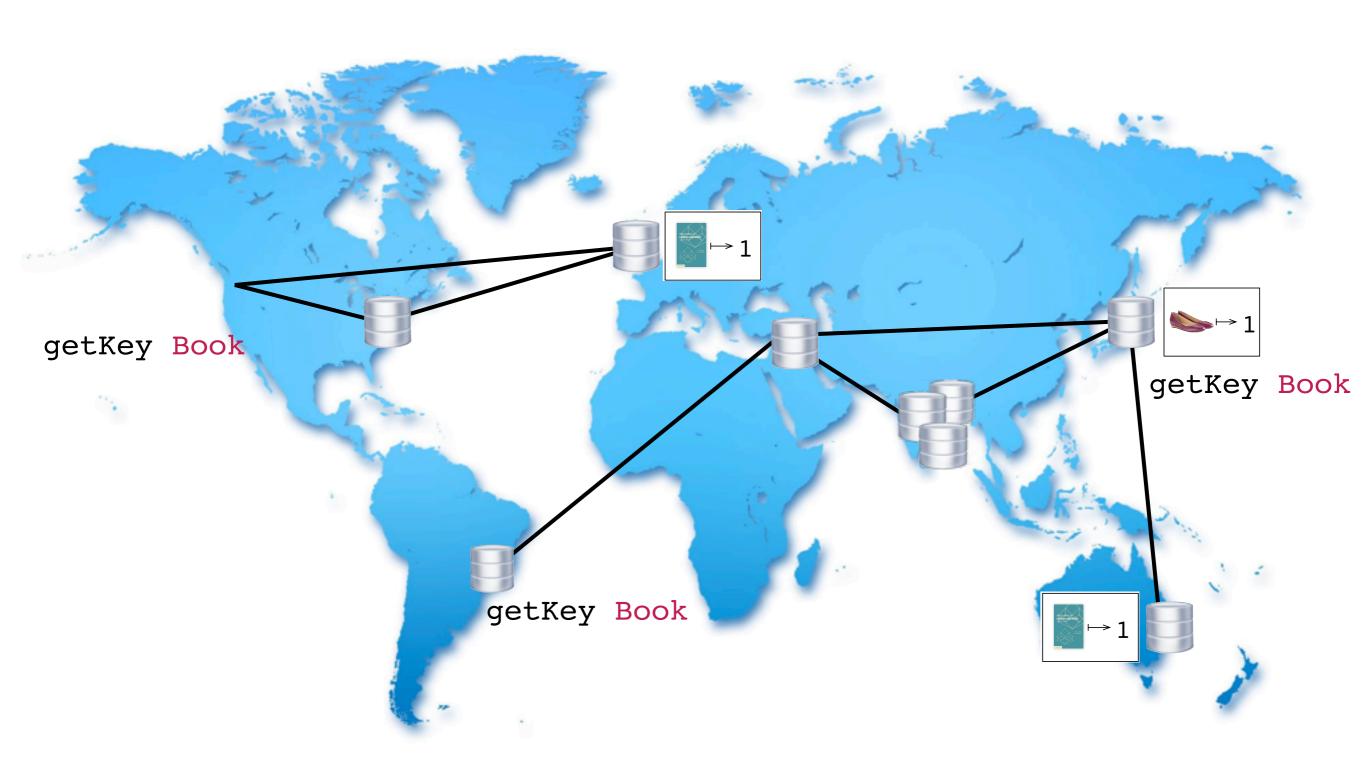




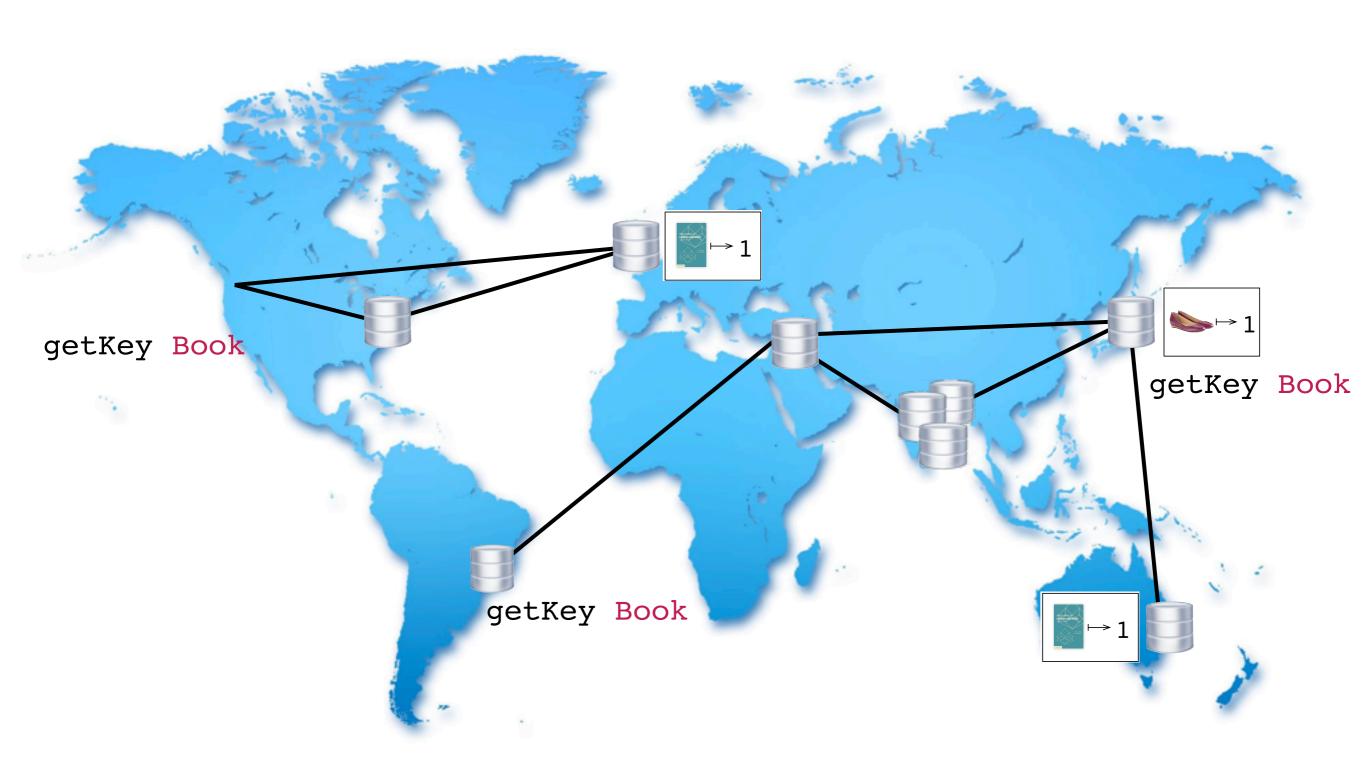




Eventual consistency.



Eventual consistency. How?



Dynamo: Amazon's Highly Available Key-value Store

Giuseppe DeCandia, Deniz Hastorun, Madan Jampani, Gunavardhan Kakulapati, Avinash Lakshman, Alex Pilchin, Swaminathan Sivasubramanian, Peter Vosshall and Werner Vogels

Amazon.com

ABSTRACT

Reliability at massive scale is one of the biggest challenges we face at Amazon.com, one of the largest e-commerce operations in the world; even the slightest outage has significant financial consequences and impacts customer trust. The Amazon.com platform, which provides services for many web sites worldwide, is implemented on top of an infrastructure of tens of thousands of servers and network components located in many datacenters around the world. At this scale, small and large components fail continuously and the way persistent state is managed in the face of these failures drives the reliability and scalability of the software systems.

This paper presents the design and implementation of Dynamo, a highly available key-value storage system that some of Amazon's core services use to provide an "always-on" experience. To achieve this level of availability, Dynamo sacrifices consistency under certain failure scenarios. It makes extensive use of object versioning and application-assisted conflict resolution in a manner that provides a novel interface for developers to use.

Categories and Subject Descriptors

D.4.2 [Operating Systems]: Storage Management; D.4.5 [Operating Systems]: Reliability; D.4.2 [Operating Systems]: Performance;

General Terms

Algorithms, Management, Measurement, Performance, Design, Reliability.

1. INTRODUCTION

Amazon runs a world-wide e-commerce platform that serves tens of millions customers at peak times using tens of thousands of servers located in many data centers around the world. There are strict operational requirements on Amazon's platform in terms of performance, reliability and efficiency, and to support continuous growth the platform needs to be highly scalable. Reliability is one of the most important requirements because even the slightest outage has significant financial consequences and impacts customer trust. In addition, to support continuous growth, the platform needs to be highly scalable.

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SOSP'07, October 14–17, 2007, Stevenson, Washington, USA. Copyright 2007 ACM 978-1-59593-591-5/07/0010...\$5.00. One of the lessons our organization has learned from operating Amazon's platform is that the reliability and scalability of a system is dependent on how its application state is managed. Amazon uses a highly decentralized, loosely coupled, service oriented architecture consisting of hundreds of services. In this environment there is a particular need for storage technologies that are always available. For example, customers should be able to view and add items to their shopping cart even if disks are failing, network routes are flapping, or data centers are being destroyed by tornados. Therefore, the service responsible for managing shopping carts requires that it can always write to and read from its data store, and that its data needs to be available across multiple data centers.

Dealing with failures in an infrastructure comprised of millions of components is our standard mode of operation; there are always a small but significant number of server and network components that are failing at any given time. As such Amazon's software systems need to be constructed in a manner that treats failure handling as the normal case without impacting availability or performance.

To meet the reliability and scaling needs, Amazon has developed a number of storage technologies, of which the Amazon Simple Storage Service (also available outside of Amazon and known as Amazon S3), is probably the best known. This paper presents the design and implementation of Dynamo, another highly available and scalable distributed data store built for Amazon's platform. Dynamo is used to manage the state of services that have very high reliability requirements and need tight control over the tradeoffs between availability, consistency, cost-effectiveness and performance. Amazon's platform has a very diverse set of applications with different storage requirements. A select set of applications requires a storage technology that is flexible enough to let application designers configure their data store appropriately based on these tradeoffs to achieve high availability and guaranteed performance in the most cost effective manner.

There are many services on Amazon's platform that only need primary-key access to a data store. For many services, such as those that provide best seller lists, shopping carts, customer preferences, session management, sales rank, and product catalog, the common pattern of using a relational database would lead to inefficiencies and limit scale and availability. Dynamo provides a simple primary-key only interface to meet the requirements of these applications.

Dynamo uses a synthesis of well known techniques to achieve scalability and availability: Data is partitioned and replicated using consistent hashing [10], and consistency is facilitated by object versioning [12]. The consistency among replicas during updates is maintained by a quorum-like technique and a decentralized replica synchronization protocol. Dynamo employs

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Conflict-Free Replicated Data Types*

Marc Shapiro^{1,5}, Nuno Preguiça^{1,2}, Carlos Baquero³, and Marek Zawirski^{1,4}

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 CITI, Universidade Nova de Lisboa, Portugal
 Universidade do Minho, Portugal
 UPMC, Paris, France
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Abstract. Replicating data under Eventual Consistency (EC) allows any replica to accept updates without remote synchronisation. This ensures performance and scalability in large-scale distributed systems (e.g., clouds). However, published EC approaches are ad-hoc and error-prone. Under a formal Strong Eventual Consistency (SEC) model, we study sufficient conditions for convergence. A data type that satisfies these conditions is called a Conflict-free Replicated Data Type (CRDT). Replicas of any CRDT are guaranteed to converge in a self-stabilising manner, despite any number of failures. This paper formalises two popular approaches (state- and operation-based) and their relevant sufficient conditions. We study a number of useful CRDTs, such as sets with clean semantics, supporting both add and remove operations, and consider in depth the more complex Graph data type. CRDT types can be composed to develop large-scale distributed applications, and have interesting theoretical properties.

Keywords: Eventual Consistency, Replicated Shared Objects, Large-Scale Distributed Systems.

1 Introduction

Replication and consistency are essential features of any large distributed system, such as the WWW, peer-to-peer, or cloud computing platforms. The standard "strong consistency" approach serialises updates in a global total order [10]. This constitutes a performance and scalability bottleneck. Furthermore, strong consistency conflicts with availability and partition-tolerance [8].

When network delays are large or partitioning is an issue, as in delay-tolerant networks, disconnected operation, cloud computing, or P2P systems, eventual consistency promises better availability and performance [17]21]. An update executes at some replica, without synchronisation; later, it is sent to the other

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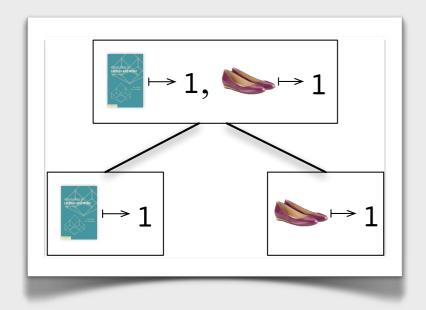
[Shapiro et al., SSS '11]

Two "styles" of Conflict-Free Replicated Data Types:

"Convergent"

CVRDTs

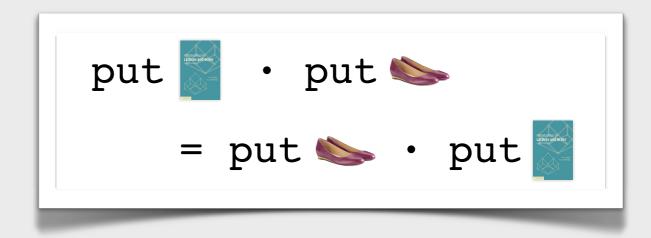
"state-based"



"Commutative"

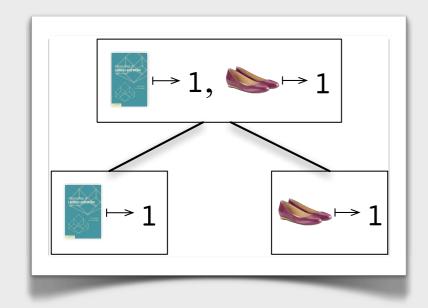
CmRDTs

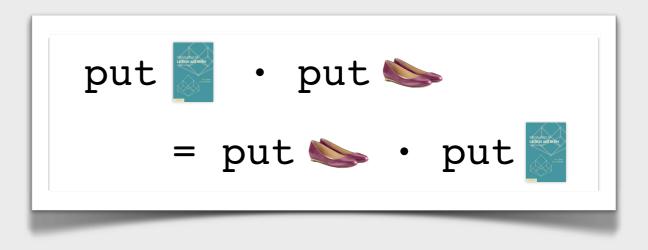
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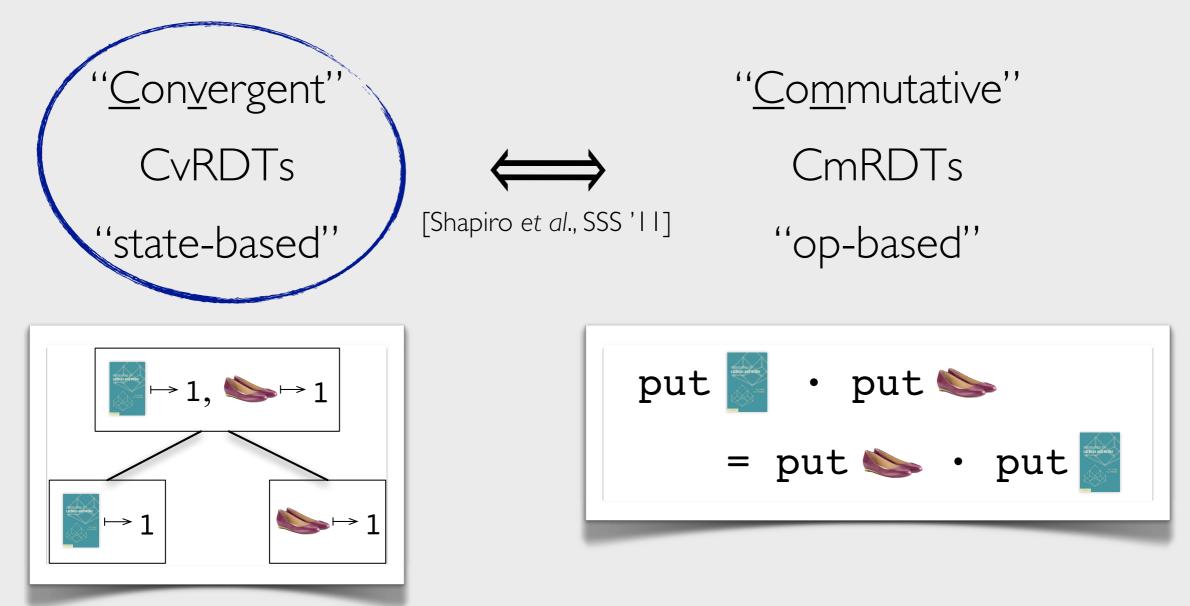
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Two "styles" of Conflict-Free Replicated Data Types:



Threshold reads (deterministic)

Ordinary reads (nondeterministic)

Least-upper-bound writes (every write computes a join)

General inflationary writes (only *merges* must be joins)

Shared memory

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One framework for reasoning about both eventual and strong consistency

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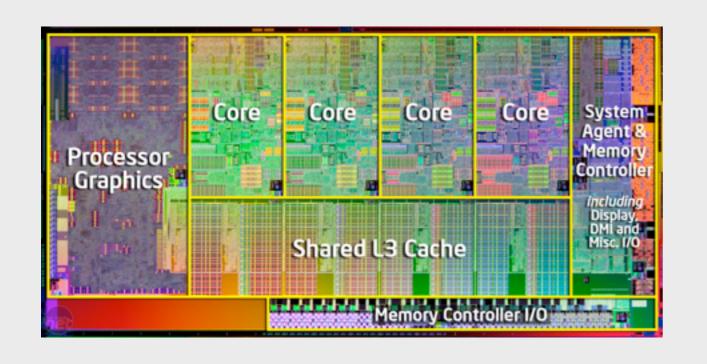
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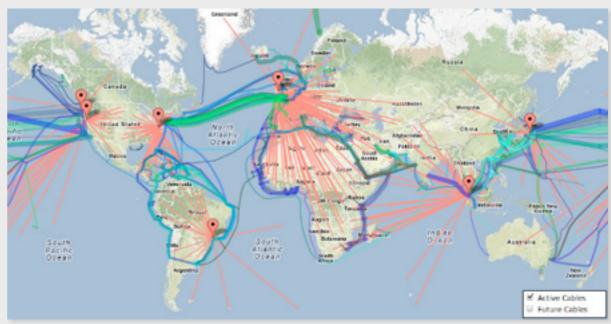
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 One framework for reasoning about both eventual and strong consistency
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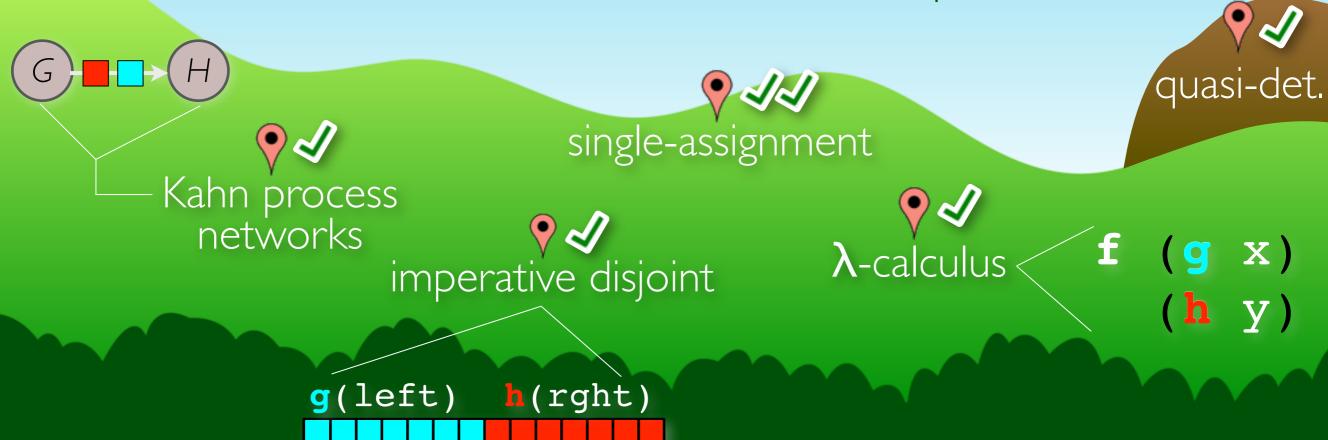




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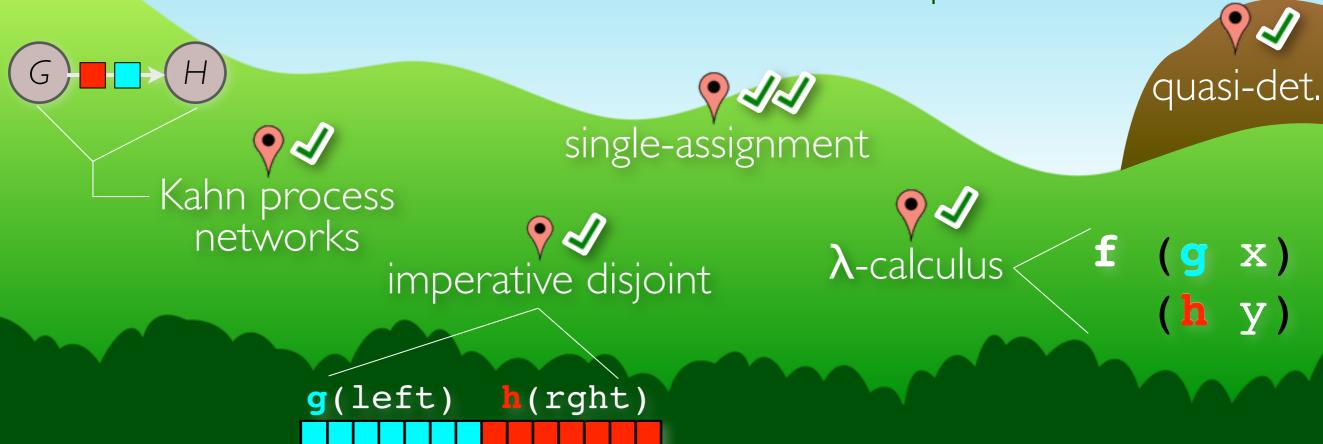








LVars and LVish across the landscape:











Thank you!

Email: lkuper@cs.indiana.edu LVars project repo: github.com/iu-parfunc/lvars Code from this talk: github.com/lkuper/lvar-examples Papers: cs.indiana.edu/~lkuper Research blog: composition.al